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Pinning in Type II Superconductors

A. I. Larkin and Yu. N. Ovchinnikov
L. D. Landau Institute for Theoretical Physics, Academy of Sciences of the USSR, Moscow

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Large and randomly arranged pinning centers cause a strong deformation of a flux line lattice, so that each pinning center acts on the lattice with a maximum force. The average force for such single-particle pinning can be inferred from a simple summing procedure and has a domelike dependence on magnetic field. Pinning centers of average force, such as clusters of dislocations, strongly deform the flux line lattice only in weak fields and in fields close to the critical field, where there is a peak in the dependence of the critical current on magnetic field. In the range of intermediate fields there is a weak collective pinning. A large concentration of weak centers leads to collective pinning in all fields. In this case, near the critical field a critical current peak should be observed. To explain this peak and to define the boundaries between the regions of collective and single-particle pinning the possible break-off of the flux line lattice from the lines of magnetic force should be taken into consideration, which leads to extra softening of the lattice.

1. INTRODUCTION

Type II superconductors in a sufficiently strong magnetic field transfer to a mixed state which is a flux line lattice.[1] The critical current in this state is determined by the pinning force, i.e., by the flux line lattice interaction with inhomogeneities of the sample.[2] There are two types of pinning-force dependence on the magnetic field. In the case of strong pinning this dependence has a smooth, wide maximum at fields $(0.3–0.5)H_{c2}$. In the case of weak pinning usually in a wide range of fields the pinning force weakly depends on magnetic field and only near H_{c2} does it have a narrow and very high maximum.[3–6] In the present paper this interesting "peak effect" is explained theoretically.

To find an average pinning force it is essential that in the presence of pinning centers there is no long-range order present in the flux line lattice.[7] In a certain volume V_c there is a short-range order and vortices are arranged almost periodically. With increasing distance this periodicity is disturbed.

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When a current below the critical value is passing, each of the volumes V_c displaces independently under the Lorentz force for a distance less than the size ξ, so that the pinning force that arises compensates the Lorentz force. Since inside the volume the lattice is almost regular and the pinning centers are randomly distributed, the pinning forces acting upon the lattice from either side of each center compensate each other. The maximum pinning force acting upon the volume V_c is equal to $fN^{1/2}$, where f is the force of interaction of an individual center with a lattice, and N is the number of pinning centers in the volume V_c. In a magnetic field B of the order of H_{c2} each center interacts with the lattice with a force of the order of f; therefore, $N = nV_c$, where n is the density of the pinning centers. The critical current density j is expressed through the pinning force acting upon a unit volume and can be found by the formula

$$Bj = fN^{1/2}V_c^{-1} = f(n/V_c)^{1/2} \tag{1}$$

In the region of magnetic fields where elastic deformations of a flux line lattice are important, the volume V_c has been found in Ref. 7. The order of magnitude of this volume can be found from simple energy considerations. The energy of interaction of one center with the flux line lattice is equal to fa, where a is the lattice parameter. The sign of this energy is determined by the position of the pinning center in a flux line lattice. Therefore, for the randomly distributed inhomogeneities the interaction energy for the volume V_c is equal to $faN^{1/2}$. The volume boundaries are displaced for a distance of the order of a, which leads to an increase in the elastic energy. Thus, the energy change per unit volume δF caused by inhomogeneities is equal to

$$\delta F = C_{66}(a/R_c)^2 + C_{44}(a/L_c)^2 - faN^{1/2}V_c^{-1} \tag{2}$$

where R_c and L_c are the transverse and longitudinal sizes of the region in which there is a short-range order. C_{66} and C_{44} are elastic moduli of the flux line lattice. In formula (2) the term with the compression modulus C_{11} is omitted, since due to the large value of C_{11} the shear deformations of the lattice exceed the compression displacement. Inserting $N = nV_c$ and $V_c = R_c^2 L_c$ into Eq. (2), we find R_c and L_c from the minimum condition of expression (2):

$$R_c = 32^{1/2}\frac{C_{66}^{3/2}C_{44}^{1/2}a^2}{nf^2}; \qquad L_c = \frac{8a^2C_{66}C_{44}a^2}{nf^2}; \qquad V_c = \frac{256a^6C_{44}^2C_{66}^4}{n^3f^6} \tag{3}$$

Substituting this value of V_c into formula (1), we find the expression for the critical current density

$$Bj = n^2f^4/16a^3C_{44}C_{66}^2 \tag{4}$$

Formula (4) has been obtained in Ref. 8. In fields close to H_{c2}, C_{44} tends to a constant limit, $C_{66} \sim (1-b)^2$ $(b = B/H_{c2})$, and the pinning force f in many cases is proportional to Δ^2, i.e., $1-b$. As a result, the critical current depends only slightly on magnetic field.

On approaching H_{c2} two physical effects become important: they limit the region of applicability of elasticity theory and of Eq. 4, which based upon this theory. Both these effects lead to a high maximum in the critical current. One effect arises when the correlation length R_c is compared with the effective penetration depth of the magnetic field $\lambda_{\text{eff}} = \lambda(1-b)^{-1/2}$. Short-scale distortions of the flux line lattice cannot be described by elasticity theory, since the magnetic field does not change at distances lower than its penetration depth and the flux line lattice breaks away from magnetic force lines. As a result, there arises spatial dispersion of the elastic modulus[9] C_{44} and an effective softening of the lattice. In this region V_c decreases exponentially and the critical current thus increases.[10] It stops increasing when the dimension R_c becomes of the order of the lattice parameter a. The transverse dimension of the region is still large and can be found by minimizing the energy (2), but R_c should be substituted for a in this expression, and the modulus C_{44} for its effective value with the spatial dispersion taken into account $\tilde{C}_{44} = C_{44}(1—b)\varkappa^{-2}$, where $\varkappa$ is the Ginzburg–Landau parameter. Then we have

$$V_c = L_c a^2 = a^2(4\tilde{C}_{44}a^2/fn^{1/2})^{2/3} \tag{5}$$

In this formula $\tilde{C}_{44}$ and f are proportional to $(1-b)$, and the volume V_c weakly depends on the magnetic field, whereas the critical current determined by formula (1) tends to zero by a linear law. Such an explanation of the peak effect is valid, provided in the volume V_c determined by formula (5) the number of pinning centers $nV_c \gg 1$ is large.

