JOHN CRAVEN

LIBERALISM AND INDIVIDUAL PREFERENCES

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1. Sen's interpretation [3] of liberalism is that a single individual has the right to determine that the social preference over some pair of states of society is identical to his own preference over that pair. Of course, the use of the term *liberalism* implies that the distinction between the two states is some set of features that affect the particular individual in some personal way. If I were to have the right to choose between two states whose distinction was whether or not my neighbour were to be executed without trial, then, whilst the logic of collective choice theory could be applied, we should not wish to refer to this right as liberal.

2. Sen shows that if two individuals have some liberal right, if the Pareto unanimity rule holds, and if the individuals' preferences may be of any complete and transitive form, then a social 'preference cycle' may appear. His example has two individuals who have liberal rights such that a is socially preferred to b if and only if individual I so prefers and b is socially preferred to c if and only if II so prefers. Then if I prefers c to a to b and II prefers b to c to a, the liberal society prefers a to b and b to c. If all other individuals in society prefer c to a, unanimity gives a social preference for c over a, completing the cycle.

3. Among the proposed resolutions of the paradox is that of Suzumura [4] who shows the liberal rights and the Pareto principle are consistent if there is one 'liberal individual' in society. Such an individual "... claims only those parts of his preferences to count which are compatible with others' preferences over their respective protected spheres [i.e., the pairs over which they are to decide the social preference]". If II were a liberal individual in Sen's example, he would not seek to include his preference for b over a in the

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determination of the social preference, and hence he could not seek to include his preference for c over a given that he prefers b to c. Then the Pareto principle could not be applied to c over a. The paradox can also be resolved if there is some third individual who is liberal in this sense, because he would not want a preference for c over a to count since it is incompatible with the rights and preferences of I and II.

4. This resolution avoids Sen's paradox by restricting the domain of some individual's preferences. The restrictions imply that some individual responds to the existence of liberal rights, and so gives some support to the implementation of these rights. In short, some individual is taking a 'liberal view' towards others in the formation of his own preferences. This recognition that liberalism may be reflected in individuals' preferences leads us to a more radical departure from Sen's propositions. We consider the abandonment of the interpretation of liberalism as a right to determine the social preference, and instead view liberalism as the source of an externality between individuals' preferences. So we are replacing the interpretation of liberalism as a property of the collective choice rule by its interpretation as a restriction on the possible preferences that individuals hold.

5. If I take the view that my neighbour should decide whether he is to read book a or book b, I say in effect that if he prefers a to b then I too will express a preference for a over b. We denote i's preference ordering by R_i with P_i denoting strict preference and I_i denoting indifference. If individual i is liberal towards individual j over a pair of states (a, b), then aP_jb implies aP_ib and bP_ja implies bP_ia. We write this agreement as $iA(a, b)j$, and refer to it as a liberal view, subject to the *caveat* of the next paragraph.

6. Just as in Sen's definition of liberalism, we should wish to apply the term *liberal* to such an agreement only if the distinction between the states a and b is something which affects j (and no other) personally. The agreement of preferences which characterises our view of liberalism might also characterise i's behaviour as a fanatical follower of j who wants to execute k without trial. The logic of social choice may be applied to both cases, but it is again reasonable to interpret only one as liberalism.

7. The definition given for i being liberal towards j over (a, b) leaves open the question of i's reaction if j is indifferent between a and b. It may be that i follows j in this also, but it may not be unreasonable to suppose that if j 'does not care' then i will see no reason for j to determine the social preference and will make up his mind on other grounds. If j is indifferent between reading book a and book b, i may express a preference based on his own view of the books' merits. In this case i is putting his liberalism towards j lexicographically prior to other considerations in the determination of his own strict preference since he agrees with j provided that the latter expresses a strict preference. If i is liberal towards j over (a, b) but aP_ib when aI_jb, there is, in a sense, a discontinuity in the connection between R_i and R_j: as j moves from aP_jb to bP_ja through aI_jb, i moves from aP_ib to bP_ia without the intermediate aI_ib. This possibility arises because we assume that i has no way of measuring the *intensity* of j's preference, and so he cannot combine his agreement with j with his other views according to the strength of j's preference. In so far as liberalism is a *fundamental* attribute of individual's relationships with their fellow citizens, it may indeed be lexicographically prior to other considerations, and so we should not need to measure preference intensity to incorporate liberalism into collective choice theory in this way.

