

# Anisotropic Gap Distortion Due to Superflow and the Depairing Critical Current in Superfluid $^3\text{He-B}^*$

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*We have calculated the pair-breaking critical current for superfluid  $^3\text{He-B}$  at all temperatures by taking into account the anisotropic distortion of the order parameter in the presence of superflow. We find that the component of the order parameter along the flow  $\Delta_{\parallel}$  is strongly reduced, while  $\Delta_{\perp}$ , the component perpendicular to the flow, slightly increases. The superfluid density  $\rho_s$  has also been determined as a function of the superfluid velocity  $v_s$ . Fermi liquid corrections are explicitly included and it is shown that these corrections lead to a drastic change in the functional dependence of  $\Delta_{\parallel}$ ,  $\Delta_{\perp}$ ,  $\rho_s$ , etc., on  $v_s$ . In contrast to this, the pair-breaking critical current is shown to be independent of the Fermi liquid corrections.*

## 1. INTRODUCTION

Recent experiments<sup>1</sup> designed to measure critical currents in superfluid  $^3\text{He-B}$  have led to a renewed interest in the theoretical aspects of such critical currents. Calculations of depairing critical currents for the superfluid A and B phases of  $^3\text{He}$  have previously been performed by Fetter,<sup>2</sup> who specialized to the Ginzburg-Landau regime, and by Vollhardt and Maki<sup>3</sup> (VM), who not only included Fermi liquid corrections, but also allowed for a general temperature dependence. However, considering the similarities between the BCS superconductor and the B phase<sup>4</sup> of superfluid  $^3\text{He}$ , the latter authors assumed a large, uniform current (corresponding to

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$v_s \approx 10^{-3} - 10^{-2} v_F$ ) to produce only an isotropic change of the magnitude of  $\Delta(T)$ , the order parameter. That is, an explicit distortion of the order parameter due to the superflow was not included. The purpose of the present paper is therefore to improve on this point by taking into account the anisotropic deformation of the order parameter in the presence of a uniform superflow (in the  $z$  direction). For this the (three) gap equations for  ${}^3\text{He-B}$  in the presence of superflow have been treated self-consistently.

It is very interesting to note that the present problem and the one of  ${}^3\text{He-B}$  in a strong magnetic field<sup>2,5</sup> have several features in common. Indeed, the gap equations prove to be very similar in both situations, i.e., the effects of a strong magnetic field and of superflow on the  ${}^3\text{He-B}$  order parameter resemble each other. In both cases the order parameter is anisotropically distorted, the functional dependence of this distortion on the respective pair-breaking parameter being almost identical.

The calculations employ a microscopic approach where, as before,<sup>3</sup> the uniform current is introduced into the single-particle Green's function via a frequency shift as was done by Maki and Tsuneto.<sup>6</sup> Furthermore, we explicitly include Fermi liquid corrections, which prove to be of decisive importance for our results. The effects of the Fermi liquid corrections are incorporated by renormalizing the quasiparticle mass and the superfluid velocity as shown by Leggett.<sup>7</sup>

The present calculations employ the weak coupling model for simplicity. In spite of the fact that strong coupling corrections seem to be necessary to explain some deviations from the weak coupling results (see, for example, the results of recent measurements by Archie *et al.*<sup>8</sup>), the weak coupling model is quite adequate for obtaining functional dependences and qualitative features if we exclude the problem of the stability of the A phase. This is also the reason why we will not elaborate on the phase transition from the B-phase state to the axial state (in fact, to the "planar" state; however, in the weak coupling model and in the absence of a magnetic field both states are degenerate), which will eventually occur when the superflow is increased beyond the pair-breaking critical current, if this is at all possible experimentally. It has been shown by Fetter<sup>2</sup> that this transition sensitively depends on strong coupling corrections.

The calculations have been performed for the following values of the Fermi liquid correction parameter  $F_1$ :  $F_1 = 0$ ,  $F_1 = 6.04$  (corresponding to zero pressure<sup>9</sup>), and  $F_1 = 15.66$  (corresponding to melting pressure<sup>9</sup>). The results we are going to present will concentrate on the first two cases only, because the recent experiments<sup>1</sup> were done at saturated vapor pressure (i.e., essentially zero pressure) and also because the results for  $F_1 = 15.66$  are qualitatively very similar to those for  $F_1 = 6.04$ .

