# PARAMETRIC RELATIONS FOR THE ATMOSPHERIC BOUNDARY LAYER

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Abstract: Some parametric relations for the atmospheric planetary boundary layer (PBL) are suggested for possible use in the various atmospheric circulation and air quality models, as well as in other applications. These are for parameterizing the mean wind and temperature profiles, the vertical fluxes of momentum, heat and moisture, the variances of velocity fluctuations and length and time scales in the PBL. The parametric relations for the PBL height, the vertical velocity at the top of the PBL and the total energy dissipation in the PBL are also discussed. Experimental and/or theoretical bases for the various parametric relation are given. Some of the suggested parameterizations should be considered as tentative, until they are properly validated.

## 1. INTRODUCTION

In various applications, ranging from the local air quality modeling to the global climate simulations, some aspects or properties of the atmospheric planetary boundary layer (PBL) must be incorporated through simple parametric relations which may have been developed earlier on the basis of observations, theory, or numerical modeling of the PBL. For example, in a sophisticated dispersion or air quality model, one may have to prescribe the vertical distributions of mean wind speed and direction, variances of velocity components, scales of turbulence and the boundary layer depth, as functions of stability, surface roughness, geostrophic winds, etc. In most of the global general circulation models, the surface fluxes of momentum, heat and moisture must be parameterized. Other large scale models of the atmosphere may require the parameterization of the mean vertical motion (Ekman pumping) at the top of the PBL, or the total energy dissipation in the PBL. The mesoscale models of the atmosphere, in general, must accurately parameterize most aspects of the PBL, because mesoscale circulations are intimately related to and usually have their roots in the PBL which may not be adequately resolved in such models.
Such parameterizations are reviewed here, which should be quite relevant to atmospheric modeling over the world's oceans and flatter parts of the continents.

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Parameteric relations for the PBL may also be used for inferring certain turbulent quantities, such as variances and fluxes, from more easily measured mean winds, temperatures, etc. These may be used for extrapolating information at other levels from measurements at one or two levels in the PBL.

In view of the above applications and needs, we have conducted a thorough search of the parametric relations that have been proposed for the atmospheric boundary layer. Based on this, our recommended relations or schemes for parameterizing the various properties of the PBL over a homogeneous and flat terrain are presented in the following sections. These parametric relations should be considered as tentative, and may be replaced by better and more proven relations as they become available.

## 2. MEAN WIND AND TEMPERATURE PROFILES

For this, the PBL of depth h may be divided into a surface layer of depth $h_s = 0.1\ h$ and an outer layer. In unstable and convective conditions, the outer layer may be further sub-divided into a mixed layer, which extends up to the base of capping inversion, and the stable transition layer at the top.

### 2.1 Surface layer (SL)

Mean wind ($\bar{u}$) and temperature ($\bar{\theta}$) profiles in the fully developed region of the surface layer, well above the tops of roughness elements, in which the vertical fluxes remain nearly constant with height, are considered to be well established from micrometeorological observations at homogeneous sites. These are best represented in the framework of the Monin-Obukhov similarity theory as

$$\frac{\bar{u}}{u_*} = \frac{1}{\kappa}\left[\ln\frac{z}{z_o} - \psi_m\left(\frac{z}{L}\right)\right]$$
$$\frac{\bar{\theta}-\bar{\theta}_o}{\theta_*} = \frac{\alpha}{\kappa}\left[\ln\frac{z}{z_o} - \psi_h\left(\frac{z}{L}\right)\right] \quad (1)$$

for $1.5\ h_o < z \leq h_s$. Here, $h_o$ is the representative height of roughness elements, $z_o$ the roughness parameter, $\bar{\theta}_o$ the mean surface temperature, $u_*$ the friction velocity, $\theta_* = -Q_o/u_*$ the temperature scale defined from the kinematic surface heat flux $Q_o$, $L = -u_*^3/\kappa\beta Q_{vo}$ the Obukhov length, $\kappa$ von Karman constant, $\beta$ the buoyancy parameter, $Q_{vo}$ the virtual heat flux, $\alpha$ an empirical constant, and $\psi_m$ (z/L) and $\psi_h$ (z/L) are the stability-dependent similarity functions related to the dimensionless wind shear $(\kappa z/u_*)\partial\bar{u}/\partial z = \phi_m$ (z/L) and the dimensionless temperature gradient $(\kappa z/\theta_*)\partial\bar{\theta}/\partial z = \phi_h(z/L)$.

