DETERMINATION OF VERTICAL TEMPERATURE PROFILES FOR THE ATMOSPHERIC BOUNDARY LAYER BY GROUND-BASED MICROWAVE RADIOMETRY

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Abstract. A useful method for remote sensing of vertical temperature profiles in the atmospheric boundary layer is described. From angular measurements of brightness temperature at 58 GHz, profiles have been inferred up to an altitude of 700 m. Calculations were done with an iterative inversion procedure (Smith et al., 1972) using Twomey-type smoothing (Twomey, 1963). It is shown how an initial-guess profile can be directly derived from the radiation measurements using a nomogram.

1. Introduction

Radiation measurements in the region of the 0_2-band between 50 and 70 GHz have been used to determine vertical temperature profiles for years. Measurements can be done multispectrally or in the case of a fixed frequency, by changing the zenith angle.

The first method is applied generally in the satellite measuring technique or in measurements from objects with a non-stable position (e.g., buoys). The second method is of importance for ground-based measurements on a stable platform, and is the one used in the experiment described below. The bending of the earth's surface is of no influence because the 0_2 emission at 58 GHz comes from relatively short distances. Furthermore, the influence of water vapour can be neglected (Figure 1). Since about 90% of the emission occurs in the lower 600 m, only low-level clouds cause interference. In the following calculations, the influence of clouds is not considered.

2. The Radiative Transfer Equation

Under the assumption of local thermodynamic equilibrium and with the Rayleigh–Jeans approximation of the Planck function, one gets from the radiative transfer equation (Goody, 1964) the brightness temperature T_B as a function of zenith angle θ:

$$T_B(\nu) = \int_0^\infty T(h)\,\alpha(\nu, h) \sec\theta \exp\left[-\int_0^h \alpha(\nu, h') \sec\theta \,\mathrm{d}h'\right] \mathrm{d}h \tag{1}$$

where ν = frequency, h = altitude, $T(h)$ = temperature, and $\alpha(\nu, h)$ = absorption coefficient.

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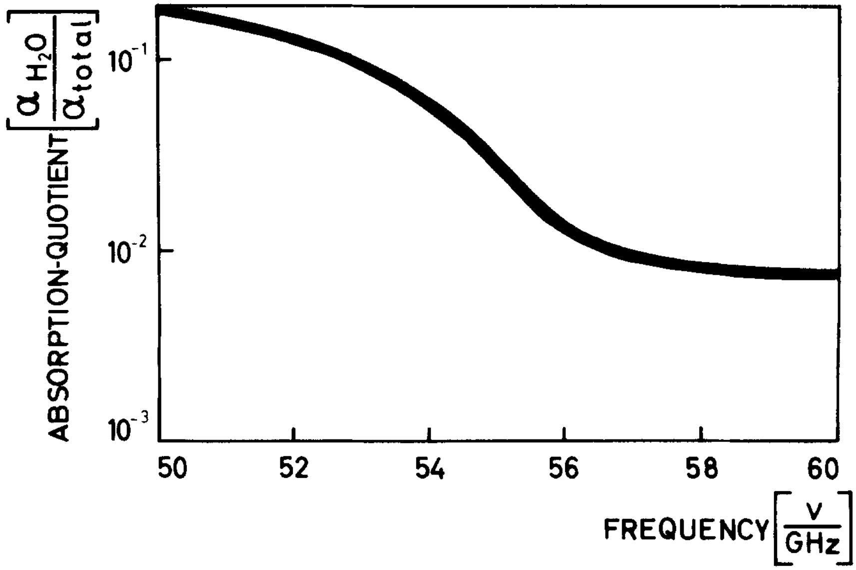

Fig. 1. Relation of water vapour absorption to total absorption in the atmosphere as a function of frequency

Equation 1 is a Fredholm integral equation of the first kind which is generally written as $\phi(x)=\int \psi(y)\,A(x, y)\,dy$ with the kernel

$$A(x, y)=\alpha(\nu, h)\sec\theta \exp\left[\int_0^h \alpha(\nu, h')\sec\theta \,dh'\right]. \qquad (2)$$

The kernel is often called a weighting function. As there is only a very small temperature dependence of $A(x, y)$, this function can be calculated as a function of altitude and zenith angle (Liebe *et al.*, 1977; Rosenkranz, 1975).