## 2. FORMULATION

The single-particle Green's function for  ${}^3\text{He-B}$  in the Nambu representation for zero current is given by

$$G_0(\mathbf{p}, \omega_n) = (i\omega_n - \xi\rho_3 - \sigma_2\rho_1\mathbf{\Delta} \cdot \boldsymbol{\alpha})^{-1} \quad (1)$$

where

$$\xi = p^2/2m^* - \mu, \quad \hat{p} = \mathbf{p}/p_F, \quad \boldsymbol{\alpha} = (\sigma_1\rho_3, \sigma_2, \sigma_3\rho_3) \quad (2)$$

$m^*$  is the effective mass of a quasiparticle,  $\mu$  is the chemical potential, and  $\sigma_i$ ,  $\rho_i$  are Pauli matrices which operate on the spin and the particle-hole space, respectively. Here  $\mathbf{\Delta}$  is the order parameter vector; in the absence of current (isotropic case) it can be written as

$$\mathbf{\Delta}(\mathbf{p}) = \Delta \cdot \mathbf{p} \quad (3)$$

but in the presence of superflow  $\mathbf{\Delta}(\mathbf{p})$  will become anisotropic (self-consistency of the gap equations!) and is then instead given by

$$\mathbf{\Delta}(\mathbf{p}) = (\Delta_{\perp} p_1, \Delta_{\perp} p_2, \Delta_{\parallel} p_3) \quad (4)$$

$\Delta_{\perp}$  and  $\Delta_{\parallel}$  are the magnitudes of the order parameter perpendicular and parallel to the external flow, which is supposed to be along the  $z$  direction. For zero current we have  $\Delta_{\perp} = \Delta_{\parallel}$  and Eq. (4) reduces to Eq. (3).

As discussed by VM, the effects of a uniform current are now introduced into the above Green's function by replacing  $i\omega_n$  by  $i\omega_n + \mathbf{v}_s \cdot \mathbf{p}$ , where  $\mathbf{v}_s$  is the superfluid velocity and  $\mathbf{p}$  is the quasiparticle momentum. The reason lies in a gaugelike transformation of the order parameter  $\mathbf{\Delta}(\mathbf{r}) \rightarrow \exp(2im\mathbf{v}_s \cdot \mathbf{r})\mathbf{\Delta}(\mathbf{r})$ , where  $m$  is the mass of a  ${}^3\text{He}$  atom.

As has been shown by Leggett,<sup>7</sup> the interaction between quasiparticles leads to a polarization of the liquid, which in turn modifies the effect of the current. In other words, the polarization results in a mean-field screening for the superfluid velocity  $\mathbf{v}_s$ , so that the system does not feel the bare velocity  $\mathbf{v}_s$ , but rather an effective velocity  $\mathbf{v}_s^*$ , with

$$\mathbf{v}_s^* = \mathbf{v}_s / (1 + \frac{1}{3}F_1\phi) \quad (5)$$

The function  $\phi$  is a generalized Yosida function, which will be derived within the context of the calculation of the superfluid current and the superfluid density. Consequently, the Fermi liquid corrections are accounted for by (i) using  $m^*/m = 1 + \frac{1}{3}F_1$  in the expression for the density of states, etc., and (ii) by replacing  $\mathbf{v}_s$  in the Green's function by  $\mathbf{v}_s^*$ . In the presence of a uniform current the proper Green's function is then given by

$$G(\mathbf{p}, \omega_n) = G_0(\mathbf{p}, \omega_n - is \cos \theta) \quad (6)$$

where

$$s = v_s^* p_F \quad (7)$$

and  $\theta$  is the angle between  $\mathbf{v}_s$  and  $\mathbf{p}$ , and  $p_F$  is the Fermi momentum.

The gap equations in the weak coupling limit are given by

$$\Delta_i(\mathbf{p}) = -3g_i T \sum_{\omega_n} \int \frac{d^3 p'}{(2\pi)^3} (\mathbf{p} \cdot \mathbf{p}')^{\frac{1}{2}} \text{Tr} \{ \sigma_2 (\rho_1 + i\rho_2) \alpha_i G(\mathbf{p}', \omega_n) \} \quad (8)$$

This leads to a set of two equations for  $\Delta_{\perp}$  and  $\Delta_{\parallel}$ , i.e., the components of the order parameter perpendicular and parallel to the flow. Depending upon whether we perform the  $p'$  integration or the  $\omega_n$  summation, we obtain

$$\frac{1}{\lambda} = 3\pi T \sum_{\omega_n}^{\omega_c} \text{Re} \int_0^1 dz f_i(z) [(\omega_n - isz)^2 + \Delta_{\parallel}^2 z^2 + \Delta_{\perp}^2 (1 - z^2)]^{-1/2} \quad (9a)$$

or

$$\frac{1}{\lambda} = \frac{3}{4} \int_0^{\omega_c} d\xi \int_0^1 dz f_i(z) \frac{\tanh [(E + sz)/2T] + \tanh [(E - sz)/2T]}{E} \quad (9b)$$

where  $i = 1, 2$  and

$$f_1(z) = 1 - z^2, \quad f_2(z) = 2z^2, \quad E^2 = \xi^2 + \Delta_{\parallel}^2 z^2 + \Delta_{\perp}^2 (1 - z^2) \quad (10)$$

Furthermore, we use  $\lambda = N(0)|g_i|$ ,  $\omega_c \equiv \omega_{n_0} = (2n_0 + 1)\pi T$ , where  $N(0) = 3Nm^*/p_F^2$  is the density of states at the Fermi level and  $\omega_c$  is the cutoff frequency.