The Monin-Obukhov (M-O) similarity functions have been empirically determined from micrometeorological flux-profile measurements by many investigators (Brutsaert, 1982). Particularly simple and adequate for many practical applications are the so-called Businger-Dyer formulas (Businger, 1973) with $\alpha=1$

$$\phi_h = \phi_m^2 = (1-15\frac{z}{L})^{-\frac{1}{2}}, \text{ for } \frac{z}{L} < 0$$
$$\phi_h = \phi_m = 1 + 5\frac{z}{L}, \text{ for } \frac{z}{L} \geq 0 \quad (2)$$

which, after integration, yield

$$\psi_m = \ln\left(\frac{1+x^2}{2}\right) + 2\ln\left(\frac{1+x}{2}\right) - 2\tan^{-1}(x) + \frac{\pi}{2}$$
$$\psi_h = 2\ln\left(\frac{1+x^2}{2}\right), \text{ for } \frac{z}{L} < 0 \quad (3)$$
$$\psi_m = \psi_h = -5\frac{z}{L}, \text{ for } \frac{z}{L} \geq 0$$

where $x = (1-15\frac{z}{L})^{\frac{1}{4}}$.

The M-O similarity parameter z/L is simply related to and can be determined from the gradient Richardson number Ri as (Arya, 1982)

$$z/L = Ri, \text{ for } Ri \leq 0$$
$$z/L = Ri/(1-5Ri), \text{ for } Ri > 0 \quad (4)$$

## 2.2 Outer layer

The wind and temperature profiles in the outer layer are highly dependent on stability. Under stable condition, these are characterized by strong gradients with substantial turning of wind with height. Due to essential decoupling of various layers and the sensitivity of profiles to even small surface slope, changes in the net radiation with height and other factors, there is not a strong basis for generalizing the mean profile in the stably stratified nocturnal PBL (Brost and Wyngaard, 1978; Caughey et al., 1979; Nieuwstadt, 1983). There is some observational evidence (Wetzel, 1982; Wesely and Coulter, 1983), however, for considering the mean wind speed ($\bar{V}$) and potential temperature profiles as approximately linear up to the level of the low-level jet, which often coincides with the top of the PBL. Therefore, we can tentatively recommend the use of parametric relations.

$$\bar{V} = \bar{V}_s + \frac{z-h_s}{h-h_s}(\bar{V}_h - \bar{V}_s)$$
$$\bar{\theta} = \bar{\theta}_s + \frac{z-h_s}{h-h_s}(\bar{\theta}_h - \bar{\theta}_s) \quad (5)$$

for $h/L > 0$, where the subscripts s and h refer to the top of the surface layer and the PBL, respectively. $\bar{V}_s$ and $\bar{\theta}_s$ can be obtained from the surface layer profile relations.

Under stable and convective conditions, the mean velocity and temperature distributions can be considered more or less uniform, independent of height, in the mixed layer. Following Garratt et al. (1982), we recommend the relations

$$\frac{\bar{u}_m}{u_*} = \frac{1}{\kappa} [\ln \frac{h}{z_o} - \frac{1}{2} \ln | \frac{h}{L} | - 2.3]$$

$$\frac{\bar{v}_m}{u_*} \simeq 0 \qquad (6)$$

where $\bar{u}_m$ and $\bar{v}_m$ are the layer-averaged velocity components in the mixed layer, with x-axis oriented in the direction of the surface wind or shear. In the free-convective limit ($-h/L>1000$), Wyngaard et al. (1974) and Arya (1978) obtained slightly different relations

$$\frac{\bar{u}_m}{u_*} = \frac{1}{\kappa}[\ln (- \frac{L}{z_o})-a]$$

$$\frac{\bar{v}_m}{u_*} = 0 \qquad (7)$$

in which the constant $a \simeq 0$ (it actually is weakly dependent on $h_s/L$).

The mixed-layer potential temperature $\bar{\theta}_m$ is an evolving function of time and can be predicted from the rate equations for $\bar{\theta}_m$ and h in the context of an appropriate slab model (Driedonks, 1982; Arya and Byun, 1983). The simplest encroachment model gives

$$\bar{\theta}_m (t) = \bar{\theta}_m (t_o) + \gamma_\theta [h(t) - h (t_o)] \qquad (8)$$

where $\gamma_\theta \equiv \partial\bar{\theta}_+/\partial z$ is the potential temperature gradient in the free atmosphere. Eq. (8) should be adequate for most practical applications.