The overlapping of the weighting functions (Figure 2) demonstrates the interdependency of the information (described by Twomey (1966)). In order to determine $T(h)$ in equation 1, a finite-difference form of the equation must be used. The quadrature form most used is

$$\phi_i = \sum_{j=1}^{n} A_{ij}\chi_j \qquad i=1, 2 \ldots m.$$

The atmosphere is divided into thin homogeneous layers, 8 layers being chosen in our case, each with a thickness of 200 m. To determine the measuring angles, constant values of weighting are taken from figure 2 and converted to polar coordinates. In practice, angles corresponding to the weighting factor 10^{-2} are used

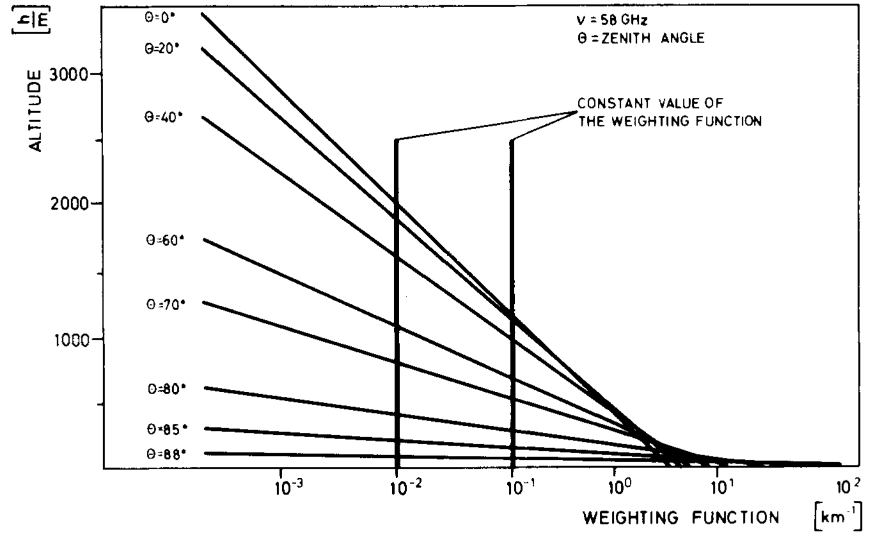

Fig. 2. Weighting functions used for upwardlooking radiation measurements

(Figure 3). The information difference between two chosen zenith angles corresponds to one layer thickness. For the respective angles, computations show that the influence of brightness temperature from layers behind that line can be neglected. In principle, therefore, calculations can be done with a triangular weighting

