THE COMPUTATION OF THE FRICTION VELOCITY u_* AND THE TEMPERATURE SCALE T_* FROM TEMPERATURE AND WIND VELOCITY PROFILES BY LEAST-SQUARE METHODS

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Abstract. A method is given to calculate the surface layer parameters: u_* (friction velocity) and T_* (temperature scale) from wind speed and temperature profiles.

The problem is formulated as a minimization of a least-square function, which is constructed from the difference between the measured profiles and the well-known Kansas profile relations.

The wind speed and temperature profiles are treated simultaneously in this procedure. All the available wind speed and temperature measurements are used in order to reduce the effect of measurement errors.

Estimates of the goodness of fit and confidence limits on the estimated parameters are discussed.

The method has been applied to data obtained during experiments in a wide variety of conditions: Project Prairie Grass, experiments over Lake Flevo and experiments at the meteorological tower at Cabauw, the last two in the Netherlands.

1. Introduction

In the atmospheric surface layer, the friction velocity u_* and the heat flux $\overline{wT}$ are important parameters, e.g., Busch (1973). From these parameters the so-called temperature scale T_* and the Obukhov-length L can be defined as

$$T_* = -\overline{wT}/u_*$$
$$L = -u_*^3\bar{T}/kg\overline{wT}, \tag{1}$$

where $\bar{T}$ is the average surface layer temperature, g the constant of gravity and k the 'von Kármán' constant. These parameters can only be obtained directly from extensive and difficult turbulence measurements. When these measurements are not available, other methods must be found to estimate them.

Frequently wind and temperature profiles are available. From simultaneous measurements of turbulence and wind speed and temperature profiles, empirical relationships have been established between the mean profiles and the surface layer parameters under stationary and homogeneous conditions. By fitting the measured profiles to an empirical profile relationship, an estimate can be obtained for the surface layer parameters. Several methods have been published.

Krügermeyer (1975) fits the measured profiles to a specified function of height. The surface-layer parameters are then found from a comparison of the differential of this function with the empirical relation for the gradients. The disadvantage is that the approximation of the profile gradients is relatively inaccurate.

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Klug (1967) uses a method by which the surface-layer parameters are found from a least-square fit of only the wind profile to a wind profile relation.

Paulson (1970) uses both the wind and the temperature profiles to estimate the surface-layer parameters. In this method, however, the estimate of u_* is primarily derived from the wind profile and the estimate of T_* from the temperature profile.

In the method introduced here, both profiles at all measuring heights will be used simultaneously to estimate u_* and T_*, so that all the information contained in both profiles is used. Also an indication of the accuracy of the estimates can be obtained.

2. Calculation of the Friction Velocity (u_*) and the Temperature Scale (T_*) from Profile Relations

A. FORMULATION OF THE EQUATIONS

The profile relations which result from the well-known Kansas experiments (Businger *et al.*, 1971) will be used here.

$$u(z, u_*, T_*) = \frac{u_*}{k}\{\ln(z/z_0) - \Psi_1(z/L)\}$$

$$\theta(z, u_*, T_*) - \theta_0 = \frac{0.74T_*}{k}\{\ln(z/z_0) - \Psi_2(z/L)\} \tag{2}$$

$$L<0:\ \Psi_1 = 2\ln\left(\frac{1+x}{2}\right) + \ln\left(\frac{1+x^2}{2}\right) - \operatorname{arctg}(x) + \frac{\pi}{2}. \qquad x = \left(1 - 15\frac{z}{L}\right)^{1/4}$$

$$\Psi_2 = 2\ln\left(\frac{1+y}{2}\right); \qquad y = \left(1 - 9\frac{z}{L}\right)^{1/2}$$

$$L>0:\ \Psi_1 = -4.7z/L$$

$$\Psi_2 = -6.4z/L\,.$$

The temperature scale T_* and the Obukhov-length L are defined in (1). In accordance with the results of Businger *et al.* (1971), the k is set equal to 0.35. The z_0 is defined as the roughness length. The potential temperature at this height is θ_0. A zero-plane displacement has been neglected, because the profile relations will be applied at heights much larger than this displacement height.