In the opposite limiting case where the concentration of the pinning centers is small $nV_c \ll 1$, the peak effect arises from other causes. On approaching H_{c2}, the elastic moduli C_{66} and $\tilde{C}_{44}$ decrease and the elastic deformation caused by an individual center increases. At a certain value of the field this deformation becomes of the order of the lattice parameter. In large fields each pinning center plastically deforms the lattice and holds it with a maximum force f. The critical current is thus proportional to the defect concentration

$$Bj = fn \tag{6}$$

In the magnetic field in which plastic deformation appears first, the current determined by formula (6) much exceeds the critical current in weaker fields determined by formula (4). In order to obtain the critical value of the magnetic field corresponding to the arising of plastic deformation and the critical current maximum, one should find the value of deformation **u** caused by one pinning center. Transverse sizes of the region in which deformation is strong are of the order of the lattice parameter a. The longitudinal size and the value of the deformation **u** are found from the minimum condition of the free energy δF per pinning center

$$\delta F = \left(\frac{C_{66}}{a^2} + \frac{\tilde{C}_{44}}{L^2}\right) u^2 a^2 L - fu$$
$$L \sim a(\tilde{C}_{44}/C_{66})^{1/2}; \qquad u \sim f a^{-1}(\tilde{C}_{44} C_{66})^{-1/2} \tag{7}$$

Plastic deformation and the peak effect arise when u becomes of the order of a. Expression (6) for the critical current is valid under the condition that

$$f > a^2 (\tilde{C}_{44} C_{66})^{1/2} \tag{8}$$

Without taking account of the spatial dispersion of the modulus C_{44} the condition (8) was obtained by Labusch.[11]

In the case that condition (8) is satisfied for all fields, the critical current is a smooth, domelike function of the magnetic field and the peak effect is absent. If the force f is small and proportional to $1-b$, the condition (8) is fulfilled only at fields close to H_{c2}, or in weak fields. In this case the peak effect is observed both near H_{c2} and at fields $B \ll H_{c2}$. In some cases the condition (8) is never fulfilled. In such a case the critical current over the entire range of magnetic field is determined by formula (4) and the peak effect is absent.

2. FLUX LINE LATTICE INTERACTION FORCE WITH THE PINNING CENTERS

There are various physical reasons for the arising of inhomogeneities in a superconductor. Particles of another phase, dislocation clusters, grain boundaries in a polycrystalline sample, or an inhomogeneous distribution of impurities may serve as inhomogeneities. As a result, such physical quantities as the electron–phonon interaction constant, the electron free path length, and the density of states of the Fermi surface are random coordinate functions.

Let us first investigate superconductors with random electron interaction

$$g_{(\mathbf{r})}^{-1} = \langle g_{(\mathbf{r})}^{-1} \rangle + g_1(\mathbf{r}) \tag{9}$$

The variation of the free energy δF to first order with respect to g_1 is equal to

$$\delta F = \nu \int d^3\mathbf{r}\, g_1(\mathbf{r}) |\Delta(\mathbf{r})|^2 \tag{10}$$

where $\nu = mp/2\pi^2$ is the density of states on the Fermi surface.

If the distance between the pinning centers exceeds their linear dimensions, it is convenient to introduce the interaction force $\mathbf{f}$ of the flux line lattice with an individual pinning center

$$\mathbf{f}(\mathbf{r}) = \nu \int d^3\mathbf{r}_1 \, g_1(\mathbf{r}_1) \frac{\partial |\Delta(\mathbf{r}+\mathbf{r}_1)|^2}{\partial \mathbf{r}} \tag{11}$$

For pinning centers of size r_0 smaller than the vortex size ξ, we find from formula (11)

$$\mathbf{f}(\mathbf{r}) = \nu \frac{\partial |\Delta(\mathbf{r})|^2}{\partial \mathbf{r}} \int d^3\mathbf{r} \, g_1(\mathbf{r}) \tag{12}$$

For pinning centers of large sizes $r_0 \gg \xi$ the force $\mathbf{f}$ depends essentially on the smoothness of the function $g_1(\mathbf{r})$. For smooth inhomogeneities the force $\mathbf{f}$ is exponentially small. For instance, for $g_1(\mathbf{r}) \sim \exp[-(r/r_0)^2]$, the force $f \sim \exp(-4\pi^2 r_0^2/3a^2)$, where a is the distance between vortices in a triangular lattice. In a real case when grain boundaries are pinning centers, a sharp variation of superconductor parameters is observed at the boundary. Since $|\Delta(\mathbf{r})|^2$ is a periodic coordinate function, the main contribution to the integral in formula (11) produces a layers of vortices tangent to the grain surface. The amplitude of the pinning force $\mathbf{f}$ is proportional to the area of this layer, provided the layer involves many vortices, or to the linear dimension, if there is only one vortex in the layer. The order of magnitude of the force $\mathbf{f}$ is given by

$$f = \nu g_1 \Delta^2 \xi l_z (1 + l_\parallel / a) \tag{13}$$

For ellipsoidal grains

$$l_z \sim l_\parallel \sim (r_0 \xi)^{1/2} \tag{14}$$

If the boundaries of the grains are flat, the angular points serve as pinning centers. In this case

$$l_z = \left(r_0^{-1} + \frac{\cos\theta_1}{\xi} \right)^{-1}; \quad l_\parallel = \left(r_0^{-1} + \frac{\cos\theta_2}{\xi} \right)^{-1} \tag{15}$$

where θ_1 is the angle between the direction of the normal to the surface and the direction of induction, and θ_2 is the angle between the normal to the surface and the elementary cell vector. Formula (11) is valid if the electron interaction is a random quantity. When the other parameters affecting the superconducting transition temperature vary, formula (11) is valid near T_c or near H_{c2} provided account is taken of the fact that g is the effective dimensionless constant of the electron–electron interaction.

In case the random quantity does not lead to a variation of the superconducting transition temperature, the variation of the free energy is connected only with gradients of the order parameter Δ. For instance, with a random variation of the electron free path the addition to the free energy near the transition temperature has the form

$$\delta F = \frac{\pi\nu}{8T} \int d^3\mathbf{r}\, \delta(\eta D)|\partial_-\Delta|^2 \tag{16}$$

where $D = vl_{\text{tr}}/3$ is the diffusion coefficient and

$$\eta(T) = 1 - \frac{8T\tau_{\text{tr}}}{\pi}\left[\psi\left(\frac{1}{2}+\frac{1}{4\pi T\tau_{\text{tr}}}\right) - \psi\left(\frac{1}{2}\right)\right]; \qquad \partial_- = \frac{\partial}{\partial\mathbf{r}} - 2ie\mathbf{A}$$

In the vicinity of H_{c2} for superconductors with a small electron free path ($T_c\tau_{\text{tr}} \ll 1$) the addition to the free energy is

$$\delta F = \frac{\nu}{4\pi T}\psi'\left(\frac{1}{2}+\frac{eBD}{2\pi T}\right) \int d^3\mathbf{r}\, \delta D\, |\partial_-\Delta|^2 \tag{17}$$

In order of magnitude, formula (11) is also valid for a random variation of the electron free path, provided the following substitution is performed:

$$g_1 \to g_1 + \tau\frac{\delta(D\eta)}{D\eta} \to -eD\eta\frac{\delta H_{c2}}{T_c}; \qquad \tau = 1 - \frac{T}{T_c} \tag{18}$$

