AVERAGING PROCEDURES FOR FLOW WITHIN VEGETATION CANOPIES

M. R. RAUPACH

CSIRO Division of Environmental Mechanics, P.O. Box 821, Canberra City ACT 2601, Australia

and

R. H. SHAW

Department of Land, Air and Water Resources, University of California, Davis, California 95616, U.S.A.

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Abstract. Most one-dimensional models of flow within vegetation canopies are based on horizontally averaged flow variables. This paper formalizes the horizontal averaging operation. Two averaging schemes are considered: pure horizontal averaging at a single instant, and time averaging followed by horizontal averaging. These schemes produce different forms for the mean and turbulent kinetic energy balances, and especially for the 'wake production' term describing the transfer of energy from large-scale motion to wake turbulence by form drag. The differences are primarily due to the appearance, in the covariances produced by the second scheme, of dispersive components arising from the *spatial* correlation of time-averaged flow variables. The two schemes are shown to coincide if these dispersive fluxes vanish.

1. Introduction

The airflow within and just above a vegetation canopy is strongly three-dimensional, because it is mechanically and thermally influenced by the complex geometry of the canopy element array. Nevertheless, a one-dimensional framework is always used in both theoretical and experimental studies of the vegetation-atmosphere interaction. An operation of horizontal averaging is therefore implicit in all such theories. Experimentalists are usually forced to assume that flow properties measured at one point are equal to those of the horizontally averaged flow field.

The horizontal averaging operation is important because it must account for the appearance, within a one-dimensional framework, of various inherently three-dimensional canopy effects: form drag, viscous drag, property emission or absorption, and the generation of wake turbulence at length scales determined by the canopy elements. Wilson and Shaw (1977) pointed out that the last of these effects is incorrectly described by the traditional approach of introducing form drag as an extra body force term in the momentum equation: such a term suppresses turbulent kinetic energy (TKE) within the canopy, whereas the effect of form drag is to convert mean kinetic energy (MKE) and large-scale TKE to TKE at element scales. Part of the inertial eddy cascade process is thereby short-circuited. Wilson and Shaw suggested that all these canopy effects can be correctly described, without recourse to additional appended terms, by properly horizontally averaging the conservation equations. They offered two horizontal averaging schemes: in the first, the instant-

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aneous flow field is horizontally averaged over a plane large enough to eliminate variations due to both the canopy structure and the largest length scales of the turbulent flow. In the second scheme, the three-dimensional flow in the canopy is first time-averaged in the normal way, and then averaged horizontally over a plane large enough only to eliminate variations in canopy structure. We will call these schemes I and II, respectively. Under the usual assumption of a uniform, level canopy above which the flow is stationary and horizontally homogeneous, both schemes lead to well-defined, time-independent results. Wilson and Shaw asserted that the results are identical.

This paper formalizes the horizontal averaging operator and its properties, and then considers the effect of both averaging schemes on the conservation equations for mean quantities and second moments in a canopy flow. The schemes are shown to produce different results at second order, in a way which makes explicit the assumptions involved in a one-dimensional theory, and which helps to clarify the nature of wake influence on canopy flow.

2. Properties of the Horizontal Averaging Operator

Let angle brackets denote the horizontal average of a flow variable, and double primes the departure therefrom; thus, the velocity field may be written $u_i(\mathbf{x}, t) = \langle u_i \rangle + u_i''(\mathbf{x}, t)$. If the flow is stationary, $\langle u_i \rangle$ is independent of t and depends only on the vertical coordinate z, provided the averaging area is large enough. Single-point time averages, and fluctuations therefrom, will be respectively denoted by overbars and single primes, so that $u_i(\mathbf{x}, t) = \bar{u}_i(\mathbf{x}) + u_i'(\mathbf{x}, t)$. [The position vector will be denoted, according to convenience, by any of three representations: $x_i = \mathbf{x} = (x, y, z)$. Similarly, the velocity vector is either u_i or (u, v, w). The mean wind is aligned with the x-axis.]