The coupling constant  $\lambda$  can be eliminated from Eq. (9) in the weak coupling limit by subtracting the respective equations at  $T = T_c$  and at  $T = 0$  and  $v_s = 0$ . For the latter case Eq. (9b) reduces to

$$\frac{1}{\lambda} = \int d\xi \frac{1}{E_0} \quad (11)$$

where  $E_0^2 = \xi^2 + \Delta_{00}^2$  and  $\Delta_{00} = \Delta_{\text{BCS}}(T = 0)$ .

For further treatment (and the numerical analysis) of Eq. (9) we will use the form given in Eq. (9b) containing the tanh functions. After elimination of  $\lambda$  we obtain

$$\int_0^{\infty} d\xi \int_0^1 dz f_i(z) \left\{ \frac{\tanh [(E + sz)/2T] + \tanh [(E - sz)/2T]}{E} - \frac{2}{E_0} \right\} = 0 \quad (12)$$

where the convergence of the integral is guaranteed. Equation (12) determines the order parameters in the presence of a uniform current.

We now turn to the superfluid mass current  $\mathbf{j}_s$ , associated with the superfluid velocity  $\mathbf{v}_s$ , which is given by

$$\mathbf{j}_s = \rho \mathbf{v}_s - \mathbf{j} \quad (13)$$

where  $\mathbf{j}$  is the mass current of quasiparticle excitations due to the superfluid velocity:

$$\mathbf{j} = T \sum_{\omega_n} \int \frac{d^3 \rho}{(2\pi)^3} \mathbf{p} \frac{1}{4} \text{Tr} G(\mathbf{p}, \omega_n) \quad (14)$$

Here  $\rho = Nm$  is the mass density of  $^3\text{He}$ .

We now want to simplify the expression for  $|\mathbf{j}|$ . The summation and integration have to be done very carefully to avoid difficulties with convergence.<sup>10</sup> Depending upon whether we perform the  $\xi$  integration or the  $\omega_n$  summation, we obtain [here we use  $j^0 = j/N(0)p_F$ ]

$$j^0(s, T) = \frac{1}{3}s - \pi T \sum_{\omega_n} \int_0^1 dz \text{Re} \frac{z(i\omega_n + sz)}{[(\omega_n - isz)^2 + \Delta_{\parallel}^2 z^2 + \Delta_{\perp}^2 (1 - z^2)]^{1/2}} \quad (15a)$$

or

$$j^0(s, T) = \int_0^{\infty} d\xi \int_0^1 dz \frac{z}{2} \left( \tanh \frac{E + sz}{2T} - \tanh \frac{E - sz}{2T} \right) \quad (15b)$$

As in the case of the gap equations, we will use Eq. (15b) for the necessary numerical treatment, except near  $T_c$ , the transition temperature.

The superfluid current  $j_s$  can be written as

$$\mathbf{j}_s = \rho_s \mathbf{v}_s \quad (16)$$

where  $\rho_s$  is the superfluid density, which has the general form

$$\rho_s/\rho = (1 - \phi)/(1 + \frac{1}{3}F_1\phi) \quad (17)$$

$\phi$  is a generalized Yosida function, which appeared in the reduction factor for the superfluid velocity and which accounts for the polarization of the fluid. Inserting Eq. (17) into Eq. (16) and using the definition of  $s$  in Eq. (7), we can write

$$j_s = (1 - \phi)s\rho/p_F \quad (18)$$

Hence Eq. (18) yields a consistent definition of  $\phi$  by means of the superfluid current:

$$\phi = 1 - \tilde{j}_s/s \quad (19)$$

where  $\tilde{j}_s = j_s p_F/\rho$ . We can now express  $j_s$  fully in terms of  $s$  rather than the

bare  $v_s$ . First we replace  $v_s$  by means of Eq. (5) and use  $N(0) = 3\rho(1 + \frac{1}{3}F_1)p_F^{-2}$  to obtain

$$\tilde{j}_s = s(1 + \frac{1}{3}F_1\phi) - (1 + \frac{1}{3}F_1)j^0 \quad (20)$$

Now we substitute  $\phi$  by means of Eq. (19) and divide by the common factor  $1 + \frac{1}{3}F_1$ . This finally leads to

$$\tilde{j}_s = s - 3j^0 \quad (21)$$

It is important to realize that  $j_s$  only depends on the Fermi liquid corrections through  $s$ , the effective superfluid velocity.

