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THE OUTRANKING APPROACH AND THE FOUNDATIONS OF ELECTRE METHODS

ABSTRACT. In the first part of this paper, we describe the main features of real-world problems for which the outranking approach is appropriate and we present the concept of outranking relations. The second part is devoted to basic ideas and concepts used for building outranking relations. The definition of such outranking relations is given for the main ELECTRE methods in Part 3. The final part of the paper is devoted to some practical considerations.

Keywords: Preference modelling, pairwise comparisons, outranking relations, ELECTRE methods, concordance, discordance, credibility index

1. INTRODUCTION TO THE OUTRANKING APPROACH

The concept of outranking relations was born of difficulties encountered with diverse concrete problems (see Abgueguen (1971), Bètolaud and Février (1973), Buffet *et al.* (1967), Charpentier and Jacquet-Lagrèze (1976), Laffy (1966)). Since then, numerous applications of the concept have been developed. Among the most recent ones, we can mention: Barda *et al.* (1990), Climaco *et al.* (1988), Martel and Nadeau (1988), Maystre and Simos (1987), Parent and Schnäbele (1988), Rabeharisoa (1988), Renard (1986), Roy *et al.* (1986), Slowinski and Treichel (1988). Many others will be found in Jacquet-Lagrèze and Siskos (1983), de Montgolfier and Bertier (1978), Roy (1985), Schärlig (1985).

1.1. Preliminary Notations and Definitions

In order to understand what the outranking approach is and what kind of real-world problems it refers to, it is necessary to specify what is supposed to be given initially.

(a) A set A of *potential actions* (see Roy, 1990) (or alternatives) is considered. Such actions are not necessarily exclusive, i.e., they can be put into operation jointly.

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(b) A consistent *family* F of n *criteria* g_j has been defined (see Bouyssou, 1990). This means that preferences of actors involved in the decision process are formed, argued and transformed by reference to points of view adequately reflected by criteria of F.

$g_j(a)$ is called the jth *performance* of a. It is not restrictive to suppose that:

- $g_j(a)$ is a real number (even if it reflects a qualitative assessment);
- $\forall a' \in A$ and $a \in A$, $g_j(a') \geqslant g_j(a) \Rightarrow a'$ is at least as good as a if we consider only the point of view reflected by the jth criterion.

(c) Let us emphasize on a given criterion, for instance the kth. The imprecision, and/or the uncertainty, and/or the inaccurate determination of performances (see Roy, 1988) may lead some actor to judge:

- a' indifferent to a when $g_j(a') = g_j(a)$ $\forall j \neq k$ even if $g_k(a') \neq g_k(a)$;
- a' strictly preferred to a when $g_j(a') = g_j(a)$ $\forall j \neq k$ only if the difference $g_k(a') - g_k(a)$ is sufficiently significant.

The problem considered here is that of the significance of the criterion g_k. The two possibilities above underline the fact that maps are not territories: the *vector of performances* $g(a) = [g_1(a), \ldots, g_n(a)]$ is like a map of that territory which is the action a. We want to compare actions, i.e., territories, not maps. These comparisons are made by means of maps, and we have to avoid working as if maps do not differ from territories.

(d) Let us consider now, at the comprehensive level, the comparison of a' and a on the basis of $g(a')$ and $g(a)$. The actors involved in the decision process may not all have exactly the same judgement. To give meaning to a comprehensive model of preferences, we will refer to a particular actor D called the *decision-maker*. This one may be viewed either as a real person for whom or in the name of whom decision-aid is provided, or as a mythical person whose preferences can be used to enlighten the decision-aid problem. The comprehensive model of preferences in question does not, consequently, pretend to be an accurate description of well-stated preferences thought to be firmly fixed in the mind of a clearly identified decision-maker D. If D is a mythical, inaccessible or vaguely defined decision-maker, the model is only a system of preferences with which it is possible to work in order to bring forward elements of a response to certain questions.

Under these conditions, the comprehensive model of preferences should allow us to take into account hesitations between two of the three following cases:

$a'Ia$: a' indifferent to a,
$a'Pa$: a' strictly preferred to a,
aPa': a strictly preferred to a'.

According to the types of hesitations, we will speak of:

- *weak preference*[1] (relation Q):

  $a'Qa$ if the hesitation is between $a'Ia$ and $a'Pa$ (being sure that not aPa'),
  aQa' if the hesitation is between $a'Ia$ and aPa' (being sure that not $a'Pa$);

- *incomparability* (relation R): $a'Ra$ if the hesitation deals with $a'Pa$ and aPa' (at least).