The transition layer above the mixed layer is generally characterized by strong velocity and temperature gradients. The mean profiles in this layer are often assumed to be linear between the mixed layer values at the bottom and the free-atmospheric values (e.g. geostrophic winds) at the top of the layer (Garratt et. al,1982). The thickness of the transition layer $\Delta h$ is assumed to be infinitesimal in certain jump models of the convective PBL, but in reality, $\Delta h$ is in the range 0.1h to 0.3h.

## 3. SURFACE FLUXES AND RESISTANCE LAWS

The surface fluxes of momentum, heat and moisture ($\tau_o$, $H_o$, $E_o$) are usually parameterized in terms of mean wind speed ($\bar{V}_r$) potential temperature ($\bar{\theta}_r$) and specific humidity ($\bar{q}_r$) at some reference level ($z_r$) in the PBL and the 'representative' surface temperature $\bar{\theta}_o$ and specific humidity $\bar{q}_o$, through the following bulk transfer relations:

$$\begin{aligned} \tau_o &= \rho C_D \bar{V}_r^2 \\ H_o &= -\rho c_p C_H \bar{V}_r(\bar{\theta}_r - \bar{\theta}_o) \qquad (9) \\ E_o &= -\rho C_E \bar{V}_r(\bar{q}_r - \bar{q}_o) \end{aligned}$$

Here, $\rho$ is the density, $c_p$ is the specific heat and $C_D$, $C_H$ and $C_E$ are the bulk transfer coefficients which in general depend on the reference height, PBL height, surface roughness, bulk stability and possibly other parameters. In crude parameterizations of the PBL in certain GCM's, $C_D$, $C_H$ and $C_E$ are assumed equal and assigned fixed numerical values. In some cases, different values (typically, $2 \times 10^{-3}$) are assigned for land (typically, ($3 \times 10^{-3}$) and ocean (typically, $1.5 \times 10^{-3}$) surfaces and some allowance made for different stability conditions. In more accurate parameterizations of the surface fluxes, the bulk transfer coefficients should be specified on the basis of our improved understanding of the PBL which has resulted from systematic experiments and sophisticated numerical models.

### 3.1 Parameterizations based on the surface-layer similarity

The best known region of the atmospheric PBL is the surface layer for which the observations represented in the framework of M-O similaarity theory have yielded some universal flux-profile relations. The parameterizations based on these relations will also be expected to be most accurate and reliable. For these to be applicable, the reference level $z_r$, which would normally be the first interior grid level in a model, should fall within the surface layer. Very few models would have such a fine resolution, however. This method of parameterization can still be used in models with coarser resolution, if mean variables at a fixed level in the SL are calculated, for example, from the dynamic forecast equations for this level (Benoit, 1976).

The parametric relations based on the M-O similarity theory can be expressed in the form of Eqs. (9) (Arya, 1977), with the bulk transfer coefficients given as

$$\begin{aligned} C_D &= C_D\left(\frac{z_r}{z_o}, Ri_b\right) \\ C_H &= C_H\left(\frac{z_r}{z_o}, Ri_b\right) \qquad (10) \\ C_E &= C_E\left(\frac{z_r}{z_o}, Ri_b\right) \end{aligned}$$

where

$$Ri_b = \beta z_r(\bar{\theta}_{vr} - \bar{\theta}_{vo})/\bar{V}_r^2 \qquad (11)$$

is a bulk Richardson number. The virtual potential temperatures $\theta_{vr}$ and $\theta_{vo}$ are used to account for the buoyancy effect of water vapor.

The bulk transfer coefficients $C_D$, $C_H$, etc., are unique functions of $z_r/z_o$ and $Ri_b$ whose forms depend on the particular flux-profile relations adopted for the SL. The coefficients based on the flux-profile relations of Businger et al (1971) are given by Arya (1977). These show a monotonic decrease of $C_D$ and $C_H$ with increasing stability, reducing to zero as $Ri_b \to 0.21$. Benoit (1976) has proposed a more elaborate and refined scheme in which extreme conditions of free convection on the one side and very stable stratification approaching the critical Richardson number on the other, are specially taken into account. In the former, the transfer coefficients are held constant at their maximum values, which are presumably reached at the transition between forced and free convection and are given as functions of the ratios $z_r/z_o$ and $z_r/h$. The values of $C_D$, $C_H$ etc., in very stable conditions are also recommended to be held constant for $Ri_b \gtrsim 0.15$.