8. Our interpretation of liberalism leads to a restriction on the domain of the collective choice rule since some combinations of preferences are excluded. It also implies that there is no conflict between the Pareto principle, which is a property of the collective choice rule, and liberalism, which is a property of individual's preferences. The existence of liberal views makes it more likely that the Pareto criterion can be applied in the generation of social preferences over some pairs of alternatives since some disagreements between individuals' preferences are excluded. Similarly the domain restrictions implied by liberalism may help to avoid the problem of Arrow's impossibility theorem [1]. More formally, the probability of no majority winner (as calculated by Garman and Kamien [2] for the case of unrestricted domain with equiprobable strict preference orderings) can be decreased by liberal views. With three states and three individuals (without liberal views) the probability of no majority winner is 1 in 18. The typical case is aP_1bP_1c, bP_2cP_2a, cP_3aP_3b; the alternatives and individuals in the typical case can be permuted to give 12 cases in $216 = 6^3$. If individual 2 is liberal towards 1 on (b, c) and 3 is liberal

towards 2 on (a, b) then there are 54 possible preference orderings, of which only 2 (aP_1cP_1b, cP_2bP_2a, bP_3aP_3c; and bP_1cP_1a, aP_2bP_2c, cP_3aP_3b) yield no majority winner. Hence the probability is reduced to 1 in 27.

9. Our view of liberalism as a connection between preferences contrasts with Sen's interpretation of liberalism as a set of rights that are enshrined in the collective choice rule. For Sen's view to hold the constitution or political process which is the practical manifestation of the collective choice rule is established without regard to individuals' preferences. The liberal rights are presumably assigned to individuals because those who control the political process wish to have a society that is, in some sense, liberal. These rights are maintained whatever individuals' preferences may be. By contrast, there is no liberal right in our version of liberalism; an individual's preference determines the social preference if and only if there are sufficient others with a liberal view towards that individual (or who are his followers, see paragraph 6 above). The number of individuals that is sufficient to give this support itself depends on the collective choice rule; if the social preference on (a, b) is determined by a simple majority vote, then individual 1 determines the social choice on (a, b) provided that everyone in some subset containing at least one half of society is liberal towards 1 over (a, b). Alternatively, if the constitution gives a dictator absolute power, it is he who must be liberal towards 1 if that individual's preference is to prevail. Thus our interpretation of liberalism would seem to model more closely the undoubted fact that the rights of a group in society can be maintained only as long as sufficient people wish to maintain them, but that these rights are lost as soon as the political process gives sufficient power to those who oppose that group's rights. We therefore take the view that liberalism is endogenous in that society can be liberal only if there are individuals who are able and willing to defend liberalism (see also paragraph 13 below), and we reject Sen's view that liberal rights can be sustained despite powerful opposition that is manifested in non-liberal preferences.

10. We have argued that liberalism may be viewed as a connection between individuals' preference orderings and that other forms of behaviour, such as fanatical following, may be treated similarly. Other plausible forms of behaviour can be modelled in a similar way. For example, strong disagreement

implies that we should observe opposite preferences: hatred, jealousy and naive political opposition might fall into this category.