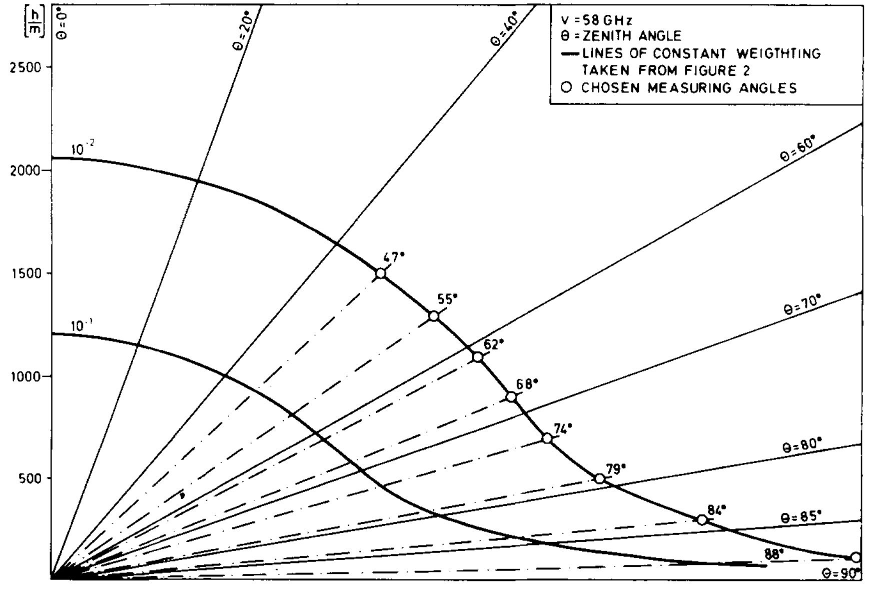

Fig. 3. Determination of the measuring angles

matrix. The loss of information caused by measurements near the zenith not being used must be tolerated.

3. Method of Solution

The iterative method for calculating a temperature profile uses a universal algorithm based on the principle of component smoothing (Faddejew and Faddejewa, 1964). It is a modified form of the Twomey solution (Twomey, 1963).

The basic equation for an iterative method is generally

$$\mathbf{X}^{(k)} = \mathbf{X}^{(k-1)} + h^{(k)}(A)[\mathbf{F} - A\mathbf{X}^{(k-1)}],$$

where k = step of iteration, $\mathbf{X}$ = solution vector, $\mathbf{F}$ = measuring vector, A = coefficient matrix (weighting matrix), and $h^{(k)}(A)$ = polynomial (eventual of order zero).

$\mathbf{X}^{(0)}$ is a convenient initial guess. For the experiment described here in which eight layers correspond to eight measuring angles, the system is mathematically well-determined. The coefficient matrix however is ill-conditioned, i.e., it has several eigenvalues close to zero. In the case given here, one eigenvalue is near one and the remaining seven eigenvalues are between 10^{-3} and 10^{-6}. Since the measuring vector, and to a certain extent even the coefficients of the weighting matrix, have inherent uncertainties, a direct solution produces no physically significant result.

Starting with the Twomey solution

$$\mathbf{X}^{(k)} = \mathbf{X}^{(k-1)} + QA^T[\mathbf{F} - A\mathbf{X}^{(k-1)}] \tag{3}$$

with

$$Q = (A^TA + \gamma I)^{-1},$$

and T = matrix transposition, γ = Lagrangian multiplier (smoothing parameter), and I = identity matrix, the following equation results

$$A^TA\mathbf{X}^{(k)} - A^TA\mathbf{X}^{(k-1)} + \gamma I\mathbf{X}^{(k)} - \gamma I\mathbf{X}^{(k-1)} = A^T\mathbf{F} - A^TA\mathbf{X}^{(k-1)}$$

which finally leads to

$$\mathbf{X}^{(k)} = \mathbf{X}^{(k-1)} + (\gamma I)^{-1}A^T[\mathbf{F} - A\mathbf{X}^{(k)}]. \tag{4}$$

The implicit $\mathbf{X}^{(k)}$ existing in the right side of the equation can be substituted by $\mathbf{X}^{(k-1)}$ without any essential degeneration of the iteration.

The matrix product $(\gamma I)^{-1}A^T$ was replaced by the weighting matrix A and a diagonal matrix Z found empirically. Thus, the iterative method can do without matrix inversion and transposition. The basic equation of the realized iteration method then is

$$\mathbf{X}^{(k)} = \mathbf{X}^{(k-1)} + ZA[\mathbf{F} - A\mathbf{X}^{(k-1)}] \tag{5}$$