Formulas (12) and (13) are valid if g_1 is small and the order parameter Δ differs only slightly from its value in a homogeneous sample. Such a situation is probably realized with pinnings on dislocations or on grain boundaries in polycrystals, if the field is not too close to H_{c2}. The value g_1 is not small if particles of other phases fall out of the superconductor matrix. In case H_{c2} particles are smaller than H_{c2} matrices, the suppression of the order parameter leads to a decrease of the effective interaction. For particles of large sizes with $r_0 \gg \xi(T)$, the following replacement should be done in formula (13):

$$g_1^{-1} \to g_{\text{eff}}^{-1} = g_1^{-1} + (\pi^2 T_c^2/\Delta^2) \tag{19}$$

For metal particles of small dimensions with $r_0 < \xi(T)$ the suppression of superconductivity may occur in a volume of the order of $\xi^3(T)$. In this case in formula (12) the following substitution should be carried out:

$$\left(\int g_1\, d^3\mathbf{r}\right)^{-1} \to (g_1 V_0)^{-1} + [S_0\xi(T)(\tau g_1)^{1/2}]^{-1} + \frac{\pi^2 T_c^2}{\Delta^2\xi^3(T)} \tag{20}$$

where V_0 and S_0 are the volume and surface area of a metal particle. If, in the superconductor matrix, particles with a larger value of H_{c2} fall out—dielectric particles or those with low boundary transparency—then superconductivity is stimulated at the boundary. For particles with an increased value of H_{c2} one should substitute in (12) and (13)

$$\Delta^2 \to \Delta_{\text{eff}}^2 = \Delta^2 + g_1 \pi^2 T^2 \frac{V_0}{[V_0 + \xi^3(T)]} \tag{21}$$

The same substitution is also valid for dielectric particles or for particles with low boundary transparency, provided that in formulas (12), (13), and (21) g_1 is substituted for τ.

Thus, the temperature and field force dependences are determined by the type of inhomogeneities and their sizes. If this force is sufficiently large, then a great number of centers act upon the superconductor with the force **f**. A one-particle pinning occurs in this case and the average current is proportional to the force **f**. If the force **f** is small, then a weaker collective pinning takes place. To calculate the average force and, consequently, the transport current, and also to define the range of the one-particle pinning, it is necessary to find the flux line lattice deformation.

3. EQUILIBRIUM EQUATIONS FOR A DEFORMED FLUX LINE LATTICE

Smooth deformations of the flux line lattice can be described with the help of elasticity theory. However, the spatial dispersion of the elastic moduli is important for small wave vectors $K \sim \lambda_{\text{eff}}^{-1}$.[9] This dispersion is connected with the fact that magnetic force lines cannot bend at distances less than penetration depths of λ_{eff}. Therefore, for $K > \lambda_{\text{eff}}^{-1}$ the displacement of the flux line lattice **u** and the correction to the vector potential $\mathbf{A}_1$ should be considered as independent variables. In order to obtain the equations for these quantities let us present the order parameter $\Delta(\mathbf{r})$ in the following form:

$$\Delta(\mathbf{r}) = \Delta_0(\mathbf{r} - \mathbf{u}) \exp [2ie(\mathbf{u}\mathbf{A}_0](1 + S + i\chi)$$
$$\Delta(\mathbf{r}) = \Delta_0(\mathbf{r}) + \Delta_1$$
$$\Delta_1(\mathbf{r}) = -u(\partial/\partial\mathbf{r} - 2ie\mathbf{A}_0)\Delta_0 + \Delta(S + i\chi) \tag{22}$$

The variation of the free energy δF caused by the deformation of the flux line lattice is, to second order with respect to Δ_1 and $\mathbf{A}_1$,

$$\delta F = \frac{\nu}{2} \int d^3\mathbf{r} \int \frac{d\Omega\mathbf{p}}{4\pi} \left\{ \Delta_1^* \left[-\Delta_1 \ln\left(\frac{T_c}{T}\right) + 2\pi T \sum_{\omega > 0} \left(\frac{\Delta_1}{\omega} - \beta_1\right)\right] \right.$$
$$\left. + \Delta_1 \left[-\Delta_1^* \ln\left(\frac{T_c}{T}\right) + 2\pi T \sum_{\omega > 0} \left(\frac{\Delta_1^*}{\omega} - \beta_1^*\right)\right]\right\}$$
$$+ \frac{1}{8\pi} \int d^3\mathbf{r}\, \mathbf{A}_1 \cdot \{\text{rot rot } \mathbf{A}_1 - 4\pi \mathbf{j}_1\} + \int d^3r\, \langle \mathbf{j}_1 \rangle \mathbf{A}_1 \tag{23}$$

with

$$\mathbf{j}_1 = -\frac{iep}{4\pi} T \sum_{\omega} \int \frac{d\Omega \mathbf{p}}{4\pi} \mathbf{p} \alpha_1(\mathbf{p})$$

where $\alpha_1(\mathbf{p})$ and $\beta_1(\mathbf{p})$ are linear corrections with respect to Δ_1 and $\mathbf{A}_1$ to the Green's functions integrated by the energy variable ξ.[12–14] Inserting expression (22) for Δ_1 into formula (23), after averaging over the cell, we get

$$\begin{aligned}\delta F = \frac{k_h^2}{8\pi} \int d^2\mathbf{r} \Big\{ & \frac{1}{k_h^2} (\text{rot } \mathbf{A}_1)^2 + \left(\mathbf{A}_1 + [\mathbf{Bu}] - \frac{1}{2e} \frac{\partial \chi}{\partial \mathbf{r}} \right)^2 \\ & + a_1 \left(A_z - \frac{1}{2e} \frac{\partial \chi}{\partial z} \right)^2 + a_2 \left[\left(\frac{\partial S}{\partial \mathbf{r}} \right)^2 + k_\psi^2 S^2 \right] + a_3 \left(\frac{\partial S}{\partial z} \right)^2 \\ & - a_4 \left(S \frac{\partial \mathbf{u}}{\partial \mathbf{r}} \right) - a_5 S \left(\frac{\partial}{\partial \mathbf{r}} [\mathbf{BA}_1] \right) \Big\} + \int d^3\mathbf{r} \Big\{ \langle \mathbf{j}_1 \rangle \mathbf{A}_1 \\ & + \frac{C_{66}}{2} \frac{\partial u_\alpha}{\partial r_\beta} \frac{\partial u_\alpha}{\partial r_\beta} + \frac{a_6}{2} \left(\frac{\partial \mathbf{u}}{\partial z} \right)^2 \Big\}; \qquad \alpha, \beta = 1, 2 \end{aligned} \tag{24}$$

The coefficients k_h^2, k_ψ^2, and a_i are functions of magnetic field, temperature, and electron free path. In deriving formula (24) the magnetic field has been assumed to vary only slightly over the cell size. Therefore, expression (24) for the free energy holds either for fields close to H_{c2}, or at a large value of the parameter $\varkappa$ for fields $\mathrm{B} > \mathrm{H}_{c1}$. It should be noted that in fields of the order of H_{c1} the spatial dispersion of the elastic moduli is unimportant.