11. It is also plausible that an individual may wish to agree (or disagree) with his view of the collective decision of a group. If a set K of individuals have preferences R_i ($i \in K$), individual j (not in K) may have his own group choice rule D^j_K (R_i, $i \in K$) for the group K over some pair (a, b). Then j can generate a group preference relation (not necessarily complete, but at least defined over (a, b)) $R^j_K = D^j_K\ (R_i, i \in K)$ and then agree with the group preference as he has defined it, so that $aP^j_K b$ implies $aP_j b$ and $bP^j_K a$ implies $bP_j a$. For example the rule $D^j_K(\ )$ may require a simple majority of K to determine R^j_K over (a, b). Alternatively j may require that a strict preference be determined by a greater majority, with $aI^j_K b$ if neither state a nor state b commands the required majority of the members of K.

12. The examples of connections between preferences that we have given define i's preference over (a, b) according only to others' preferences over (a, b). Clearly this pair-wise independence may be violated by some forms of behaviour, such as a conditional agreement by i that he will agree with j over some pair (a, b) that concerns j only if j has a particular preference $cP_j d$ over some pair (c, d) that concerns i. So $aP_j b$ implies $aP_i b$ and $bP_j a$ implies $bP_j a$ if and only if $cP_j d$.

13. We have discussed liberalism in previous paragraphs as straightforward agreement between preferences. However, a liberal view may be manifest in a stronger way than this. Voltaire, among others, is alleged to have voiced an opinion of the form: "I disapprove of what you say, but I will defend to the death your right to say it",[1] which is a much stronger expression of liberalism than mere preparedness to agree. To avoid the complication of deciding whether a dead Voltaire's view should be incorporated in a social decision, let us reiterate the statement in two parts as

(a) i prefers yellow socks to green socks,

(b) i would be prepared to lose two teeth in a fight to protect a society in which j can wear whatever colour socks j prefers.

Statement (a) presumably applies to one or more of the following circumstances: i's preferences for his own socks; i's preferences for those towards whom he is not 'sock liberal'; i's preferences if j is indifferent on his own sock colour. Statement (b) concerns the following four states of the world, where $Y = j$ wears yellow socks, $G = j$ wears green socks, and 32 or 30 refers to the number of teeth left to i: $a = (Y, 32)$, $b = (G, 32)$, $c = (Y, 30)$, $d = (G, 30)$. We assume that, *ceteris paribus*, i prefers more teeth to fewer, so that aP_ic and bP_id. Individual i's liberalism towards j over (a, b) and (c, d) imply that if j prefers, *ceteris paribus*, G to Y, then bP_ia and dP_ic. Statement (b) further implies that, in these circumstances dP_ia, and so $bP_idP_iaP_ic$. If j prefers, *ceteris paribus*, Y to G, the same militantly liberal views imply $aP_icP_ibP_id$.

14. All of these examples require that when individual i observes a list of preferences $R_1, R_2, \ldots, R_{i-1}, R_{i+1}, \ldots, R_m$ for the $m - 1$ other members of society M, his own preference ordering R_i is restricted to lie in some admissible set $S_i(R_j, j \in M, j \neq i)$ which is a proper subset of the set of all possible orderings of the states. For any sensible theory of collective choice based on individual preferences we must assume that each individual's views are *coherent*; that is for each i, and for any list of others' preferences $S_i(\ )$ must be nonempty. Thus, for example, i cannot be liberal towards g over (a, b), h over (b, c) and j over (a, c) since if aP_gb, bP_hc and cP_ja, i has no preference ordering consistent with his liberal views.

15. Although the existence of connections between preferences may help to resolve some paradoxes in the application of collective choice rules, they can on the other hand give rise to a separate set of difficulties that are logically prior to Arrow-type impossibility results. As a simple example, suppose that i is liberal towards j over (a, b), and also aI_jb implies aI_ib. At the same time j disagrees with i over (a, b) so that aP_ib implies bP_ja and bP_ia implies aP_jb, and also, aI_ib implies aP_jb. Then there is no pair of preference orderings for i and j which satisfy these views simultaneously, although the individuals' views are both coherent. We say that there is no *equilibrium list* of orderings, since, whatever list of orderings is proposed, at least one of the individuals would seek to change his ordering. Thus we may have an 'impossibility' imposed by individuals' views that arises before any discussion of the compatibility of proposed properties of the collective choice rule.