### 3. LIMITING CASES

The existence of two order parameter components and hence of two coupled gap equations makes an analytic solution for the energy gaps and  $j_s$  generally impossible. Only at  $T = 0$  and in the Ginzburg–Landau regime can one obtain analytic results, although even at  $T = 0$  the gap equations cannot be solved in closed form. Therefore numerical methods generally have to be employed. However, some expressions can be evaluated for the above-mentioned limiting cases.

#### 3.1. $T = 0$

In this case the integrals appearing in the gap equations can be solved; nevertheless, the equations become unmanageable and do not simplify the analysis. Only the expressions for  $j_s$  (and hence  $\phi$ ) can be evaluated easily. We obtain

$$\tilde{j}_s = s - \theta(s - \Delta_{\parallel}) \frac{(s^2 - \Delta_{\parallel}^2)^{3/2}}{s^2 + \Delta_{\perp}^2 - \Delta_{\parallel}^2} \quad (22)$$

and

$$\phi = \theta(s - \Delta_{\parallel}) \frac{(s^2 - \Delta_{\parallel}^2)^{3/2}}{s(s^2 + \Delta_{\perp}^2 - \Delta_{\parallel}^2)} \quad (23)$$

where  $\theta(x)$  is a step function with

$$\theta(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$$

The two results imply that at  $T = 0$  and for  $s < \Delta_{\parallel}$  we obtain  $j_s = s\rho/p_F$  and  $\phi = 0$ . Indeed, it is easy to see that  $s = \Delta_{\parallel}$  is the pair-breaking condition for the superfluid velocity, i.e., pairs only start to be broken for  $s > \Delta_{\parallel}$ .

### 3.2. $T \approx T_c$

In this case the order parameter and hence  $s$  are small compared to  $T_c$ , so that we can expand the gap equation (9a) and  $j_s$ , Eq. (15a), in terms of  $\Delta_\perp$ ,  $\Delta_\parallel$ ,  $s \ll T_c$ . Eliminating  $\lambda$  from the gap equation by subtracting the respective equation at  $T = T_c$ , we find (up to second order)

$$\frac{1}{5}(4\Delta_\perp^2 + \Delta_\parallel^2) = \Delta_0^2 - \frac{2}{5}s^2 \quad (24a)$$

$$\frac{1}{5}(2\Delta_\perp^2 + 3\Delta_\parallel^2) = \Delta_0^2 - \frac{6}{5}s^2 \quad (24b)$$

where

$$\Delta_0(T) = \left( \frac{8\pi^2 T_c^2}{7\zeta(3)} \right)^{1/2} \left( 1 - \frac{T}{T_c} \right)^{1/2}$$

is the order parameter of  $^3\text{He-B}$  near  $T_c$  in the absence of superflow. The two gap equations can easily be solved for  $\Delta_\parallel$  and  $\Delta_\perp$  and yield

$$\Delta_\parallel = \Delta_0^2 - 2s^2, \quad \Delta_\perp^2 = \Delta_0^2 \quad (25)$$

We see that near  $T_c$  the component of the order parameter perpendicular to  $v_s$ ,  $\Delta_\perp$ , is not affected by the flow at all up to this order of the expansion, while  $\Delta_\parallel$  is strongly reduced.

In a similar way the expansion of Eq. (15a) for  $j_s$  yields

$$\tilde{j}_s = \frac{21}{20} \frac{\zeta(3)}{(\pi T_c)^2} s \left( \frac{2}{3} \Delta_\perp^2 + \Delta_\parallel^2 \right) \quad (26)$$

Substituting  $\Delta_\perp$ ,  $\Delta_\parallel$  by means of Eq. (25), we obtain

$$j_s = \frac{21}{20} \frac{\zeta(3)}{(\pi T_c)^2} s \left( \frac{5}{3} \Delta_0^2 - 2s^2 \right) \quad (27)$$

In general, however, the gap equations, etc., have to be treated numerically. For this we have first solved the gap equations self-consistently for given  $s$  and  $T$ . Then  $j_s$  and  $\phi$  (and  $\rho_s/\rho$  for given  $F_1$ ) have been evaluated at the same point of  $(s, T)$  by means of Eqs. (19) and (21). Once this was done we calculated  $v_s$  via Eq. (5) for given  $F_1$  to account for the Fermi liquid corrections. In this way we obtained the values for  $\Delta_\perp$  and  $\Delta_\parallel$  as functions of  $v_s$  for various reduced temperatures as shown in Figs. 1a and 1b. In the former we show the results for  $F_1 = 0$  and in the latter for  $F_1 = 6.04$ , corresponding to zero pressure.<sup>9</sup>

The functional dependence of  $\Delta_\perp$  and  $\Delta_\parallel$  on  $v_s$  is almost identical to that of the order parameters of  $^3\text{He-B}$  in a strong magnetic field, where a similar distortion of the energy gaps occurs. Both  $v_s$  and the magnetic field  $H$  act as a pair-breaking parameter. One of the differences is, however, that at  $T = 0$