Suppose the measured wind velocity u_m is given at N_u heights and the measured temperature θ_m at N_T heights. The functions

$$\Phi_u = \sum_{i=2}^{N_u} [(u_m(z_i) - u_m(z_1) - u(z_i, u_*, T_*) + u(z_1, u_*, T_*)]^2$$

$$\Phi_T = \sum_{j=2}^{N_T} [(\theta_m(z_j) - \theta_m(z_1) - \theta(z_j, u_*, T_*) + \theta(z_1, u_*, T_*)]^2 \tag{3}$$

express the difference between the measurements and the profile relations (2). This form has been chosen to eliminate the dependence on z_0 and θ_0 from (3).

The parameters u_* and T_* in (3) then are found from the condition that the function Φ reaches a minimum.

$$\Phi(u_*, T_*) = g_u \Phi_u + g_T \Phi_T. \tag{4}$$

The weight factors g_u and g_T are defined as

$$g_u = 1/\Delta u^2, \qquad g_T = 1/\Delta T^2, \tag{5}$$

where Δu and ΔT are the measuring errors respectively in u_m and θ_m.

The assumptions on which (4) is based are: (Bard, 1974) (a) The difference between the measurements and the profile relations (2) are caused only by random measurement errors; (b) these errors are not dependent on the measurement height; (c) the measurement errors in the temperature and wind profiles are not correlated.

The solution for u_* and T_* is then found from the nonlinear equations

$$\frac{\partial \Phi}{\partial u_*} = 0, \qquad \frac{\partial \Phi}{\partial T_*} = 0. \tag{6}$$

With the solutions obtained for u_* and T_*, estimates for z_0 and θ_0 can be found by fitting the wind and temperature profiles (2) separately to the measured data. Because of the logarithmic dependence of the wind profile on z_0, the estimate of this parameter is not very accurate.

When from other sources the value of z_0 is known, it can be incorporated in the approximation procedure by substituting into $\Phi_u(3)$: $u_m = 0$ for $z_1 = z_0$. Whenever possible the latter procedure should be used, because the estimates for u_* and T_* will then improve. This is confirmed by the results of Ling (1976).

B. NUMERICAL SOLUTION

The equations (6) can be solved only by iterative methods. For a summary of the several techniques which are available, see Bard (1974).

The well-known Gauss-Newton method has failed here, because a convergent iteration sequence could be obtained only when the starting values were chosen very close to the actual solution. Of the alternative methods that were investigated, the best results were found with the method of Marquardt (1963), which is a mixture of the steepest descent and the Gauss-Newton technique.

In our calculations, solutions with the method of Marquardt generally took only about five iteration steps.

C. INTERPRETATION OF THE ESTIMATES

The accuracy of the estimates for u_* and T_* can be related to the magnitude of the residuals, which are defined as the difference between the measured data and the profile relations (2). The normalized sum of the residuals (4) indicate how well the measured profiles are approximated by the relations (2).

For conditions in which these profile relations apply, the residuals should be of the order of the measuring errors. If the assumptions concerning the error distribution given in Section 2a can be made, one can derive (Bard, 1974) that Φ (4) is distributed as a χ^2-distribution with $N_T + N_u - 4$ degrees of freedom. From this known distribution, a criterion for Φ can be obtained.

A confidence region for the estimates u_* and T_* can be obtained by linearizing (4). The confidence intervals expressed as δu_* and δT_* can be found from the inequality (Bard, 1974):

$$\frac{\partial^2 \Phi}{\partial u_*^2} \delta u_*^2 + 2 \frac{\partial^2 \Phi}{\partial u_* \, \partial T_*} \delta u_* \, \delta T_* + \frac{\partial^2 \Phi}{\partial T_*^2} \delta T_*^2 \leq \frac{2}{N_u + N_T - 4} \Phi(u_*, T_*) F_\alpha(2, N_u + N_T - 4) , \tag{7}$$

where F_α is the α point probability of the F-distribution with 2 and $N_u + N_T - 4$ degrees of freedom.