The formal definition of the horizontal average is

$$\langle \Psi \rangle = \frac{1}{A} \iint_R \Psi(x)\,\mathrm{d}x\,\mathrm{d}y \tag{1}$$

where Ψ is a scalar field defined in the air but not at points occupied by canopy elements, and A is the area of a region R of the xy plane. Within a canopy, R is multiply connected because it is intersected by plant parts (see Figure 1).

This operator satisfies all but one of the commutation properties required of a turbulence averaging operator (known as the Reynolds conditions; see Monin and Yaglom, 1971, p. 207). The exception concerns the commutation of the horizontal averaging and horizontal spatial differentiation operators, which is not always assured within the canopy. A general rule is: *if Ψ is constant at the air-element interfaces, then horizontal averaging and spatial differentiation commute, so that $\langle \partial\Psi/\partial x_i \rangle = \partial\langle \Psi \rangle/\partial x_i$ (for $i = 1, 2$). Otherwise, they do not commute; in particular, $\langle \partial\Psi''/\partial x_i \rangle \neq 0$.* To demonstrate this, consider an averaging region R (part of a horizontal plane within the canopy) with a rectangular outer boundary C_0 which

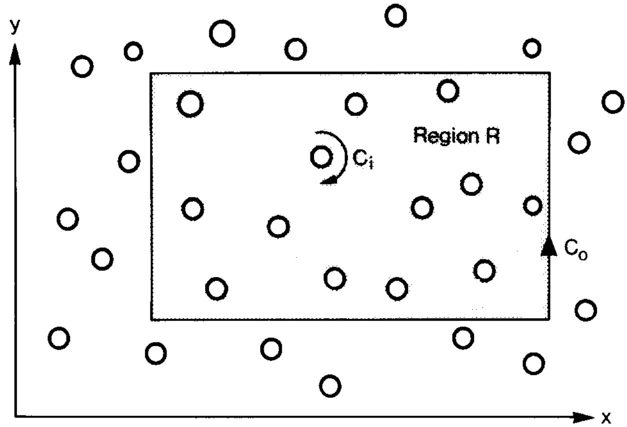

Fig. 1. A horizontal plane within the canopy, showing the averaging region R.

intersects no plant parts, and a series of inner boundaries C_i at the air-element interfaces (see Figure 1). Without losing generality, we may examine only the horizontal average of $\partial\Psi''/\partial x$. By Green's theorem and Equation (1),

$$\left\langle \frac{\partial \Psi''}{\partial x} \right\rangle = \frac{1}{A} \iint_R \frac{\partial \Psi''}{\partial x} \, \mathrm{d}x \, \mathrm{d}y = \frac{1}{A} \left[\int_{C_o} \Psi'' \, \mathrm{d}y + \sum_i \int_{C_i} \Psi'' \, \mathrm{d}y \right]. \tag{2}$$

The integral around C_o vanishes because of the overall homogeneity of the flow, but those around C_i vanish only if Ψ'' is constant at the interfaces.

Some simple examples may clarify this. First, consider the time-averaged pressure field for the flow over and about a series of impermeable fences lying across the wind (see Figure 2). A pressure differential exists across each fence because form drag takes place there; therefore, in the space between fences, $\partial\bar{p}/\partial x = \partial\bar{p}''/\partial x > 0$. A horizontal average (within the fluid only) gives $\langle \partial p''/\partial x \rangle > 0$. However, $\partial\langle\bar{p}''\rangle/\partial x = 0$ by definition, so the operators are non-commutative.

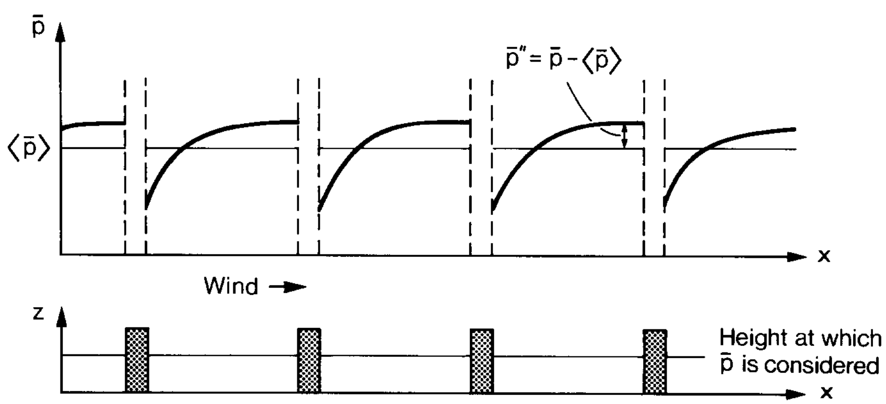

Fig. 2. Schematic pressure field about a series of impermeable fences lying across the wind.