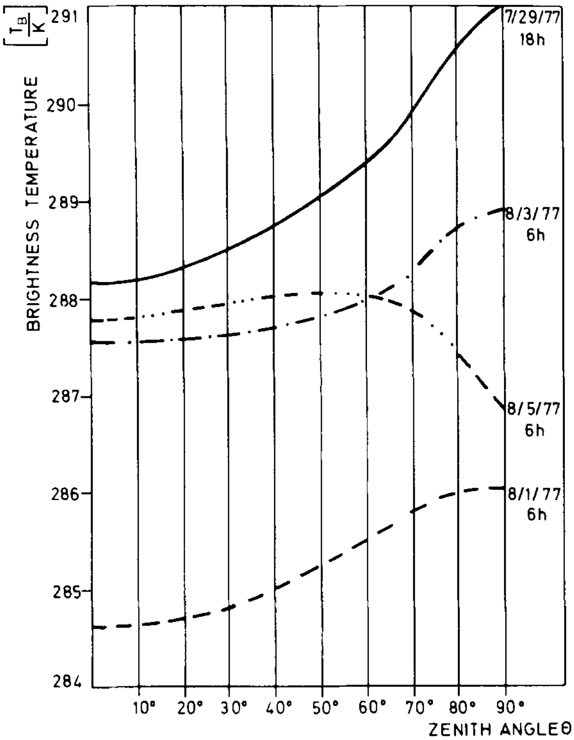

Fig. 4. Measured brightness temperatures with a 58 GHz radiometer

$$Z = \begin{pmatrix} 1.5 & & & & & & & 0 \\ & 0.45 & & & & & & \\ & & 1.6 & & & & & \\ & & & 5.0 & & & & \\ & & & & 13.5 & & & \\ & & & & & 32.0 & & \\ & & & & & & 62.5 & \\ 0 & & & & & & & 108.0 \end{pmatrix}$$

A similar iteration has been given by Smith et al. (1970), i.e., with

$$\mathbf{X}^{(k)} = \mathbf{X}^{(k-1)} + DA^T(\mathbf{F} - A\mathbf{X}^{(k-1)}] . \quad (6)$$

D is a diagonal matrix with the coefficients

$$d_{jj} = 1 \sum_{i=1}^{n} a_{ij} \qquad j = 1, 2 \ldots m$$

and corresponds to the $(\gamma I)^{-1}$ of the modified Twomey solution in equation 4.

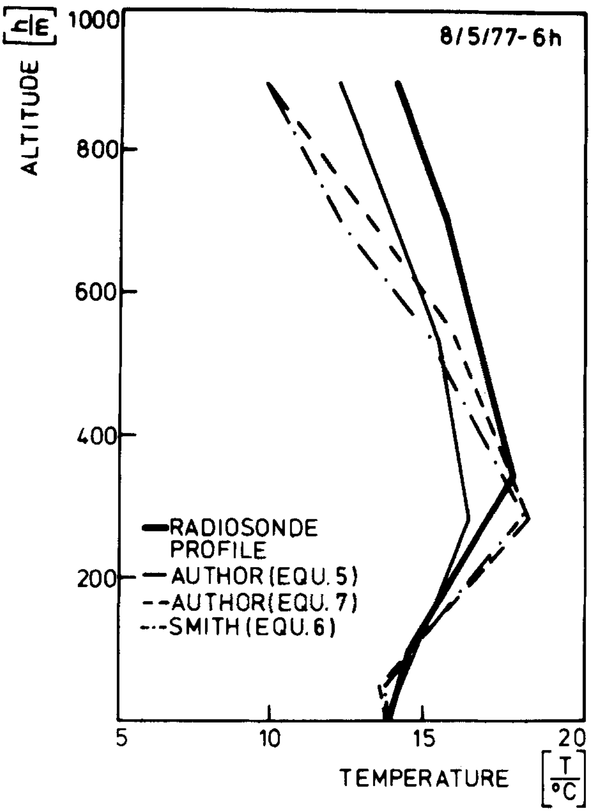

Fig. 5. Comparison between radiosonde profile and 3 iterative methods of profile retrieval for a relatively strong inversion at about 300 m altitude

A further possible iteration method is to replace the matrix product ZA, respectively, DA^T by a single diagonal matrix

$$\mathbf{X}^{(k)} = \mathbf{X}^{(k-1)} + H[\mathbf{F} - A\mathbf{X}^{(k-1)}] \tag{7}$$

with

$$h_{ij} = \begin{cases} 1 & \text{if } i=j=1 \\ \dfrac{1}{65.0-p^2} & \text{if } i=j=2,3,\dots 8 \\ & p=i=j \end{cases}$$

A comparison of results is given in the following section.