Deviations between the measured profiles and the relations (2) may also result from the fact that due to physical reasons the relations (2) are not a suitable model to use in the approximation procedure. In this case, the residuals can no longer be considered as random variables, so that the discussion given above concerning the error limits is no longer valid. In that case an objective interpretation of the estimates for u_* and T_* can no longer be derived.

3. Comparison with Experimental Results

A. PRAIRIE GRASS EXPERIMENTS

During the very extensive diffusion experiment 'Prairie Grass' (Barad, 1958), the wind velocity and the temperature profiles averaged over 20 min were measured along masts between 0.25 and 16 m. The wind velocity was available at 5–6 heights and the temperature at 6–7 heights. For a detailed description of these measurements, see Barad (1958).

The roughness of the experimental area was estimated to be $z_0 = 0.8$ cm (Pasquill, 1974). This value was used as the zero wind speed level as described in Section 2a.

The measured wind speed and temperature data could be very well approximated by the profile relations (2). The residuals were of the order of the measuring errors. The 90% confidence limits for u_* and T_* were found to be approximately 5×10^{-3} m s^{-1} and 1×10^{-2} K.

During the experiments, turbulent fluxes were measured with five bivanes (Barad, 1958). The u_* was calculated as an average of the results from at least two instruments. Negative stress values were excluded. A comparison with the values of u_* obtained from the profile approximation is given in Figure 1. The agreement is good.

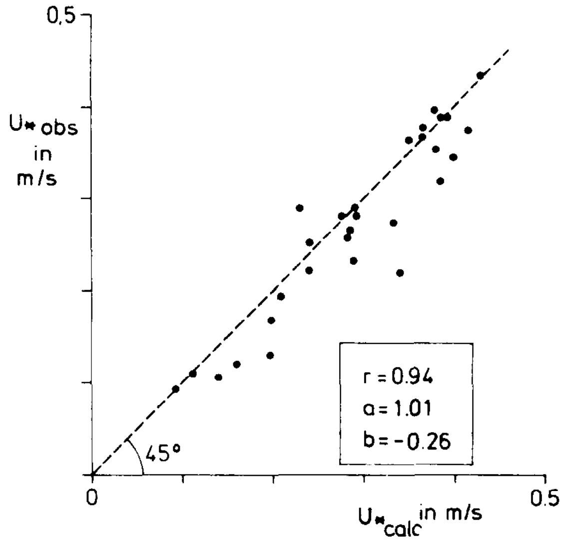

Fig. 1. Scatter diagram of the observed friction velocity u_{*obs} and the calculated value u_{*calc} for the Prairie Grass experiments. The r denotes the correlation coefficient, the a and b denote the slope and intercept, respectively, of the linear regression line $u_{*obs} = au_{*calc} + b$.

No direct measurements of the heat flux $\overline{wT}$ were made. However, it was determined from the temperature and wind profiles in order to calculate the surface energy balance (Barad, 1958).

These results are compared with the estimates of the heat flux by the profile approximation in Figure 2. The very good agreement especially with the results of