The above parametric relations can also be used for estimating surface fluxes from observations of mean wind, temperature, etc. at one height in the SL and the appropriate surface properties $z_o$, $\bar{\theta}_o$, etc.

### 3.2 Parameterizations based on similarity matching

In most large scale atmospheric models with coarse vertical resolution, no information is available on flow properties in the SL. The first interior grid level may lie near the top of the PBL or even outside the PBL. The information available is at best representative of the PBL as a whole. In such cases, the following drag and other transfer relations based on the matching of the surface and outer-layer similarity profiles are found to be useful (Deardorff, 1972; Arya, 1977, 1978).

$$\begin{aligned}
\kappa\bar{u}_m/u_* &= -(\ln \hat{z}_o + A_m) \\
\kappa\bar{v}_m/u_* &= -B_m \text{ sign } f \\
\kappa(\bar{\theta}_m-\bar{\theta}_o)/\theta_* &= -(\ln\hat{z}_o + C_m) \\
\kappa(\bar{q}_m-\bar{q}_o)/q_* &= -(\ln\hat{z}_o + D_m)
\end{aligned} \qquad (12)$$

Here, f is the Coriolis parameter, $\hat{z}_o$ is the roughness parameter normalized by the scale height h or $u_*/f$, $A_m$, $B_m$, etc. are some universal functions of stability parameter ($h/L$ or $u_*/fL$) and, possibly, some other (baroclinity, entrainment, etc.) parameters (Arya, 1977, 1978). Detailed comparisons with observations in unstable and convective PBLs (Garratt et al., 1982) have indicated that

$$\begin{aligned}
A_m &\simeq \frac{1}{2}\ \ln(-h/L) + 2.3 \\
B_m &\simeq 0, \text{ for } h/L < -2
\end{aligned} \qquad (13)$$

and that these are little affected by baroclinity, the scale-height ratio, advection and entrainment. One would expect $C_m$ and $D_m$ to be similar to $A_m$ and not be overly sensitive to these other processes.

The dependence of $A_m$, $B_m$, etc., on the stability parameter h/L, is much stronger in stably stratified PBL (Arya, 1977; Yamada, 1976). One can justifiably ignore any weak dependence of these similarity parameters, on baroclinity, advection, etc. Furthermore, numerical models of the stationary, stable PBL predict the forms of the above functions to be linear for h/L>2 (Arya, 1977), i.e.,

$$A_m = a_0 - a_1(h/L)$$
$$B_m = b_0 + b_1(h/L) \qquad (14)$$
$$C_m = c_0 - c_1(h/L)$$

which are also consistent with observations (Yamada, 1976). The values of the various constants $a_0, a_1$, etc., are far from being well established, however (Arya, 1977).

For near-neutral conditions ($-2 \leq h/L \leq 2$), suitable interpolations may be used between the values of $A_m$, $B_m$, etc., at $h/L = \pm 2$.

The implicit relations (12) can be easily inverted and expressed into more convenient explicit forms of bulk transfer relations

$$\tan^{-1}(\bar{v}_m/\bar{u}_m) = \alpha_m(h/z_o, Ri_m)$$
$$\tau_o = \rho C_D(h/z_o, Ri_m)\bar{V}_m^2$$
$$H_o = \rho c_p C_H(h/z_o, Ri_m)\bar{V}_m(\bar{\theta}_o - \bar{\theta}_m) \qquad (15)$$
$$E_o = \rho C_E(h/z_o, Ri_m)\bar{V}_m(\bar{q}_o - \bar{q}_m)$$

where the angle ($\alpha_m$) between the surface stress and the layer-averaged velocity, and the transfer coefficients $C_D$, $C_H$, etc. are functions of $h/z_o$ and the layer averaged bulk Richardson number

$$Ri_m = \beta h(\bar{\theta}_{vm} - \bar{\theta}_{vo})/\bar{V}_m^2 \qquad (16)$$

which can be evaluated and represented in the forms of nomograms (Arya, 1977), knowing the forms of the similarity functions $A_m, B_m$, etc.