In all calculations the iteration was stopped when the residuals had reached the noise equivalent brightness temperature of the radiometer (0.1 K).

4. Results

The results are derived from measurements which were carried out in cooperation

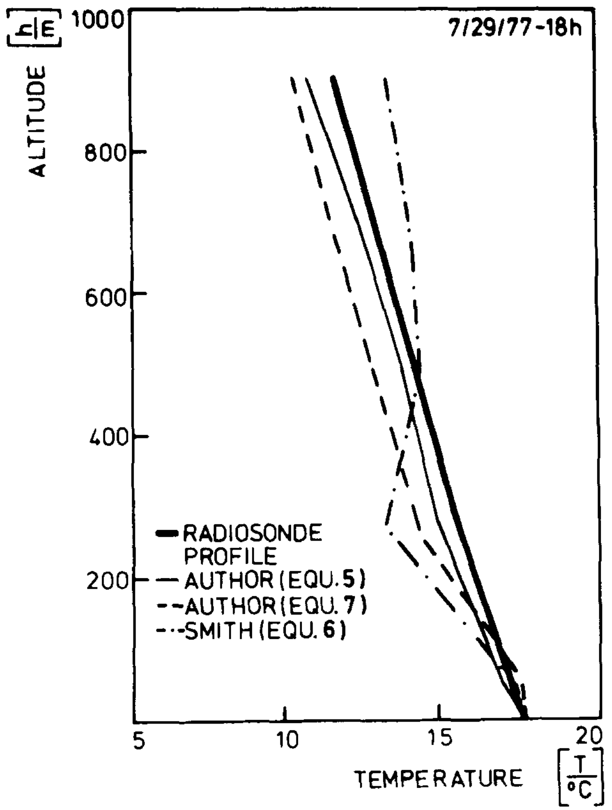

Fig. 6. Comparison between radiosonde profile and 3 iterative methods of profile retrieval for a dry adaiabatic lapse rate

with the measuring group of the 'Geophysikalischer Beratungsdienst der Bundeswehr' at Kiel–Holtenau in August 1977. Radiosonde ascents took place at 6 h and 18 h GMT.

Radiation measurements for 8 zenith distances between horizon and zenith takes about 8 min. The curves of brightness temperatures shown in Figure 4 are values averaged over one hour, distributed symmetrically about the mean of the radiosonde values.

Even without using iterative or statistical solution methods, statements concerning the appearance of the temperature profile can be made. For instance, in the case of inversions close to the ground, an increase of brightness temperature with decreasing zenith angle is observed (Figure 4, Figure 5); and for a dry adiabatic lapse rate, the brightness temperature decreases monotonically with decreasing zenith angle (Figure 4, Figure 6). This information is used for determining the initial-guess in the iteration procedure.

Profile retrieval can be made only up to an altitude of about 700 m because inversions above 500 m can hardly be identified in the respective curve of brightness