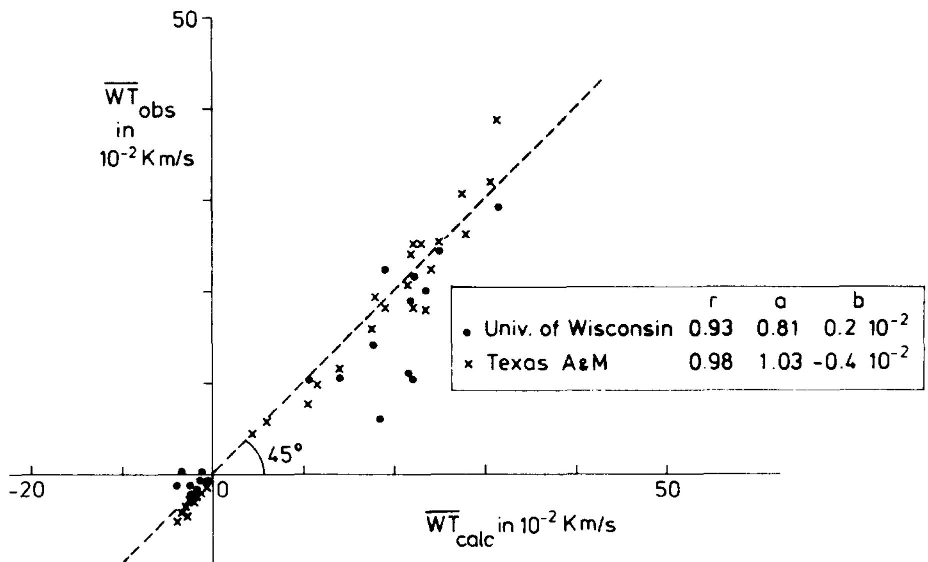

Fig. 2. Scatter diagram of the observed heat flux $\overline{wT}_{obs}$ and the calculated value $\overline{wT}_{calc}$ for the Prairie Grass experiments. The r, a and b are defined as in Figure 1.

the Texas A & M group, is partly based on the fact that the same profile data were used.

B. FLEVO EXPERIMENTS

In 1967 extensive meteorological experiments were performed over Lake Flevo in the Netherlands. Apart from turbulence measurements resulting in value for u_* and $\overline{wT}$, the wind velocity u at three points, the temperature θ and the specific humidity q at two points were measured between 2 and 8 m. These measurements are 20-min averages. For a full description, see Wieringa (1972, 1973).

Over a water surface, the stability is influenced by the humidity (Busch, 1973). This effect can be described by using the virtual temperature θ_v in the temperature profile (2).

$$\theta_v = \theta(1+0.61q)\,. \tag{8}$$

The calculated values of u_* are compared with the measured values in Figure 3. The agreement is good.

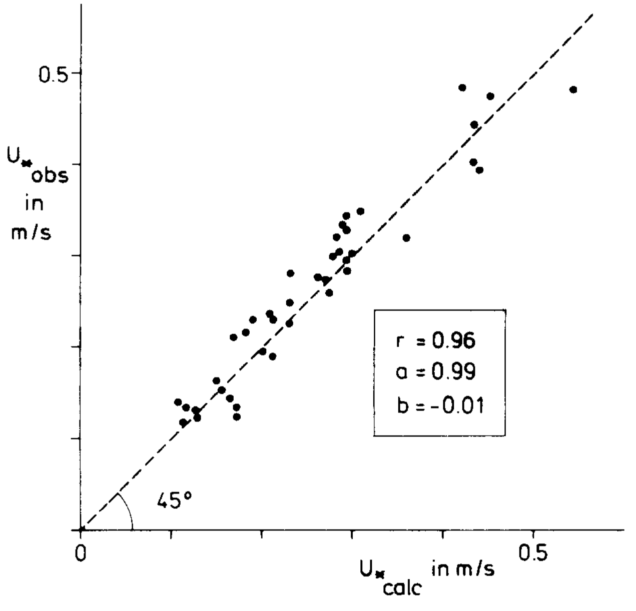

Fig. 3. Scatter diagram of $u_{*\mathrm{obs}}$ and $u_{*\mathrm{calc}}$ for the Lake Flevo experiments. The r, a and b are defined as in Figure 1.

Because the calculations are performed with the virtual temperature, the result of the approximation procedure is the virtual heat flux $\overline{wT_v}$:

$$\overline{wT_v} = \overline{wT} + 0.61\bar{T}\overline{wq}\,, \tag{9}$$

where $\overline{wq}$ is the humidity flux. The Obukhov-length corrected for humidity L_v is then found by substituting $\overline{wT_v}$ into (1) instead of $\overline{wT}$.