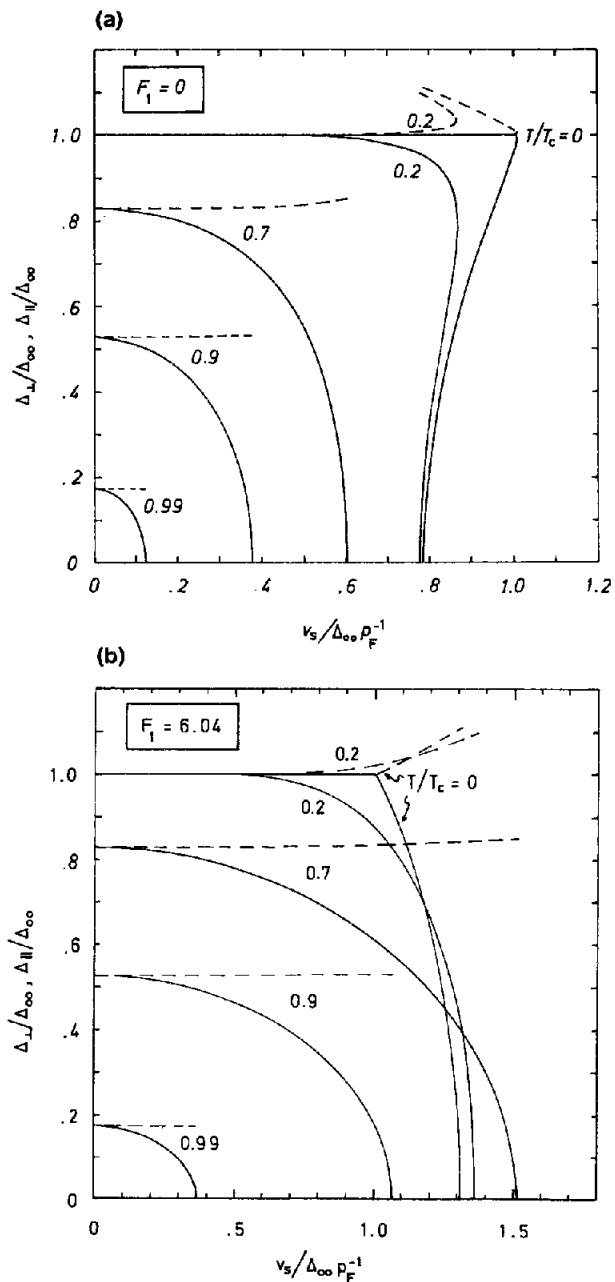


Fig. 1. The components of the order parameter of  $^3\text{He-B}$  parallel and perpendicular to the superflow,  $\Delta_{\parallel}$  and  $\Delta_{\perp}$ , as functions of the superfluid velocity  $v_s$ , for several reduced temperatures  $T/T_c$ . Here  $\Delta_{\parallel}$  is represented by a full and  $\Delta_{\perp}$  by a broken line, where  $\Delta_{00} = \Delta_{\text{BCS}}(T=0)$  and  $p_F$  is the Fermi momentum. (a) No Fermi liquid corrections ( $F_1 = 0$ ), (b)  $F_1 = 6.04$ , corresponding to zero pressure.