Note that over the expected ranges of $h/z_o$ and $Ri_m$, the drag and other transfer coefficients can vary over more than a hundred-fold range and are quite sensitive to the stability parameter $Ri_m$. Therefore, constant assigned values of these for all the terrain and stability conditions would hardly seem justifiable.

The alternative forms of flux parameterizations, involving surface geostrophic winds, or mean variables at the top of the PBL, have also been proposed (Clarke and Hess, 1974); Meljarejo and Deardorff, 1974; Arya, 1975; Zilitinkevich, 1975). But these are not recommended here, because of the similarity functions involved in them are much more sensitive to baroclinity, the scale-height ratio, entrainment and advection, as compared to the similarity functions in Eqs. (12) (Arya and Wyngaard, 1975; Arya, 1978; Garratt, et al., 1982). However, the geostrophic drag relations may be used for specific applications in which the information on geostrophic rather than actual winds is more readily available. For the barotrophic PBL, in particular, the simplest geostrophic drag law, which can be derived from the equations of motion (Deardorff, 1972; Brost and Wyngaard, 1978) is

$$\tau_o = c\rho f h G_o \sin \alpha_o \qquad (17)$$

in which $G_o$ is the geostrophic wind, $\alpha_o$ is the angle between the surface wind and geostrophic wind and c is a weak function of stability; c = 1 for the moderately unstable and convective conditions ($h/L<-2$) and $c \simeq 0.6$ for neutral and stable conditions.

The parametric relations based on mean variables at z = h might be more appropriate for large scale atmospheric models with the first interior grid level $z_1$ above the PBL in which $\bar{u}_h$ , $\bar{\theta}_h$, etc. could be extrapolated from the computed variables at $z_1$ and $z_2$ at each time step.

## 4. TURBULENT FLUXES AND VARIANCES IN THE PBL

In fine resolution models, there may be several grid points lying within the PBL. In these, the vertical fluxes at all the interior grid levels will have to be parameterized. The parameterization of flux profiles in the PBL should also be useful in other applications. Ideally, it should be based on observations taken under various stability conditions.

In unstable and convective PBLs, the fluxes at the top of the mixed layer can be determined in terms of the entrainment velocity $W_e$ and the jumps ($\Delta\bar{u}$, $\Delta\bar{v}$, etc.) in the mean variable across the transition layer, using the well-known entrainment relations (Driedonks, 1982):

$$\begin{aligned} (\overline{u'w'})_h &= -W_e\Delta\bar{u} \\ (\overline{v'w'})_h &= -W_e\Delta\bar{v} \\ (\overline{\theta'w'})_h &= -W_e\Delta\bar{\theta} \\ (\overline{q'w'})_h &= -W_e\Delta\bar{q} \end{aligned} \qquad (18)$$

Furthermore, the heat and moisture fluxes within the mixed layer are known to have approximately linear variation between the values at the surface and the top of the mixed layer. For the momentum fluxes, the linear profiles are only expected under barotropic conditions; in the presence of baroclinity the profiles may develop significant curvatures leading to large flux values in the middle of the PBL (Wyngaard et al., 1974; Kaimal et al., 1976).

There is limited observational evidence also suggesting linear variation of fluxes in the stably stratified PBL (Caughey et al., 1979). But more recently, Nieuwstadt (1983) has demonstrated, from comparisons between Minnesota and Cabauw observations, that due to the effects of nonstationarity, gravity waves and radiation, no generalizable similarity flux profiles may be expected in the nocturnal PBL.

A more satisfactory scheme for parameterizing the fluxes in the stably stratified PBL might be the use of the gradient transport relations

$$\begin{aligned} \overline{u'w'} &= -K_M\ \partial\bar{u}/\partial z \\ \overline{v'w'} &= -K_M\ \partial\bar{v}/\partial z \\ \overline{\theta'w'} &= -K_H\ \partial\bar{\theta}/\partial z \\ \overline{q'w'} &= -K_E\ \partial\bar{q}/\partial z \end{aligned} \qquad (19)$$

in which one may prescribe $K_M = K_H = K_E \simeq 0.15\ \lambda_{mw}\ \sigma_w$, where $\lambda_{mw}$ and $\sigma_w$ are the length and velocity scales based on vertical velocity fluctuations whose parameterization will be discussed later.

The variances or standard deviations of velocity fluctuations in PBL must be known for realistic modeling of turbulent transport and diffusion in the PBL. These are normally scaled by friction velocity ($u_*$) or the convective velocity ($w_*$), depending on the stability regime of the PBL.

