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A spatial model with party activists: implications for electoral dynamics*

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1. Introduction

The purpose of this paper is to develop a multidimensional spatial model akin to the standard one of elections but also to study here the impact of a second category of decisions open to citizens: the decision to become a partisan activist. The model developed here is meant to be compatible with standard election models (i.e., it rests on essentially identical assumptions). The difference here is that citizens can choose to "join" one of the two parties (i.e., become a partisan activist) with the resulting implication that there will be some distribution of activists of the two parties in the N-dimensional space. The decision to join a party is assumed to parallel the voting decision in form. However, it is then shown that, unlike voting decisions, activists' decisions lead to equilibria distributions for the political parties (as sets of activists) in a multidimensional policy space. The presumption

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(but one not studied here) is that, with a "stable" two-party system and with a tie between candidates and their party's activists, a parties-and-elections spatial model will induce some regularity for candidates that, as McKelvey (1979) and Schofield (1978) have so elegantly demonstrated, is ordinarily absent in a purely electoral spatial model.

The model presented here is a generalization to multidimensional spaces of a unidimensional model I proposed earlier (Aldrich, 1982a). Unidimensional spatial models differ from their N-dimensional counterparts, of course, over the existence of equilibria. The question that arose in the earlier paper, therefore, concerned the location of parties in space. There it was shown that the parties ordinarily will be fairly cohesive internally and moderately divergent from each other in policy terms. This finding seemed to reflect what Page called "party cleavages" (in 1978) in his study of presidential compaigns and seemed to parallel (and thus provide a justification for) the assumption in Aranson and Ordeshook's (1972) model of sequential elections (i.e., candidates attempting to win nomination through appeal to partisans and then office from appeal to the general electorate). The findings presented here tend to support the generalization of "party cleavages" to N-dimensional spaces of a particular sort, as well as to open up avenues for the study of longer-term electoral dynamics. This last possibility requires the building of results such as those presented in the first sections of this paper. The last section examines the dynamics. The next section introduces notation and the decision calculus of potential partisan activists.

2. Notation and the calculus of activism

Assume that there is a multidimensional space $X = R^N$. The point (vector) $x_i \epsilon X$ denotes the ideal point of the i^{th} citizen; $x_i = (x_{i1}, x_{i2},..., x_{in})'$. Following the "standard Davis-Hinich-Ordeshook" spatial model (as in 1970 or Riker and Ordeshook, 1973), all citizens possess quadratic utility functions and have a common A matrix of weights in that function. Then, the utility i associates with any particular vector of policy positions, say y, if λ - $(x_i - y)'$ $A(x_i - y)$, where λ is a scalar. Since all share the same A matrix, there is a transformation of the space to make A equal to the identity matrix. Without loss of generality, therefore, I will assume the transformation so that the utility function can be written as $U_i(y) = \lambda - (x_i - y)'(x_i - y)$.

The two parties, say θ and ψ, will be defined as the set of citizens who choose to become active in the respective parties. That is, we are concerned with the set of citizens – and their policy preferences -- who are members of the two parties. Let θ_i and ψ_i denote party members. The notation θ_i, ψ_i is meant to parallel x_i, i.e., θ_i, $\psi_i \epsilon X$, indicating the ideal points of the members of the two parties. Of particular interest is the ideal point (vector) of the "average" activist in θ and ψ, the center of mass of the distribution of activists being denoted $\bar{\theta}$ and $\bar{\psi}$.

Finally, let $f(x)$ denote the density of ideal points for all citizens across X. While spatial models of elections have studied $f(x)$ in terms of modality (especially unimodal and bimodal distributions), modality has turned out to play a relatively small role. Here, modality plays a much more crucial role. For notational simplicity, if $f(x)$ is unimodal, I will set the mode to be $x = 0$. I also may assume that $f(x)$ is symmetric, defined as usual. Thus, $f(x)$ is symmetric about a point, say y, if $f(\|y + x\|) = f(\|y - x\|)$ for all x, where $\|\cdot\|$ denotes distance (in the transformed space, distance is strict Euclidean). If $f(x)$ is unimodal and symmetric, $y = 0$, so $f(x) = f(-x)$.

With the above notation, let me introduce the ideas behind the "calculus of activism" (these are elaborated in Aldrich, 1982a). The basic concept is to make an analogy between spatial voting and spatial activism. Insofar as possible, assumptions will be kept identical to voting in the spatial model. The implication is that activism is a kind of contribution of scarce resources to a candidate or party (contributing, for example, time, effort, or money), just as voting is a contribution of a vote to a candidate. Thus, I am not considering the activity of a political leader or entrepreneur, nor that of the professional politician. The more appropriate referent would be to the "grass roots" activist, the kind who can be counted on to work for a variety of candidates and offices from time to time with regularity. These activists are those who make up the bulk of computerized mailing lists these days. In general, each citizen is a potential activist who, therefore, calculates whether or not to contribute to (or "join") a party. For simplicity, I assume here that the decision is strictly dichotomous; one is either active in a party or inactive.

I assume that the decision to make a contribution as a party activist is like that of the spatial voter. The prime difference, therefore, lies in two areas. First, the contribution of time, effort, and/or money is more costly to the individual than is casting a vote. There are, of course, the variety of benefits of a particularized sort that help to offset the higher costs (so-called "solidary benefits," social experiences, rewards such as autographed photos, positions of minor responsibility and prestige, the feeling of contributing to a worthy cause not unlike performance of a citizen's duty, etc.). Yet, for most people, most of the time, the costs outweigh such benefits. I assume this is true universally, so it therefore is posited as a necessary (but far from sufficient) condition for becoming a partisan activist that the public-policy positions of the political party be valued sufficiently to outweigh net costs (i.e., costs less particularized benefits). This line of reasoning leads to the second difference between spatial party activists and voters. Voters in the spatial model evaluate the policy positions of the two candidates. Here, I assume that potential activists evaluate the policy positions of the two parties. This assumption has two further implications. First, people are motivated to become party activists not by particularly attractive candidates but by their perception of where the parties are in policy terms. It is in this sense that I am modeling the regular (potential) contributor. The second implication, then, is to make it necessary to define what I mean by the "party position on policy." In a manner consistent with the general thrust of the model, I assume that the "party" means the set of current activists (and, in particular, their spatial ideal preferences). The specific measure for evaluation, I assume, is just the spatial location of the average ideal point of party activists, $\bar{\theta}$ and $\bar{\psi}$. Therefore, the utility any individual i associates with party θ is simply $U_i(\bar{\theta})$. As usual, I assume that all i accurately perceive the mean party positions at all times.

The above assumptions make it possible to define decision rules for when a citizen will choose to become active in a party. The simplifying assumptions mean that the only remaining variable is the utility associated with a party. It is easier to define the decision rules in several steps. First, let us define the parallels to "alienation" and "indifference" in the typical spatial voting model.

A citizen is alienated from a party unless the utility he or she associates with the party (i.e., $U_i(\bar{\theta})$) outweighs the (net) cost of contributing time, effort, and/or money.

Since the variable in the utility function is the distance separating the citizen from the center of the party on policy, and since I am assuming (unrealistically but for purposes of development) that all i face the same net costs, let c denote the net costs ($c > 0$), and assume that an i is alienated from a party, say θ, if $\|x_i - \theta\| \leq c$. In the utility function, this implies that i is alienated from θ if $U_i(\theta) \leq \lambda - c^2$. In the transformed policy space, a citizen is alienated if he or she is c units or more from $\bar{\theta}$; thus, the nonalienated citizens are those whose ideal points are within a spheroid with radius c centered at $\bar{\theta}$ (or $\bar{\psi}$). Since costs outweigh benefits, I assume alienated citizens become active in that party with zero probability.

Indifference may be defined similarly. That is, i is indifferent between becoming an activist in θ and ψ if the difference between the two parties, relative to i's ideal point, does not generate benefits that outweigh the costs. A natural definition of indifference is, therefore, that i is indifferent if $|U_i(\bar{\theta}) - U_i(\bar{\psi})| \leq c^2$. An indifferent citizen becomes an activist with zero probability. Note that while the cost-benefit interpretation of indifference and alienation leads to the same constant of costs, c^2, and while I use c^2 in both terms, there may be reason to assume a difference between the two cases of (lack of) activism. In what follows, the results would differ only in algebraic detail if there were some difference between the calculation of alienation and indifference (the same statement holds in terms of comparisons between parties; however, the study of the "algebraic details" of different costs of alienation and/or indifference between the two parties may be substantively more rewarding).

The inference is that those who are neither alienated from, say, party θ, nor indifferent to it will become activists with a positive probability. A reasonable assumption about the form of this probability of activism is that it is related to the distance between x_i and the party center. In general, I assume that the probability of activism decreases, or does not increase, as x_i moves from the party center. Formally, if $p_{i\theta}$ denotes the probability that a citizen whose ideal point is at x_i is active in party θ, then:

$$p_{i\theta} = \begin{cases} g(\|x_i - \bar{\theta}\|), \text{ if } i \text{ is neither alienated from } \theta \text{ nor indifferent between } \theta \\ \quad \text{and } \psi, \text{ where it is assumed that } 0 < p_{i\theta} \leqslant 1; \\ 0, \text{ otherwise.} \end{cases}$$

The term $g(\|x_i - \bar{\theta}\|)$ denotes a probability that is a monotone-decreasing function of distance between x_i and θ. Obviously, $p_{i\psi}$ is defined similarly (and, of course, i can be active only in the closer party). Note that equiprobability contours of $p_{i\theta}$ are spheroidal (in the transformed space) about $\bar{\theta}$, until they touch the boundary of indifference (if they do).