A second, opposite example is that of the first spatial derivative of the velocity field. Consider an ordinary plant canopy: the no-slip condition ensures that $u_i = 0$ at element interfaces, so $u_i'' = u_i - \langle u_i \rangle$ is constant there. First-order spatial differentiation and horizontal averaging therefore commute for velocity (and, similarly, for higher velocity moments), so that

$$\frac{\partial u_i}{\partial x_i} = \frac{\partial \bar{u}_i}{\partial x_i} = \frac{\partial u_i'}{\partial x_i} = \frac{\partial \langle u_i \rangle}{\partial x_i} = \frac{\partial u_i''}{\partial x_i} = \frac{\partial \bar{u}_i''}{\partial x_i} = 0 \tag{3}$$

by the continuity condition for an incompressible fluid.

A third important case is the Laplacian of the velocity field, of which the term $\partial^2 \bar{u}/\partial y^2$ is representative. By considering a typical element consisting of a vertical flat plate parallel to the mean flow, it is clear that

$$\left\langle \frac{\partial^2 \bar{u}}{\partial y^2} \right\rangle \neq \frac{\partial}{\partial y} \left\langle \frac{\partial \bar{u}}{\partial y} \right\rangle$$

because $\partial \bar{u}/\partial y$ is not constant at the element interface. (In this example $\partial \bar{u}/\partial y$ is oppositely signed on opposite sides of the plate; see Figure 3). Hence Laplacian and horizontal averaging operators do not commute for velocity.

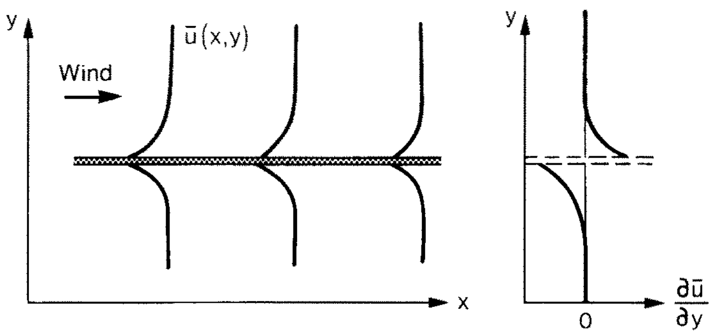

Fig. 3. Schematic field of $\bar{u}$ about a vertical flat plate parallel to the mean flow.

3. The Equation of Motion

In the absence of Coriolis forces and buoyancy effects, both of which we ignore for simplicity, the equation of motion for flow in and above the canopy is the Navier-Stokes equation

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \nabla^2 u_i \tag{4}$$

where ρ and ν are the density and kinematic viscosity of air, respectively. We mix vector and tensor notation for convenience. Its form under averaging scheme I is found by inserting $u_i = \langle u_i \rangle + u_i''$ and $p = \langle p \rangle + p''$, averaging spatially, and then using the rules given above. The result is

$$\frac{\partial\langle u_i\rangle}{\partial t} + \langle u_j\rangle\frac{\partial\langle u_i\rangle}{\partial x_j} + \frac{\partial}{\partial x_j}\langle u_i''u_j''\rangle = -\frac{1}{\rho}\frac{\partial\langle p\rangle}{\partial x_i} - \frac{1}{\rho}\left\langle\frac{\partial p''}{\partial x_i}\right\rangle + \nu\nabla^2\langle u_i\rangle + \nu\langle\nabla^2 u_i''\rangle. \tag{5}$$