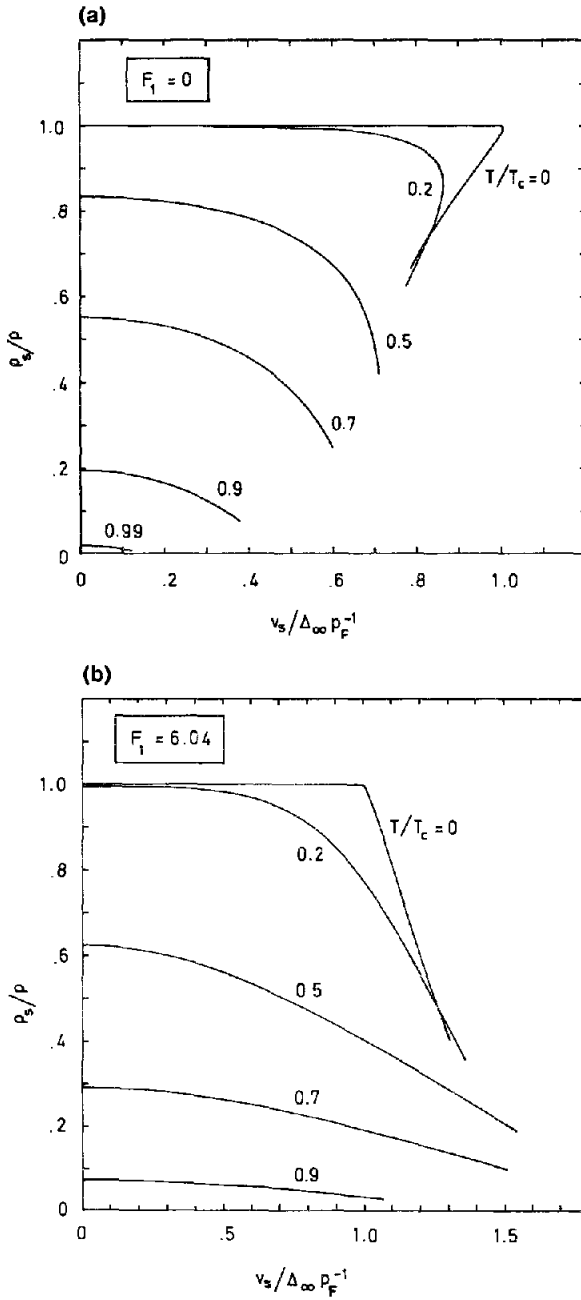


Fig. 2. The superfluid density  $\rho_s$  vs the superfluid velocity  $v_s$  for several reduced temperatures. (a)  $F_1 = 0$ , (b)  $F_1 = 6.04$ .

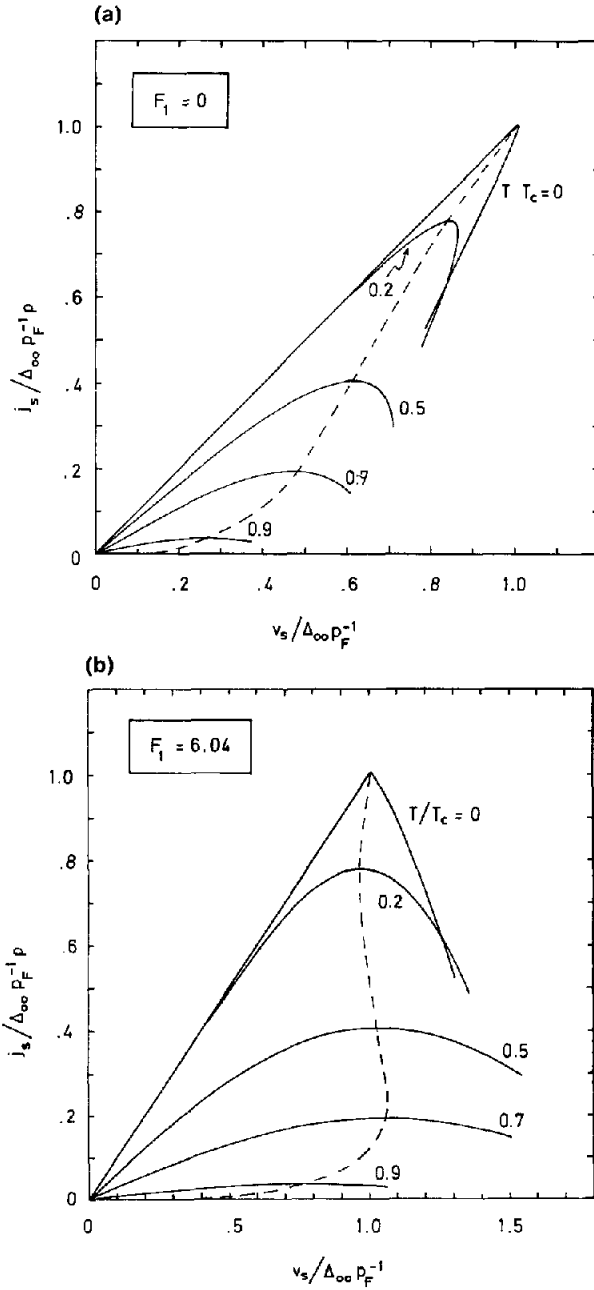


Fig. 3. The superfluid mass current  $j_s$  vs  $v_s$  for several reduced temperatures. The broken lines represent the positions of the maxima of the  $j_s$  curves throughout the whole temperature range. (a)  $F_1 = 0$ , (b)  $F_1 = 6.04$ .