While the main justification for this calculus is its similarity to the spatial voting calculus, a few more words of justification are in order. The first point is an elaboration of the main justification. While the contribution of money or labor is more costly than the comparable contribution of a vote, the typical activist nonetheless still is in a relatively low cost-low benefit calculation, since most contribute quite small sums of money or time. But second, joining a party is like joining an interest group and therefore faces the same kinds of problems that Mancur Olson so clearly delineated (1971). The typical interest-group member joins primarily (or even exclusively) because of the selective incentives (or failure to understand Olson's logic). Even more "political" models of group membership (e.g., Moe, 1980a, b) still conclude that selective incentives are necessary. Here, people join a party not to change its goals but to help in their realization (if public-policy concerns do matter). That is, they assume that $\bar{\theta}$ or $\bar{\psi}$ are the goals of the party, and their activities will help (or not hurt) the chances of the various candidates selected and supported by the party. Thus, the "p" term (or probability that one's contribution will make a difference) or its counterpart that is implicit in the formulation of the calculus is much more broadly distributed over the various elective offices and units of our system, as well as over time. Thus, the presumption that one's contribution may have an effect is not as implausible as in the strict voting calculus applied to a single office. Moreover, I have argued elsewhere (1982b) that in a situation in which two or more groups contest over goals, selective incentives no longer are sufficient, in most cases, most of the time, to generate even low-level contributions such as those modeled here. Rather, the mixture of public-good incentives (such as trying to implement $\bar{\theta}$) and private incentives that come only with membership are together necessary and sufficient. Neither is sufficient alone. Finally, in a special case quite similar to that developed here, I argue that public goals ordinarily will weigh at least as heavily in the individual's decision to contribute as do selective incentives and generally will weigh more heavily. This conclusion follows from the presumption that groups competing for members will tend to have comparable resources and thus will be able to offer comparable "packages" of selective incentives. The choice of which group or party to join, therefore, will hinge on public-policy concerns.

3. The problem of equilibrium

The central "social" or "aggregate" question in this model is the impact of the various decisions of individual citizens as they choose or eschew partisan activism. In particular, one is interested in the distribution of activists in each of the two parties. Here, I will focus on the locations of the vectors $\bar{\theta}$ and $\bar{\psi}$, since the "joining" rules prescribe the ranges of ideal points of citizens who have nonzero probabilities of joining the parties. Thus, we are led to search for equilibrium values for the mean partisan ideal points, say $\bar{\theta}^*$ and $\bar{\psi}^*$. The problem is this: as people join a party, they contribute to the location of the center of the party and thereby will shift its location unless they happen to have an ideal point at the mean. But, by shifting the location of the mean, their entry into the ranks of partisan activists may induce others to join or drop out of activity in partisan affairs. Further, the shift in the location of one party may make some in the other party indifferent and/or make some who were indifferent no longer so, inducing them to become active in the other party and thereby shifting the location of the center of the other party as well. So, what is interesting to study is the location of $\bar{\theta}$ and $\bar{\psi}$ in the policy space and the dynamics of these locations. The next section analyzes the effect of the alienation part of the calculus, while the subsequent section examines the addition of indifference.

4. The effect of alienation

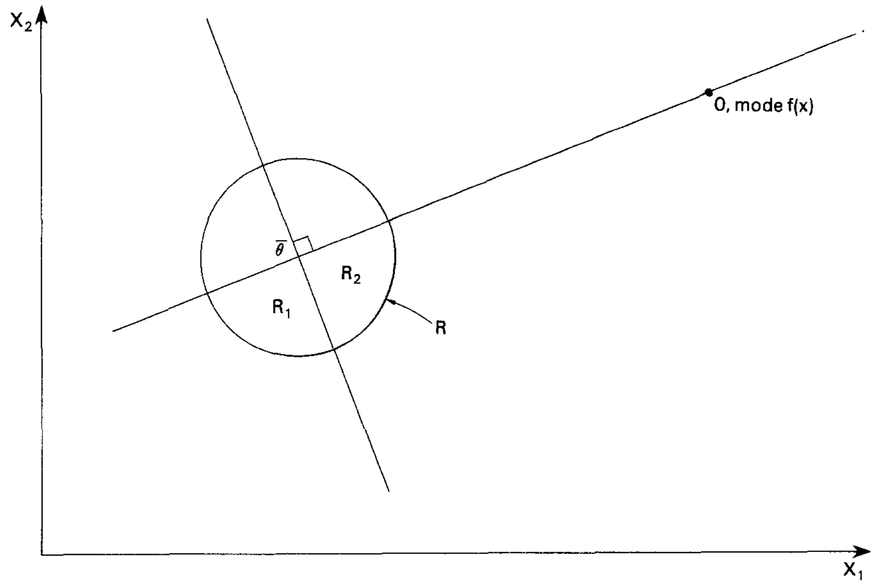
First, I will examine the role of alienation to show that it tends to make the distribution of citizens in a party "converge" towards modes. Here, I will examine a single party, θ, in isolation. Suppose that $f(x)$ is strictly unimodal. Unimodality has been defined in the literature in two ways. The more common way is to define unimodality by requiring that equidensity contours of $f(x)$ enclose convex sets (cf., McKelvey, 1975). Hinich and Ordeshook require that $f(x)$ decrease (strictly, here) along all rays emanating from the mode of $f(x)$ (cf., Hinich and Ordeshook, 1969). Clearly, the latter formulation is less restrictive (i.e., the first implies that the second definition holds, but not vice versa), so I will use it here. Then, if $f(x)$ is strictly unimodal, the following proposition holds, where the mode of $f(x)$ is set to be at $x = 0$.

Proposition 1. If $f(x)$ is strictly unimodal over the space $X = R^N$, and if people choose to join the (single) party θ if they are not alienated, and if probability $p_{i\theta}$ is defined as above, then the distribution of activists in the party will be such that $\overline{\theta}$ is within at least c units of the mode (i.e., if $\overline{\theta}$ is more than c units of the mode, activists will join or drop out of the party such that $\overline{\theta} \rightarrow 0$, at least until $\|\overline{\theta} - 0\| < c$).

Proof: Alienation implies that the distribution of ideal points of activists in θ, i.e., those for whom $p_{i\theta} > 0$, defines a spheroid about $\overline{\theta}$ with radius of c units. If $\overline{\theta} \neq 0$, there is a line connecting the point (vectors) $\overline{\theta}$ and 0. Bisect the spheroid by passing the N-1 dimensional hyperplane perpendicular to the line passing through $\overline{\theta}$ and 0 through the point $\overline{\theta}$. This divides the region (say R), into two symmetric, half-spheroids, (say R_1 and R_2), where R_2 consists of points closer to the mode of $f(x)$. See Figure 1 for a two-dimensional illustration.

The first step will be to show that every point in R_1 on an arbitrary ray from the mode of $f(x)$ can be matched uniquely with a point in R_2. I will show this for two dimensions (as illustrated in Figure 2) and then show that it generalizes. Consider Figure 2. The (unique) ray from 0 through $\overline{\theta}$ is line A. The length of A in R_1 is equal to that in R_2 (namely, being c units). Clearly, then, there is a one to one mapping of such points in R_1 with those in R_2 along A. All other rays, such as B, will form an acute angle with A.

Figure 1

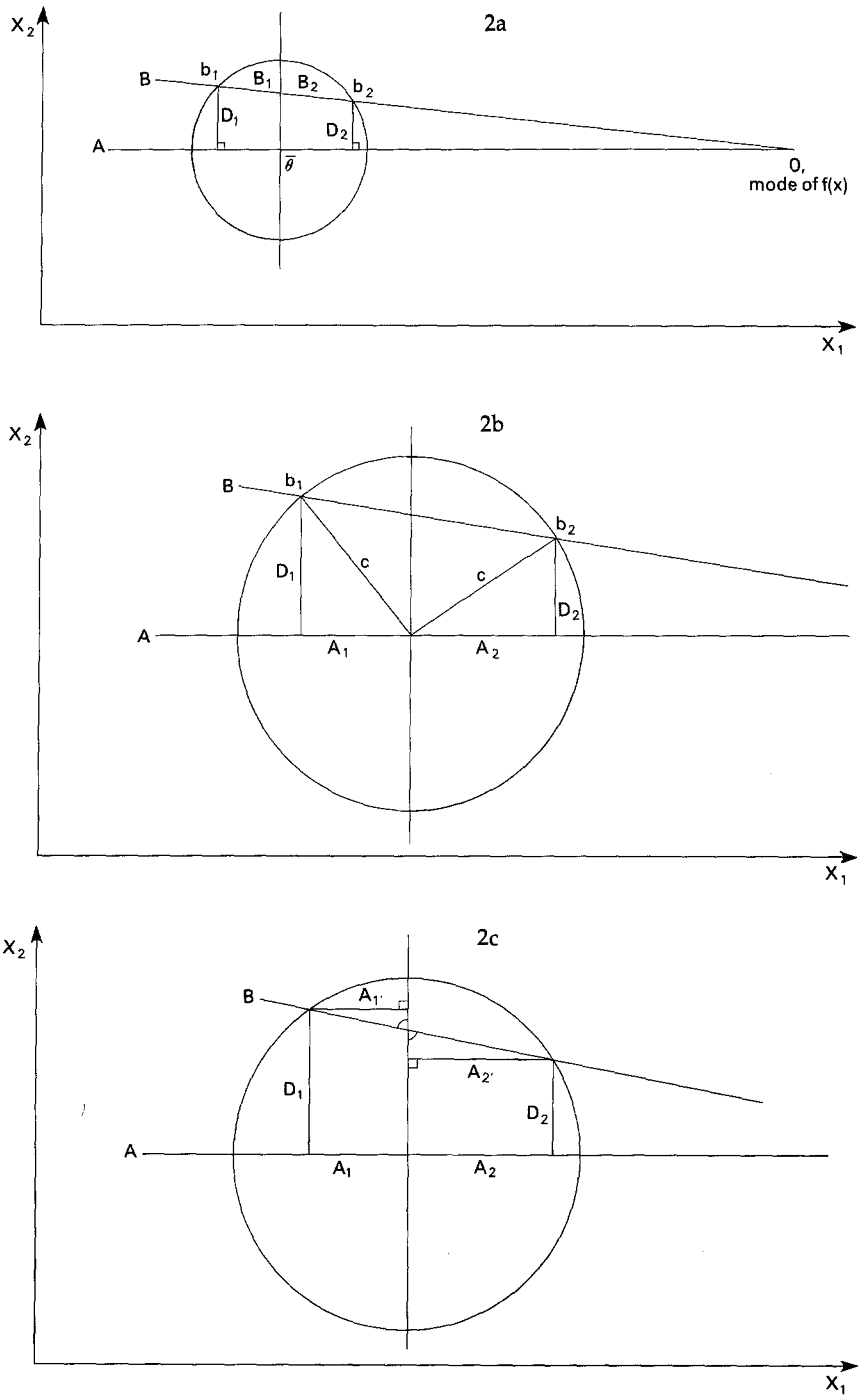


We will consider all such angles, i.e., "rotate" B through all the rest of the circle (and, later, spheroid) until it touches the points of intersection of the circle and bisector perpendicular to A through $\bar{\theta}$, thus passing through each point in R_2 (but not all in R_1). Each point will fall on one such ray like B or on ray A. With that set of rays, what I now want to show is that the line B_1 (i.e., B in R_1) is strictly shorter than the segment B_2 in R_2 (and is equal only if $B = A$). If so, it will be possible to match each point in R_1 with one on the same ray in R_2 uniquely.