This equation has been discussed by Wilson and Shaw (1977), who pointed out in particular that the second and the fourth terms on the right-hand side (both of which arise through non-commutativity, as indicated above) represent the form and viscous drag forces imposed by the canopy. In stationary, horizontally homogeneous conditions with negligible mean horizontal pressure gradient, Equation (5) simplifies to

$$\left.\begin{aligned}\frac{\partial}{\partial z}\langle u''w''\rangle &= -\frac{1}{\rho}\left\langle\frac{\partial p''}{\partial x}\right\rangle + \nu\langle\nabla^2 u''\rangle\\ &= f_D + f_V\end{aligned}\right\} \tag{6}$$

where f_D and f_V are the forces per unit mass of air exerted by form and viscous drag, respectively. Both are negative in practice, since they are oppositely directed to the mean flow.

To average Equation (4) under scheme II, first write the time-averaged equation of motion at a single point, and then substitute $\bar{u}_i = \langle\bar{u}_i\rangle + \bar{u}_i''$, and likewise for other time-averaged variables. This gives

$$\frac{\partial\langle\bar{u}_i\rangle}{\partial t} + \langle\bar{u}_j\rangle\frac{\partial\langle\bar{u}_i\rangle}{\partial x_j} + \frac{\partial}{\partial x_j}\langle\bar{u}_i''\bar{u}_j''\rangle + \frac{\partial}{\partial x_j}\langle\overline{u_i'u_j'}\rangle = -\frac{1}{\rho}\frac{\partial\langle\bar{p}\rangle}{\partial x_i} - \frac{1}{\rho}\left\langle\frac{\partial\bar{p}''}{\partial x_i}\right\rangle + \nu\nabla^2\langle\bar{u}_i\rangle + \nu\langle\nabla^2\bar{u}_i''\rangle \tag{7}$$

which differs from Equation (5) only in the form of the Reynolds stress terms. The extra Reynolds stress $\langle\bar{u}_i''\bar{u}_j''\rangle$ is a *dispersive* covariance, meaning a covariance arising from the spatial correlation of quantities averaged in time but varying with position. It combines with the usual single-point, time-averaged covariance to produce a total, spatially averaged covariance:

$$\langle u_i''u_j''\rangle = \langle\bar{u}_i''\bar{u}_j''\rangle + \langle\overline{u_i'u_j'}\rangle. \tag{8}$$

For example, a dispersive contribution arises in the total momentum flux $\langle u''w''\rangle$ if points of time-averaged updraught or downdraught correlate spatially with departures of $\bar{u}$ from its spatial mean.

Although dispersive fluxes arise naturally from the superposition of two averaging processes, they have so far eluded direct measurement. Antonia and Luxton (1971) proposed that this contribution is responsible for observations of anomalous profiles of $|\overline{u'w'}|$ close to rough surfaces in wind tunnels, where several studies using crossed hot-wire anemometry found a layer below the constant-stress region in which $|\overline{u'w'}|$ decreased with decreasing height (e.g., Mulhearn and Finnigan, 1978).

However, Mulhearn (1978) was unsuccessful at directly measuring the dispersive flux over two-dimensional bar roughness, and concluded that the anomaly must have another explanation. A similar conclusion was suggested by the measurements of Raupach *et al.* (1980) over several three-dimensional rough surfaces.

4. Equations for Second Moments

The equation for the single-point second central velocity moment $\overline{u_i' u_k'}$ is usually written (Hinze, 1975) as

$$\frac{\partial \overline{u_i' u_k'}}{\partial t} + \bar{u}_j \frac{\partial \overline{u_i' u_k'}}{\partial x_j} = -\overline{u_k' u_j'} \frac{\partial \bar{u}_i}{\partial x_j} - \overline{u_i' u_j'} \frac{\partial \bar{u}_k}{\partial x_j} - \frac{\partial}{\partial x_j} \overline{u_i' u_j' u_k'} - \frac{1}{\rho}\left(\frac{\partial \overline{p' u_k'}}{\partial x_i} + \frac{\partial \overline{p' u_i'}}{\partial x_k}\right) + \overline{\frac{p'}{\rho}\left(\frac{\partial u_k'}{\partial x_i} + \frac{\partial u_i'}{\partial x_k}\right)} - 2\nu \overline{\frac{\partial u_i'}{\partial x_j}\frac{\partial u_k'}{\partial x_j}} + \nu \nabla^2 \overline{u_i' u_k'} \tag{9}$$