More precisely, the hesitations mentioned above may come from:

- the existence in D's mind (if D is a real person) of zones of uncertainty, half-held belief or conflicts and contradictions;
- the vaguely defined quality of the decision-maker;
- the fact that the scientist who built the model ignores, in part, how D compares a' and a;
- the imprecision, uncertainty, inaccurate determination of the maps $g(a')$ and $g(a)$ by means of which a' and a are compared.

(e) Approaches of the AHP[2] and MAUT[3] type base the comprehensive model of preferences on the explicitation of a value function or a utility function $V(a)$:

$$V(a) = V[g_1(a), \ldots, g_n(a)]$$

aggregating the n criteria in such a way that, in D's mind:

$$a'Ia \text{ iff } V(a') = V(a)\,,$$
$$a'Pa \text{ iff } V(a') > V(a)\,.$$

The assumptions made above do not seem readily compatible with such a way of modelling. This is one (but not the only) reason which leads us to formulate the outranking concept.[4]

1.2. *Outranking Concept: Level of Preferences Restricted to the g_j Criterion*

To each criterion, g_j, it is possible to associate a *restricted outranking relation* S_j. By definition, S_j is a binary relation: $a'S_ja$ holds if the values of the performances $g_j(a')$ and $g_j(a)$ give a sufficiently strong argument for considering the following statement as being true in a D's model of preferences:

> 'a', with respect to the jth criterion only, is at least as good as a'.

Let us point out that 'at least as good as' must be considered as 'not worse than'.

Let us consider for instance the case in which an *indifference threshold* q_j is associated with g_j. By definition, q_j is a real positive number such that:

$$a'I_ja \text{ iff } \; |g_j(a') - g_j(a)| \leqslant q_j\,.$$

In this case:

$$a'S_ja \text{ iff } g_j(a') \geqslant g_j(a) - q_j\,.$$

This formula can easily be generalized so as to take into account thresholds which are not constant (for example which vary with $g_j(a)$).

1.3. Outranking Concept: Level of Comprehensive Preferences

Taking into account the whole family of criteria, it is possible to define a *comprehensive outranking relation* S. By definition, S is a binary relation: $a'Sa$ holds if the values of the performances entering into $g(a')$ and $g(a)$ give a sufficiently strong argument for considering the following statement as being true in a D's model of preferences:

> 'a', with respect to the n criteria, is at least as good as a'.
>
> (Here again, 'at least as good as' is synonymous with 'not worse than'.)

For illustrating this concept, let us consider the following numerical example defined in Table I. Quite obviously, the assertions bSa, aSb and cSd hold. Apart from very strange cases (in which the last criterion would have an enormous importance), the assertion cSa holds. But, in the absence of an aggregating rule and additional information, none of the following assertions would seem to be accepted: cSb, bSc, dSb, bSd.

This example shows that, for defining a comprehensive outranking relation S on A, it is necessary to formulate a set of appropriate conditions which, when they are satisfied, can be viewed as sufficiently strong arguments for justifying the assertion $a'Sa$.

1.4. Fundamental Properties of Outranking Relations

Let us consider the n restricted outranking relations S_j linked to the n criteria of F and a binary relation S aggregating the same criteria. So

TABLE I

Numerical example.

A \ F	g_1	g_2	g_3	g_4	g_5
a	50	50	50	50	50
b	55	46	48	54	55
c	90	90	90	45	42
d	90	90	90	10	10

$\forall j \in F: q_j = 5$ and $0 \leq g_j(a) \leq 100$

that S can be viewed as a comprehensive outranking relation (in conformity with the meaning given above to this concept), it seems natural to us to demand that S verifies the following properties:

(a) S is reflexive ($aSa\ \forall a \in A$) and, for $a' \neq a$, the meaning of each configuration shown in Figure 1. is that presented in that figure.

Let us remark that:

- $a'Sa$ and not aSa' cannot be interpreted, without some precautions, as 'a' is strictly preferred to a';
- S is not necessarily a transitive binary relation.

(b) Let us denote by Δ_F the dominance relation defined by:

$$a\Delta_F \text{ iff } g_j(a) \geq g_j(b)\ \forall j \in F .$$

S has to verify, whatever the potential actions a, a', b and b' are:

$$a'Sa \text{ and } a\Delta_F b \Rightarrow a'Sb ,$$

$$b'\Delta_F a' \text{ and } a'Sa \Rightarrow b'Sa .$$

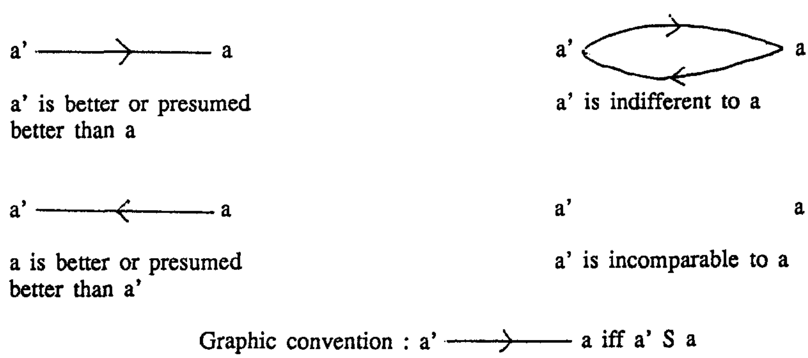

Fig. 1. Meaning of the four possible configurations in the comparison of two potential actions a and a'.