For unstable and convective PBLs, the following parametric relations may be suggested on the basis of observations by Panofsky et al., (1977), Kaimal, et al., (1976) and Caughey (1982).

$$\begin{aligned} \frac{\sigma_u}{u_*} = \frac{\sigma_v}{u_*} &= (12 - 0.5\ \frac{h}{L})^{1/3}, \quad \text{for } \frac{z}{h} \leq 0.1 \\ \frac{\sigma_w}{u_*} &= 1.3(1-3\ \frac{z}{L})^{1/3}, \quad \text{for } \frac{z}{h} \leq 0.1 \\ \frac{\sigma_u}{w_*} = \frac{\sigma_v}{w_*} = \frac{\sigma_w}{w_*} &\simeq 0.6, \quad \text{for } 0.1 < \frac{z}{h} < 0.8 \end{aligned} \qquad (20)$$

In the upper part of the mixed layer and the interfacial transition layer, the values of $\sigma_u$, $\sigma_v$ and $\sigma_w$ are expected to depend on the entrainment parameter $W_e/w_*$.

For the stable PBL, the limited Minnesota data presented by Caughey et al. (1979), in the framework of the PBL similarity theory, suggests the linear relations

$$\frac{\sigma u}{u_*} \simeq 2.4(1 - \frac{z}{h})$$
$$\frac{\sigma w}{u_*} \simeq 1.6(1 - \frac{z}{h}). \quad (21)$$

$\sigma_v/u_*$ may be assumed to lie between $\sigma u/u_*$ and $\sigma_w/u_*$.

More recently, Nieuwstadt (1983) has suggested on the basis of Cabauw data that $\sigma_u$, $\sigma_v$ and $\sigma_w$ bear constant ratios with the local friction velocity $u_{*L} = [(\overline{u'w'})^2 + (\overline{v'w'})^2]^{1/4}$. However, the parameterizations based on height-dependent velocity and temperature scales require the knowledge of momentum and heat fluxes in the PBL, which are more difficult to get than the variances.

The neutral stability being a rare condition in the atmosphere, there are virtually no observations of turbulence in a neutral PBL. On the basis of observations in near-neutral surface layer and the results of the large eddy simulation model of Deardorff (1970), we suggest the following parametric relations:

$$\sigma u/u_* = 2.5 \exp |-3fz/u_*|$$
$$\sigma v/u_* = 1.8 \exp |-3fz/u_*| \quad (22)$$
$$\sigma w/u_* = 1.3 \exp |-3fz/u_*|$$

## 5. LENGTH AND TIME SCALES

In statistical models of diffusion, the integral time or length scales of motion are required to be specified. Related scales are the frequencies $n_m$ or the wavelengths $\lambda_m = \bar{V}/n_m$ at which the variance spectra of fluctuating velocity components have their peaks.

Spectral measurements in the convective PBL (Kaimal et al., 1976; Caughey and Palmer, 1979) have given the peak wave lengths $\lambda_{mu}$, $\lambda_{mv}$ and $\lambda_{mw}$, normalized by the PBL height h, as functions of z/h, which can be approximated as

$$\lambda_{mu}/h \simeq \lambda_{mv}/h \simeq 1.5$$
$$\lambda_{mw}/h = 5.9z/h, \text{for } z/h \leq 0.1 \quad (23)$$
$$\lambda_{mw}/h = 1.8[1-\exp(-4z/h)-0.0003\exp(8z/h)], \text{for } z/h > 0.1$$

For the nocturnal stable PBL, Caughey et al., (1979) have represented the peak wave lengths in the various spectra as function of z/h. Hanna (1982) has suggested that the following simple power laws can be fitted to these data from the Minnesota experiment:

$$\lambda_{mu}/h = 1.5\ (z/h)^{0.5}$$

$$\lambda_{mv}/h = 0.7\ (z/h)^{0.5} \qquad (24)$$

$$\lambda_{mw}/h = 1.0\ (z/h)^{0.8}$$

These should be considered as highly tentative, since the validity of similarity scaling in the nocturnal PBL has not been established (Nieuwstadt, 1983).