Consider Figure 2a. Lines D_1 and D_2 are the perpendiculars dropped from points b_1 and b_2 to A. The two triangles they form are similar (right) triangles (sharing the angle between A and B) and thus D_2 is necessarily smaller than D_1, the triangle formed by D_2 being similar and interior to that formed by D_1. In Figure 2b, there are triangles formed from D_2, C_2, and the relevant portion of A, and similarly for D_1, C_1 and the portion of A. Both are right triangles, D_2 is less than D_1, while $C_1 = C_2 = c$. By the Pythagorean Theorem, therefore, A_1 is strictly less than A_2.

In Figure 2c, I have drawn rectangles with sides of length A_1 and D_1, A_2 and D_2.

Figure 2


2a
X_2
B
b_1
B_1
B_2
b_2
D_1
D_2
A
$\bar{\theta}$
O,
mode of f(x)
X_1
2b
X_2
B
b_1
b_2
c
c
D_1
D_2
A
A_1
A_2
X_1
2c
X_2
B
$A_{1'}$
$A_{2'}$
D_1
D_2
A
A_1
A_2
X_1

Consider the triangles formed by A_1', B_1, and the perpendicular bisector; A_2', B_2 and the bisector. Since they are both right triangles and the marked angles are equal (i.e., those formed by the chord crossing the bisector), the two triangles are similar. But, since A_1' is strictly shorter than A_2', B_1 must be strictly shorter than B_2.

Now, for more than two dimensions, each ray is a line, intersecting the boundary of the spheroid exactly twice, at a point in R_1 and at one in R_2. These two points and $\bar{\theta}$ will define a plane through the spheroid and, since it passes through the center point, $\bar{\theta}$, the resultant section will be a circle. Therefore, the same analysis will hold, while the set of rays involved will be, simply, that set that is required to pass through each point (once) in R_1. Hence, the two-dimensional analysis generalizes to N dimensions.

Since $B_1 < B_2$, every point on B_1 can be paired with one on B_2, such as pairing, say, b_1 with b_2. Hence, each point in R_1 is paired with one on the same ray in R_2, and no point in R_2 is paired with more than one point in R_1. By the definition of unimodality used here, $f(b_1) < f(b_2)$ for all such pairs. For the variable $p_{i\theta}$, each equiprobability contour of $p_{i\theta}$ is circular, thereby crossing the ray exactly twice, once in R_1, once in R_2. The pairing of points, therefore, is like that of (b_1, b_2), i.e., a point on B_1 in R_1 is paired with its equiprobability companion on the same ray in R_2. Therefore, the inequality below must hold:

$$\underset{R_1}{\int \dots \int} p_{i\theta}\, f(x)dx < \underset{R_2}{\int \dots \int} p_{i\theta}\, f(x)dx$$

That is, the density of activists in R_1 is strictly less than the density of activists in R_2. Thus, the "center of gravity" of the party activists' distribution is not at $\bar{\theta}$ but at some point interior to R_2. The mean value theorem of calculus implies that there is some point, $\bar{\theta}'$; interior to R that is the center of gravity or mean density point, i.e., for which:

$$f(\bar{\theta}') = 1/R \int \dots \int p_{i\theta}\, f(x)\, dx$$

(where R denotes the "area" in X from which citizens are not alienated).

Then, equilibrium occurs when $\bar{\theta} = \bar{\theta}'$. A necessary (but not sufficient) condition for that to hold is that for any particular bisection of R through $\bar{\theta}$, both halves must have the same "weight," i.e., the above inequality in R_1, R_2 must be an equality.

If $\bar{\theta}$ is exactly c units from 0, then, for the first time, an $x_i = 0$ is at the border between those alienated and those not so, and the above inequality still holds. That is, all r_2 points in the pairing will still be closer to the mode than their r_1 pair. As $\bar{\theta}$ is less than c units from the mode, however, some points in R_2 may be (and eventually will be) at a "lower" equidensity contour than their paired point in R_1. Thus, the inequality is not necessarily correct; it may become an equality, it may be reversed. Until that time, however, $\bar{\theta}$ must approach the mode. Q.E.D.

The point of the above proposition is the simple one that alienation as a part of the activist's decision calculus makes the distribution of activists converge towards the mode of $f(x)$. Or, in other words, the party will tend to become dominated by activists whose preferences are most widely shared in the electorate. It is important to note that the proposition did not rely on global unimodality. That is, if $x = a$ is a mode for a region in X such that the ideal point of the just alienated citizen farthest from a (i.e., c units away from $\bar{\theta}$ and farthest from a on the line passing through $\bar{\theta}$ and a) is still in the local range of unimodality, then $\bar{\theta}$ will tend to converge toward that mode. Also note that Proposition 1 implies that the range of activists in the party will include the mode (i.e., a citizen with ideal point at the mode will not be alienated from the party).

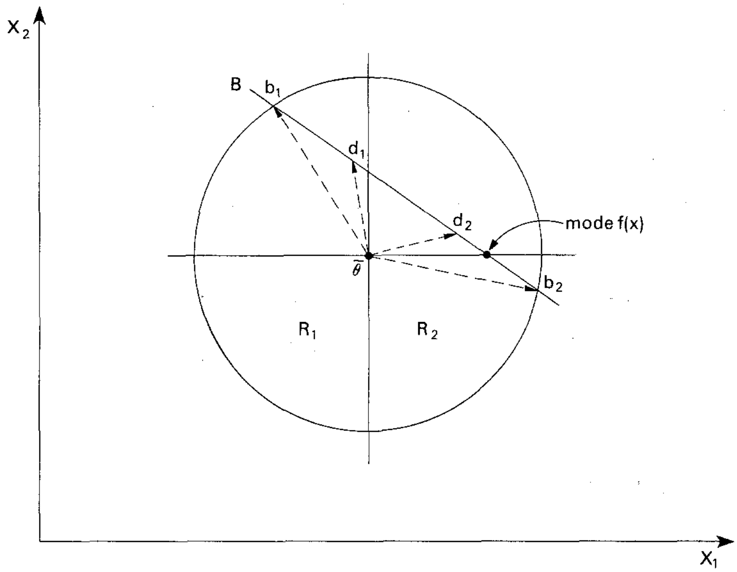
What I have shown so far is convergence toward modes, not equilibrium. If just alienation is operative, it really does not matter much exactly where the party ends up, or even if the distribution is in equilibrium, as long as we know that the party will remain near the (or a) mode. Nonetheless, equilibrium is useful.

Proposition 2. If $f(x)$ is symmetric and unimodal and if alienation is the only reason for a zero probability of activism (i.e., indifference is not relevant), than there is an equilibrium at the mode of $f(x)$, and it is unique.

Proof: I will show existence of an equilibrium at $x = 0$ (the mode) first. If $\bar{\theta}$ is at the mode, the just alienated zone will be a concentric circle at c units from the mode. Party θ may be bisected into the R_1 and R_2 of before by any N-1 dimensional hyperplane passing through $\bar{\theta}$. Each point in R_1, say y_i, can be paired with its symmetric opposite, i.e., y_i with $-y_i$. By the probability assumption $p_{i\theta}(y_i) = p_{i\theta}(-y_i)$. By symmetry $f(y_i) = f(-y_i)$. Therefore, $p_{i\theta}\ (y_i)f(y_i) = p_{i\theta}\ (-y_i)f(-y_i)$. With every point balanced with its opposite, the mean value of activists in θ will be $\bar{\theta}' = 0$. (Note this proves that an equilibrium, $\bar{\theta}^*$, exists if $f(x)$ is symmetric about a point, say, a, within a region of at least c units about a. Obviously a will be at least a local mode unless $f(x)$ is at a plateau c units about a.)

For uniqueness, suppose $\bar{\theta}$ is between 0 and c units of the mode (having covered already all points more than c units from the mode and showing, inter alia, that they are not in equilibrium). If $f(x)$ is unimodal, and if the mode is interior to, say, R_2 (as in Figure 3), then it can be shown that the rays and their symmetric projections beyond the mode follow the same properties as in Prop. 1. The line through $\bar{\theta}$ and 0 is, as before, of equal length in R_1 as in R_2. Lines like B can be shown easily to be shorter in R_1 than R_2 (B_2

Figure 3



Note : Vectors from $\bar{\theta}$ to b_1 and b_2 are same length, as are those from $\bar{\theta}$ to d_1 and d_2.

must be longer than c units, while B_1 must be less). Each point in R_1, therefore, can be paired with one in R_2 on the same ray or its opposite projection, following the same pairing rules as before that keep $p_{i\theta}$, for example, constant in the pair. Each point in R_2 is closer to the mode than its pair in R_1. Therefore, the appropriate inequalities hold. Q.E.D.