Consequently, we have:

$$a\Delta_F b \Rightarrow aSb\ .$$

(c) Finally, it is natural to demand:

$$\text{if } g_j(a') = g_j(a)\ \forall j \neq k, \text{ then } a'Sa \text{ iff } a'S_k a\ .$$

This condition can be reinforced as follows:

$$\text{if } a'I_j a\ \forall j \neq k, \text{ then } a'Sa \text{ iff } a'S_k a\ .$$

From this last property combined with the previous one, we can deduce (for more details, see Roy and Bouyssou, 1987a):

$$a'S_j a\ \forall j \in F \Rightarrow a'Sa\ .$$

2. BASIC CONCEPTS FOR BUILDING OUTRANKING RELATIONS

2.1. General Considerations

The formal expression and even the nature of the conditions which must be satisfied to validate the assertion $a'Sa$ can be influenced by many factors. The most important ones seem to be:

- the degree of significance of criteria taken into account in F;
- the nature of basic concepts used: concordance, discordance, substitution rate, intensity of preference, etc.;
- the nature of the inter-criteria information required (for a full definition of this term, see Roy and Bouyssou, 1987b);
- the strength of the arguments required: the strongest argument we can imagine for validating $a'Sa$ is clearly 'a' dominates a' but weaker arguments can be sufficient; it is the reason why S is usually much richer than Δ_F.

The variety of possible options relevant to each of the four preceding factors explains why there is not a single 'best' way of formulating the conditions to be satisfied to validate $a'Sa$. Henceforth, in this paper, we shall present only the way in which these conditions are formulated, within the framework of ELECTRE methods, in order to build the outranking relations upon which they are based. This is why we shall study only those cases in which:

- the degree of significance of each criterion g_j is reflected by means of two thresholds[5] q_j and p_j in conformity with the model of the pseudo-criterion presented in 2.2;
- the basic concepts are those of concordance and discordance (see 2.2 below);
- the inter-criteria information is synthesized in, at most, two kinds of data: for each criterion g_j, its importance coefficient k_j and its veto threshold v_j.

We would like to point out that, by slightly modifying the framework defined by the preceding options but maintaining the central role given to the concept of concordance, Brans and Mareschal (1990) and Brans *et al.* (1984), on the one hand, and Vansnick (1986, 1990), on the other hand, have proposed comprehensive preference models based on binary relations which are not necessarily transitive, leaving room for incomparability; however, although their approach was inspired by the outranking approach and shares many points in common with it, the binary relations they refer to are not, strictly speaking, outranking relations (for more details on this point, see Roy and Bouyssou, 1989).

Let us come back to the last of the four variety factors mentioned at the beginning of 2.1. Of course, it is difficult to fix a minimum degree of strength so that the assertion $a'Sa$ is accepted if and only if the strength of the arguments which justify it are at least equal to this minimum. For this reason, two types of modelling are envisaged:

First type: a set of r ($r \geqslant 1$) outranking relations is introduced so as to model D's preferences:

$$S_1 \subset S_2 \subset \cdots \subset S_r,$$

the increasing of the index from 1 through r corresponds to a decreasing strength of the arguments required for validating $a'Sa$.

Second type: instead of one or more crisp outranking relations for modelling D's preferences, it is a fuzzy outranking relation which is introduced; this means that we associate with each ordered pair (a', a) a real number $\sigma(a', a)$ $(0 \leq \sigma(a', a) \leq 1)$ characterizing the degree of strength of the arguments and allowing us (on the basis of the two vectors $g(a')$, $g(a)$ and additional inter-criteria information) to validate the assertion $a'Sa$. $\sigma(a', a)$ is called the *credibility index* of the outranking $a'Sa$.

The preference model is of type 1 in ELECTRE I, II and IV but of type 2 in ELECTRE III and A. If in practice these two types of modelling differ significantly, from a theoretical point of view, they are nearly equivalent: the only difference between them comes from the fact that, in the second type, the size r of the sequence of crisp outranking relations is not pre-determined.

2.2. *Concordance and Discordance Concepts*

(a) Concordant criterion

By definition, the jth *criterion is in concordance* with the assertion $a'Sa$ if and only if $a'S_ja$.