If one assumes that the transfer coefficients for heat K_H and for humidity K_q are equal, the humidity profile will follow the same relation as the temperature profile (2). A value for the heat flux can then be found by substituting into the temperature profile (2) the value for L_v and by fitting this profile to measured temperature data with T_* as a free parameter. The heat flux then follows from (1). The results for $\overline{wT}$ are compared with the measurements in Figure 4. The calculated heat flux is systematically lower than the observed heat flux. This may be caused by the fact that the assumption $K_H = K_q$ is not correct, which is still a matter of controversy.

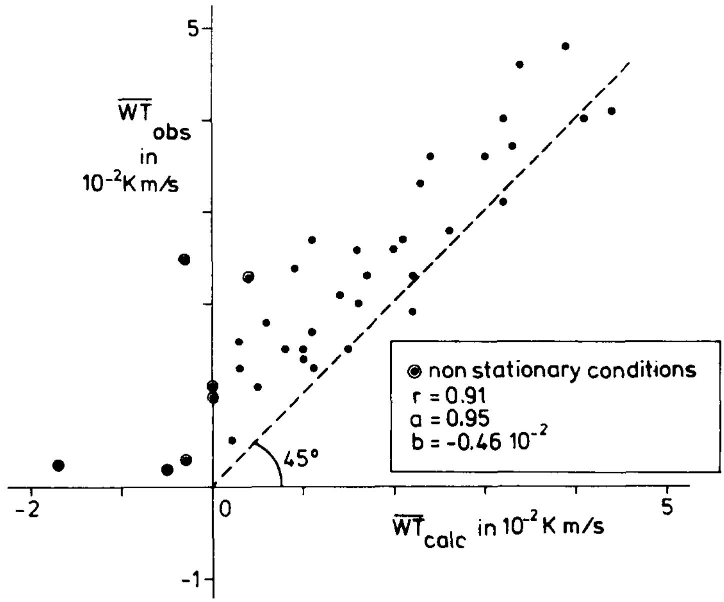

Fig. 4. Scatter diagram of $\overline{wT}_{\text{obs}}$ and $\overline{wT}_{\text{calc}}$ for the Lake Flevo experiments. The r, a and b are defined as in Figure 1.

The circles in Figure 4 indicate that during the measuring period, conditions were non-stationary. Under these circumstances, large deviations between the observed and calculated values are possible, because the profile relations (2) are valid only for stationary conditions.

The accuracy of the estimated values for u_* and $\overline{wT}$ could not be considered because of the small number of measuring heights that were available.

C. CABAUW EXPERIMENTS

In the centre of the Netherlands, a 200-m meteorological tower is situated, on which extensive wind and temperature profiles have been measured. For a description of the tower, its instrumentation and its surroundings, see Van Ulden *et al.* (1976).

The profile method was applied for those periods when direct measurements of the u_* and $\overline{wT}$ were available. These measurements are described in more detail by

Driedonks (1977). All the measurements were obtained as 30-min averages at a height of 20 m.

The roughness length z_0 as given by Van Ulden *et al.* (1976) was applied as the zero wind speed level as described in Section 2a.

According to the wind direction, the calculation has been subdivided into two cases:

(1) *Wind Direction from South to West*

The terrain in this direction is relatively flat ($z_0 \sim 7$ cm) and homogeneous.

The wind profile was available at five heights and the temperature profile at eight heights between 2 and 200 m. Most of the experiments were done under near-neutral conditions and with a mixing height of at least 1000 m. In these conditions