At temperatures close to T_c

$$\begin{gathered} a_1 = a_3 = a_5 = 0; \qquad a_6 = \mathbf{B}(\mathbf{H} - \mathbf{B})/4\pi; \qquad a_2 = 1/4e^2; \qquad a_4 = B/e \\ k_h^2 = \frac{2e^2 p^2 v \tau_{\text{tr}} \eta}{3T} \langle |\Delta|^2 \rangle; \qquad k_\psi^2 = k_h^2 \left[1 - \frac{\partial H}{\partial B} \right]^{-1} \end{gathered} \tag{25}$$

In fields close to H_{c2} for superconductors with a small electron free path we get

$$\begin{gathered} k_h^2 = \frac{8\pi e \nu \langle |\Delta|^2 \rangle}{B} \gamma_1; \qquad k_\psi^2 = \frac{eB \langle |\Delta|^2 \rangle}{2\pi^2 T^2} \frac{\gamma_2}{\gamma_1} \\ a_2 = 1/4e^2; \qquad a_1 = 4e^2 a_3 = \gamma_3/\gamma_1; \qquad a_4 = B/e \\ a_5 = \gamma_3 (eB\gamma_1)^{-1}; \qquad a_6 = \mathbf{B}(\mathbf{H} - \mathbf{B})/4\pi \\ C_{66} = 0.48 \frac{\nu \langle |\Delta^2| \rangle^2 \gamma_2}{32\pi^2 \beta_A T^2} \left\{ 1 - \frac{8e^2 p^2 v^3 \tau_{\text{tr}}^2}{g\pi\gamma_2} \left[\psi' \left(\frac{1}{2} + \frac{eBD}{2\pi T} \right) \right]^2 \right\} \end{gathered} \tag{26}$$

where

$$\begin{aligned}
\gamma_1 &= \psi\Big(\frac{1}{2}+\frac{3eBD}{2\pi T}\Big)-\psi\Big(\frac{1}{2}+\frac{eBD}{2\pi T}\Big) \\
\gamma_2 &= -\beta_A\psi''\Big(\frac{1}{2}+\frac{eBD}{2\pi T}\Big)-\frac{8e^2p^2v^3\tau_{tr}^2}{g\pi}(\beta_A-1)\Big[\psi'\Big(\frac{1}{2}+\frac{eBD}{2\pi T}\Big)\Big]^2 \\
\gamma_3 &= \frac{eBD}{\pi T}\psi'\Big(\frac{1}{2}+\frac{eBD}{2\pi T}\Big)-\gamma_1 \\
\langle|\Delta|^2\rangle &= \frac{8\pi T}{\gamma_2}eD(H_{c2}-B)\psi'\Big(\frac{1}{2}+\frac{eBD}{2\pi T}\Big)
\end{aligned} \tag{27}$$

A complete change of the free energy consists of two parts

$$\delta F = \delta F_n + \delta F_u \tag{28}$$

determined by formulas (10), (15), (17) and (24). Varying the free energy (28) over the parameters χ, S, $\mathbf{A}_1$ and $\mathbf{u}$, we get a system of equations for these quantities. Excluding the phase χ and modulus S of the order parameter, and also the vector potential $\mathbf{A}_1$, we get the equation for the value of the displacement of $\mathbf{u}$:

$$\begin{aligned}
&C_{66}\mathbf{K}_\perp^2\mathbf{u}+C_{44}(\mathbf{K})K_z^2\mathbf{u}+(C_{11}(\mathbf{K})-C_{66})\mathbf{K}_\perp(\mathbf{K}\mathbf{u}) \\
&\quad=(2\pi)^3\delta(\mathbf{K})[\langle\mathbf{j}\rangle\mathbf{B}]+\nu\int d^3\mathbf{r}\exp(-i\mathbf{K}\mathbf{r})g_1(\mathbf{r})\Big\{\frac{\partial|\Delta(\mathbf{r}-\mathbf{u})|^2}{\partial\mathbf{r}} \\
&\quad+\frac{4ieB\mathbf{K}_\perp\cdot|\Delta(\mathbf{r}-\mathbf{u})|^2}{\mathbf{K}^2+K_\psi^2-K_z^2\gamma_3/\gamma_1-k_h^2\mathbf{K}_\perp^2(\gamma_3/\gamma_1)^2(\mathbf{K}^2+k_h^2)^{-1}} \\
&\quad\times\Big(1+\frac{\gamma_3}{\gamma_1}\frac{k_h^2}{\mathbf{K}^2+k_h^2}\Big)^2\Big\}
\end{aligned} \tag{29}$$

where

$$\begin{aligned}
C_{44}(\mathbf{K}) &= \frac{B^2}{4\pi}\frac{k_h^2(1+\gamma_3\gamma_1)}{\mathbf{K}^2+k_h^2+(K_z^2+k_h^2)\gamma_3/\gamma_1}+\frac{\mathbf{B}(\mathbf{H}-\mathbf{B})}{4\pi} \\
C_{11}(\mathbf{K})-C_{66} &= \frac{B^2k_h^2}{4\pi}\Big\{\frac{\mathbf{K}^2+k_h^2(1+\gamma_3/\gamma_1)}{(\mathbf{K}^2+k_h^2)(\mathbf{K}^2+k_h^2+(K_z^2+k_h^2)\gamma_3/\gamma_1)} \\
&\quad-\frac{[1+k_h^2\gamma_3\gamma_1^{-1}(\mathbf{K}^2+k_h^2)^{-1}]^2}{\mathbf{K}^2+K_\psi^2+K_z^2\gamma_3\gamma_1^{-1}-k_h^2\mathbf{K}_\perp^2(\gamma_3/\gamma_1)^2(\mathbf{K}^2+k_h^2)^{-1}}\Big\}
\end{aligned} \tag{30}$$

In the vicinity of the transition temperature the expression for the moduli coincides with the corresponding expressions obtained by Brandt.[9] The modulus C_{11} is large compared to C_{66} and the compression deformation is thus small in comparison with the shear deformation. Below we consider only the transverse component $\mathbf{u}_\mathbf{K}$, $(\mathbf{K}\mathbf{u}_\mathbf{K})=0$. If inhomogeneities present a number of separate pinning centers, then Eq. (29) acquires the form

$$C_{66}\mathbf{K}_\perp^2\mathbf{u}+C_{44}(\mathbf{K})K_z^2\mathbf{u}=(2\pi)^3\delta(\mathbf{K})[\langle\mathbf{j}\rangle\mathbf{B}]+\sum_i \mathbf{f}_i \exp(-i\mathbf{K}\mathbf{r}_i) \tag{31}$$

where the force $\mathbf{f}$ is given by formulas (12) and (13).