According to the definition of S_j (see 1.2) and to the introduction of an indifference threshold q_j, the jth criterion is in concordance with the assertion $a'Sa$ iff:

$$g_j(a') \geq g_j(a) - q_j .$$

The subset of all criteria of F which are in concordance with the assertion $a'Sa$ is called the *concordant coalition* (with this assertion). It is denoted by $C(a'Sa)$.

(b) Discordant criterion

By definition, the jth *criterion is in discordance* with the assertion $a'Sa$ if and only if aP_ja'.

According to the pseudo-criterion model:[6]

$$aP_ja' \text{ iff } g_j(a) > g_j(a') + p_j .$$

This means that the strict preference restricted to the jth criterion of a over a' is significantly established only when the difference $g_j(a) - g_j(a')$ is sufficiently large considering the imprecision, uncertainty and inaccurate determination of the performances.

The subset of all criteria of F which are in discordance with the assertion $a'Sa$ is called the *discordant coalition* (with this assertion). It is denoted by $C(aPa')$ since it can also be viewed as the concordant coalition with the assertion aPa'.

(*c*) *Consequences*

From the above definitions, we have:

$$C(a'Sa) \cap C(aPa') = \emptyset \text{ and } C(a'Sa) \cup C(aPa') \subset F .$$

Let us emphasize that we can have:

$$C(a'Sa) \cup C(aPa') \neq F .$$

This inequality holds if and only if there exists at least one criterion which is neither concordant nor discordant with the assertion $a'Sa$. This case appears iff:

$$g_j(a) - p_j \leq g_j(a') < g_j(a) - q_j \quad (p_j > q_j) .$$

The subset of F defined by the criteria satisfying this last condition will be denoted by $C(aQa')$. Consequently, we have:

$$j \in C(aQa') \text{ iff } aQ_ja',$$

the binary relation Q_j modelizes the weak preference situation (see 1.1 d)) restricted to the jth criterion.

In practice, $C(aQa')$ is empty for a large number or ordered pairs of potential actions.

Finally, with each ordered pair (a', a), a partition of F into three subsets is associated:

$$C(a'Sa),\ C(aQa'),\ C(aPa') .$$

It is on the basis of this partition that the assertion $a'Sa$ is (or is not) validated in ELECTRE methods. Before specifying the conditions of validation, it is important to remark that the partition defined above is robust in the sense given to this term by Roberts (1979). This means that the partition remains invariant when any criterion g_j is replaced by:

$$g'_j = \chi(g_j) \text{ with } \chi \text{ monotonous increasing function.}$$

This supposes, of course, that the initial thresholds q_j and p_j would be replaced by new thresholds (not necessarily constant) having the same meaning but taking into account the fact that the way the jth performance is defined has been modified.

2.3. Concordance and Discordance Indices

(a) The notion of importance of each criterion

For validating a comprehensive outranking relation S, it is necessary to take into account the fact that the role which has to be devoted to each criterion in the aggregation procedure is not necessarily the same. In other words, we need to characterize what is usually referred to as 'the greater or lesser importance' given to each criterion of F. When the aggregation procedure leads to a weighted sum (as in AHP for instance), this notion of importance is taken into account by means of constant substitution rates (currently called weights) assigned to each criterion. In more sophisticated models (MAUT for instance), those substitution rates can vary with performances. In both cases, the coefficients (tradeoffs) so defined are not intrinsic, i.e., they do not depend only on the axis of significance of the criterion g_j to which they refer. They also depend on the way g_j is defined: if g_j is replaced by $\chi(g_j)$ (χ monotonous increasing function), then each weight (or any substitution rate) has to be modified.

In ELECTRE methods, the importance of the jth criterion is taken into account by means of, at most, two characteristics:

– its *importance coefficient* k_j ($\geqslant 0$) which is intrinsic: the k_j's only

intervene in the definition of the concordance degree (see (b) below); moreover, these coefficients do not exist in ELECTRE IV;
- its *veto threshold* v_j ($\geq p_j$)[7]: v_j only intervenes in the definition of the discordance degree of criterion g_j (see (c) below).