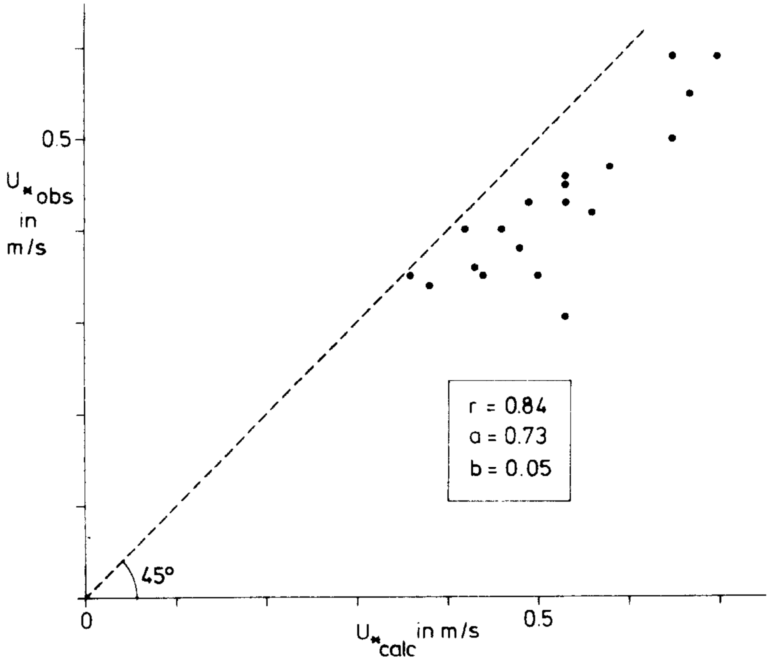

Fig. 5. Scatter diagram of u_{*obs} and u_{*calc} for the Cabauw experiments (wind direction south-west). The r, a and b are defined as in Figure 1.

the validity of the profile relations might extend over a larger part of the atmospheric boundary layer than the surface layer (Tennekes, 1973). Therefore it was assumed that for this case the profile relations (2) could be applied to 200 m, in order to take into account as many measuring heights as possible.

A comparison with the results of turbulence measurements at 20 m is given in Figures 5 and 6. The calculated value of u_* and $\overline{wT}$ are slightly higher than the observed values. This might be attributed to the value of z_0, which is believed to be

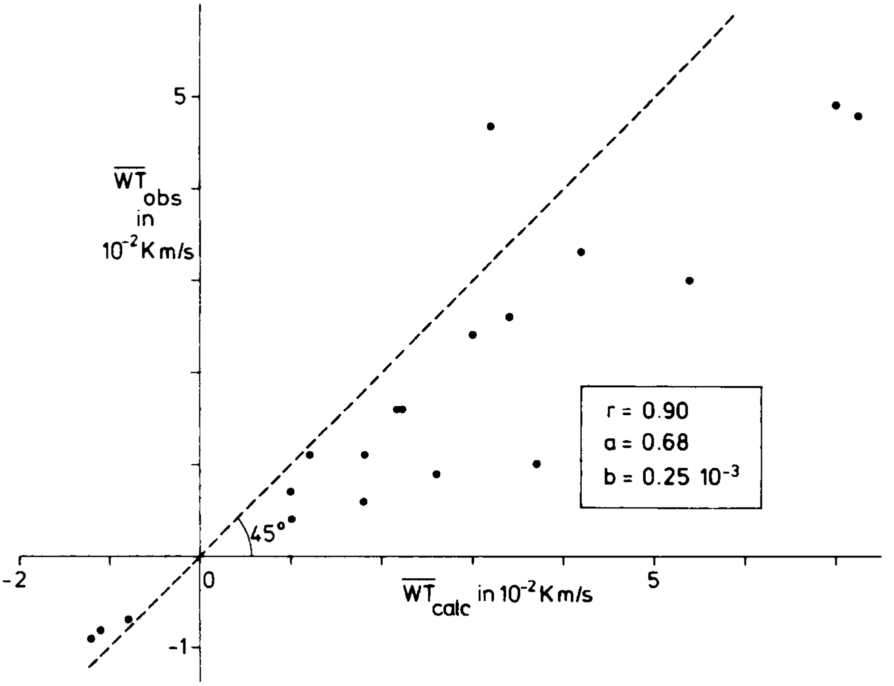

Fig. 6. Scatter diagram of $\overline{wT}_{obs}$ and $\overline{wT}_{calc}$ for the Cabauw experiments (wind direction south-west). The r, a and b are defined as in Figure 1.

somewhat too high. A calculation in which only the measurements up to a height of 80 m were used did not give better agreement.