The Eulerian integral time scale $T_E$, being inversely proportional to the frequency $n_m$, is related to $\lambda_m$ as (Hanna, 1981)

$$T_E = 0.17\ \lambda_m/\bar{V}. \qquad (25)$$

Most needed in dispersion calculations is the Lagrangian integral time scale $T_L$, which is approximately related to $T_E$ as (Pasquill, 1974)

$$T_L/T_E = \beta \simeq b/i \qquad (26)$$

where $i = \sigma_{u,v,w}/\bar{V}$ is the appropriate turbulence intensity and b is a constant. From the simultaneous Eulerian and Lagrangian measurements of turbulence in the day time convective PBL, Hanna (1981, 1982) has estimated $\beta \simeq 1.7$ and $b \simeq 0.6$ (this is substantially higher than the recommended value of $b = 0.4$ by Pasquill (1974) on the basis of older and less accurate observations).

Substituting from Eq. (25) into (26).

$$T_L = 0.10\ \lambda_m/\sigma \qquad (27)$$

which in conjunction with Eqs. (23) and (24) would lead to the parameteric relations for $T_L$, as given by Hanna (1981, 1982).

In most of the parametric relations discussed earlier. the PBL height h is an important length scale which must be given or parameterized. In the daytime unstable and convective conditions, the PBL height over a land surface rapidly grows with time in response to the integrated sensible heat flux from the surface, mean vertical motion at the top of the PBL, mechanical mixing and other factors governing the entrainment rate at the top of the PBL.

A variety of theoretical models have been proposed in the literature for predicting the depth of the unstable PBL over a flat and homogeneous terrain. These are based on the rate equations for h and other associated parameters, such as the potential temperature in the mixed layer $\bar{\theta}_m$, temperature jump at the inversion base $\Delta\bar{\theta}$, etc. The models differ mainly in their choice of closure assumptions and representation of the transition layer above the mixed layer. It has been shown (Driedonks, 1982, Arya and Byun, 1983) that there are not very significant differences in the predicted h by various models, except for the crudest encroachment type models. The constant heat-flux-ratio model proposed by Tennekes (1973) and others might be adequate for parameterizing the daytime mixed layer height. The rate equations to be solved are:

$$\frac{\partial h}{\partial t} = C\,\frac{Q_o}{\overline{\Delta\theta}} + \bar{w}_h$$

$$\frac{\partial \overline{\Delta\theta}}{\partial t} = Q_o\left(\frac{C}{\overline{\Delta\theta}}\,\frac{\partial\theta_+}{\partial z} - \frac{1+C}{h}\right) \quad (27)$$

where $C \simeq 0.2$.

The evolution of the stable nocturnal PBL is rather slow and it is not clear whether a simple diagnostic relation for h might be adequate or one must carry an appropriate rate equation for the same. Arya (1981) and Garratt (1982) have recommended and provided considerable observational support for the similarity relation

$$h = d(u_* L/|f|)^{1/2} \quad (29)$$

in which $d \simeq 0.4$. On the other hand, Nieuwstadt (1981), Wetzel (1982) and Nieuwstadt and Tennekes (1981) have argued against the use of a diagnostic equation, such as (29), and instead have proposed a rate equation of the form

$$\frac{\partial h}{\partial t} = -\frac{h-h_e}{T_D} \quad (30)$$

in which $h_e$ is the equilibrium PBL height and $T_D$ is a time scale of PBL evolution which is shown to increase monotonically with time after the evening transition. Accordingly, the PBL height evolves at a decreasing rate and may never quite attain its equilibrium height which is given by Eq. (29) or a similar diagnostic relation. A rate equation for h may be preferrable during the initial hour or two, but after that, since h may not deviate from $h_e$ by more than 10-15%, one may use Eq. (29) as well. More recently, Nieuwstadt (1984) has also recommended the use of Eq. (29) on the basis of its satisfactory agreement with Cabauw observations.

## 6. VERTICAL VELOCITY AT THE TOP OF THE PBL

Ignoring the local and advective accelerations, and assuming that the momentum fluxes vanish at z = h, the integration of the equations of mean motion and continuity with respect to z from 0 to h yields

$$\frac{\tau_o}{\rho f} = h(\bar{v}_m - \bar{v}_{gm})$$

$$0 = h(\bar{u}_m - \bar{u}_{gm}) \quad (31)$$

$$\bar{w}_h = -\frac{\partial}{\partial x}(h\bar{u}_m) - \frac{\partial}{\partial y}(h\bar{v}_m) + \bar{u}_h \frac{\partial h}{\partial x} + \bar{v}_h \frac{\partial h}{\partial y}.$$

Differentiating the first of Eqs. (31) w.r.t.y and the second w.r.t.x., and substituting in the third, we have

$$\bar{w}_h = -\frac{\partial}{\partial y}\left(\frac{\tau_o}{\rho f}\right) + (\bar{u}_h - u_{gm})\frac{\partial h}{\partial x} + (\bar{v}_h - v_{gm})\frac{\partial h}{\partial y}$$

$$- h\left(\frac{\partial u_{gm}}{\partial x} + \frac{\partial v_{gm}}{\partial y}\right) \qquad (32)$$

where $u_{gm}$ and $v_{gm}$ are the layer-averaged components of geostrophic wind.