(*b*) *The concordance index*

By definition, the *concordance index* $c(a', a)$ characterizes the strength of the positive arguments able to validate the assertion $a'Sa$. The strongest among them come from the criteria of $c(a'Sa)$ since they are all in favor of the assertion considered. They contribute one part to $c(a', a)$: $c_1(a', a)$. Some weaker positive arguments can also come from criteria of $c(aQa')$ since such criteria reflect a hesitation between the two following possibilities: $a'I_ja$ (which is in favor of $a'Sa$) and aP_ja' (which is not in favor of $a'Sa$). They contribute a second part to $c(a', a)$: $c_2(a', a)$. Consequently:

$$(2.1) \qquad c(a', a) = c_1(a', a) + c_2(a', a) .$$

By definition:

$$(2.2) \qquad c_1(a', a) = \frac{1}{k} \sum_{j \in C(a'Sa)} k_j \text{ with } k = \sum_{j \in F} k_j ,$$

$$(2.3) \qquad c_2(a', a) = \frac{1}{k} \sum_{j \in C(aQa')} \varphi_j \cdot k_j \text{ with } \varphi_j = \frac{p_j + g_j(a') - g_j(a)}{p_j - q_j}$$

Let us now explain how these formulae are justified.

According to the definition, it is natural to set:

$$0 \leq c(a', a) \leq 1 ,$$

$$c(a', a) = 0 \text{ if } C(aPa') = F ,$$

$$c(a', a) = 1 \text{ if } C(a'Sa) = F .$$

The ratio k_j/k reflects, by definition, the relative strength (in F) assigned to each g_j when this criterion is concordant with $a'Sa$. In other words, k_j can be viewed as the number of representatives supporting the point of view synthesized by the jth criterion in a voting procedure. If $j \in C(a'Sa)$, the whole strength k_j (or all the k_j representatives) contributes to $C(a', a)$. On the contrary, this contribution is null if $j \in C(aPa')$. If $j \in C(aQa')$, it is only a fraction φ_j of k_j which contributes to $C(a', a)$. This fraction φ_j must evidently increase from 0 to 1 when $g_j(a')$ increases from $g_j(a) - p_j$ to $g_j(a) - q_j$. In other words, the more hesitation occurs in favor of $a'I_ja$, the more 'the number $\varphi_j \cdot k_j$ of voters' who defend the assertion $a'Sa$ increases. It might seem arbitrary to modelize this growth with a linear formula. Any other formula would be just as arbitrary and would not offer the same simplicity.

(c) Veto effect and discordance index

Let us now consider the effect, on the validation of $a'Sa$, of any discordant criterion. Obviously, such a criterion is against the assertion in question but the strength of this opposition can be more or less compatible with the acceptance of the assertion. For reflecting the capacity given to the jth criterion when it is discordant, for rejecting the assertion $a'Sa$ without any help of other criteria, a *veto threshold* v_j is defined as follows:

> $g_j(a) - g_j(a') > v_j$ is incompatible with the assertion $a'Sa$ whatever the other performances are, i.e. even if $c(a', a) = 1 - k_j/k$.

We can also admit that this veto effect can occur for a difference $g_j(a) - g_j(a')$ smaller than v_j when $c(a', a) < 1 - k_j/k$. This leads to reinforce the veto effect all the more as $c(a', a)$ decreases.

The veto effect defined above works on the principle of all or nothing. Consequently, it is appropriate for defining one or more crisp outranking relations. If we consider now the second type of modelling introduced in 2.1 based on a fuzzy outranking relation, it is useful to modulate from 0 to 1 the strength of the opposition to $a'Sa$, according to the position of the difference $g_j(a) - g_j(a')$ on the interval $[p_j, v_j]$. This explains the definition of the *discordant index* (of criterion g_j with respect to the assertion $a'Sa$):

$$(2.4) \qquad d_j(a', a) = \begin{cases} 0 \text{ if } g_i(a) - g_j(a') \leq p_j \\ \dfrac{g_j(a) - g_j(a') - p_j}{v_j - p_j} \text{ if } p_j < g_j(a) - g_j(a') \leq v_j \\ 1 \text{ if } g_j(a) - g_j(a') > v_j \,. \end{cases}$$

Here again, it is in the interest of simplicity that a linear formula has been chosen.

Let us end this paragraph by emphasizing that, in a certain sense, the criterion g_j is all the more important when v_j is close to p_j. Yet this way of tackling the notion of a criterion's importance (through a veto effect) is fundamentally different from that which prevails when our reasoning is based on positive arguments in the context of concordance (importance coefficient k_j). It is clear that the two criteria rankings according to (i) decreasing values of $v_j - p_j$ and (ii) increasing values of k_j are not unrelated. Despite this, without reflecting any incoherence whatsoever, they may be significantly different.