(2) *Wind Direction from North to South-east*

In these wind directions the terrain in the neighbourhood of the tower is rather inhomogeneous and rough ($z_0 \sim 30$ cm).

The conditions ranged from very stable to highly unstable. The measured wind and temperature profiles were used up to a height of 40 m at, respectively, three and four measuring heights.

The calculated and observed values of u_* and $\overline{wT}$ are compared in Figures 7 and 8. The agreement is good.

In both cases it can be questioned whether the profile relations could be applied. It is believed that the reasonable agreement with the observed values can be attributed mainly to the incorporation of z_0 as the lowest wind-speed level. Omission of z_0 from the approximation procedure resulted in poor estimates for u_* and $\overline{wT}$ (in general too low compared with the observed values).

The confidence limits on u_* and T_* did not relate to the quality of the estimates. This is caused by the fact that the assumptions concerning the residuals as described in Section 2c are not met here.

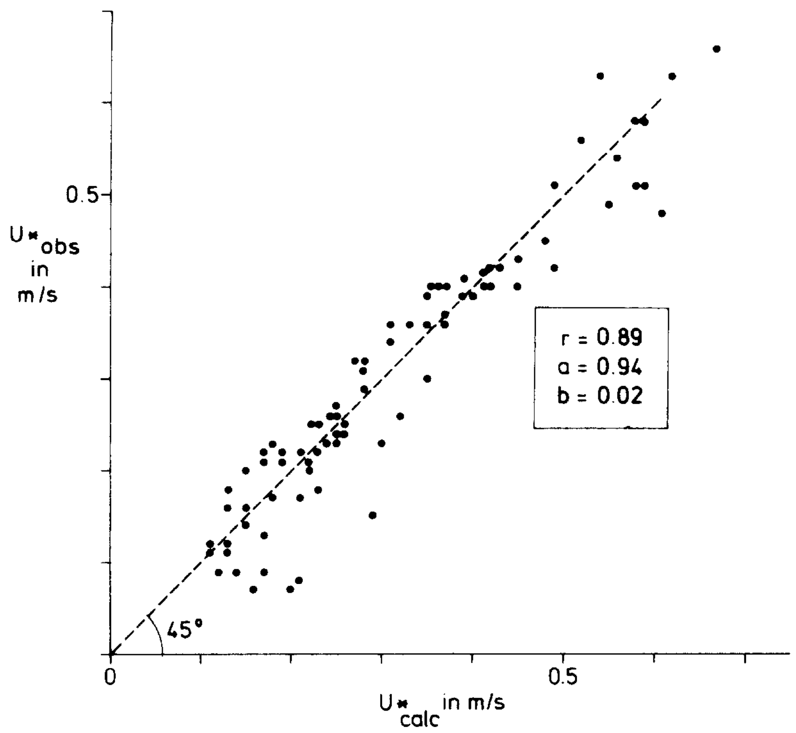

Fig. 7. Scatter diagram of u_{*obs} and u_{*calc} for the Cabauw experiments (wind direction north-south/east). The r, a and b are defined as in Figure 1.

4. Conclusion

The values of u_* and T_* were estimated from measured wind and temperature profiles by fitting these profiles simultaneously to the well-known Kansas profile relations.

The method has been extensively tested against experimental results. In all cases a reasonable agreement between the calculated and measured parameters was obtained.

It has been found that incorporation of the roughness parameter into the approximation procedure improves the quality of the estimates.