5. The effect of indifference

If alienation has the effect of driving a party towards a mode, then it is useful to characterize a space and distribution of the electorate, $f(x)$, by the number of modes. For example, if $f(x)$ is biomodal, then we may find a party being dominated by activists drawn from one mode, while the other party is "captured" by activists whose preferences are shared by those at the other mode. The distribution of activists may or may not be in equilibrium, but even if not, the party would be centered near the mode. Moreover, each party would be, in a sense, independent of the other. That is, citizens would be active in a party, or not active, based only (or primarily) on their proximity to that party. It is only when, from a citizen's perspective, the two parties are similar enough to consider indifference that an explicit comparison of the two parties becomes relevant. The most obvious instance is when two parties are being driven toward the same mode (whether $f(x)$ is strictly unimodal or whether it is multimodal). As the calculus of activism leads both parties to converge to the same mode, indifference becomes relevant. While alienation is, in a sense, a converging force (e.g., both parties converging toward the mode if $f(x)$ is unimodal), indifference will prove to be a diverging force, one that keeps the parties from converging too closely together.

To begin, the definition of indifference presented earlier leads to the following formulas for calculating the location of the ideal point locations of the citizens just indifferent between points $\bar{\theta}$ and $\bar{\psi}$:

$$\text{For } x\text{; closer to } \theta\text{; } 2x'(\bar{\theta} - \bar{\psi}) = c^2 + \bar{\theta}'\bar{\theta} - \bar{\psi}'\bar{\psi}$$
$$\text{For } x\text{; closer to } \psi\text{; } 2x'(\bar{\psi} - \bar{\theta}) = c^2 + \bar{\psi}'\bar{\psi} - \bar{\theta}'\bar{\theta}$$

The equations are linear in x and define N-1 dimensional hyperplanes perpendicular to the line connecting $\bar{\theta}$ and $\bar{\psi}$ (call the hyperplanes H_θ and H_ψ, respectively). H_θ and H_ψ are parallel to each other, and all x falling between the two are indifferent.

The size of this "zone" of indifference depends, of course, on the magnitude of c, but its size also depends on the distance separating $\bar{\theta}$ and $\bar{\psi}$. Or:

Proposition 3. If $c^2 > 0$, then hyperplanes H_θ and H_ψ will be perpendicular to the line passing through $\bar{\theta}$ and $\bar{\psi}$; between H_θ and H_ψ there will be a "zone of indifference" from which neither party will draw activists, and the distance separating H_θ and H_ψ will increase the closer θ is to ψ.

Proof: I will prove the last point, the other parts of the proposition being obvious. Without loss of generality, let the (unique) midpoint between the two parties be zero, i.e., $(\bar{\theta} + \bar{\psi})/2 = 0$. Thus, the question is how far the "just indifferent" ideal point is from the midpoint between the two parties (which may vary in location in X, of course, as the party centers shift). I will show that as $\bar{\theta} \to 0$ (i.e., the two parties centers converge, since $\bar{\theta} = -\bar{\psi}$) the location of the "just indifferent" ideal gets farther from 0.

The just indifferent ideal point, x, for x closer to $\bar{\theta}$ than to $\bar{\psi}$, is found by solving:

$$2x'(\bar{\theta} - \bar{\psi}) = c^2 + \bar{\theta}'\bar{\theta} - \bar{\psi}'\bar{\psi}$$

Since $\bar{\theta} = -\bar{\psi}$, substituting $-\bar{\theta}$ for $\bar{\psi}$ yields:

$$2x'(\bar{\theta} - (-\bar{\theta})) = c^2 + \bar{\theta}'\bar{\theta} - \bar{\theta}'\bar{\theta} \qquad \text{or}$$

$$x'\bar{\theta} = c^2/4$$

Since $c^2/4$ is a constant, as θ approaches zero, x must become farther from zero. Q.E.D.

Table 1 gives some algebraic details for $c^2 = 1$. The key points are that at a $\bar{\theta}$ of infinity, the location of the citizen just indifferent to joining $\bar{\theta}$ is at $x = 0$, or converges to the midpoint between the candidates. It begins to pull away from $x = 0$ toward $x = \bar{\theta}$ as $\bar{\theta}$ converges toward $\bar{\psi}$. At $\bar{\theta}$ equally approximately 1.207 (i.e., $\bar{\theta}$ and $\bar{\psi}$ separated by about 2.414 units, when $c = c^2 = 1$), the just indifferent citizen closest to $\bar{\theta}$ and $\bar{\psi}$ (i.e., on the line passing through $\bar{\theta}$ and $\bar{\psi}$) is also just alienated. That is, until that point, all who are indifferent are also alienated. For shorter distances separating $\bar{\theta}$ and $\bar{\psi}$, some who are indifferent are not alienated also; indifference is now relevant and Propositions 1 and 2 are no longer sufficient. At the above mentioned point, $x = 0.2071$ (which follows since that x is 1 – or c – units from $\bar{\theta}$ and thus "just alienated"). The location of this just indifferent ideal point gets farther and farther away from zero until $\bar{\theta} = .5$, where $x = .5$, also. At that point a person whose ideal point is at the party center is indifferent to joining the party. Parties any closer together would result in the center of the party being in the "zone of indifference," an obvious contradiction. In sum, indifference becomes relevant only in the range when the two parties are about 2.414 units apart; given that $c = 1$, more and more ideal point locations are indifferent as the parties converge, and the parties can be no closer than c^2 unit apart.

Table 1. **Assume that distance is measured from the midpoint between the two parties and that $c = c_2 = 1$. Then x_θ denotes the position on H_θ that intersects the line passing through $\bar{\theta}$ and $\bar{\psi}$.**

If $\bar{\theta}$ (= $-\bar{\psi}$) is at:	x_θ (= $-x_\psi$) is at:	Distance between $\bar{\theta}$ and $\bar{\psi}$	Distance between X_θ and X_ψ	Location of the ideal point of a citizen "just alienated" from θ
0.49	0.51	0.98	1.02	-0.51
0.50	0.50	1.00	1.00	-0.50
0.60	0.42	1.20	0.83	-0.40
0.70	0.36	1.40	0.71	-0.30
0.80	0.31	1.60	0.63	-0.20
0.90	0.28	1.80	0.56	-0.10
1.00	0.25	2.00	0.50	0.00
1.10	0.23	2.20	0.45	0.10
1.20	0.21	2.40	0.42	0.20
1.2071	0.2071	2.4142	0.4142	0.2071
2.0	0.13	4.00	0.25	1.00
3.0	0.08	6.00	0.17	2.00
4.0	0.06	8.00	0.12	3.00
∞	0	∞	0	∞

The obvious "corollary" to Proposition 3 is that indifference will keep the two parties from converging to the same point. So, for example, if $f(x)$ is unimodal and symmetric, both parties, if they started far enough apart (greater than 2.414 units of c^2, for example) would converge toward the mode but would not converge completely to the mode due to indifference. It is obvious that the case of unimodal $f(x)$ is most important here, and the rest of the section will relate to this. I will begin with an existence proof and than a stability result.

Proposition 4. If $f(x)$ is unimodal with equidensity contours forming concentric ellipsoids about the mode, then there are equilibrium positions for the two parties.

Proof: The proof is of existence. The mode is at $x = 0$. Suppose $\bar{\theta}$ and $\bar{\psi}$ start at a point far enough away that indifference may be ignored, and suppose that $\bar{\theta}$ is on an axis of $f(x)$ (any one will do) while $\bar{\psi}$ is set at $-\bar{\theta}$ and, by construction, is kept so (thus $\frac{1}{2}(\bar{\theta} + \bar{\psi})$ is at the mode throughout). The axis of $f(x)$, therefore, is also the line passing through $\bar{\theta}$ and $\bar{\psi}$. Both $\bar{\theta}$ and $\bar{\psi}$, as a consequence, are symmetric about the axis, as is $f(x)$. Thus, $\bar{\theta}$ and $\bar{\psi}$ fall someplace along the axis. Also, since H_θ and H_ψ are hyperplanes perpendicular to the axis, when indifference becomes relevant, $\bar{\theta}$ and $\bar{\psi}$ will remain symmetric about the axis (and mirror images of each other).

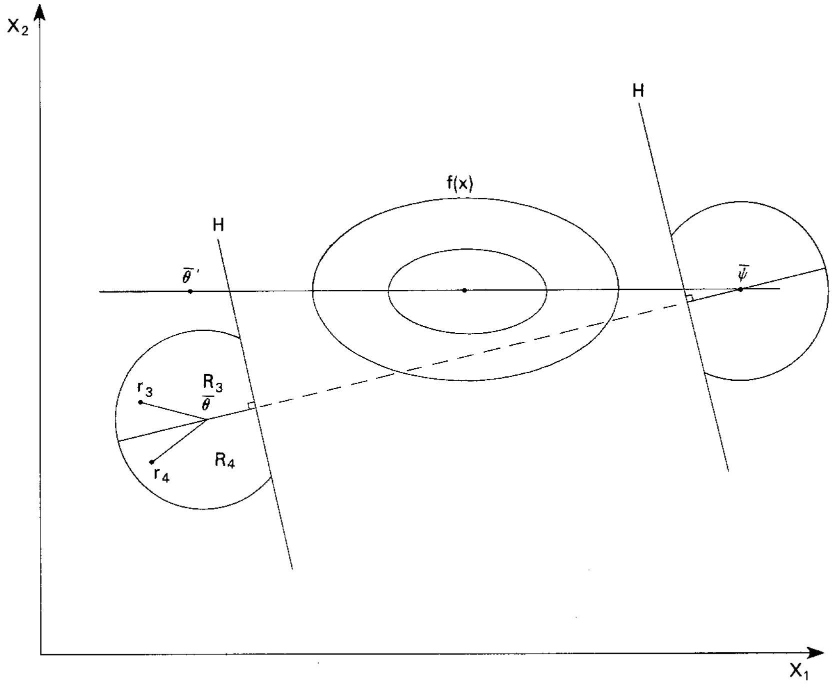
By Proposition 4, $\bar{\theta}$ (and therefore $\bar{\psi}$) will converge towards the mode, since the (now two, equal) equations will be a strict inequality:

$$\underset{R_{1\theta}}{\int \ldots \int} p_{i\theta}\, f(x)dx < \underset{R_{2\theta}}{\int \ldots \int} p_{i\theta}\, f(x)dx \text{ (and similarly for } \psi)$$

When indifference becomes relevant, the "exterior" boundary of $R_{2\theta}$, i.e., the one defined by the hyperplane through $\bar{\theta}$, will be moving toward the mode, while H_θ will be moving away from the mode. Thus, for a $\bar{\theta}'$ closer to the mode than a $\bar{\theta}$, $R_{2\theta}'$ will be strictly interior to $R_{2\theta}$ and the right-hand side will be strictly decreasing, while the left-hand side of the above equation is strictly increasing. Since $R_{2\theta}$ will collapse to a hyperplane (when $\bar{\theta}$ is at the point equivalent to .5 in Table 1) and then disappear entirely, the left-hand side must equal the right-hand side at some unique point. By construction, the same is true for $\bar{\psi}$ at the same point. Finally, since $\bar{\theta}$, $\bar{\psi}$, and $f(x)$ are symmetric about the axis, the center of gravity will stay on the axis (see Proposition 5) and thus be equal to $\bar{\theta}$ (and $\bar{\psi}$) at some point, i.e., the two parties will be in equilibrium. Q.E.D.