3. DEFINITION OF OUTRANKING RELATIONS IN ELECTRE METHODS

3.1. ELECTRE IS

In ELECTRE IS, the assertion $a'Sa$ is considered valid iff the two following conditions are satisfied:

$$(3.1) \qquad c(a', a) \geq a, 1/2 < s \leq s^* \text{ with } s^* = 1 - \frac{1}{k} \min_{j \in F} k_j \,,$$

$$(3.2) \qquad g_j(a') + v_j \geq g_j(a) + q_j \cdot w(s, c)$$

(more rigorously, in the above formula, q_j should be replaced by $\min\{q_j, v_j - p_j\}$) with

$$w(s, c) = \frac{1 - c(a', a) - k_j/k}{1 - s - k_j/k} \,.$$

The first condition (concordance condition) simply expresses the fact that the value of the concordance index $c(a', a)$ (see (2.1), (2.2) and (2.3)) must be high enough to validate $a'Sa$: s is a parameter called the *concordance level*. This means that a sufficiently high majority of criteria has to be in favor of the assertion; that is why $s > 1/2$. Furthermore, it is easy to prove that if we give, to the concordance level, a value $s > s^*$, then:

- (3.1) is satisfied iff $C(aPa') = \emptyset$,
- when (3.1) is satisfied, so is (3.2).

Consequently, to set $s > s^*$ means that we want to impose $a'S_ja$ $\forall j \in F$ for validating $a'Sa$. The outranking relation so defined generalizes (for $q_j \neq 0$) the dominance relation (see 1.4b)). Requiring such unanimity constitutes an extreme case which generally has no interest in practice. In practical applications it seems natural to have the s parameter vary between 3/5 and 4/5.

The condition (3.2) expresses that, for each criterion, the veto effect does not occur. The coefficient $w(s, c)$ is used to modelize the reinforcement of the veto effect introduced in 2.3(c) above.

$$w(s, 1 - k_j/k) = 0\text{: no reinforcement},$$

$$w(s, s) = 1\text{: maximum reinforcement (magnitude } q_j).$$

3.2. *ELECTRE III*

The outranking relation in ELECTRE III is a fuzzy binary relation (see 2.1, second type). The credibility index $\sigma(a', a)$ which defines it makes the concordance index $c(a', a)$ intervene again. Moreover it brings in the discordance indices $d_j(a', a)$ (see 2.4) for those discordant criteria verifying $d_j(a', a) > c(a', a)$. In the absence of such discordant criteria, $\sigma(a', a) = c(a', a)$. This credibility value is reduced in the presence of one or more discordant criteria when $d_j(a', a) > c(a', a)$. This reduction is all the greater as $d_j(a', a)$ approaches 1. In conformity with the veto effect, $\sigma(a', a) = 0$ if $d_j(a', a) = 1$ for at least one criterion. More precisely, we have:

$$(3.3) \qquad \sigma(a', a) = c(a', a) \cdot \prod_{j \in D_c(a', a)} \frac{1 - d_j(a', a)}{1 - c(a', a)}$$

with

$$D_c(a', a) = \{ j / j \in F, d_j(a', a) > c(a', a) \}$$

(The explanations which justify this formula are presented in Roy and Bouyssou, 1989; in the interest of brevity, we will not develop them again here.)

3.3. Other ELECTRE Methods

Up to now, we have not mentioned either ELECTRE I or ELECTRE II. These methods have been supplanted by ELECTRE IS and ELECTRE III respectively. They are, nevertheless, still interesting from both a pedagogical and historical standpoint.

Let us remember that ELECTRE I (see Roy, 1968) was the first decision-aid method using the concept of outranking relation. The idea of modulating the credibility of the outranking insertion was introduced in ELECTRE II (see Roy and Bertier, 1973) where two models of preferences are taken into account: the first one being relatively poor but strongly justified and the second one richer but less defensible.

ELECTRE IV is a method in which no k_j is introduced. This does not mean that each criterion has exactly the same 'weight'. ELECTRE IV is appropriate for cases in which we are not willing or able to introduce information on the specific role (i.e. importance) devoted to each criterion in the aggregation procedure. A sequence of nested outranking relations is introduced:

$$S^1 \subset S^2 \subset S^3 \subset S^4 \subset S^5 .$$

Each S^i is defined by referring to concordance and discordance concepts (for an exhaustive definition of these five binary relations, see

Roy and Bouyssou, 1989). An application of this method to a ranking problem of suburban line extension projects is presented in Roy and Hugonnard (1982).

Let us mention finally a new ELECTRE method (ELECTRE A, A for Assignment) which has been built to solve some specific problems in the banking sector. It is now used routinely but is not publishable for reasons of confidentiality. The general orientation is that indicated in Moscarola and Roy (1977) and Roy (1981).