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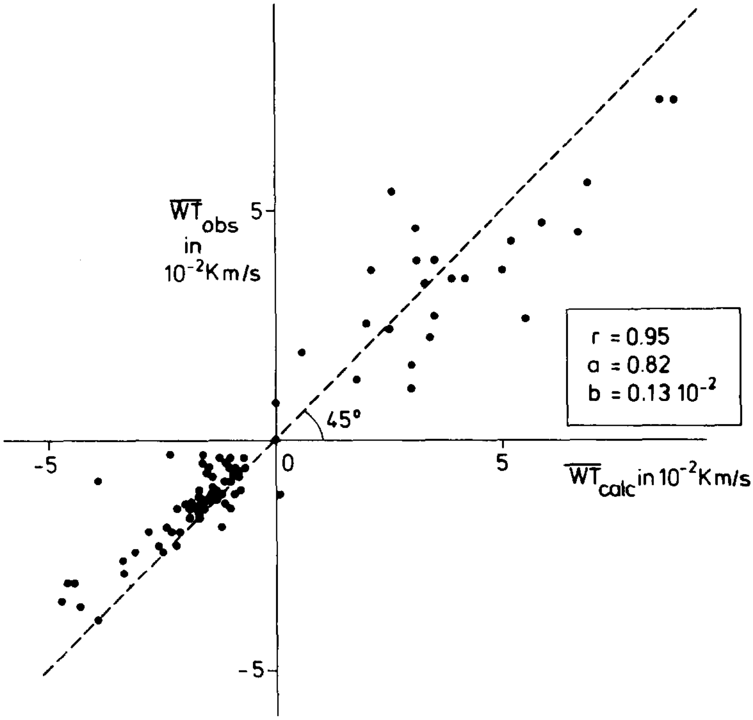

Fig. 8. Scatter diagram of $\overline{wT}_{obs}$ and $\overline{wT}_{calc}$ for the Cabauw experiments (wind direction north-south/east). The *r*, *a* and *b* are defined as in Figure 1.

References

Barad, M. L.: 1958, 'Project Prairie Grass, a Field Program in Diffusion', (three Volumes), Geophysical Research Papers 73, AFCRC-TR-58-235.

Bard, Y.: 1974, *Nonlinear Parameter Estimation*, Academic Press.

Busch, N. E.: 1973, 'On the Mechanics of Atmospheric Turbulence', *Workshop on Micrometeorology*, Am. Meteorol. Soc., pp. 1–65.

Businger, J. A., Wyngaard, J. C., Izumi, Y., and Bradley, E. F.: 1971, 'Flux Profile Relationships in the Atmospheric Surface Layer', *J. Atmospheric Sci.* **28**, 181–189.

Driedonks, A.: 1977, 'Case Studies of Atmospheric Turbulence during Early Morning Inversion Rise', (to be published).

Klug, W.: 1967, 'Determination of Turbulent Fluxes of Heat and Momentum from the Wind Profile', *Quart. J. Roy. Meteorol. Soc.* **93**, 101–104.

Krügermeyer, L.: 1975, 'Vertikale Transporte von Impuls, sensibler und latenter Wärme aus Profilmessungen über dem tropischen Atlantik während APEX', Berichte des Inst. für Radiometeorologie und maritime Meteorologie, Hamburg.

Ling, C.: 1976, 'On the Calculation of Surface Shear Stress Using the Profile Method', *J. Geophys. Res.* **81**, 2581–2582.

Marquardt, D.: 1963, 'An Algorithm for Least-Squares Estimation of Nonlinear Parameters', *J. Soc. Indust. Appl. Math.* **11**, 431–441.

Paulson, C. A.: 1970, 'The Mathematical Representation of Wind Speed and Temperature Profiles in the Unstable Atmospheric Surface Layer', *J. Appl. Meteorol.* **9**, 857–861.

Tennekes, H.: 1973, 'The Logarithmic Wind Profile', *J. Atmospheric Sci.* **30**, 234–238.

van Ulden, A. P., van der Vliet, J. G., and Wieringa, J.: 1976, 'Temperature and Wind Observations at Heights from 2 to 200 m at Cabauw 1973' Roy. Neth. Met. Inst., Scientific Report 76-7.

Wieringa, J.: 1972, 'Tilt Errors and Precipitation Effects in Trivane Measurements of Turbulent Fluxes over Open Water', *Boundary-Layer Meteorol.* **2**, 406–426.

Wieringa, J.: 1974, 'Comparison of Three Methods for Determining Strong Wind Stress over Lake Flevo', *Boundary-Layer Meteorol.* **7**, 3–19.