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DEONTIC LOGIC WITHOUT DEONTIC OPERATORS

ABSTRACT. The usual axioms and inference rules of deontic logic employ as a new primitive term an operator for 'obligatory' or for 'permitted'. These axioms and inference rules are here derived from a language which instead of the operator contains a predicate 'admissible' defined on the set of state descriptions of an assertoric language. This approach eliminates the problem of constructing a deontic formalism of its own. The predicate version requires fewer and weaker decisions and is closer to intuitive notions than the operator version. The solutions of some open problems of deontic logic flow automatically from the well-known rules of the assertoric predicate calculus. Special attention is given to the relations between deontic and assertoric statements.

The proposed formulation of deontic logic suggests a natural generalization by proceeding from two to any number of degrees of admissibility. When there is a continuum of degrees of desirability, deontic logic becomes identical with the calculus of utilities.

1. THE OPERATOR FORMULATION OF DEONTIC LOGIC

The operator formulation of deontic logic may be exemplified by von Wright's (1968, p. 17) axiom system. An operator '§' is applied to atomic or molecular assertoric sentences 'p'. The result '§p' is an atomic deontic sentence read 'p is permitted'. Atomic deontic sentences can be negated and combined by the connectives of the propositional calculus to form molecular pure deontic sentences. Later on von Wright (1968, pp. 82f) also admits mixed sentences, i.e. compounds of pure deontic and pure assertoric sentences formed by the connectives of the propositional calculus.

The deontic operator is subject to the following axioms:

(1) $\S(p \vee q) \equiv \S p \vee \S q$

(2) $\S p \vee \S - p$

Often another deontic operator is introduced by the following definition:

(3) $!p \equiv - \S - p$

'$!p$' is read 'p is obligatory'. Applying the transformation rules of the propositional calculus, (2) can be written in the more familiar form

(4) $!p \supset \S p$

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'$!-p$' says '$-p$ is obligatory', i.e. 'p is forbidden'.

Inference rules may be stated for systems of pure deontic sentences (von Wright 1968, p. 17) and for mixed systems (von Wright 1968, pp. 82f; Bergström 1962, pp. 37f). On the other hand, Rescher (1966, p. 100) leaves open the problem of inference rules for mixed systems.

Some of the problems arising in connection with the operator formulation of deontic logic are:

(1) What is the contradictory of '$!p$'? (Bergström, 1962, pp. 22ff; Ross, 1968, pp. 169f).

(2) Should obligation and permission be interdefinable according to (3)? (Von Wright, 1968, p. 17 n. 1; Frey, 1965, pp. 380ff).

(3) Should '$!p$' imply '$!(p \vee q)$'? (Von Wright, 1968, pp. 20ff; Ross, 1968, pp. 160f).

(4) What should the inference rules for sentence systems containing deontic expressions be like? (Bergström, 1962, pp. 37–51; Rescher, 1966, pp. 99f; von Wright, 1968, pp. 17, 82f; Weinberger, 1970, pp. 217–221.) It is just the most recent of these studies that clearly shows a great deal of uncertainty concerning deontic inference.

The predicate version of deontic logic will furnish a solution to each of these problems.

2. OPERATOR-FREE DEONTIC LOGIC

2.1. *State Descriptions*

Let us think of a language system containing n atomic assertoric sentences $p_1, p_2, \ldots, p_n$.

We define:

> A state description is a conjunction of the n atomic sentences each of which may or may not be preceded by a negation.

Thus a state description is a strongest consistent statement. It conveys information about the truth values of all the atomic sentences of the language system. There are 2^n possible state descriptions.

Any statement is logically equivalent to a disjunction of state descriptions (which is nothing but its full disjunctive normal form). An analytic statement is equivalent to the disjunction of all 2^n state descriptions. A contradiction is equivalent to the empty disjunction containing no state description at all.

We define:

> The range of p is the set of state descriptions whose disjunction is equivalent to p.
> The content of p is the complement of its range, i.e. the set of state descriptions with which p is incompatible.

The content of p is identical with the range of $-p$. The range of an analytical statement is the universal set, its content the empty set. The range of a contradiction is the empty set, its content the universal set.

Let us introduce an individual variable s whose values are state descriptions, and predicates $P, Q, \ldots$ defined on the set S of state descriptions where Ps says that s is in the range of p ($P, Q, \ldots$ being the capital letters corresponding to $p, q, \ldots$).