in which it is customary to identify terms representing advection, production, turbulent transport, pressure transport, pressure strain, dissipation and molecular transport. The analogous equations under averaging schemes I and II are best discussed by considering only the kinetic energy $\frac{1}{2}u_i u_i$, of which the single-point turbulent component, $\frac{1}{2}\overline{u_i' u_i'}$, is described by contracting Equation (9) over i and k.

Under averaging scheme I, the total kinetic energy may be decomposed into a mean part (the MKE) and a turbulent part (the TKE) thus:

$$\tfrac{1}{2}\langle u_i u_i \rangle = \tfrac{1}{2}\langle u_i \rangle \langle u_i \rangle + \tfrac{1}{2}\langle u_i'' u_i'' \rangle. \tag{10}$$

Budgets for each part may be derived by the usual technique of multiplying the equations for $\langle u_i \rangle$ and u_i'' by $\langle u_i \rangle$ and u_i'', respectively, and then averaging. With due regard for commutation of operators, one obtains, for the MKE budget,

$$\left(\frac{\partial}{\partial t} + \langle u_j \rangle \frac{\partial}{\partial x_j}\right)\frac{\langle u_i \rangle \langle u_i \rangle}{2} = \langle u_i'' u_j'' \rangle \frac{\partial \langle u_i \rangle}{\partial x_j} - \frac{\partial}{\partial x_j}\left(\langle u_i \rangle \langle u_i'' u_j'' \rangle + \frac{\langle p \rangle \langle u_j \rangle}{\rho}\right) + \nu \langle u_i \rangle \langle \nabla^2 u_i'' \rangle - \frac{1}{\rho}\langle u_i \rangle \left\langle \frac{\partial p''}{\partial x_i} \right\rangle, \tag{11}$$

and for the TKE budget*

$$\left(\frac{\partial}{\partial t} + \langle u_j \rangle \frac{\partial}{\partial x_j}\right)\frac{\langle u_i'' u_i'' \rangle}{2} = -\langle u_i'' u_j'' \rangle \frac{\partial \langle u_i \rangle}{\partial x_j} - \frac{\partial}{\partial x_j}\left(\frac{\langle u_i'' u_i'' u_j'' \rangle}{2} + \frac{\langle p'' u_j'' \rangle}{\rho}\right) + \nu \langle u_i'' \nabla^2 u_i'' \rangle + \frac{1}{\rho}\langle u_i \rangle \left\langle \frac{\partial p''}{\partial x_i} \right\rangle. \tag{12}$$

* To derive Equation (12), the pressure term is treated thus:

$$\left\langle u_i'' \frac{\partial p''}{\partial x_i} \right\rangle = \left\langle u_i \frac{\partial p''}{\partial x_i} \right\rangle - \langle u_i \rangle \left\langle \frac{\partial p''}{\partial x_i} \right\rangle = \frac{\partial}{\partial x_i}\langle p'' u_i'' \rangle - \langle u_i \rangle \left\langle \frac{\partial p''}{\partial x_i} \right\rangle.$$

The operators in the first term on the right commute because of the no-slip condition at element surfaces, which ensures that $p'' u_i = 0$.

The right-hand side of each equation contains four terms: (i) a shear production term which converts MKE to TKE and which therefore appears with opposite sign in each equation, (ii) a transport term with inertial and pressure components, (iii) a viscous term and (iv) a 'wake production' term. The first two terms are quite conventional.

The third term represents the direct conversion of MKE or TKE to heat. In the MKE equation it would normally be negligible, but it is significant in the canopy because it represents the rate of working of the mean flow against viscous drag forces*. In the TKE equation, the viscous term incorporates not only the usual processes of molecular transport and viscous dissipation of TKE at high wave-numbers [cf. Equation (9)], but also the direct conversion of TKE to heat in the laminar boundary layers of individual canopy elements.