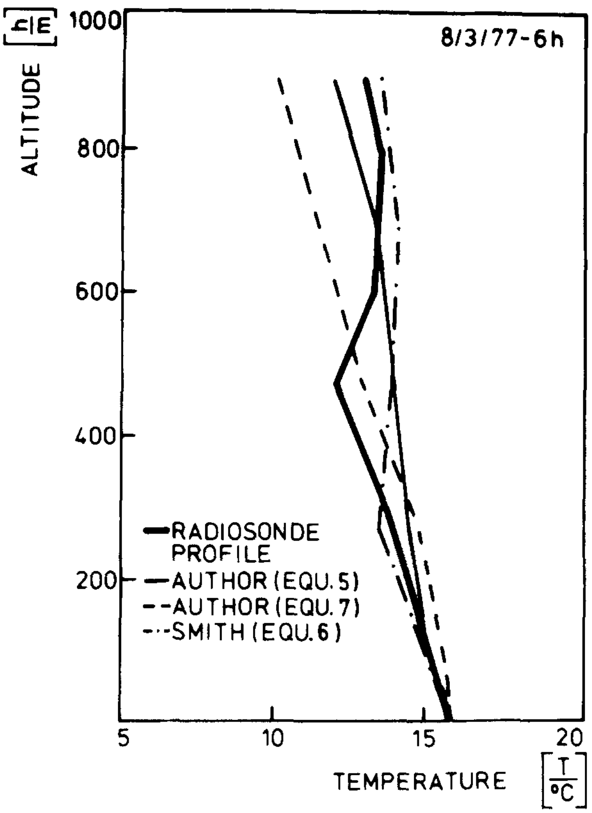

Fig. 7. Comparison between radiosonde profile and 3 iterative methods of profile retrieval for an inversion above 500 m

temperature. This is shown in Figure 7 and by the corresponding brightness temperature in Figure 4.

A further auxiliary quantity is the ground temperature at the time of radiosonde ascent. The ground temperature was determined with an Assmann–Aspiration-psychrometer at a height of 2 m. This temperature serves as a basis for the initial-guess. As the value of brightness temperature measured radiometrically at the horizon level always includes an error, e.g., caused by the finite-beam angle of the antenna, the nature of the ground, and the ground temperature, profile sounding was started with the smallest possible measuring angle for which the main lobe of the antenna did not touch the ground. In the experiment described here, the main lobe has a beam angle of 1.2 deg, so that 88 deg was chosen as the lowest measuring angle. The brightness temperatures measured at this angle correspond to the temperature at an altitude of about 30 m. In the above mentioned calculations, the influence of the side lobes had been neglected, being 30 dB less than the main lobe; the temperatures detected by them are not very different from the value detected by the main lobe of the antenna.

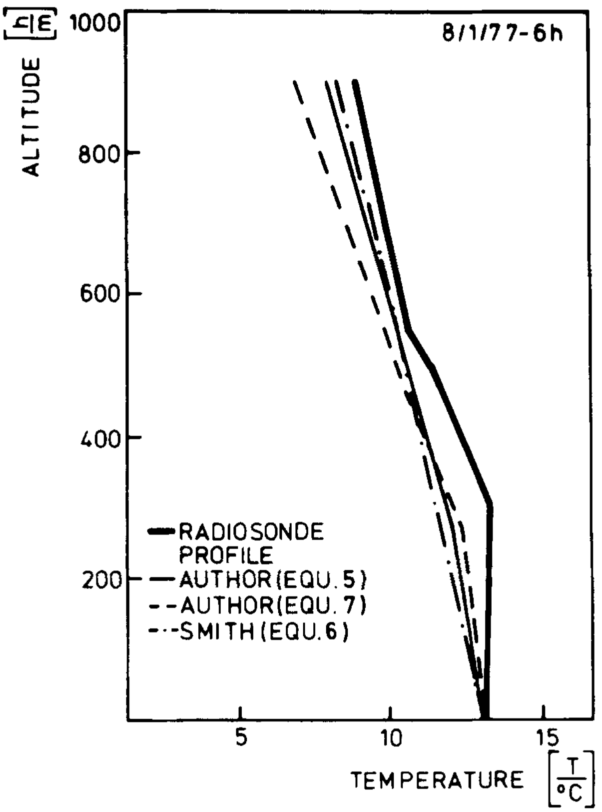

Fig. 8. Comparison between radiosonde profile and 3 iterative methods of profile retrieval for an almost isothermal atmosphere in the first 300 m

With the nomogram of Figure 9, it is possible to derive temperature profiles directly from the measured brightness temperatures without mathematical effort. Therefore, the theoretically expected brightness temperatures have to be calculated from a given set of profiles.