The concept of state description is defined analogously for the predicate calculus. If the language system contains m primitive predicates and n individuals, a state description is a conjunction of mn sentences each of which assigns one of the predicates or its negation to one of the individuals. The number of possible state descriptions is 2^{mn}. The definitions of range and content are the same as above.

2.2. *Assertoric Statements in Terms of State Descriptions*

The set S of state descriptions consists of consistent statements only. Those analytic statements which are meaning postulates (definitions of extra-logical terms) exclude certain state descriptions from S as inconsistent. E.g., the definition of 'bachelor' excludes those state descriptions by which some individual is said to be a bachelor and married or female.

A synthetic statement p says that the true state description is in P, the range of p, so the state descriptions in $-P$ are said not to be true. If p is the premiss of an assertoric inference (when there are several premisses, 'the' premiss is their conjunction), then going from the premiss to the conclusion is equivalent to replacing S by P. The conclusion then is analytically valid within P, i.e., in P there is no state description that lies outside the range of the conclusion statement. Because we are interested in synthetic assertoric statements in so far as they are premisses in (mixed) inferences, we may adopt the convention that a synthetic statement excludes certain state descriptions from S as empirically false, just as an analytic statement, as far as it is a meaning postulate, excludes certain state descriptions from S as inconsistent. So we have for all assertoric statements:

(5) $p \equiv (s)\, Ps$

When p is analytic in the narrower sense (e.g. if it is $q \vee -q$), (5) is also valid: it then expresses a property that the members of S possess in any case, instead of being a condition that excludes certain s's.

The connectives of the propositional calculus translate into state descriptions language as follows:

(6) $-p \equiv (s) - Ps$

(7) $p \,\&\, q \equiv (s)\, (Ps \,\&\, Qs)$

In general, if $f(p, q, \ldots)$ is a molecular expression of the propositional calculus made up of the atomic sentences $p, q, \ldots$, then

(8) $f(p, q, \ldots) \equiv (s)\, f(Ps, Qs, \ldots)$

2.3. *The Deontic Predicate 'Admissible'*

Now we construct a deontic logic by introducing the predicate A defined on S where 'As' says that s is admissible, or permitted (not, of course, in the sense of the formation rules of the assertoric language, but in the sense of some evaluation to be expressed by the deontic language). A may be true of any number of state descriptions; it may be true of all of them (that would mean that everything is permitted). But a reasonable deontic system should not forbid everything; so we may accept as an axiom:

(9) $(Es)\, As$

i.e., at least one state is permitted. But note that a deontic system in which (9) is not true, i.e. in which everything is forbidden, would not be formally inconsistent in terms of the rules of the predicate calculus.

Now we *define the deontic operators in terms of A*:

(10) $!p \equiv (s)\, (As \supset Ps)$

(11) $\S p \equiv (Es)\, (As \,\&\, Ps)$

In words: 'p is obligatory' means that all admissible state descriptions are within the range of p, or: there is no admissible state description outside the range of p. 'p is permitted' means that there is at least one admissible state description within the range of p.

As for the latter definition, one might ask whether it should not rather read: all state descriptions within the range of p are admissible. But this would mean that whenever p is realized, an admissible state is realized, no matter which further statements are true; and this would not correspond to the natural language meaning of 'p is permitted' which is: it is permitted to realize p, but if we at the same time realize q, this may not be permitted. E.g., it is permitted to smoke cigarettes, but it is not permitted to smoke cigarettes in a gasoline station.

On this basis we can *prove axiom* (3) *which defines one deontic operator in terms of the other:* Substituting (10) and (11) in (3) we get:

$$(s)\,(As \supset Ps) \equiv -(Es)\,(As\ \&\ -Ps)$$

Transforming the right side of the equivalence, we get:

$$(s)\,(As \supset Ps) \equiv (s) - (As\ \&\ -Ps)$$

$$(s)\,(As \supset Ps) \equiv (s)\,(As \supset Ps), \quad \text{a tautology.}$$

This transformation works in reverse order as well.

So problem 2 (Section 1) is settled for our deontic system. Of course one may debate the adequacy of axiom (3). Those who do not accept it usually argue that many things are neither explicitly allowed nor forbidden by the law or some moral system. But from (3) we get by substituting $-q$ for p:

(12) $\quad !-q \equiv -\S q$

whence it follows that if q is not permitted, it is forbidden. If we understand 'not permitted' as 'not explicitly permitted', then this consequence of (12) (and (3)) is at variance with practice: the law does not have to permit things explicitly, but it is understood that anything that is not forbidden is permitted.