Proposition 5. Under the conditions of Proposition 4, if $f(x)$ has a major axis, that axis will dominate (i.e., perturbations off it will lead to convergence back to it), any minor

Figure 4



axes equilibria will be unstable, while there will be infinitely many equilibria if two or more "major" axes have spheroidal equidensity contours of $f(x)$.

Proof: Consider the two-dimensional example in Figure 4. There, $\bar{\theta}$ and $\bar{\psi}$ are in equilibria along the major axis of $f(x)$, but $\bar{\theta}$ is perturbed off the major axis. The hyperplanes H_θ and H_ψ are no longer perpendicular to the axis, rather they are perpendicular to the line connecting $\bar{\theta}$ and $\bar{\psi}$. (We do not know yet where $\bar{\psi}$ is. We may assume initially that it, temporarily, remains fixed in its old location.) The area of activists' ideal points in θ is symmetric about the line through $\bar{\theta}$ and $\bar{\psi}$. Let that line (or hyperplane in N dimensions) bisect θ into, say, R_3 and R_4 where R_3 is the area that is generally closer to the major axis. Then pair each point in R_3 with one in R_4 (and vice versa, a one-to-one mapping that can be maintained no matter where $\bar{\theta}$ and $\bar{\psi}$ are in space), such as r_3 and r_4 in Figure 4. These are points that would touch if $\bar{\theta}$ were "folded" along the bisector. In general, they are points that have vectors of the same length from $\bar{\theta}$ and where the vectors form the same-sized angle (and "orientation" in higher dimensions) with the bisector. Since points such as r_3 and r_4 are equidistant from $\bar{\theta}$, $p_{i\theta}(r_3) = p_{i\theta}(r_4)$ (and so on for all such r_3's and r_4's) under either probability of activism assumption. Then, we need look only at $f(r_3)$ and $f(r_4)$. If $f(r_3) = f(r_4)$ for all r_3's and r_4's, $\bar{\theta}$ is in equilibrium. If $f(r_3) < f(r_4)$ throughout, then $\bar{\theta}$ will tend to swing farther from the axis, while if $f(r_3) > f(r_4)$ it will tend to swing back.

A key point, therefore, is the orientation of the bisector. If ψ remains fixed, then the bisector will be oriented between parallel to the original axis and the line from $\bar{\theta}$ to the mode of $f(x)$. If it were parallel to the axis, then, that $f(r_3) > f(r_4)$ follows straightforwardly. In particular, the line connecting r_3 and r_4 is perpendicular to the axis, and any point (such as r_3) closer to the axis than another (e.g., r_4) will fall on a higher equidensity contour of $f(x)$. Therefore, $\bar{\theta}$ will tend back to the original axis. (Note, the assumption that the bisector is parallel to the axis implies that ψ has moved an equal distance off the axis in the same direction; thus it, too, will tend back to the axis.) If the bisector is on the line from $\bar{\theta}$ to the mode (hence, meaning that $\bar{\psi}$ has moved so that it, too, falls on this line), then each (r_3, r_4) pair is equidistant from the mode. Given the regularity of an ellipsoidal $f(x)$, this implies that $f(r_3) = f(r_4)$ only if this is true for all such pairs. The equality is exact only if they fall on the same equidensity contour, but this implies further that the equidensity contours are circular. If the original axis were the major axis, this implies that $f(r_3) > f(r_4)$ (i.e., the equidensity contours are at a "smaller" angle than a circle near the major axis). Thus, the density of activists in R_3 is greater than in R_4, and the center of gravity of activists in $\bar{\theta}$ (and, by the symmetry involved, in $\bar{\psi}$, too) will be closer to the major axis than $\bar{\theta}$, and thus, the party (both $\bar{\theta}$ and $\bar{\psi}$) will tend back to the major axis. The same reasoning shows that if the acis were minor, $f(r_3) < f(r_4)$ and there will be a tendency to swing away from a more minor axis in the direction of a more major axis.

Next, we need to consider $\bar{\psi}$. If $\bar{\psi}$ is unperturbed, while $\bar{\theta}$ is, then the main thing that happens to $\bar{\psi}$ is that H_{ψ} angles from perpendicular to the axis to be perpendicular to the line through $\bar{\theta}$ and $\bar{\psi}$. If follows that $\bar{\psi}$ will be off the axis toward the line that passes through $\bar{\theta}$ and the mode of $f(x)$. However, we have seen already that if it moves to that line, it will tend back toward the main axis, while $\bar{\theta}$ will have a similar tendency throughout.

So far, I have shown that a major equilibrium is stable, at least in the sense that a perturbation off the major axis leads to parties returning to the axis. When the equidensity contours are spheroidal, I have shown that if $\bar{\psi}$ is perturbed in the opposite direction from the axis of equilibrium (since "major axis" is not meaningful here) as $\bar{\theta}$ and at the same (angular) "distance" (i.e., so the line through $\bar{\theta}$ and $\bar{\psi}$ also passes through the mode of $f(x)$), then the parties will be in a new equilibrium. But, this simply "shows" that every line through the mode is a "major" axis. It also follows from the logic of the above argument that if $\bar{\psi}$ remains fixed, $\bar{\theta}$ would tend to return to the original "axis," but (as above) $\bar{\psi}$ will tend to move in the opposite direction, generating a new equilibrium pair (all of which are exactly similar). For the minor axis equilibrium, it was shown that a perturbation leads to $\bar{\theta}$ diverging from the minor axis. Since $\bar{\psi}$ will be "perturbed" definitionally by indifference, it will swing in the opposite direction toward the major axis. Thus, minor axes equilibria are unstable in the face of perturbations toward the major axis.

The argument did not depend on any particular dimensionality (in effect, all pairs of dimensions could be considered). And since all axes of an ellipsoidal distribution can be put into a weak order of major to "most minor" axes, all cases are covered. Q.E.D.

The substantive interpretation is that the two parties will diverge from each other on the major axis of $f(x)$ but will be centered at the same point on all nonmajor axes. The final remaining point, therefore, concerns the uniqueness of the major axis equilibria. In effect, there is a single remaining dimension along which multiple, stable equilibria could be found. Due to the similarity to the unidimensional case, I offer the following conjecture and refer the reader to (1982a) for details:

Conjecture: The major axis equilibrium derived in Proposition 5 is either unique (and thus the only stable equilibrium) or the others are in a close proximity to that one, along the major axis (say $\bar{\theta}^* + \epsilon$ along that axis, where ϵ is a function of c and the "steepness" of $f(x)$). (See Aldrich, 1982a.)

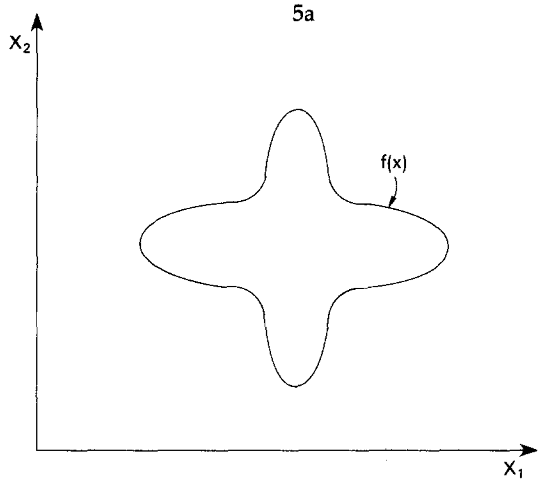
Thus, we reach a point where some general observations can be drawn. If, for example, $f(x)$ is bimodal, there are two major possibilities. Either the two parties will go toward each separately, or they might "compete" over a single mode. In the former case, indifference might be irrelevant (if, for example, the two modes are far enough apart that all who are indifferent are also alienated), in which case the analysis of the preceding section holds straightforwardly. If indifference is relevant, its most obvious effect is to "push" the parties farther away from each other (and thus the modes) than otherwise. Except for that, the rest of the analysis should generally parallel the former case. The latter case is, in effect, analogous to a unimodal distribution. Obviously, if $f(x)$ is multimodal, the analysis is similar, except each mode could serve to "capture" a party (also the extension to a multiparty system might provide opening leverage to a spatial modeling of those sorts of elections).

The unimodal case, where both alienation and indifference become relevant, is the more interesting case. Alienation induces convergence toward the most dense concentrations of ideal points, while indifference keeps the parties from converging too close to each other. In multidimensional spaces, the "diverging" effect of indifference keeps both parties from climbing "all the way up the mountain" to the mode of $f(x)$ (as alienation leads to "hill climbing"), but "directional rotation" about the mode is possible. Here, the result of the last proposition suggests that if "inward" pressures of convergence were stopped, the parties would tend to become dominated by the axis of densest concentration of ideal points. Thus, the parties would tend to rotate along the major axis (if "hill climbing" is stopped, they would tend to "climb the steepest ridge"). It appears, that is, that the symmetry of Proposition 5 is of less importance, while the existence of a dominant "axis" is more important. Therefore, it seems reasonable to conjecture that the parties will diverge (somewhat) along the "axis" of densest concentration of ideal points, should one exist, with less divergence along more minor "axes."