4. SINGLE-PARTICLE PINNING

The interaction force of an individual pinning center with the flux line lattice depends on the difference $\mathbf{r}-\mathbf{u}$, where $\mathbf{r}$ is the coordinate of the pinning center relative to the nondeformed lattice and $\mathbf{u}$ is the displacement of the lattice at the site of the pin location. From formula (31) the equation for the value of this displacement follows:

$$\mathbf{u}=C^{-1}\mathbf{f}(\mathbf{r}-\mathbf{u}); \qquad C^{-1}=\int\frac{d^3\mathbf{K}}{(2\pi)^3}[C_{66}\mathbf{K}_\perp^2+C_{44}(\mathbf{K})K_z^2]^{-1} \tag{32}$$

The limits of integration in formula (29) depend on the size of the pinning center. For a small pinning center the integration with respect to the wave vector $\mathbf{K}$ proceeds over the cell volume in an inverse lattice. When the pinning force $\mathbf{f}$ is weak or the rigidity of the lattice C is strong, the displacement $\mathbf{u}$ is small and uniquely determined by the coordinate $\mathbf{r}$. In this case the force $\mathbf{f}(\mathbf{r}-\mathbf{u})$ does not differ from the force $\mathbf{f}(\mathbf{r})$ in magnitude and is a periodic function of $\mathbf{r}$. A typical graph of this function is given in Fig. 1. The critical current is determined by the value of the function $\mathbf{f}(\mathbf{r}-\mathbf{u})$ averaged over positions of pins $\mathbf{r}$. In an approximation of independent pinning centers this force is zero. As is shown below, the average force in this case is defined by collective effects and is much less than $\mathbf{f}$. Another case is realized for a large force $\mathbf{f}$ or for a low rigidity of the lattice C. If the maximum value of the displacement $\mathbf{u}$ exceeds some critical value u_c, the dependence of $\mathbf{u}(\mathbf{r})$ and, consequently, of $\mathbf{f}(\mathbf{r}-\mathbf{u})$ on $\mathbf{r}$ becomes multivalued. A graph of this function is indicated in Fig. 2 and can be easily obtained from the graph shown in Fig. 1 by shifting each point along the abscissa by a value proportional to the ordinate. In this case the average force depends on prehistory, i.e., there is a hysteresis. For instance, when the lattice shifts to the left, the force dependence on the position of the center is depicted by the solid curve in Fig. 2. The average force is positive and proportional to the number of pins n_{eff} in metastable states:

$$Bj=f_{\max}n_{\text{eff}} \tag{33}$$

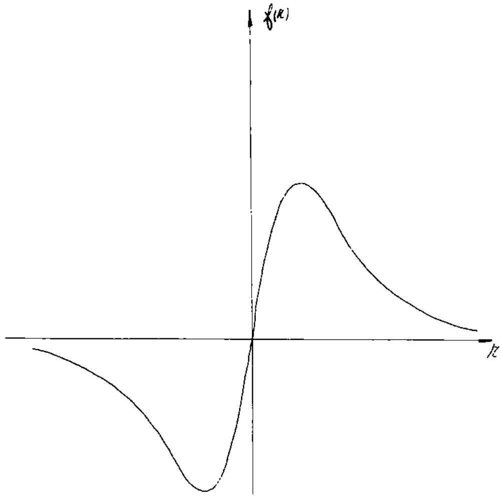

Fig. 1

The critical value of the displacement u_c with the nonzero n_{eff} is equal in order of magnitude to the radius of the pinning force action r_f. Therefore, the condition of applicability of formula (33) is the following:

$$u > r_f; \qquad f > Cr_f \tag{34}$$

If the spatial dispersion of the modulus C_{44} is not taken into account in the

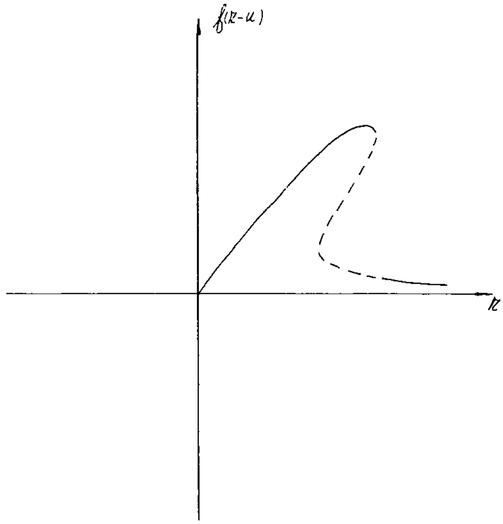

Fig. 2

evaluation of the integral (32) for C, condition (34) coincides with the Labusch criterion.[11] However, $K \sim a^{-1}$ is important in the integral. Thus, in all cases, besides requiring fields close to H_{c1}, we have $C_{44}(\mathbf{K}) \ll C_{44}(0)$ and condition (34) is weaker than the Labusch criterion. For pinning centers whose size is less than ξ, $K_\perp \sim a^{-1}$ is important in integral (32). Substituting expression (30) for the coefficiennt C_{44} into formula (32) and performing the integration over K, we get

$$C = a^2 B k_h C_{66}^{1/2} [\ln (a/\xi)]^{1/2}/\pi \tag{35}$$

In magnetic fields $B \sim H_{c2}$ expression (35) for the coefficient C coincides with the corresponding expression in Ref. 15. If spatial dispersion is not taken into account, condition (34) is not fulfilled for small particles.[16] The spatial dispersion leads to a strong decrease of the modulus C_{44} and condition (34) can be fulfilled for small particles in weak fields $B < H_{c2}$ at least. If superconductivity is suppressed in the particles, then condition (34) is fulfilled at $b < b_p$,

$$b_p \ln\left(\frac{2.7}{b_p^{1/2}}\right) = \left[1 + \frac{\tau \xi^3}{V_0 g_1} + \frac{\xi^2}{S_0}\left(\frac{\tau}{g_1}\right)^{1/2}\right]^{-2} \tag{36}$$

If superconductivity is stimulated by the inhomogeneity, the vortices may flow around the inhomogeneity. In this case the single-particle pinning for small particles is apparently absent. The dielectric particles or small void attract vortices in magnetic fields which are not too close to H_{c2}; $V\xi^{-3} < 1 - b$. Therefore, condition (34) may be fulfilled only in weak fields:

$$b < b_p = (V/\xi^3)^2 / \ln (\xi^3/V) \tag{37}$$

For large defects the region of applicability of formula (33) is considerably extended. Each such defect captures and holds many vortices. This leads to an increase of the the force $\mathbf{f}$ determined by formula (13). The value of the displacement $\mathbf{u}$ can be found from Eq. (29). For large defects both the shear deformations and the compression deformations should be taken into account:

$$\mathbf{u} = \int\int\int \frac{dK_\parallel \, dK_\perp \, dK_z}{(2\pi)^3} \left\{ \frac{K_\parallel^2}{K_\parallel^2 + K_\perp^2} [C_{66}(K_\perp^2 + K_\parallel^2) + C_{44}(\mathbf{K}) K_z^2]^{-1} \right.$$
$$\left. + [C_{11}(\mathbf{K})(K_\perp^2 + K_\parallel^2) + C_{44}(\mathbf{K}) K_z^2]^{-1} \right\} \mathbf{f}(\mathbf{K}) \tag{38}$$

On the right-hand side of Eq. (29) distances of the order of ξ are important for the direction normal to the plane tangent to the defect, and distances l_z and $l_\parallel$ in the directions lying in this plane. Therefore, the integrals with respect to K in Eq. (38) should be cut off by $K_z \sim \pi/l_z$ and $K_\parallel \sim \pi/(a + l_\parallel)$.