The fourth term represents the rate of working of the mean flow against form drag, being a scalar product of mean velocity and form drag force. It may be called the 'wake production' term, since it converts MKE to TKE in the turbulent wakes of canopy elements. Like the shear production term, it is oppositely signed in Equations (11) and (12), and so neither creates nor destroys total kinetic energy. Wake production generates TKE at length scales characteristic of the elements, which are much smaller than the typical length scales of the shear-generated eddies constituting the dominant turbulent motion in the canopy. Because of its small length scale, the wake energy is dissipated rapidly to heat by the eddy cascade process.

In passing, it should be emphasised that although Equations (11) and (12) account explicitly for the effects of form and viscous drag forces on the MKE, they do not describe the effects of these forces on the TKE. Form drag transforms large-scale, shear-generated TKE into small-scale TKE in element wakes, thereby short-circuiting part of the normal eddy cascade and accelerating the dissipation rate for large-scale TKE in the canopy. This process cannot be represented in an equation like (12), which is averaged over all frequencies in the turbulence spectrum. Viscous drag provides a direct sink to heat for TKE, which appears in Equation (12) as part of the horizontally averaged dissipation term.

Under averaging scheme II, the decomposition of total kinetic energy into MKE and TKE is

$$\tfrac{1}{2}\langle u_i u_i \rangle = \tfrac{1}{2}\langle \bar{u}_i \bar{u}_i \rangle + \tfrac{1}{2}\langle \overline{u_i' u_i'} \rangle. \tag{13}$$

The budgets for each part are derived by averaging the appropriate single-point equations; the resulting TKE budget is

* The ratio of the rate of working against viscous drag forces to the total rate of working against drag is equal to the ratio of the viscous drag force to the total drag force. Measurements by Thom (1968) on a model bean leaf showed that this ratio was between 0.2 and 0.4 for typical wind speeds and leaf angles. In canopies with smaller elements the ratio is expected to be higher; for example, Stewart and Thom (1973) assumed it to be 0.5 in a pine forest.

$$\left(\frac{\partial}{\partial t} + \langle \bar{u}_j \rangle \frac{\partial}{\partial x_j}\right)\frac{\langle \overline{u_i'u_i'} \rangle}{2} = -\langle \overline{u_i'u_j'} \rangle \frac{\partial \langle \bar{u}_i \rangle}{\partial x_j} - \frac{\partial}{\partial x_j}\left(\frac{\langle \overline{u_i'u_i'u_j'} \rangle}{2} + \frac{\langle \overline{u_i'u_i'}'' \bar{u}_j'' \rangle}{2} + \frac{\langle \overline{p'u_j'} \rangle}{\rho}\right) + \nu \langle \overline{u_i' \nabla^2 u_i'} \rangle - \left\langle \overline{u_i'u_j'}'' \frac{\partial \bar{u}_i''}{\partial x_j} \right\rangle. \tag{14}$$

The four terms on the right are analogous to those in Equation (12). The first and third terms are essentially unchanged; the second (transport) term differs only in the appearance of a dispersive turbulent kinetic energy flux analogous to the dispersive Reynolds stress $\langle \bar{u}_i'' \bar{u}_j'' \rangle$ in Equations (7) and (8). The main difference is in the form of the wake production term, which here appears as a product of local Reynolds stress and velocity gradient perturbations (cf. Raupach and Thom, 1981). These perturbations are the wake shadows of individual canopy elements, within which the fourth term in Equation (14) produces TKE in the same way as does the shear production term on a larger scale.