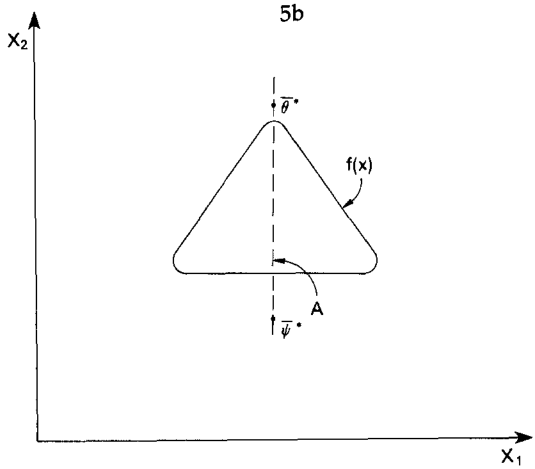
An interesting special case is a unimodal $f(x)$ like that in Figure 5. In Figure 5a, there are, essentially, two "ridge lines" approximately perpendicular to each other. It seems likely that each "ridge line" acts as a major axis in Proposition 5, so that two pair of relatively stable equilibrium positions exist, one pair along each axis (if $f(x)$ is symmetric and convex and the "valleys" between the "ridges" sufficiently deep, Proposition 5 can be applied to this case with small enough perturbations, and, of course, Proposition 4 applies straightforwardly). In 5b, I have drawn an $f(x)$ that is asymmetric (in the sense of "radial symmetry" used in the formal definition). There will be an equilibrium pair of positions

Figure 5

5a



5b



for the two parties, roughly as drawn, along line A. Party θ, centered along an "angle" of $f(x)$ (which is but a triangle with rounded corners to preserve continuity), will be in stable equilibrium using the same analysis as in Proposition 5. Party ψ is also evidently in equilibrium, the comparable regions to R_3 and R_4 of above clearly being equal, albeit in unstable equilibrium. Thus, an equilibrium exists in a unimodal but not (pair-wise) symmetric $f(x)$, showing the important point that symmetry is not a necessary condition for equilibrium. (A more cluttered example of asymmetric stable equilibrium can be drawn, as well.) Clearly, the number of such special cases can be multiplied indefinitely. It is worth detailed investigation of them only if they have substantive import. I turn to consider one such case.

6. Some electoral dynamics

In this section, I would like to examine a variable in the model that has gone unexamined to this point. In particular, the analysis has assumed that the policy space has been transformed such that, in the transformed space, all indifference contours of $U_i(x)$ are circular for all i, and, in effect, salience has been treated as a constant. Here, I want to examine the dynamic introduced into the model by variations in the relative weightings of the dimensions (I will examine exactly two dimensions in this section). Obviously, changing any of the variables in the analysis (notably $f(x)$) would lead to one form or another of "electoral dynamics." The selection of this particular variable is motivated by a substantive concern.

The literature about United States parties and elections includes a major interest in what are known as "realignments" of the political parties (e.g., the rise of the Republican Party before the Civil War, the ascendance of the Republican Party from competitive balance to dominance in the 1890s, the rise of the New Deal Democratic majority, and perhaps one around the time of the initial election of Andrew Jackson). The basic idea is that there is a sharp, relatively rapid, and durable change not just (and possibly not even) in support of one party, but that the party system reorders itself, usually by division along a new line of party cleavages on policy.

James Sundquist gives a sixteen-point description of the process of realignment after a historical and theoretical study of this type of electoral dynamics (his Chapter 13, "The Realignment Process: An Amplified Statement," 1973, pp. 275-298). What I will argue is that a variant of the case covered in Proposition 5 provides a formalized rationalization of the key points of his statement. The key points are quoted from the original:

> 1. A realignment is precipitated by the rise of a new political issue (or cluster of related issues). (p. 275)
> 2. To bring about a realignment, the new issue must be one that cuts across the existing line of party cleavage. (p. 276)
> 3. To bring about a major realignment, the new issue must also be one powerful enough to dominate political debate and polarize the community. (p. 277)
> 5. Whether a new issue becomes dominant depends not only upon intrinsic power but also on the extent to which the older issues [i.e., issue or cluster of related issues] underlying the party system have faded with the passage of time. (p. 281)
> 7. The normal response of both major parties at the outset is to straddle the new issue. (p. 283)
> 11. A realignment crisis is precipitated when the moderate centrists lose control of one or both of the major parties – that is, of party policy and nomination – to one or the other of the polar forces [i.e., those in the party who are relatively extreme on the new issue]. (p. 290)

The only time when a "realignment crisis" does not lead to a realignment, per se, is if one of the new "poles" is dominant within both parties (and, thus, insofar as we can tell, in the electorate as a whole). In that case, there is no residual controversy of significance in society along the new dimension, the preferred policy is enacted, and the only remaining controversy is over the old line of cleavage. Otherwise, a realignment occurs. The rest of his points concern details of the process (e.g., whether a new party arises and an old one disappears or whether the old party adopts that new position).

To reconstruct this process, assume there are two policy dimensions of concern. These might be dimensions as usually interpreted in the spatial literature, or they might be two "evaluative dimensions" in the sense of Hinich, et al.'s new works (e.g., Hinich and Pollard, 1981; Enelow and Hinich, 1980). The latter interpretation seems to fit better the idea of a "cluster of related issues" (although, if using that interpretation, I will assume, nonetheless, that citizens perceive with certainty, etc.; thus, I will "act as if" it operates as a usual spatial dimension).

Sundquist indicates throughout that there are only two issues or two clusters of issues of concern. Indeed his drawings (cf; Chapter 2, "Some Hypothetical Scenarios," 1973, pp. 11-25) are all two-dimensional. Thus, the assumption of two dimensions is not problematic (at least in reconstructing his -- and the usual – interpretations of realignments). My understanding of his points 1 and 5 is that an original dimension (say, X_1) is dominant at the outset but begins to decline, while points 2 and 3 mean that the new dimension, X_2, arises and increases in importance relative to that of X_1, in fact coming to dominate. I assume that "power," "dominance," and "importance" of a dimension refer to its salience, and so I assume that the main dynamic is the decrease in salience of X_1 and increase (relative to X_1) of the salience of X_2. From beginning to end, the obvious interpretation is that X_1 is more salient than X_2 but the situation reverses. It seems reasonable to assume that there is a continuous change in the ratio of weights attached to X_1 and X_2.

Point number 2 is that the new issue cuts across the old line of cleavage. Sundquist makes clear that by "line of cleavage" he means what separates those in one party from those in the other in policy terms. Indeed, his drawings cited above are exactly the same as a line parallel to the lines of indifference, H_θ and H_ψ, probably running through the midpoint between the two parties. Of course, this interpretation is the standard one of a line of party cleavage (see also Page, 1978, for example). By having the new issue cut across the old-line cleavage, Sundquist clearly means (and draws) a new (potential) line of cleavage nearly perpendicular to the old one through the center of the space. While he points out that the new line is unlikely to be exactly perpendicular, I will assume that we are so fortunate as to find it exactly perpendicular. The only interpretation I can make of this is that preferences along one dimension are unrelated to preferences along the second. That is, the A matrix has (as argued above) variable "main diagonal" entries but is a diagonal matrix.

Clearly, the whole thrust of this paper has been to study the consequences of different distributions of ideal points in the policy space. Since I now want to examine variations in saliency, I will hold $f(x)$ constant. To "neutralize" its effects, I will assume that in the original utility metric (i.e., before transforming the quadratic utility function to be circular), $f(x)$ is distributed such that it is unimodal and its equidensity contours are circular. Since equidensity contours of $f(x)$ will remain elliptical after any transforming of X (e.g., to make indifference contours of $U_i(x)$ circular), all propositions (notably number 5) can be employed. This assumption "neutralizes" $f(x)$ in the sense that there is no major axis (prior to transformation). Obviously, any impact of variable saliency would be modified in general by any effects of $f(x)$ (e.g., citizens being distributed more broadly along, say, X_1 and X_2, or whatever).

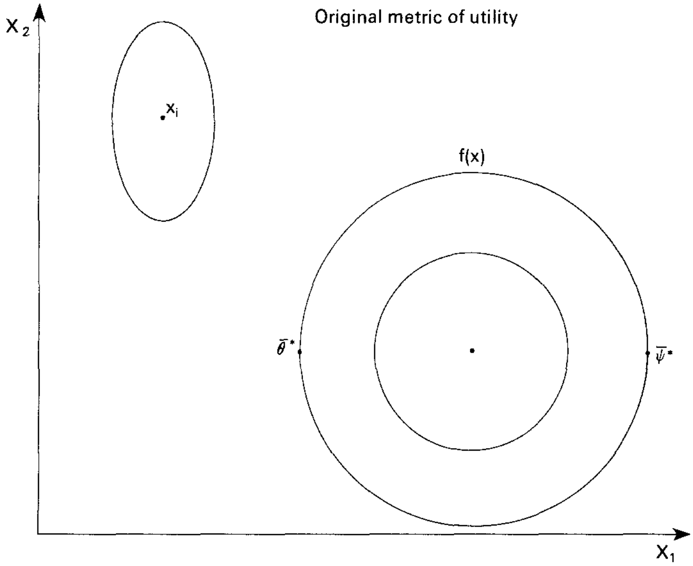
If $U_i(x)$ is quadratic, and if (at the outset) X_1 is weighted more heavily than X_2 and X_1 and X_2 are "uncorrelated," then indifference contours of $U_i(x)$ are elliptical, in two dimensions, with major axis parallel to X_2 and minor axis parallel to X_1 (i.e., utility drops off faster for a one-unit change from x_i along X_1 than for a one-unit change along X_2). This point and $f(x)$ are exemplified in Figure 6a below. If we then transform X such that $U_i(x)$ has circular indifference contours, $f(x)$ will be transformed such that its equidensity contours are elliptical rather than circular, with major axis parallel to X_1. In the transformed space, equiprobability contours of $p_{i\theta}$ will be circular (up to the effect of indifference, and the distributions will be as in Proposition 5). By Proposition 5, then, the only stable equilibrium points for θ and ψ are distributed along X_1, with one "conservative" and one "liberal" party. (See Figure 6b.)