Performing the integration, we get

$$u = f\left\{\frac{1}{2\pi C_{66} l_z}\ln\left[1+\frac{l_z(4\pi C_{66})^{1/2}}{B(l_\parallel + a)}\left(1+\frac{\pi}{k_h\{l_\parallel + a[\ln(a/\xi)]^{1/2}\}}\right)\right]\right.$$
$$\left.+\frac{4\pi}{B^2 k_h^2 a(l_\parallel + a)}\frac{[K_\psi^2+(\pi/a)^2]^{1/2}}{K_\psi a[\ln(a/\xi)]^{1/2}}\tan^{-1}\frac{a[\ln(a/\xi)]^{1/2}[K_\psi^2+(\pi/a)^2]^{1/2}}{l_z K_\psi}\right\} \tag{39}$$

For the normal metal particles the force f is determined by formulas (13) and (19). Using this expression, we can be convinced that the condition (34) is fulfilled, provided the dimension of particles $l > \varkappa\xi(1-b)^{-1}$. For particles with the dimension $\xi < l < \varkappa\xi$ condition (34) is fulfilled in fields $B \ll H_{c2}$. In fields $B \sim H_{c2}$ formula (39) gives $u \sim \xi$. Apparently, metastable states and single-particle pinning occur in this case. But only numerical calculations can provide a final answer to this question.

If the dielectric phase falls out, or voids are formed, or there is a low transparency of the grain boundaries, then condition (34) is fulfilled in all magnetic fields, if the size of the grains is $l > a$.

In all cases when condition (34) is fulfilled, in a wide range of magnetic fields, the dependence of the value Bj on magnetic field has the form of a flat dome. In fields of the order of H_{c2} the effective concentration n_{eff} is of the order of the total concentration of the pinning centers. Therefore, with the domelike dependence of the critical current, there is a maximum at $B \sim (0.3\text{–}0.5)H_{c2}$, which, irrespective of the model, is equal to

$$jB \sim \frac{\nu\Delta^4 l^2}{\pi^2 T_c^2} n = nl^2 \frac{H_{c2}^2}{\pi^3 \varkappa^2} \tag{40}$$

For the particles of a normal metal the decrease of current while approaching H_{c2} is connected with decreasing Δ and $j \sim (1-b)^2$. For the dielectric particles Δ does not tend to zero while approaching H_{c2}. In this case the decrease of the current is determined by vortices flowing around the defects. With decreasing field the average force diminishes, which is connected with a simultaneous decrease of both the effective concentration n_{eff} and the force of interaction f with one of the pinning centers. At a large distance between the vortices some pinning centers are free of vortices. Thus,

$$n_{\text{eff}} = n\left[1+\frac{a^2}{u(\xi + l_\parallel)}\right]^{-1} \tag{41}$$

For the large-radius particles $n_{\text{eff}} < n$ only in very weak fields. For such particles a decrease of jB in weak fields is caused by the diminishing of the number of vortices confined by one pinning center. From formulas (13), (19), (33), and (41) we find

$$jB = \frac{H_{c2}^2}{\pi^4 \varkappa^2} l_z \left(l_\parallel + \frac{\pi\xi}{b^{1/2}} \right) b^{1/2} n \left[1 + \frac{\xi}{l_z l_\parallel} \left(\frac{1}{\varkappa\xi} + \frac{1}{l_\parallel + \pi\xi b^{-1/2} [\ln(\pi/b^{1/2})]^{1/2}} \right)^{-1} \right]^{-1} \tag{42}$$

Let us consider the pinning caused by dislocation clusters. In such clusters the effective electron free path is less than that in a nondeformed sample. Therefore, it is possible to assume that pinning centers are large regions $r_0 > \xi$ with an increased value of the critical magnetic field H_{c2}. The value g_1 determined from formula (18) is small in this case ($g_1 < \tau$). The condition (34) is fulfilled at any magnetic field only for the case of a large dislocation cluster. In case the size of these clusters satisfies the inequality

$$\xi < l < \varkappa\xi\tau g_1^{-1} \tag{43}$$

the inequality (34) is fulfilled only in weak fields and in those close to H_{c2}. In an intermediate region of magnetic fields, where condition (34) is not fulfilled, the critical current is small.

Using expression (39) for the displacement u, we find the values of the magnetic field b_p when the current has a maximum near H_{c2}:

$$1 - b_p = \frac{g_1}{\tau} \left[1 + \left(\frac{\xi^2}{l^2} \frac{\tau}{g_1} \right)^{1/3} \right]^{-1} \tag{44}$$

From expression (33) we find the value of the maximum critical current

$$jB \approx n \frac{H_{c2}^2}{\pi^2 \varkappa^2} \left(\frac{g_1}{\tau} \right)^2 l_z l_\parallel \tag{45}$$

If the pinning is connected with the free path inhomogeneity, the value g_1 in formulas (44) and (45) is proportional to $T_c - T$. In this case the critical current is proportional to $(T_c - T)^2$. For sufficiently large clusters the position of the maximum determined by formula (44) is temperature independent. For smaller defects the position of the maximum is displaced toward the side of large fields. In a narrow vicinity of T_c the variation of the effective interaction may turn out to be important. In this case there is a temperature-independent term in g_1. As a result, with increasing temperature the maximum in the current is displaced to the side of lower fields. Both types of temperature dependence of the maximum position of the critical current have been observed in experiment.[5,6,17] In superconductors with a large $\varkappa$, a peak may be observed in weak fields $H_{c1} < B \ll H_{c2}$. The position of the peak is also determined by formula (39):

$$b_p = \frac{g_1}{\tau} \left(1 + \frac{l}{\varkappa\xi} \right) \left[1 + \frac{\tau}{g_1} \frac{\pi^2 \xi^2}{l^2} \ln \left(\frac{\pi^2 \tau \xi}{g_1 l} \right) \right]^{-1} \tag{46}$$

The critical current at the maximum point is equal to

$$jB = n(H_{c2}^2/g\pi\varkappa^2)(g_1/\tau)l_{\|}l_z(l_{\|}+a)(\xi/a)(l_{\|}+a^2/\xi)^{-1} \tag{47}$$

The critical current between maxima is small and determined by collective effects.