even very small magnetic fields suffice to break pairs, while in the case of superflow the superfluid velocity  $v_s$  has to exceed  $v_s^0 = (\Delta_{\parallel}/\Delta_{00})p_F$  to satisfy the pair-breaking condition. It is interesting to notice how the Fermi liquid corrections change the functional dependence of the energy gaps on  $v_s$ . For  $F_1 = 0$  and  $T/T_c \lesssim 0.5$  both  $\Delta_{\perp}$  and  $\Delta_{\parallel}$  are double-valued for  $v_s > v_{s\parallel}$ , where  $v_{s\parallel}$  characterizes the superfluid velocity at which  $\Delta_{\parallel}$  goes to zero. At this point the B-phase order parameter reduces to that of the planar state, which in our case corresponds to the A-phase state. It should be stressed here that the lower (upper) branch of  $\Delta_{\parallel}$  ( $\Delta_{\perp}$ ) for  $v_s > v_{s\parallel}$  is unlikely to be reached experimentally. The reason is that at the point where  $\Delta_{\perp}$  and  $\Delta_{\parallel}$  turn back, i.e., at the infinite slope (or even slightly before that<sup>5</sup>), the B-phase state becomes energetically unstable against the transition to the A-phase state. The inclusion of the Fermi liquid corrections—which results in a rescaling of the abscissa—removes this double-valuedness, so that the gaps now behave monotonically and are single-valued. A similar behavior has been obtained for  $\rho_s$ , the superfluid density, as shown in Figs. 2a and 2b. The  $\rho_s$  is characteristically nonlinear and all curves terminate at  $v_s = v_{s\parallel}(T)$ , i.e., where  $\Delta_{\parallel} = 0$ . One can see that  $\rho_s$  is strongly reduced by the Fermi liquid corrections except near  $T = 0$  and  $v_s = 0$ , where  $\phi$  is small and the Fermi liquid corrections are thus unimportant. Lastly, we show in Figs. 3a and 3b the superfluid mass current  $j_s$  as a function of  $v_s$ , again for  $F_1 = 0$  and  $F_1 = 6.04$ . At  $T = 0$  and  $v_s < \Delta_{00}/p_F$  no pair-breaking can take place and  $j_s$  is simply given by  $j_s = \rho v_s$ . When  $v_s$  is further increased, competition occurs between this increase of  $j_s$  and the decrease due to pair-breaking, which now sets in. Therefore  $j_s$  has a maximum *after*  $v_s = \Delta_{00}/p_F$  is fulfilled. By means of a broken curve we have indicated the locus of the critical current  $j_{sc}$ , i.e., of the maxima of  $j_s$  at different temperatures. It should be realized that the inclusion of the Fermi liquid corrections only changes the horizontal but not the vertical position of those maxima, so that  $j_{sc}$  is independent of the Fermi liquid corrections.

#### 4. DEPAIRING CRITICAL CURRENT

After we have calculated  $\Delta_{\perp}$ ,  $\Delta_{\parallel}$ ,  $j_s$ , and  $\rho_s$  as functions of  $T$  and  $s$  we now want to determine the depairing critical current  $j_{sc}$  defined by the maximal value of  $j_s$ . Hence,  $j_{sc}$  is determined by

$$\frac{dj_s}{dv_s} = \frac{dj_s}{ds} \frac{ds}{dv_s} = 0 \quad (28)$$

It is not difficult to see that  $ds/dv_s \neq 0$  for all  $v_s$ , so that Eq. (28) reduces to

$$\frac{dj_s}{ds} = \frac{\partial j_s}{\partial s} + \frac{\partial j_s}{\partial \Delta_{\perp}} \frac{\partial \Delta_{\perp}}{\partial s} + \frac{\partial j_s}{\partial \Delta_{\parallel}} \frac{\partial \Delta_{\parallel}}{\partial s} \equiv 0 \quad (29)$$

In principle this last equation can be solved exactly, because  $\partial j_s/\partial s$ ,  $\partial j_s/\partial \Delta_{\perp}$ , and  $\partial j_s/\partial \Delta_{\parallel}$  are easily calculated from Eq. (21) and  $\partial \Delta_{\perp}/\partial s$  and  $\partial \Delta_{\parallel}/\partial s$  can be obtained by taking the derivative of the gap equations. Nonetheless it is easier to determine the maxima of  $j_s$  by numerical localization of the peaks of the  $j_s$  curves. As explained above, the critical current is independent of the Fermi liquid corrections. This can also be understood from Eq. (29), which has exactly one solution for  $s$  (i.e.,  $s_{cr}$ ) for any given temperature irrespective of the Fermi liquid corrections, which are incorporated in  $s$  itself. Then  $s_{cr}$  determines  $j_{sc}$ . The corrections play a role, however, in the calculation of the critical velocity  $v_{sc}$  by means of Eq. (5).

In the vicinity of the transition temperature  $T_c$  the critical current can be determined analytically. From Eq. (27) we find that  $j_s$  has a maximum at  $s_{cr} = (5/18)^{1/2} \Delta_0$ . This leads to a critical velocity of

$$v_{sc} \simeq \left(1 + \frac{1}{3} F_1\right) \frac{2}{5} \sqrt{5} \left(\frac{\pi^2 T_c^2}{7\zeta(3)}\right)^{1/2} \left(1 - \frac{T}{T_c}\right)^{1/2} \quad (30)$$

and the depairing critical current in this limit is

$$j_{sc} = \frac{8\sqrt{5}}{9} \left(\frac{\pi^2 T_c^2}{7\zeta(3)}\right)^{1/2} \left(1 - \frac{T}{T_c}\right)^{3/2} \frac{\rho}{p_F} \quad (31)$$