Contrary to the assertion by Wilson and Shaw (1977), Equations (12) and (14) are not identical, because of the difference between the decompositions (10) and (13) of the total kinetic energy. By further decomposing (13), one obtains

$$\tfrac{1}{2}\langle u_i u_i \rangle = \tfrac{1}{2}\langle \bar{u}_i \rangle \langle \bar{u}_i \rangle + \tfrac{1}{2}\langle \bar{u}_i'' \bar{u}_i'' \rangle + \tfrac{1}{2}\langle \overline{u_i'u_i'} \rangle. \tag{15}$$

The first term on the right is the MKE under scheme I (since $\langle \bar{u}_i \rangle = \langle u_i \rangle$, provided that the horizontal averaging area is large enough) and the third term is the TKE under scheme II. The middle term, $\tfrac{1}{2}\langle \bar{u}_i'' \bar{u}_i'' \rangle$, is the kinetic energy of the time-averaged spatial variations in the velocity field; it is seen as TKE by scheme I, but as MKE by scheme II. Equation (15) shows that a budget for this 'dispersive kinetic energy' can be constructed thus:

$$\begin{aligned}\text{(dispersive KE budget)} &= (\text{MKE})_{\text{II}} - (\text{MKE})_{\text{I}} \\ &= (\text{TKE})_{\text{I}} - (\text{TKE})_{\text{II}}\end{aligned}$$

where the terms represent budget equations found with averaging schemes specified by the subscripts. A lengthy but straightforward calculation, using either the MKE or the TKE budgets, yields,

$$\left(\frac{\partial}{\partial t} + \langle \bar{u}_i \rangle \frac{\partial}{\partial x_j}\right)\langle \bar{u}_i'' \bar{u}_i'' \rangle = -\langle \bar{u}_i'' \bar{u}_j'' \rangle \frac{\partial \langle \bar{u}_i \rangle}{\partial x_j} + \left\langle \overline{u_i'u_j'}'' \frac{\partial \bar{u}_i''}{\partial x_j} \right\rangle - \frac{\partial}{\partial x_j}\left(\langle \bar{u}_i'' \overline{u_i'u_j'}'' \rangle + \frac{\langle \bar{u}_i'' \bar{u}_i'' \bar{u}_j'' \rangle}{2} + \frac{\langle \bar{p}'' \bar{u}_j'' \rangle}{\rho}\right) + \nu \langle \bar{u}_{ii}'' \nabla^2 \bar{u}_i'' \rangle + \frac{1}{\rho}\langle \bar{u}_i \rangle \left\langle \frac{\partial \bar{p}''}{\partial x_i} \right\rangle. \tag{16}$$

The five terms on the right are (i) a production term involving only the *dispersive* Reynolds stress $\langle \bar{u}_i'' \bar{u}_j'' \rangle$; (ii) the wake production term under scheme II, here a sink term; (iii) a transport term involving only dispersive fluxes of kinetic energy; (iv) a viscous term accounting for direct dissipation of $\frac{1}{2}\langle \bar{u}_i''\bar{u}_i''\rangle$ to heat; and (v) the wake production term under scheme I, here a source term. The viscous term will be negligible if the length scale of the canopy elements (and of their wakes) is much larger than the Kolmogorov microscale. If this is true, Equation (16) reduces in steady, advection-free conditions to the simple form

$$-\left\langle \overline{u_i' u_j'}'' \frac{\partial \bar{u}_i''}{\partial x_j} \right\rangle = \frac{1}{\rho} \langle \bar{u}_i \rangle \left\langle \frac{\partial \bar{p}''}{\partial x_j} \right\rangle \tag{17}$$

provided also that *all dispersive fluxes are negligible.* Only then is equality achieved between the wake production terms under averaging schemes I and II.

Physically, this implies that the small-scale pressure gradient associated with each canopy element is responsible for the production of wake turbulence only at the level of that element. The possibility of wake production at levels above the top of the canopy, which is left open by the form of the wake production term in Equation (14), is thereby excluded. In practice, this condition may well be very nearly achieved. As already mentioned in Section 3, wind tunnel experiments have failed to find dispersive momentum fluxes even in situations where they should be significant. In the remainder of this paper, we assume that dispersive fluxes are negligible and that averaging schemes I and II coincide.

5. Discussion

Although the main use of these equations is in constructing physically based second-order closure models for canopy flow (Wilson and Shaw, 1977), it is instructive to use them to make some simple, order-of-magnitude estimates of the relative importance of the various mechanisms affecting kinetic energy within the canopy.