5. COLLECTIVE PINNING

Let us consider now the case of weak pinning when an individual pinning center causes a weak deformation of the flux line lattice.[10] If a current is flowing, a large region of the flux line lattice containing many pinning centers displaces as a whole. Each such region may be considered as one large pinning center. These regions may be in metastable states, provided the relative shifts of the flux line lattice are of the order of the action force radius r_f. In order to obtain the volume of such a region we calculate the correlation function of displacements at distances much greater than the lattice period. From Eqs. (29) we find

$$\begin{gathered}\langle[\mathbf{u}(\mathbf{r})-\mathbf{u}(0)]^2\rangle=\int\frac{d^3\mathbf{K}}{(2\pi)^3}W(\mathbf{K})(1-\cos\mathbf{Kr})[C_{66}\mathbf{K}_\perp^2+C_{44}(\mathbf{K})K_z^2]^{-2}\\ W(\mathbf{K})=\nu^2\int d^3\mathbf{r}\exp(-i\mathbf{Kr})\,\varphi(\mathbf{r}_1)\left\langle\frac{\partial|\Delta(\mathbf{r})|^2}{\partial\mathbf{r}}\,\frac{\partial|\Delta(\mathbf{r}+\mathbf{r}_1)|^2}{\partial\mathbf{r}}\right\rangle\end{gathered} \tag{48}$$

where $\varphi(\mathbf{r})=\langle g_1(\mathbf{r})g_1(0)\rangle$ is the correlation function of inhomogeneities. The function $W(0)$ is expressed through the force $\mathbf{f}$ introduced earlier [formulas (11), (12), and (18)]:

$$W(0)=n\langle\mathbf{f}^2(\mathbf{r})\rangle \tag{49}$$

where n is the concentration of pinning centers.

Calculating the integrals in formula (48) with a logarithmic accuracy, we have

$$\langle[\mathbf{u}(\mathbf{r})-\mathbf{u}(0)]^2=\frac{W(0)}{4\pi^{1/2}BC_{66}^{3/2}}\left\{\left(\rho^2+\frac{4\pi C_{66}}{B^2}z^2\right)^{1/2}+\frac{1}{2k_h}\ln\left(\frac{\rho^2}{\xi^2}+\frac{4\pi C_{66}z^2}{B^2k_h^2\xi^4}\right)\right\} \tag{50}$$

Regions of the vortex lattice in which relative shifts are less than the action force radius r_f [formula (11)] will be called correlated regions.

The linear dimension $\mathbf{R}_c$ of correlated regions is determined by Eq. (50),

$$\langle[\mathbf{u}(\mathbf{R}_c)-\mathbf{u}(0)]^2\rangle=r_f^2 \tag{51}$$

In many cases of small defects, magnetic fields close to H_{c2} and defects with sharp edges, the action force radius r_f coincides with the size of the vortex ξ

in order of magnitude. Regions whose relative shifts exceed r_f can independently follow their pinning centers. Therefore, the average force acting upon the flux line lattice from the side of inhomogeneities is determined by averaging Eq. (29) over such regions with volume V_c. Inside such a region the lattice may be assumed to be nondeformed. On averaging Eq. (29), we get

$$j_c^2 B^2 \approx W(0)/V_c \tag{52}$$

where V_c is the volume of the region determined by Eq. (51). If the value of R_c determined by formula (51) satisfies the condition

$$R_c k_h \gg 1$$

then the spatial dispersion of elastic moduli in the integral (48) is unimportant. The size of the correlation region in this case is equal to

$$R_c = \frac{4\pi^{1/2} B C_{66}^{3/2} r_f^2}{W(0)}; \qquad L_c = \frac{B R_c}{(4\pi C_{66})^{1/2}} = \frac{2B^2 C_{66} r_f^2}{W(0)} \tag{53}$$

From formulas (52) and (53) we derive the expression for the critical current

$$j_c B = W_{(0)}^2 / 10 B^2 C_{66}^2 r_f^3 \tag{54}$$

Expression (54) for the critical current coincides with the results of Ref. 8.

In another limiting case

$$\xi < R_c < k_h^{-1}$$

in the integral (48), $k_\perp \gg k_h$ is important, for which the elastic modulus C_{44} is

$$C_{44}(\mathbf{K}) = \frac{B^2 k_h^2}{4\pi} \frac{1 + a_1}{\mathbf{K}_\perp^2} \tag{55}$$

In this region the correlator determined by formula (50) increases logarithmically with distance. Therefore, the correlation volume V_c and the critical current $\mathbf{j}_c$ depend exponentially on magnetic field[10]

$$j_c \sim \exp\left(-b \frac{4\pi^{1/2} B C_{66}^{3/2} k_h r_f^2}{W(0)} \right) \tag{56}$$

where b is a number of the order of unity. The current stops increasing exponentially when the correlation radius R_c becomes of the order of the lattice parameter a. Then the problem becomes one-dimensional. The integral determining the correlation radius L_c diverges in powers at small K. Evaluating L_c from the viewpoint of dimensions, we obtain

$$L_c = \pi \left[\frac{B^4 k_h^4 a^6 r_f^2}{(2\pi)^4 W(0)} \right]^{1/3}; \qquad j_c B = \frac{1}{a} \left(\frac{W(0)}{L_c} \right)^{1/2} \tag{57}$$

Let us examine now the dependence of the critical current on magnetic field determined by formulas (54), (56), and (57). The values $W(0)$, C_{66}, and k_h^2 obey the proportionalities

$$W(0) \sim b(1-b)^2; \qquad C_{66} \sim b(1-b)^2; \qquad k_h^2 \sim (1-b) \tag{58}$$

where $b = B/H_{c2}$. From formulas (58) it follows that in a wide range of magnetic fields, of the order of H_{c2}, the critical current is determined by formula (54) and depends smoothly on the magnetic field value. On approaching H_{c2} the plateau is substituted by a region of exponential growth described by formula (56). In the magnetic field determined by the equality

$$r_f^2 = W(0)/(4\pi^{1/2} B C_{66}^{3/2} k_h) \tag{59}$$

the critical size becomes of the order of the cell parameter and the current achieves a maximum value. With a further increase of field the current drops in proportion to $(1-b)$. In superconductors with large $\varkappa$ there is a wide range of weak magnetic fields in which $j_c B \sim b^{-2}$. With a further decrease of field it is possible to observe a region of exponential current growth determined by formula (56). The position of the current maximum, as in the case of large fields, is determined by formula (59). With a further decrease of field, the current, as follows from formula (57), does not change. It should be noted that in the case of a collective pinning, weak inhomogeneities are characterized by one parameter $W(0)$ only, which defines the current on the plateau, the values of the current at the maxima, and their positions.

6. PINNING IN FILMS

The peak effect in films in fields close to H_{c2} is not usually observed. In the case of a collective pinning in massive samples the peak effect is connected with the spatial dispersion of the elastic modulus $C_{44}(K)$. In a film the elasticity of the flux line lattice is described only by one modulus C_{66}, possessing weak dispersion only, and this reason for the peak effect is thus absent.