In this system of deontic logic we do not distinguish between explicit and implicit permission and prohibition; we stipulate that the predicate A *is* defined on the set S of state descriptions, i.e. that somehow for each state description there exists an evaluation as to whether it is admissible or not.

Frey (1965, pp. 380ff) has proposed an intuitionistic calculus for characterizing a 'concessional system' where everything that is not forbidden is permitted, and an 'interdictional system' where everything that is not explicitly permitted is forbidden. In my opinion it is not necessary to resort to an intuitionistic formalism. The concessional system could simply be characterized by interpreting '$!-p$' as 'p is (explicitly) forbidden', and '$\S p$' as 'p is permitted', and accepting (3); the interdictional system could be characterized by likewise accepting (3), interpreting '$\S p$' as 'p is explicitly permitted', and '$!-p$' as 'p is forbidden'. One might also think of a system in which p can be explicitly forbidden, explicitly permitted, or neither (where the pragmatic meaning of the last possibility ought to be clarified). This system could be formalized in a non-intuitionistic predicate version of deontic logic by using, instead of the predicate A, a three-membered predicate family.

Problem 1 (Section 1) is settled thus: $-!p$ is the contradictory of $!p$; $!-p$ is a contrary of $!p$. This can be easily verified by using (10).

2.4. *Derivation of the Axioms of the Operator Version of Deontic Logic*

Applying (11) to (1) we get:

$$(Es)\,(As\ \&\ (Ps \vee Qs)) \equiv (Es)\,(As\ \&\ Ps) \vee (Es)\,(As\ \&\ Qs)$$

Transforming the left side of the equivalence, we get:

$$(Es)((As\ \&\ Ps) \vee (As\ \&\ Qs)) \equiv (Es)\,(As\ \&\ Ps) \vee (Es)\,(As\ \&\ Qs)$$

$$(Es)\,(As\ \&\ Ps) \vee (Es)\,(As\ \&\ Qs) \equiv (Es)\,(As\ \&\ Ps) \vee$$
$$\vee\,(Es)\,(As\ \&\ Qs), \quad \text{a tautology.}$$

This transformation works in reverse order as well.

So by applying the definition of the permission-operator in terms of the deontic predicate to the axiom (1) of the operator language and transforming according to the rules of the predicate calculus, the axiom is seen to be equivalent to a tautology. So it is seen that axiom (1) does not express anything specifically deontic about the operator '§'.

Applying (11) to axiom (2) we get:

$$(Es)\,(As\ \&\ Ps) \vee (Es)\,(As\ \&\ -Ps)$$

or equivalently

$$(Es)\,((As\ \&\ Ps) \vee (As\ \&\ -Ps))$$

or equivalently

$$(Es)\, As$$

So axiom (2) of the operator version is equivalent to axiom (9) of the predicate version which characterizes a reasonable system of evaluations which does not forbid everything.

2.5. *'The Burnt Letter Paradox'*

It is easily seen that

$$(13) \qquad !p \supset !(p \vee q)$$

is analytic when translated by means of (10) into the predicate language:

$$(14) \qquad (s)\,(As \supset Ps) \supset (s)\,(As \supset (Ps \vee Qs)).$$

So from $!p$ we can infer $!(p \vee q)$ by modus ponens.

When we interpret 'p' as 'x mails the letter' and 'q' as 'x burns the letter', then the uneasy situation arises that the command to mail a letter implies the command to mail or burn it. In contradistinction to von Wright (1968, pp. 20ff) and Ross (1968, pp. 160f), I wish to argue that this situation is only apparently uneasy, let alone paradoxical.

In assertoric logic we have the undisputed inference from p to $p \vee q$. But $p \vee q$ cannot replace p in any context; it is true whenever p is true, but it may be true when p is false. So from '$x = 64$' follows '$x =$ some positive integer', but a teacher would not be very pleased with the latter answer to his question 'what is 2^6?'; and he would have every right to reject '$x = 12$' though it is a truth instance of '$x =$ some positive integer'.

The deontic case runs quite parallel. When John burns the letter, he can claim to have fulfilled some command following from the original command, but not the original command itself: $p \vee q$ is true because q, which is incompatible with p, is true. One should not be upset by the fact that violating a command may fulfill some command that follows from it, just as one is not upset by the fact that a statement that is incompatible with some hypothesis may imply some consequence of the hypothesis. So the paradox, which is not a specific paradox of deontic logic in any case, is completely dispelled.