For an antenna with very small beam angle and low side lobes, the measured horizontal brightness temperature corresponds to the ground temperature; therefore, the brightness temperatures and the layer temperatures are related to the ground level, the ordinate and abscissa showing the differences. For lines of constant measuring angle, a linear interpolation was made between the values obtained from the given set of profiles and the weighting factors.

Starting-point for the profile retrieval is the temperature measured with an Assman–Aspiration psychrometer at an altitude of 2 m. The measured brightness temperature for an angle of 88 deg corresponds to this value reduced (increased) by about 0.1 K if the curve of brightness temperature decreases (increases) with decreasing zenith angle. Having determined the 88-deg value, brightness temperatures for all zenith angles are known by radiometer calibration.

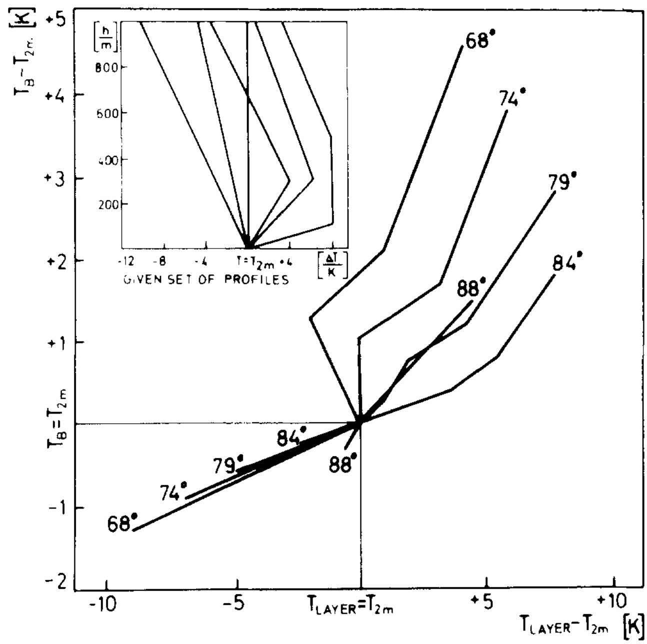

Fig. 9. Nomogram for profile retrieval. Ordinate: Difference between measured brightness temperatures and the temperature level at 2-m height. Abscissa: Difference between layer temperature and the temperature level at 2 m. Parameter: The measuring angles

As already mentioned, the measuring angles were chosen to make the weighting matrix triangular. Therefore, if one starts with the angle nearest the ground, the measured brightness temperature has information for only the first layer. For the next angle, the measured brightness temperature has information for the first and the second layers, and so on. So it is possible to obtain a relation between measuring angles and layer altitudes, and measured brightness temperatures and layer temperatures.

If one relates the measured brightness temperatures of the remaining angles to the ground temperature, positive or negative deviations are obtained. By transferring these deviations to the ordinate of the given nomogram, one will get the difference between layer temperature and ground temperature as the abscissa value. The temperature profiles of Figure 10 are derived from the brightness temperatures indicated in Figure 4 and can be compared with the profiles retrieved iteratively (Figure 5 – Figure 8).