In order to find the critical current for the case of collective pinning, we calculate the size of the region in which there is a short-range order. In a three-dimensional case this size is determined by formulas (3) and (53). Analogously, for the film we get

$$R_c = \xi C_{66} W_{(0)}^{-1/2} = C_{66} a / f n^{1/2} \tag{60}$$

where $W(0)$ is determined by formula (49), in which n is the number of defects per unit area of film. The critical current is found from formula (52) with $V_c = R_c^2$,

$$j_c B = W(0)/\xi C_{66} \tag{61}$$

This expression has been obtained in Ref. 8. With a large concentration of defects ($n\xi^2 \gg 1$), collective pinning is realized at any magnetic field. Formula (61) is valid for a wide range of magnetic fields, until $R_c > a$. In this region the quantity Bj only slightly depends on the magnetic field value. The conditions $R_c > a$ is not fulfilled only in the vicinity of H_{c2} and in the region of weak magnetic fields. In such weak magnetic fields the average current only slightly depends on the value of the magnetic field. The corresponding expression has been obtained in Ref. 18. For a low concentration of defects, $na^2 < 1$, formula (61) still holds for the intermediate region of magnetic fields, provided defects are rather weak. In weak fields and in fields close to H_{c2} the condition of single-particle pinning (34) is fulfilled. For the film this condition has the form

$$r_f = \xi < (f/C_{66}) \ln (1/n\xi^2) \tag{62}$$

In fulfilling condition (62) the critical current is determined by formula (33) of the single-particle pinning. In the vicinity of the critical field H_{c2}

$$j_c B = nf \tag{63}$$

At the point where condition (62) is first fulfilled, the critical current determined by formula (62) exceeds logarithmically the critical current determined by formula (61). Therefore, a slight maximum can be observed in the current dependence on magnetic field. With a further increase of magnetic field the current determined by formula (63) drops in proportion to $(1-b)$. In the close vicinity to H_{c2} formula (11) obtained by the perturbation theory for the force f cannot be applied and is substituted by Eq. (20). In this region vortices flow around the pinning centers and the critical current is

$$j_c B = nC_{66}\xi \tag{64}$$

On approaching H_{c2} the critical current determined by formula (64) diminishes as $(1-b)^2$.

7. CONCLUSION

The results obtained above refer to the case when all the pinning centers are approximately of the same magnitude. If there are defects with essentially differing pinning forces, then in the region of a single-particle pinning their contribution to the critical current is summed independently. In the vicinity of H_{c2} the position of the peak is defined by the force of interaction of the defect with the flux line lattice. Therefore, when there are two types of pinning centers with different concentration and force of interaction with the flux line lattice, there should be observed two peaks in the vicinity of H_{c2}. Such a phenomenon has been observed experimentally in Ref. 19.

In massive samples the collective pinning is much less than the single-particle one. Further, the collective pinning thus becomes unobservable if there is at least a small concentration of strong pinning centers in the sample. Apparently, this elucidates the experimental data analysis carried out by Kramer.[20] The experimental data agree well qualitatively with the results of the present paper. For strong pinning centers the dependence of the critical current on magnetic field has a smooth, domelike shape. For pinning centers with an intermediate force, the peak effect is observed near H_{c2}. However, the Labusch criterion recalculated for the mean pinning center is not fulfilled in most cases even if the spatial dispersion of the modulus is taken into account. In this case the single-particle pinning is likely to be determined by a small number of strong pinning centers.

There is not much difference between collective and single-particle pinning forces in a film. Therefore, for weak defects in a film, collective pinning should occur in a film over a wide range of magnetic field. This has been confirmed by experiment.[21]

In the case of collective pinning the dependence of the critical current on magnetic field and temperature is defined by the only parameter, the correlation function W. With single-particle pinning the results are more sensitive to the type of pinning centers. However, no problem of summing up exists in this case. And in each particular case when the characteristics of the defects are well known, it is possible to obtain for the critical current not only the qualitative results estimated above, but the quantitative formulas as well.

REFERENCES

1. A. A. Abrikosov, *Zh. Eksp. Teor. Fiz.* **32**, 1442 (1957).
2. P. W. Anderson, *Phys. Rev. Lett.* **9**, 309 (1962).
3. M. A. R. Le Blanc and W. A. Little, in *Proc. 7th Inter. Conf. on Low Temp. Phys.* (University of Toronto Press, 1961), p. 362; T. G. Berlincourt, R. R. Hake, and D. H. Leslie, *Phys. Rev. Lett.* **6**, 671 (1971).
4. M. Steingart, A. G. Purz, and E. J. Kramer, *J. Appl. Phys.* **44**, 5580 (1973).
5. S. Borka, I. I. Goncharov, D. Frichevski, and I. S. Khukhareva, *Fiz. Nizkikh* Temp. **3**, 716 (1977).
6. L. Ya. Vinnikov, V. I. Grigor'ev and O. V. Zharikov, *Zh. Eksp. Teor. Fiz.* **71**, 252 (1976).
7. A. I. Larkin, *Zh. Eksp. Teor. Fiz.* **58**, 1466 (1970).
8. A. I. Larkin and Yu. N. Ovchinnikov, *Zh. Eksp. Teor. Fiz.* **65**, 1704 (1973).
9. E. H. Brandt, *J. Low Temp. Phys.* **26**, 709 (1977).
10. A. I. Larkin and Yu. N. Ovchinnikov, *Pisma Zh. Eksp. Teor. Fiz.* **27**, 301 (1978).
11. R. Labusch, *Cryst. Lattice Defects* **1**, 1 (1969).
12. G. Eilenberger, *Z. Phys.* **214**, 195 (1968).
13. A. I. Larkin and Yu. N. Ovchinnikov, *Zh. Eksp. Teor. Fiz.* **55**, 2262 (1968).
14. Yu. N. Ovchinnikov, *Zh. Eksp. Teor. Fiz.* **66**, 1100 (1974).
15. R. Schmucher and E. H. Brandt, *Phys. Stat. Sol.* (*b*) **79**, 479 (1977).
16. A. M. Campbell and J. E. Evetts, *Critical Currents in Superconductors* (London, 1972).
17. K. E. Osborne and E. J. Kramer, *Phil. Mag.* **29**, 685 (1974).

18. A. I. Larkin and Yu. N. Ovchinnikov, *Zh. Eksp. Teor. Fiz.* **61**, 1221 (1971).
19. S. Borka, I. N. Goncharov, D. Frichevski, and I. S. Khukhareva, *Fiz. Nizkikh Temp.* **3**, 598 (1977).
20. E. J. Kramer, MSC Report-2857; *J. Appl. Phys.* **49**, 742 (1978).
21. A. A. Moshenskii, N. Ya. Fogel', A. S. Sidorenko, A. M. Gluchov, I. M. Dmitrienko, L. P. Tishchenko, and Ya. M. Fogel', *Fiz. Nizkikh Temp.* **2**, 685 (1976).