In some situations there is a strong pragmatic expectation that one be given the strongest true statement, not some (weaker) consequence of it (though it is also true). E.g., when someone asks 'which is the speed regulation in towns in this country?', he would feel deceived by the answer 'one must not go faster than 80 km/h' though it is a (true) consequence of the appropriate 'one must not go faster than 50 km/h'.

2.6. *Mixed Sentences Eliminated*

Let us consider a simple example of a mixed sentence:

(15) $p \supset !q$

e.g. 'when it rains, take an umbrella!'. Ross (1968, pp. 167f) argues that this is a meaningless formula, and that the intended meaning is expressed by

(16) $!(p \supset q)$

I feel the same way. (15) would say: 'when p is true, then it arises that q is obligatory'. But I do not see why the obligation should arise only after some state of affairs comes to be real. We can specify which states of affairs are admissible before we know which one will be the case. The predicate A is defined on the set of all possible state descriptions in advance. So (16) seems appropriate: p & $-q$ is inadmissible, and all the other state descriptions are admissible.

The function of pure deontic sentences is to tell something about which state descriptions have the property A. The function of pure assertoric sentences is completely different and independent of the former: Synthetic assertoric statements tell something about which state descriptions are empirically false; analytic statements either are compatible with all state descriptions, or they are meaning postulates and then tell which state descriptions are contradictory. Let us therefore propose the following formation rule for deontic language systems:

> Only pure deontic and pure assertoric sentences are well-formed expressions, with an exception to be mentioned presently.

The exception concerns inferences from sets of premisses containing both pure assertoric and pure deontic sentences. In assertoric logic a set of premisses is understood as the conjunction of the premisses. Inference rules are based upon this fact. The same thing should be laid down for mixed sets of premisses, for otherwise we could not automatically apply the inference rules of the predicate calculus. So we stipulate the following amendment to the formation rule:

> Conjunctions of pure deontic and pure assertoric sentences are well-formed expressions.

2.7. *Mixed Inference*

Rescher (1966, pp. 99f) had not been able to specify a general rule for inference from sets of premisses containing both deontic and assertoric sentences. His preliminary rule for mixed command inference (1966, p. 99) was invalidated by the following counterexample, which I here cite in slightly altered form:

(17)	$!p$	John, work!
(18)	$\underline{p \supset q}$	John in fact drinks alcohol when he works
(19)	$!q$	John, drink alcohol!

This inference is valid in our system, too:

(17′) $(s)\,(As \supset Ps)$
(18′) $(s)\,(Ps \supset Qs)$
(19′) $(s)\,(As \supset Qs)$

Why do we not like the consequence $!q$? It might be because we evaluate drinking during work negatively, or because we do not want that anybody should be commanded to drink alcohol. These evaluations, however, are not expressed in any of the premisses, so that we cannot expect them to be taken account of in the conclusion. In so far the example would not speak against the inference schema.

But even apart from the special content of q we might feel that our intention is to command p, and that it is illegitimate to infer from this some quite different command, $!q$. Here we should view the consequence $!q$ not in isolation, but as contingent upon the premisses. $!q$ is not the original evaluation of all logically possible states of affairs, but what remains of it when the range of possibilities is restricted by some synthetic premiss. The consequence $!q$ is to be understood thus: given that $p\ \&\ -q$ does not exist, all admissible states of affairs (happen to) fulfill 'q'. $!q$ is not a consequence of $!p$, but of $!p$ and $p \supset q$; it is not to be mistaken for the comprehensive evaluation of all logically possible states of affairs. The contingence of the conclusion upon the premisses is especially important when the synthetic premiss is not a consequence of laws of nature plus initial conditions that we cannot change (e.g., past conditions), so that it is physically possible for us to make the premiss false. This is the case for John's habit to drink while working; we may presume it can be changed, so that the consequence $!q$ is valid only in case $p \supset q$ happens not to be changed.

A parallel example is easily constructed in assertoric logic. From 'When the weather is bad, I do not climb on mountains' and 'During all the holidays in my life the weather is bad' follows 'During all the holidays in my life I do not climb on mountains'. This consequence, taken in isolation, would draw a distorted picture of my policy of spending holidays; but nobody would charge the assertoric inference schema for it.

Perhaps the most satisfactory analysis of the counterexample can be given on the basis of a generalization of deontic logic in the next section.