5. Summary

The first attempts to detect the temperature profile of the boundary layer radiometrically at a frequency of 58 GHz show satisfactory results. Subdivision into 200-m layers reproduces the structure of the temperature profiles to a sufficient

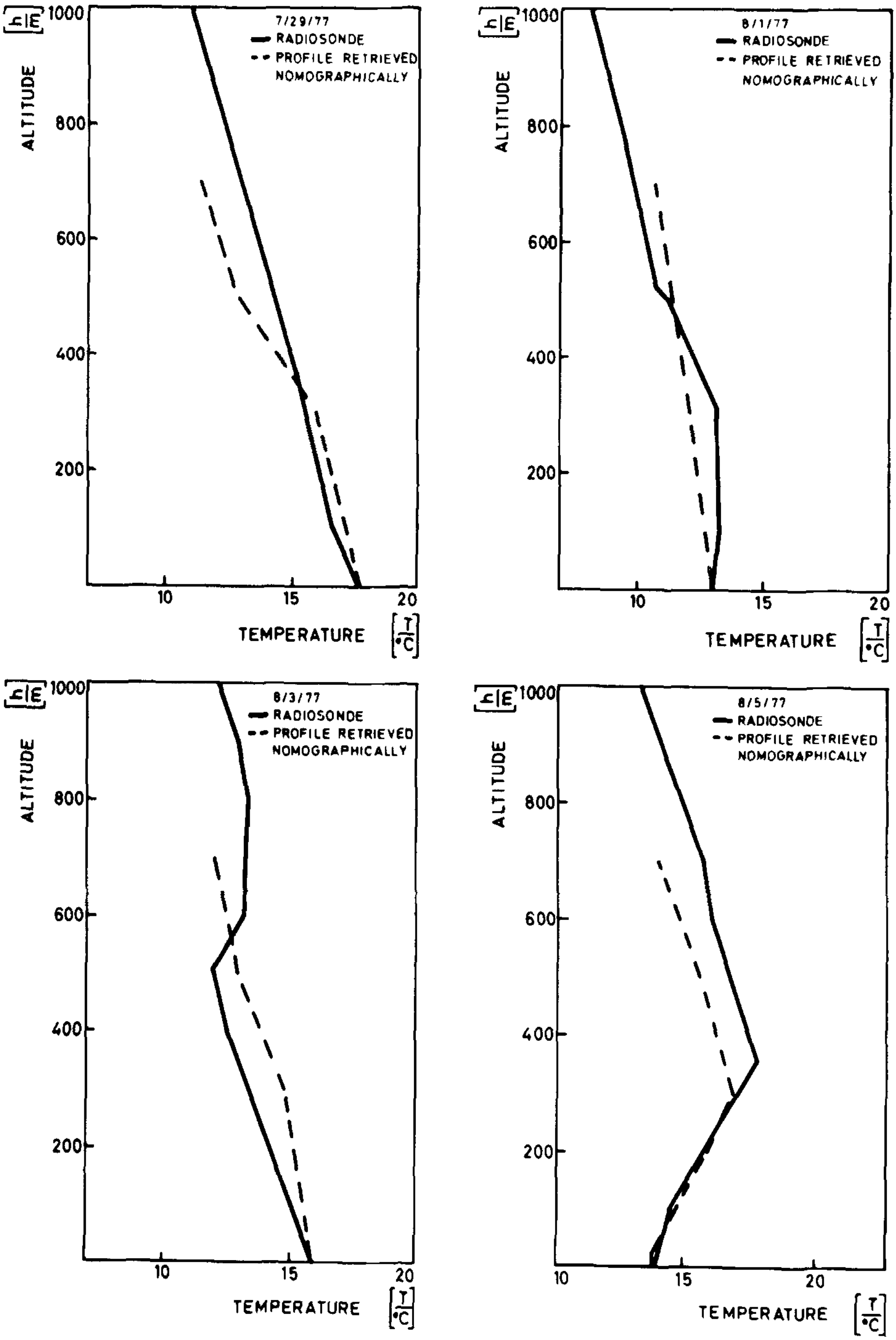

Fig. 10. Nomographically retrieved profiles from the measured brightness temperatures shown in Figure 4

extent. Even the temperature gradient – as a frequently more useful quantity – is relatively well retrieved in the altitude levels of interest. The possible detection of higher inversions and a comparison between statistical and non-statistical methods will be the subject of further investigations. The profiles obtained from the nomogram could be applied as a very useful initial-guess in a later computation.

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