First, consider the relative magnitude of the shear and wake production rates. In stationary, horizontally homogeneous conditions, the MKE budget can be written

$$U \frac{\partial \langle u''w'' \rangle}{\partial z} = U f_D + U f_V \tag{18}$$

where $f_D = -\langle \partial p''/\partial x \rangle/\rho$ and $f_V = \nu \langle \nabla^2 u'' \rangle$ are the averaged form and viscous forces, respectively (both negative), and where U is a convenient short notation for $\langle u \rangle$. This equation follows directly from Equation (6), or from Equation (11) after simplification. If

$$P_s = -\langle u''w'' \rangle \frac{\partial U}{\partial z}, \qquad P_w = -U f_D$$

are the shear and wake production rates, respectively, then their ratio is

$$\frac{P_w}{P_s} = \frac{U \partial \langle u''w'' \rangle/\partial z}{\langle u''w'' \rangle \partial U/\partial z} \left(\frac{f_D}{f_D + f_V} \right)$$

from Equation (18). To evaluate this expression, one can use the gradient-diffusion relationship $\langle u''w'' \rangle = -K\partial U/\partial z$, with K proportional to U. This should be understood here not as a model, but merely as a convenient semi-empirical relationship which is fairly well satisfied in most simple canopies (e.g., Raupach and Thom, 1981). It follows that

$$\frac{P_w}{P_s} = \frac{U\partial^2 U/\partial z^2}{(\partial U/\partial z)^2} + 1$$

where, for simplicity, we have also assumed that viscous drag is negligible in comparison with form drag. For the exponential wind profile $U(z) = U(h)\exp\{\alpha[z/h - 1]\}$, where h is the canopy height, this becomes

$$\frac{P_w}{P_s} = 2,$$

independent of the coefficient α. Although the last two steps in this argument are based on curve-fitting approximations, it is clear that wake production is at least of the same order as shear production within the canopy, and may be several times larger.

Why, then, does not wake turbulence constitute a significant fraction of the total TKE inside the canopy? Spectral analyses of canopy turbulence (e.g., Shaw *et al.*, 1974; Finnigan, 1979) do not show significant high-frequency contributions at scales determined by canopy elements. The answer is twofold: first, most larger-scale turbulence present in the canopy is not generated locally, but is transported from above by inertial or pressure transport (e.g., Maitani, 1978; Raupach, 1981). Second, wake turbulence is dissipated much more rapidly than shear turbulence because of its small length scale. An estimate of the turbulence intensity *due to wake turbulence alone* can be made thus: suppose that, only for wake turbulence, local TKE balance applies. Then

$$P_w = -Uf_D = \varepsilon_{\mathrm{wake}}$$

where $\varepsilon_{\mathrm{wake}}$ is the dissipation rate for wake turbulence alone; it will be of order q^3_{wake}/L, where q_{wake} is a velocity scale and L a length scale for the wake turbulence (Townsend, 1976, p. 61). The length scale L is of the order of that of the canopy elements. Hence,

$$\varepsilon_{\mathrm{wake}} = \frac{q^3_{\mathrm{wake}}}{L} = Uf_d = c_M a U^3$$

where c_M is an effective drag coefficient for canopy elements and a the area density. Using the approximation $a = A/h$, where h is the canopy height and A the area index, it follows that the wake turbulence intensity is

$$\frac{q_{\mathrm{wake}}}{U} = (c_M AL/h)^{1/3}.$$

Typically (say, for forest), $L/h \approx 10^{-3}$, $c_M \approx 0.2$ and $A \approx 5$. Hence $q_{\text{wake}}/U \approx 0.1$, which is much less than the typical overall turbulence intensity within a crop of unity or more (Raupach and Thom, 1981). Clearly, the high dissipation rate for wake turbulence causes its intensity inside a canopy to be low, even though its production rate is several times the (local) shear production rate.

6. Conclusions

This work has clarified the consequences of the horizontal averaging assumptions inherent in one-dimensional models of canopy flows. These assumptions are important in determining the form of the wake-production terms in the second-moment equations. The analysis has shown that dispersive covariances, if significant within the canopy, complicate the second-moment equations considerably; therefore, experimental estimates of their magnitude in real canopies are required to confirm the present assumption that they are negligible.

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