16. The possibility that the set of equilibrium lists of orderings E is empty is the least subtle form of this difficulty. For we can devise an example to show that, even where E is not empty, there may be no list of preference orderings that is consistent with all the connections between individuals' preferences and with the fact that some parts of some individuals' orderings are 'free', in the sense that the individual has no views that affect those parts of his ordering. Consider the example in which the connections between preferences are as follows:

$$aR_ib \quad \text{implies } aP_jb$$

$$bP_ia \quad \text{implies } bP_ja$$

$$(aR_jb \text{ and } cR_jd) \text{ implies } bP_ia$$

$$(bP_ja \text{ and } cR_jd) \text{ implies } aP_ib$$

$$(aR_jb \text{ and } dP_jc) \text{ implies } aP_ib$$

$$(bP_ja \text{ and } dP_jc) \text{ implies } bP_ia$$

so that j agrees with i's strict preferences on (a, b), but i agrees with j's strict preferences on (a, b) if and only if dP_jc. Then j has no view that rules out either cR_jd or dP_jc. If dP_jc, the views on (a, b) are consistent and so an equilibrium list of preferences can be found, but if cR_jd the views on (a, b) are inconsistent and so there is no equilibrium list in which cR_jd, even though this preference is not excluded by any of j's views.

17. This source of difficulty can be analysed more formally as follows. E is the set $\{(R_i\colon i \in M)/R_i \in S_i(R_j,\ j \in M,\ j \neq i) \text{ for all } i \in M\}$. If Q is a quasi-ordering (transitive but not necessarily complete) then an ordering R (transitive and complete) is a *completion* of Q if, for all (a, b), aQb implies aRb and $(aQb$, not $bQa)$ implies not bRa. We define $T(Q)$ to be the set of all completions of Q. A part of an individual's ordering is *free* if it is consistent with any list of preferences of others. Hence for each individual i we define H_i to be the set of quasiorderings Q_i of which some completion is consistent with any possible list of others preferences:

$$H_i = \{Q_i/T(Q_i) \cap S_i(R_j; j \in M, j \neq i) \neq \phi \text{ for all } (R_j; j \in M, j \neq i)\}.$$

We note that as long as i's views are coherent, H_i is nonempty since the quasiordering consisting just of reflexives such as aQ_ia is in H_i. Individual preferences are *mutually consistent* if, for any list of quasiorderings $(Q_i: i \in M)$ such that $Q_i \in H_i$ for all $i \in M$, there exists an equilibrium list of orderings $(R_i: i \in M) \in E$ such that $R_i \in T(Q_i)$ for all $i \in M$.

18. We have seen that preferences may not be mutually consistent in some cases where one individual disagrees with another. However, we can show that preferences are mutually consistent when the only connections are agreements of the form $iA(a, b)j$. So we have a *consistency theorem for a liberal society* which states that no contradiction exists between liberal views and the free parts of individuals' orderings. The proof is given in the appendix: it is constructive in that it begins with a set of quasiorderings from the H_i and uses agreements to extend these to complete orderings. The proof shows that no inconsistency can arise in this process, and that all agreements are satisfied.

19. The main conclusion of this paper is that if we accept that liberalism may be interpreted as an opinion of individuals rather than as a property of the collective choice rule, there is no paradox in the conjunction of liberalism and the Pareto principle. There is also no incompatibility between liberal views and the unrestricted domain of those parts of individuals' preferences that are unaffected by liberal views. Hence, in contrast to the difficulties raised by Sen's interpretation of liberalism, we have shown that the incorporation of liberal views does not increase the difficulties involved in collective choice.