Assume that $\bar{\theta}$ and $\bar{\psi}$ are at stable equilibrium points along X_1. (They obviously will be if X_2 is really a "new" issue, i.e., did not exist at some early time. If it did "exist"

Figure 6

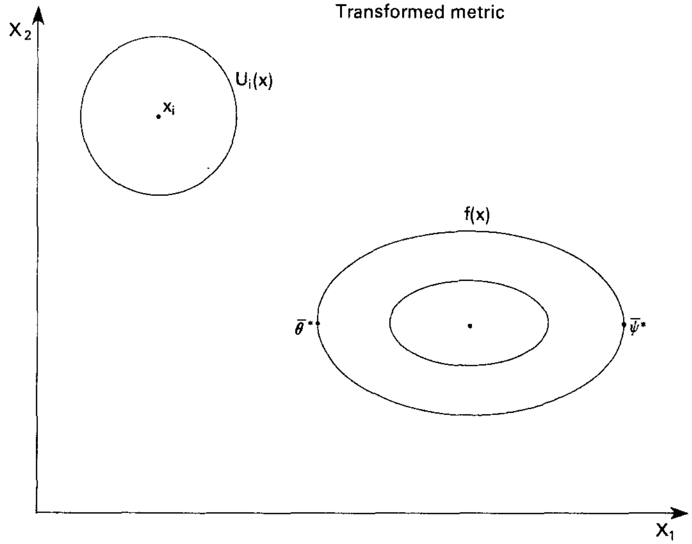
6a

Original metric of utility



6b

Transformed metric



but with very low salience relative to X_1, then $\bar{\theta}$ and $\bar{\psi}$ would almost certainly not be at an unstable equilibrium along X_2.)

As the salience of X_1 decreases relative to X_2, the indifference ellipses of $U_i(x)$ will "contract" along the major axis in the untransformed space, becoming nearly circular. When the two dimensions are equally weighted, $U_i(x)$ will have circular indifference contours. Before reaching the point of equal saliency, however, note that, even as the relative weighting of X_2 begins to approach that of X_1, Sundquist's point 7 seems appropriate. The two parties will be adopting the same position on X_2, diverging along X_1, the old line of cleavage. It seems likely, therefore, that "the normal response of both parties at the outset" will be "to straddle the new issue." That is, the center of both parties is moderate along X_2, and there are as many activists in each party who are liberal on X_2 as conservative on X_2. All activists in a party, however, tend to agree with their party's divergence on X_1, i.e., the line of cleavage between the parties cuts that dimension, not X_2.

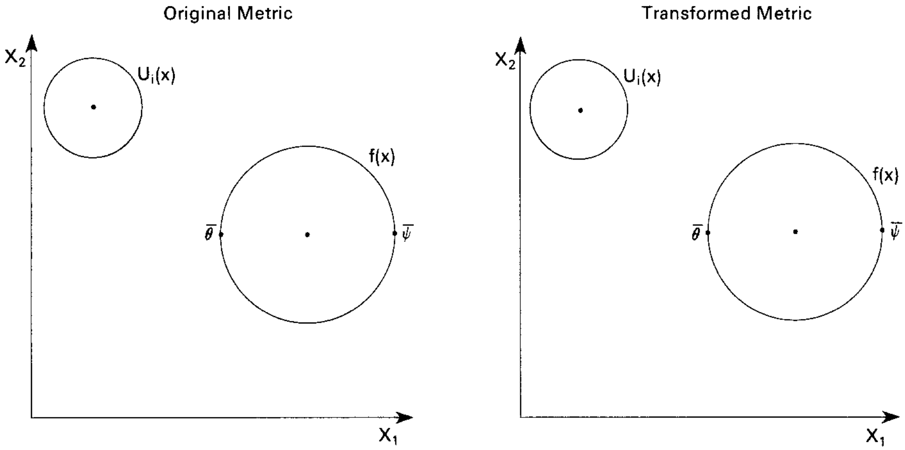
When the salience of X_1 equals that of X_2, indifference contours of $U_i(x)$ will be circular; thus the transformation will be fixed. At this point, equidensity contours of $f(x)$ will remain circular. By Proposition 5, the old equilibrium is unstable, and, indeed, all equilibria are.

As soon as X_2 becomes more salient than X_1, the transformation of $f(x)$ will make its equidensity contours elliptical again, but with the major axis parallel to X_2. Thus, the old stable equilibrium will be unstable, while the equilibrium positions will be stable along X_2 (by Proposition 5). With any perturbation, then, the parties will diverge from the unstable equilibrium and realign along the now-stable equilibrium along X_2. There will be, in other words, divisions (or a line of cleavage) between the two parties on X_2; there will no longer be any cleavage along dimension X_1.

This process is exemplified in Figures 6 and 7 for three key steps. In Figure 6a, I illustrate an early point, at which X_1 is more salient than X_2, and the parties are in stable equilibrium along X_1. In Figure 7a, the smooth transformation of relative weighting of the two dimensions has reached the point of equal salience. While the parties remain at their original equilibrium positions, that position is no longer stable. In 7b, X_2 is more salient than X_1. I have assumed that a perturbation similar to Sundquist's point 11 has occurred,

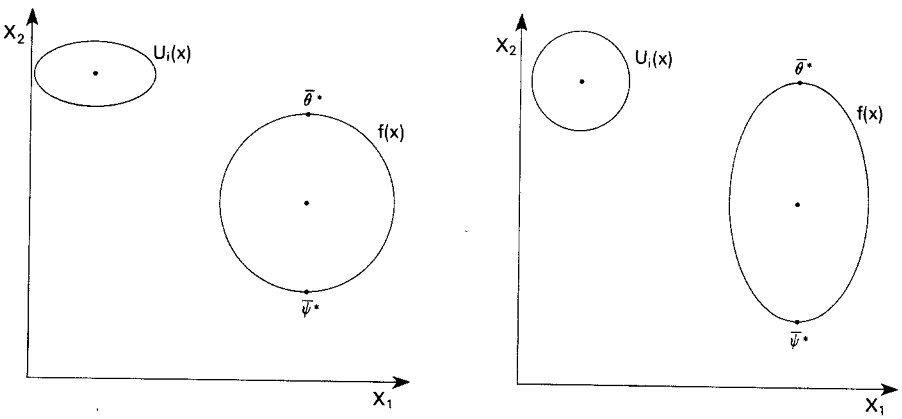
Figure 7

7a



(Parties at old, but now unstable, equilibrium)

7b



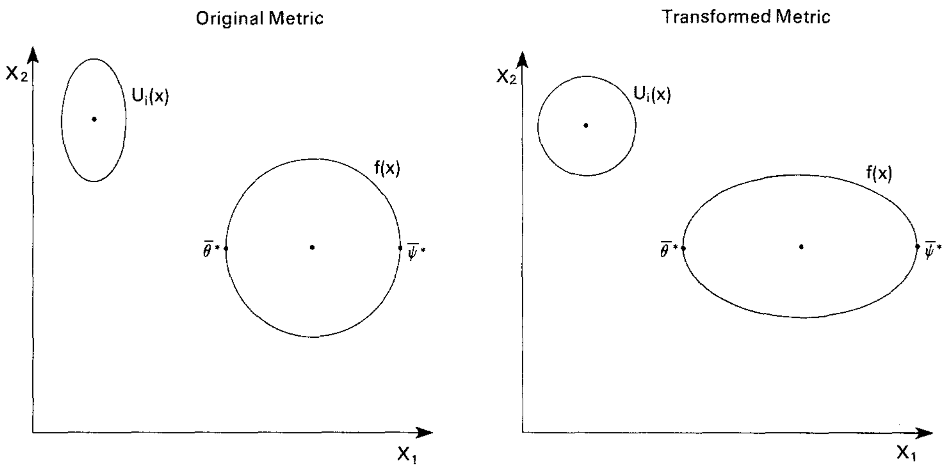
so that the parties have realigned along the newly stable equilibrium along X_2. The key point, of course, is that the realignment is a "natural" consequence of a continuous application of Proposition 5. Basically, the realignment occurs because of the disappearance of the major axis of $f(x)$ along X_1 and its appearance along X_2.

Certain features of the above example are coincidental. For example, even if $f(x)$ has ellipitical equidensity contours, there is no particular reason to assume they are circular in the untransformed space. But, that would mean only that the point of "realignment crisis" (which I take to mean at a point about like that of Figure 7a, i.e., where a heretofore stable equilibrium becomes unstable) would occur at a different balance of relative salience of the two issues. It is common to argue, for example, that the line of party cleavages is "reinforcing," that citizens tend to adopt preferences consistent with the line of partisan cleavages. If so, $f(x)$ is likely to have a major axis parallel to X_1. The consequences would be that X_2 would be more salient than X_1 before the process reaches a point like that of Figure 7a. Thus, there might be greater "agitation" about X_2 (and perhaps the refusal of parties to "take a stand" on X_2) before the realignment would occur. While this analysis has been, at best, semiformal, it could have been presented in a more formal fashion. My desire, however, was to show the close connection between the implications of this model and a consequential and substantial body of literature in the parties and election area.