There is another interpretation of the inference (17)–(19) which yields the 'Good Samaritan paradox' (Rescher, 1966, p. 100 n. 6):

> Victims of assaults should be helped
>
> Helping victims of assaults presupposes (implies) the existence of victims of assaults
>
> ---
>
> There should exist victims of assaults (i.e., victims of assaults should be brought about)

Of course one could reformulate the Samaritan command (17) as

(20) $!(q \supset p)$

i.e. when there is a victim of an assault, it should be helped,

the predicate version of which is

(20′) $(s)(As \supset (Qs \supset Ps))$

It is easily verified that (20′) together with (18′) does not yield the conclusion (19′). But this would not be a solution of the paradox, which consists in a counterexample to the inference schema (17′)–(19′). We shall be able to solve the paradox in the next section.

The point with mixed inference is that it is not necessary to devise any special rules for it at all in our system: everything is settled automatically by the inference rules of the predicate calculus.

3. GENERALIZATION OF DEONTIC LOGIC TO ANY NUMBER OF DEGREES OF ADMISSIBILITY; THE CALCULUS OF UTILITIES

Ross (1968, p. 167 n. 2) discusses the following inference:

> Everyone... shall undergo a certain vaccination
> Everyone who has undergone the vaccination shall undergo a test
> ―――――――――――――――――――――
> Everyone shall undergo the test

In symbols:

(21)	$!p$	(21′)	$(s)\,(As \supset Ps)$
(22)	$\underline{!(p \supset q)}$	(22′)	$\underline{(s)\,(As \supset (Ps \supset Qs))}$
(23)	$!q$	(23′)	$(s)\,(As \supset Qs)$

The inference (21′)–(23′) is correct according to the rules of the predicate calculus. But Ross' example throws doubt upon the validity of the deontic inference. He comments (1968, p. 167 n. 2): "Let us assume that the demand for a vaccination is a relative(ly) unimportant regulation sanctioned only by an insignificant fine; but that the demand for a subsequent test is of high importance... and therefore sanctioned by penalty of imprisonment. It would then be a serious injustice to sentence a person to imprisonment because he had neglected his obligation to undergo the vaccination."

Ross clearly refers to different degrees of undesirability of states of affairs. Consequently, I should say, deontic logic cannot deal adequately with this situation when it distinguishes only two degrees: admissibility (A) and inadmissibility ($\text{-}A$). In this case we can only say: taking the vaccination and the test is admissible, and all other states are inadmissible; but we cannot express that taking the vaccination and not taking the test is much worse than not taking the vaccination.

The natural solution to this problem is to use, instead of the predicate A, an ordered family of predicates each of which expresses a level of admissibility/inadmissibility. Equivalently we can assign integers to the state descriptions which are the rank numbers of the levels of admissibility/inadmissibility. And why should we limit ourselves to integers? We can assign real numbers to the state descriptions expressing their degree of desirability, or utility. So *deontic logic would find its natural generalization in the calculus of utilities.*

Now we can easily solve the Good Samaritan paradox: we simply say that the worst state is: an existing victim, and no helping; that a more desirable state is: an existing victim, and helping; but that it is still better if there exists no victim at all (and then, of course, no helping).

In the alcohol example we shall say that the most preferable state is John's working and not drinking; as for the rest, we can say whether we prefer him to work and drink (e.g. when John is a writer) or rather not to work (e.g. when John is a bus driver). So when, according to (18), $p \;\&\; -q$ does not exist, the consequence would be that either $p \;\&\; q$ or $-p$ is the most preferable among the remaining possibilities, and the other the least preferable. I.e., if we are ready to accept John's drinking habit, we either tell him to work (and then, as accepted, to drink), or not to work. But we need not conceal from him our premiss which implies that the most preferable state would be his working and not drinking.

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BIBLIOGRAPHY

[1] Bergström, L., 'Imperatives and Ethics', Filosofiska studier, Filosofiska institutionen vid Stockholms Universitet, Häfte 7. Stockholm, 1962.
[2] Frey, G., 'Imperativ-Kalküle', in *The Foundations of Statements and Decisions*, Internat. Colloq. on the Methods of Science, Warsaw 1961 (ed. by K. Ajdukiewicz), Polish Scientific Publishers, Warsaw, 1965.
[3] Rescher, N., *The Logic of Commands*, Routledge and Kegan Paul, London, 1966.
[4] Ross, A., *Directives and Norms*, Routledge and Kegan Paul, London, 1968.
[5] Weinberger, O., *Rechtslogik*, Springer, Wien and New York 1970.
[6] Von Wright, G. H., 'An Essay in Deontic Logic and the General Theory of Action', *Acta Philosophica Fennica*, fasc. XXI., North Holland Publishing Co., Amsterdam, 1968.