APPENDIX

A proof of the mutual consistency of preferences if all connections between them are agreements.

We assume that all individuals' views are coherent and we take a given $Q_i \in H_i$ for each $i \in M$ and define $Q_i^u (i \in M, u = 0, 1, 2 \ldots)$ according to the following rules:

(i) $Q_i^0 = Q_i$,

(ii) $aQ_i^v b$ implies $aQ_i^u b$ for $u \geqq v$,

(iii) $(aQ_i^u b, bQ_i^u c)$ implies $aQ_i^u c$,

(iv) if there exists i, j, a, b such that $iA(a, b)j$, Q_j^u defined over (a, b) and Q_i^u not defined over (a, b) then $aQ_i^{u+1}b$ if and only if $aQ_j^u b$,

(v) if for some u there exists no i, j, a, b satisfying (iv) but there exists i, j, a, b such that $iA(a, b)j$ and Q_i^u, Q_j^u not defined over (a, b) then set $aQ_j^{u+1}b$, $aQ_i^{u+1}b$, not $bQ_j^{u+1}a$ and not $bQ_i^{u+1}a$ (choosing the order of a and b arbitrarily),

(vi) if for some $u = w$ there exists no i, j, a, b satisfying either (iv) or (v) then choose $R_i \in T(Q_i^w)$ for all $i \in M$. (i), (ii), (iii) imply that Q_i^w is a quasiordering and $T(Q_i^w) \subset T(Q_i)$ so that $R_i \in T(Q_i)$ for all $i \in M$.

Rules (iv) and (v) encompass all agreements if and only if

(I) there is no i, j, a, b, u such that $iA(a, b)j$, $aQ_j^u b$, $bQ_i^u a$ and not $bQ_j^u a$; and

(II) there is no i, j, a, b, u such that $iA(a, b)j$, Q_i^u is defined over (a, b) and Q_j^u is not defined over (a, b).

Now if $bQ_i^u a$ then *either* (Ia) $bQ_i a$ *or* (Ib) there exist states $c, d, \ldots, e$, individuals $g, h, \ldots, k$ not all identical to j and integers $x, y, \ldots, z$ such that $0 \leqq x \leqq u$, $0 \leqq y \leqq u$, $\ldots$, $0 \leqq z \leqq u$, $iA(a, c)g$, $iA(c, d)h$, $\ldots$, $iA(e, b)k$, $aQ_g^x c$, not $cQ_g^x a$, $cQ_h^y d$, not $dQ_h^y c$, $\ldots$, $eQ_k^z b$ and not $bQ_k^z e$. Now (Ia) is inconsistent with $iA(a, b)j$ since no completion of $bQ_i a$ lies in $S(\ )$ when R_j is such that $aP_j b$. (Ib) is also inconsistent with $iA(a, b)j$ since i's views are not coherent if $iA(a, b)j$, $iA(a, c)g$, $\ldots$, $iA(e, b)k$, where not all $g, \ldots, k$ are identical to j. Hence I cannot arise. Similarly II cannot arise since Q_i^u is defined over (a, b) only if either Q_i is defined over (a, b) or if some sequences of states and individuals exist as in (Ib).

Q.E.D.

University of Kent at Canterbury,
Kent, England

NOTE

[1] Attributed to S. G. Tallentyre, *The Friends of Voltaire* (Smith, Elder and Co., London, 1906).

REFERENCES

[1] Arrow, K. J.: *Social Choice and Individual Values* 2nd ed. (Wiley, New York, 1963).

[2] Garman, M. and Kamien, M.: 'The Paradox of Voting: Probability Calculations', *Behavioural Science* **13** (1968), 306–16.

[3] Sen, A. K.: 'The Impossibility of a Paretian Liberal', *Journal of Political Economy* 78 (1970), 152–7.

[4] Suzumura, K.: 'On the Consistency of Libertarian Claims', *Review of Economic Studies* **45** (1978), 329–42.