In the above expression for $\bar{w}_h$ the first term represents the curl of $\tau_o/\rho f$ which is expected to be the primary term under most circumstances. The second and third terms represent the contribution to $\bar{w}_h$ due to spatial changes in the PBL height; these are zero for a horizontally homogeneous PBL. The last term in (32) involves the divergence of the layer-averaged geostrophic winds and is probably negligible for all practical purposes.

For PBL over a complex terrain, one must also consider the contribution to $\bar{w}_h$ due to the terrain slope. For a flat and homogeneous terrain, however, eq. (32) reduces to the well know expression

$$\bar{w}_h = \frac{1}{\rho} \underset{\sim}{\nabla} \times (\underset{\sim}{\tau}_o/f). \qquad (33)$$

## 7. ENERGY DISSIPATION IN THE PBL

Following Lettau (1962), one can derive an expression for the total energy dissipation $E_D$ in terms of the surface drag and the layer-averaged winds

$$E_D = \int_o^h \rho\varepsilon dz = \int_o^h \underset{\sim}{\tau}_o \cdot \frac{\partial \bar{\underset{\sim}{V}}}{\partial z} dz \qquad (34)$$

where $\varepsilon$ is the local energy dissipation . Using the relations

$$\int_o^h \frac{\partial}{\partial z}(\underset{\sim}{\tau}_o \cdot \bar{\underset{\sim}{V}})\, dz = 0 \qquad (35)$$

$$\bar{\underset{\sim}{V}} \cdot \frac{\partial \underset{\sim}{\tau}}{\partial z} = \underset{\sim}{V}_g \cdot \frac{\partial \underset{\sim}{\tau}}{\partial z}, \qquad (36)$$

one gets from Eq. (34)

$$E_D = \underset{\sim}{V}_{go} \cdot \underset{\sim}{\tau}_o + \int_o^h \underset{\sim}{\tau} \cdot \frac{\partial \underset{\sim}{V}_g}{\partial z} dz \qquad (37)$$

Here the primary term is the dot product of surface geostrophic wind and stress vectors and the second term represents the contribution of baroclinity in the PBL.

The above expression for the total energy dissipation in the PBL is expected to be valid only far away from the equator where advective accelerations can be neglected; Eq. (34) is the more generally valid expression for $E_D$.

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Questions, answers and comments

L.N. GUTMAN: Can you show us any parametric relations concerning a planetary boundary layer developing over uneven terrain?

S.P.S. ARYA: No generalisable parametric relations can be given for the PBL over complex terrain, since the terrain can vary in a variety of ways, introducing a host of new length, velocity and temperature scales. For certain classes of complex terrain features, however, it is possible to obtain the parametric relations on the basis of systematic experiments and model studies. For example, in the case of gently and uniformly sloping terrain, the suggested wind profiles and resistance laws can be generalised to include the effect of terrain slope, using the model results of Gutman and Meljarejo (1982).

L. BERKOFSKY: In your $\bar{w}_h$ equations you have terms $\bar{u}_h$, $\bar{v}_h$. How do you determine these wind components?

S.P.S. ARYA: If we assume that at the top of the PBL, winds are in geostropic balance, then $\bar{u}_h$ and $\bar{v}_h$ are determined from the geostropic wind field, i.e. , $\bar{u}_h=u_{gh}$ $\bar{v}_h=v_{gh}$.

L. BERKOFSKY: It is probable that the parametrization of $\bar{w}_h$ is not simple.

S.P.S. ARYA: The parametrization of the vertical velocity at the top of the PBL is indeed difficult and requires a good estimate of the horizontal divergence of the wind field in the PBL or the spatial variation (curl) of the surface stress vector over a flat terrain. The addition of complex topography makes such a parameterization extremely difficult.