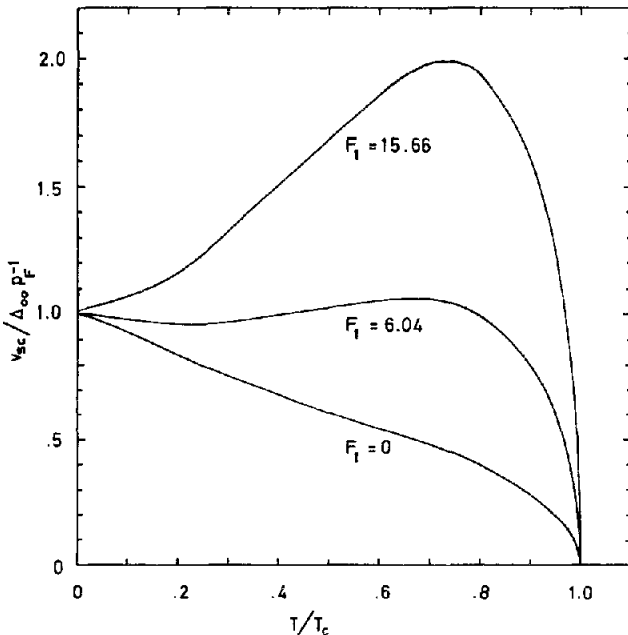


Fig. 4. The critical velocity  $v_{sc}$  as a function of the reduced temperature  $T/T_c$  for several values of the Fermi liquid correction parameter  $F_1$ .

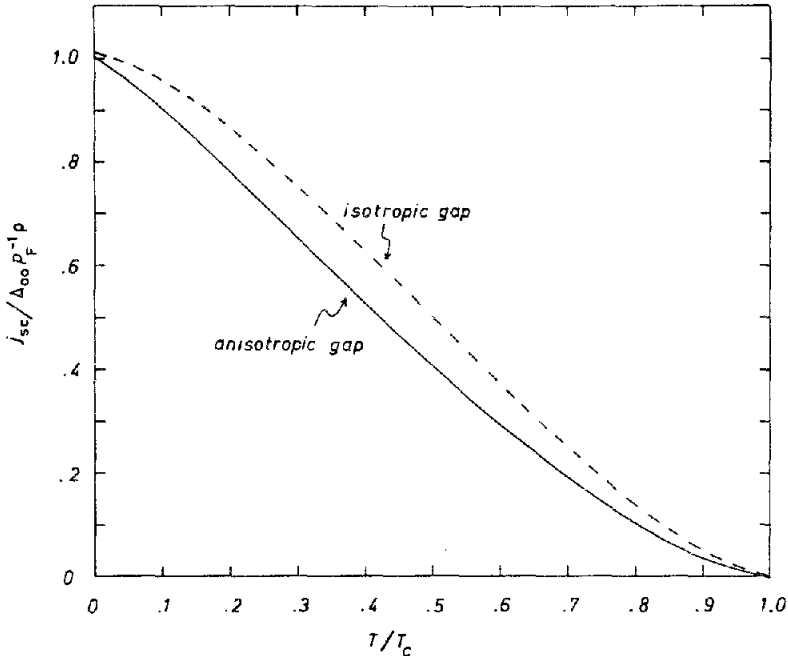


Fig. 5. The pair-breaking critical current  $j_{sc}$  vs the reduced temperature  $T/T_c$ . The solid curve shows the present results, while the broken curve represents the results obtained earlier by VM (see text).

Comparing this result with the one by VM, we see that the present result for  $j_{sc}$  is smaller by a factor  $\sqrt{5}/3 \approx 0.75$ . This value also readily follows from Fetter's earlier result,<sup>2</sup> if we take the weak-coupling limit in his corresponding expression;<sup>11</sup> it has also been recently obtained by Kleinert.<sup>12</sup>

The results for the whole temperature range are shown in Figs. 4 and 5. In Fig. 4 the critical velocity  $v_{sc}$  has been plotted as a function of  $T/T_c$  for various values of  $F_1$ . We see that  $v_{sc}$  is a nonmonotonic function of the temperature when Fermi liquid corrections are included, having a maximum around  $T/T_c = 0.7$ . In contrast to this, the critical current  $j_{sc}$  is independent of the Fermi liquid corrections and is shown in Fig. 5. For a comparison we have included the results of VM by a broken line (labeled "isotropic gap"). It is evident that the present critical current is always lower than the one obtained by leaving the gap isotropic (this result is not unexpected, because in the present calculation a restraint on the system has been removed). However, the difference is always less than 25%.

#### NOTE ADDED IN PROOF

After submitting this paper we have received a preprint of H. Kleinert with a similar calculation of  $j_{sc}$ .

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