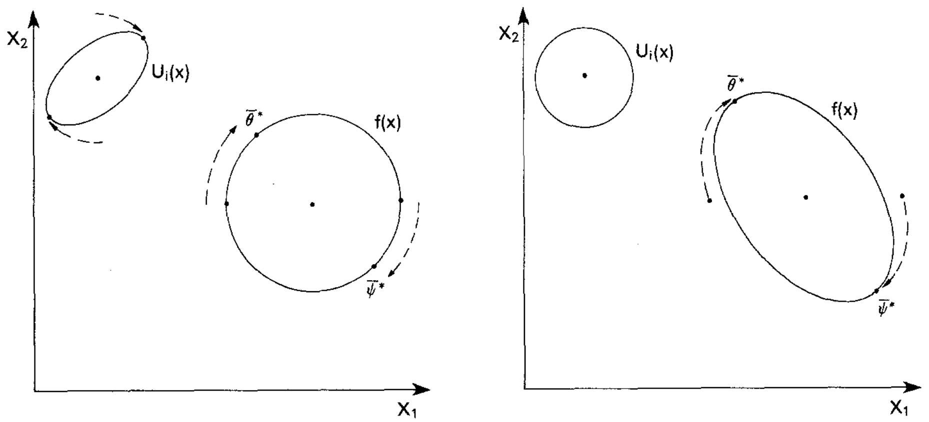
It would be unusual, of course, if an empirical A matrix were exactly diagonal, i.e., that the new dimension were orthogonal to the old. Suppose, instead, that a new dimension arises that is not orthogonal to the old in i's utility function. Assume that $f(x)$ can be described as having a major and a minor axis (in the transformed space). As we vary the magnitude of the off-diagonal elements in A relative to that of the on-diagonal elements, the effect would be to rotate the orientation of the ellipse of equidensity contours of $f(x)$. This is illustrated below in Figure 8 for the case where the original major axis of $f(x)$ is parallel to X_1 (see Figure 8a). As the "correlation" between the two dimensions in the A matrix increases (from zero), the major axis of $f(x)$ begins to rotate. In Figure 8b, there is a fairly high "correlation" between the two dimensions, so that the major axis of $f(x)$ is rotated to be oriented from upper left to lower right in the figure. By repeated application of Proposition 5, the stable equilibrium remains on the major axis of $f(x)$. The derivation

Figure 8

8a



8b



of Proposition 5 can be used to show that the party equilibrium will adjust "automatically" (i.e., without perturbation) as the orientation of the major axis rotates. Thus, here, in contrast to the previous case, the location of the stable equilibrium rotates as smoothly as does the increase in "correlation" between X_1 and X_2.

If there are high "correlations" between pairs of dimension so that the minor axis is quite minor, one interpretation is that it is what Sundquist means by a "cluster of related issues." In effect, there is an approximation of a single "liberal-conservative" dimension that has two particular policy manifestations. There are, however, various other interpretations or modifications that have substantive import.

A second interpretation that can lead to the same result is quite the opposite of the above case. In that case, $f(x)$ remained fixed, but $U_i(x)$ (and, in particular, the A matrix) varied. Indeed, $f(x)$ was "uncorrelated" across the two dimensions. Assume the opposite. That is, assume the A matrix is diagonal, but that, as the new issue "arises," it sorts those whose ideal preference on X_1 was liberal to have a greater probability of having a liberal ideal preference on X_2. The effect of this, clearly, is to create a major axis of $f(x)$ that runs from "upper left" to "lower right," just as before. If there is a dynamic, it involves that sort of shift of ideal preferences. In this case, too, then, the effect is to shift the location of the stable equilibrium locations smoothly (if the shift in $f(x)$ is smooth itself), keeping the line of cleavage along the major axis of $f(x)$.

One might argue that the civil-rights dimension over the 1950s and 1960s followed one or both of the above processes. While there was a degree of "alignment" between the New Deal economic liberalism and civil-rights liberalism in the 1950s, it was by no means obvious that the two would become as closely tied as they were by, say, the late 1960s. There were, for example, numerous prominent Republicans who were fairly liberal on civil rights in this period. The Little Rock confrontation found a Supreme Court, whose Chief Justice was a former conservative Republican governor, issue a decision enforced by a Republican president and opposed by a Democratic governor. Indeed, the main difference between Kennedy and Nixon on civil rights in 1960 was the symbolic gesture of support Kennedy gave to the Reverend King. Only later would the two dimensions become closely related.

As another example, consider what happens if the major axis of $f(x)$ rotates, and thus the center of the party shifts accordingly, as in Figure 8. Those who initially were at the center of the party (e.g., their $X_i = \bar{\theta}$) would find their ideal preference, if it remained constant, at the outer edge of the typical partisan activist preferences in the party after rotation. A possible illustration is Henry Jackson from 1960 to 1976. In 1960, it appeared that many Democrats were liberal in the New Deal-domestic policy sense and in favor of an activist foreign policy with strong national defense. Jackson, like Kennedy, typified that position, and it seemed to be at the "heart" of the party. It is reasonable to argue that Jackson's preferences did not change substantially in those terms from 1960 to his presidential campaigns of 1972 and 1976. Yet it appeared that the "heart" of the party shifted such that there was a larger proportion of those in favor of more isolationist-oriented foreign policies and in favor of less spending on defense. Thus, Jackson, perhaps without changing himself, found himself farther removed from the "heart" of the party by the time he ran for president, at least in comparison to the early 1960s when he was chairman of the Democratic National Committee. (Perhaps a movement from 8b to 8a is descriptive here.)

What I have outlined are two quite different scenarios about the shift in the location of stable distributions of activists in the two parties. The first case, flowing from the critical election/party realignment literature, is of a sharp, rapid, and substantial shift in the location of stable equilibria. It is, in the technical as well as common-usage sense, a catastrophe (formally, the disappearance of a stable point and appearance of a new stable point in a quite different – in this case orthogonal – location). The conditions for it may be considered rare (obviously, the orthogonality assumption plays a crucial role), but that is, in a substantive sense, quite good, for the phenomena described are argued to occur rarely. One caveat, of course, is that the conditions I have outlined are sufficient for realignment. They are not necessary. I think most political scientists have argued that preferences (presumably ideal preferences) are more likely to be distributed as in Figure 5. That is, while X_1 and X_2 are (nearly) orthogonal, as X_2 becomes a major dimension of concern, we find large concentrations of people divided along the old dimensions but also large concentrations divided along the new. In this case (or in the even more extreme version of four modes, one at each of the four noncentral concentrations of ideal points), it is not difficult to show that, when X_1 and X_2 are both relatively "important," there are stable locations for the two parties, one pair "cleaving" along X_1, another pair along X_2. Again, the variable saliency of X_1 and X_2 could lead to realignment from X_1 to X_2 (when X_1 becomes sufficiently less salient than X_2; realignment is certain as the salience of X_1 approaches zero).

The second scenario is one of a "smaller," noncatastrophic and more continuous alteration of the location of stable equilibria position. As the two examples tried to demonstrate, the generation of such shifts could come from changes in $f(x)$, or in the A matrix, or both. Again, these are but sufficient conditions. While I argued that there might be real-world referents in such changes, the main referent I have in mind is in the same sort of realignment literature. V.O. Key, Jr. (1959) argued that, in addition to his original statement of critical elections (aka realignment, see Key, 1955), there are "secular realignments." He claimed that these are slower, longer-term and less sharp changes that are the result of forces that "operate inexorably, and almost imperceptibly, election after election, to form new party alignments and to build new party groupings," (1959, pp. 198-199). While there is no step-by-step development of the notion of gradual realignments akin to that of Sundquist's explanation of critical realignments, the general outline in the substantive literature is at least consistent with the sort of situation offered here.

In one sense, the analysis is complete. These two types of dynamics, the rapid and the more gradual, while not necessarily exhaustive (or, if exhaustive, perhaps too general), are comprehensive of the dynamics of party orientation on policy and of the parties-and-elections substantive literature. In other senses, however, the analysis is far from complete.

From a formal standpoint, some of the assumptions beg for generalization (e.g., party activity as a more general phenomenon rather than a simple active/inactive dichotomy). Also, the section on the effects of indifference relies on a specific set of assumptions about $f(x)$, and, thus, the results refer to only a rather special case. Several assumptions in the model appear to be less important (e.g., commonly defined costs for indifference and for alienation, for party θ and for party ψ, or symmetry of $f(x)$ where that is used). Other assumptions were not used at all (e.g., $U_i(x)$ could be assumed to be "quadratic-based," as long as all i share the same base, without affecting any of the propositions). Perhaps more importantly, key questions remain unanalyzed. Notably, I have not yet addressed the relationship between a set of party activists, even if located stably in the policy space, and the candidates of their party. Does a stable set of party activists induce stability in the commonly unstable two-candidate election (if, indeed, parties are stable in asymmetric distribution of ideal points)? A second class of important, but unaddressed, questions concerns the motivation of activists. We know there are activists, but why do they contribute their time, effort, or money, and not others? Why do they not free-ride? While one can generate a list of particularized benefits, they are potential reasons, not explanations. Moreover, is it reasonable to assume that they are distributed in such a fashion to generate activism probabilities consistent with the distributions assumed? A third set of questions concerns the activity of party leaders, professionals, and political entrepreneurs and how they affect activists in the sense modeled here. In addition, are partisans motivated to take action by particularly attractive candidates, and, if so, how does that motivation affect the analysis here? Finally, this model can be taken only as a pale imitation of the institution of political parties. As institutions, they are encrusted with the usual web of rules, laws, and norms that, using Riker's apt phrase (1980), may be "congealed preferences." But, do the congealed preferences induce only inertia in what has been modeled here, or are their effects significantly different?

While this analysis can be taken simply as preliminary, I hope that the general outlines are sufficiently attractive to provide hope for a positive theory of parties and elections, to balance somewhat the rather negative findings ordinarily encountered. Even a simple model of political parties such as that posed here seems to be characterized more by equilibria or general stability than with explosive instability. The apparent importance of higher concentrations of policy preferences in the electorate provides hope for parties in a representative democracy. The kinds of results such as those in Proposition 5, if found in a reasonably large and interesting set of cases, suggest that parties might be characterized as "semi-responsible" in the sense of providing both stable and opposing viewpoints on at least the central dimension of the day. And, if this sort of model can rationalize and expand our understanding of electoral dynamics in the United States, then the thrust of this substantive literature and the importance of public opinion in democracy can be incorporated into the spatial model of parties and elections.

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