4. SOME PRACTICAL CONSIDERATIONS

4.1. How to Use Outranking Models for Decision-Aid

Let us consider a comprehensive model of preferences defined on A. Let us suppose first that this model is nothing more than a single criterion $g(a)$. The way to use it for decision-aid is quite obvious whatever the problem statement considered. As we have shown (see Roy, 1985), three basic problem statements $P.\alpha$, $P.\beta$, $P.\gamma$ must be distinguished. Briefly, we can characterize each of them by saying that decision-aid is envisaged according to the following perspective:

with $P.\alpha$: isolate the smallest subset $A_0 \subset A$ liable to justify the elimination of all actions belonging to $A \backslash A_0$;
with $P.\beta$: assign each action to an appropriate predefined category according to what we want it to become afterwards;
with $P.\gamma$: build a partial (or complete) pre-order as rich as possible on a subset A_0 of those among the actions of A which seem to be the most satisfactory.

Let us suppose now that the comprehensive model of preferences is an outranking relation S (crisp or fuzzy) or a sequence of nested outranking relations. Contrary to the preceding case, the way to proceed in order to (α) isolate A_0, (β) assign each action to a predefined category, (γ) build a partial (or complete) pre-order (according to the problem statement chosen) is not obvious. This topic is discussed by Vanderpooten (1990). Let us emphasize here the fact that each ELECTRE method combines:

(i) a given problem statement $P.\alpha$, $P.\beta$ or $P.\gamma$,
(ii) a way of defining a comprehensive model of preference (see Section 3 above).

4.2. *How to Choose among ELECTRE Methods*

Before answering such a question, we invite the reader to consider Table II. This table summarizes the main characteristics by which ELECTRE' methods can be differentiated. For selecting the most appropriate for a given decision-aid context, we suggest proceeding as follows. Consider first the problem statement chosen (see 4.1 above), then:

if $P.\alpha$: two ELECTRE methods, ELECTRE I and ELECTRE IS, can be envisaged. ELECTRE I should be selected only if it is truly essential to work with a very simple method and it is realistic to have $p_j = q_j = 0\ \forall j \in F$.
if $P.\beta$: there is presently no choice.
if $P.\gamma$: three methods, ELECTRE II, III and IV are in competition; ELECTRE II should be selected only if simplicity is required and $p_j = q_j = 0\ \forall j \in F$ is realistic; ELECTRE IV is convenient only if there exists a good reason for refusing the introduction of importance coefficients k_j.

4.3. *How to Give Numerical Values to Thresholds and Importance Coefficients*

Let us remember that the indifference and preference thresholds q_j and p_j have been introduced (see 1.2 above) so as to be able to interpret correctly differences between performances. The simplest way for giving a numerical value to such thresholds consists in coming back to their definition (see Roy, 1985, Ch. 9) and in analyzing the main sources of imprecision, uncertainty and inaccurate determination. For more details and a presentation of some more sophisticated techniques, see Bouyssou and Roy (1987) as well as Roy *et al.* (1986) for an illustration based on a concrete example, which has the advan-

Table II

Main characteristics of ELECTRE methods.

ELECTRE methods	I	IS	II	III	IV	A
Possibility for taking into account indifference and/or preference thresholds	no	yes	no	yes	yes	yes
Necessity of a quantification of the relative importance of criteria	yes	yes	yes	yes	no	yes
Number and nature of outranking relations[1]	1	1	2	1 fuzzy	5	1 fuzzy
Problem statement	α	α	γ	γ	γ	β
Final results	a kernel	a kernel with consistency and connected indices	a partial preorder	a partial preorder	a partial preorder	an assignment to predefined categories

[1] All outranking relations are based on concordance and discordance concepts; except for boxes containing 'fuzzy', the figures refer to non-fuzzy relations.

tage of using very diverse processes, given the variety of criteria in question.

In many cases it is difficult, and perhaps arbitrary, to fix a precise numerical value for some of the $q_j s$ and/or $p_j s$. We then can try to insert them between a plausible minimum and maximum value. Let us emphasize the fact that:

(i) it is often easy to give to those thresholds a value different from 0 and less arbitrary than the value 0;
(ii) and it is not easier to try to take into account the different sources of imprecision, uncertainty and inaccurate determination (see Roy, 1988) by means of probabilistic distributions (as in MAUT): choosing the form of the distribution, or giving a numerical value to its diverse characteristics (mean value, standard deviation, minimum, maximum, etc.) comprises an amount of arbitrariness as considerable as that involved in threshold evaluation.

We are confronted with similar difficulties for characterizing the specificity of the role devoted to each criterion by means of the importance coefficient k_j and the veto threshold v_j. Let us again underline that the $k_j s$ are intrinsic, i.e. they do not depend on the nature of the scale chosen for evaluating performances. This intrinsic characteristic facilitates our examination[8] of the values we can appropriately attribute to these coefficients in order to reflect the relative importance a given decision-maker will assign to different criteria bearing in mind that his ideas concerning this are often rather vague. To do this, we have developed a questioning technique which is illustrated in detail in Roy *et al.* (1986).

We would like to emphasize that this technique is not designed to 'estimate' the value of each k_j 'with maximum precision'. Indeed we consider that the very idea of estimating is without any basis at all here. The concepts of estimation and approximation refer, of course, to a quantifiable entity whose 'real value' exists somewhere. Yet this 'somewhere' can only be found in the mind of "someone", namely the decision-maker. We mentioned above (see 1.1(d)) that the latter is often difficult to identify because he is more or less mythical and when he is not, he is frequently not very accessible. When he is accessible, the idea he holds of each criterion's importance is, in most cases, neither formalised nor quantified. The role that each criterion could and should play in designing comprehensive preferences is not something factual which can be observed: it is, in the main, a reflection of a system of values, but also of more fragile opinions, which too detailed a discussion will disturb. That is why, in the questioning technique referred to above, we proceed by a comparison of actions which can be differentiated only through two or three of their performances, asking only qualitative questions. The questions may be asked simultaneously to several actors in order to set forth clearly the areas of consensus and of irreconcilable differences.

The result is a *domain* of values for k_j which are acceptable to a group of actors. This domain, by means of the non-restrictive hypothesis $k_1 \leq k_2 \leq \cdots \leq k_n$, is then rewritten in the following form:

$$m_1(k_1) \leq k_2 \leq M_1(k_1)$$

$$m_2(k_1, k_2) \leq k_3 \leq M_2(k_1, k_2)$$

$$\dots$$

$$m_{n-1}(k_1, \dots, k_{n-1}) \leq k_n \leq M_{n-1}(k_1, \dots, k_{n-1}) .$$

It is in no way restrictive to put $k_1 = 1$. We can then easily explore the domain of validity for k_js and deduce from it (when, for certain k_js the variation interest $[m_{j-1}, M_{j-1}]$ is large) a small number of contrasting sets.

Finally, the numerical value of each veto threshold v_j should be discussed on the basis of its definition (see 2.3 c)). It is often more enlightening to base our reasoning on the v_j/p_j ratio rather than on v_j alone. It is especially important to compare the way in which the criteria are ranked according to the decreasing values of this ratio to the way in which they are ranked according to increasing values of k_j. When there are differences, it is important for them to be based on clear explanations (for example: the desire to make a criterion which is not too important, nonetheless bring the veto into play for a small difference in performance). Here again, if it is difficult to give a precise value to a v_j threshold, we can assign it a variation interval.

It emerges from the preceding that it is not always possible, for the thresholds or for the importance coefficients, to match each one to a well-defined numerical value. Each time we assign a non-negligible amplitude to one or another of these parameters, it is important to explore the effect of this lack of determination. To do so, we can start by applying the ELECTRE method selecting, adopting for each threshold and importance coefficient the value which corresponds to the middle of the interval. Then we must take different combinations of extreme values into consideration. We can thus study the robustness of our conclusions in relation to incompressible margins of arbitrariness which this lack of determination intervals reflect. Examples of such analyses of robustness can be found in Roy and Hugonnard (1982), Renard (1986), Roy *et al.* (1986).

5. CONCLUSION

It seems important to us to draw attention to the fact that the difficulties mentioned above involve the assigning of numerical values essential to characterizing comprehensive preference models. Such difficulties are in no way specific to multicriteria aggregation procedures of the ELECTRE type. Difficulties of the same kind may be encountered in one way or another in all forms of modelling. We believe that they are inherent in all decision problems. It is important for the approach we adopt not to cast these difficulties into shadow but rather to highlight them.[9] The analysis of robustness should, therefore, play a central role in developing a prescription, whatever the type of modelling adopted. One of the advantages of the ELECTRE methods is precisely that they make such robustness analysis particularly easy.

NOTES

[1] For more details on this concept, see Roy (1985, Ch. 7) or Roy and Vincke (1987).
[2] Analytic Hierarchy Process (see Saaty (1980)).
[3] Multi-Attribute Utility Theory (see Keeney and Raiffa (1976)).
[4] See Roy (1990).
[5] In this paper, so as to simplify the way formulas are written, they will be given with constant thresholds. They can easily be generalized to thresholds $q_j[g_j(a)]$ and $p_j[g_j(a)]$ varying with $g_j(a)$ (see Roy and Bouyssou (1989)).

[6] For more details, see Roy (1985) or Roy and Vincke (1987).
[7] In this paper, so as to simplify the way formulas are written, they will all be given with constant thresholds. They can easily be generalized to thresholds $v_j[g_j(a)]$ varying with $g_j(a)$ (see Roy and Bouyssou (1989)).
[8] In the AHP and MAUT methods, the non-intrinsic character of the weights and substitution rates only further complicates the same examination.
[9] See Roy (1990) and Bouyssou (1988).

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