

Microscopic Derivation of the Hydrodynamic Equations for the Superfluid Fermi System: Application to the Two-Fluid Hydrodynamic Equations in Superfluid $^3\text{He-B}$

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A microscopic theory for deriving hydrodynamic equations for Fermi superfluids is developed and applied to the two-fluid hydrodynamic equations in superfluid $^3\text{He-B}$ near the transition temperature. It is shown how the two-fluid hydrodynamic equations as well as the Boltzmann-type transport equations for the Bogoliubov-Valatin quasiparticles are derived from the matrix kinetic equation. Special attention is paid to the derivation of the particle number conservation law and also to the definition of the chemical potential when the system deviates from local equilibrium.

1. INTRODUCTION

The dynamical properties of Fermi superfluids have been investigated by many authors (see Ref. 1 for a recent review). One can expect that, when the wave number q and the frequency ω of the disturbances are much smaller than, respectively, the inverse coherence length $1/\xi$ and the energy gap Δ (the so-called macroscopic limit), the thermal excitations, in this case the Bogoliubov-Valatin quasiparticles, are decoupled from the superfluid condensate; the configuration of the quasiparticles is described by Boltzmann-type transport equations. From this aspect, various transport coefficients in superfluid ^3He have been computed and show good agreement with experiment^{2-11,35}.†

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†The calculation of the transport coefficients based on other methods also has been attempted.^{12,13}

The theoretical microscopic basis for the above procedure also has been discussed by some authors. Kinetic equations for superconductors without Fermi liquid effects have been derived by Stephen¹⁴ and by Kemoklidze and Pitaevskii.¹⁵ Stephen obtained the local equilibrium solution for the matrix Green's function and derived the Landau two-fluid hydrodynamic equations. A very transparent matrix formulation which can incorporate the Fermi liquid effects has been given by Betbeder-Matibet and Nozières¹⁶ (BN), which has been applied to investigate sound propagation¹⁷ and spin dynamics¹⁸⁻²⁰ in superfluid ³He. However, BN theory is applicable only to the two extreme limits, i.e., the collisionless limit and the hydrodynamic limit. In order to treat dissipative effects, it is necessary to introduce some phenomenological form of the collision term.^{19,20}

A general theory for deriving the kinetic equation including collisions has been given by Wölfle²¹ by the use of the real time response formalism developed by Kadanoff and Baym²⁵ for a normal system. His theory is valid over all the frequency range, including the deformation of the order parameter. In fact, Wölfle²² has succeeded in explaining quantitatively the zero-sound absorption in the high-frequency range. However, his theory is restricted to the linear regime and he has not attempted to derive the whole set of two-fluid hydrodynamic equations directly from the matrix equation, although Wölfle⁶ and Einzel and Wölfle^{7,8} have computed various transport coefficients which appear in the phenomenological theory by use of the simplified version of kinetic equations.

A systematic derivation of the two-fluid hydrodynamic equations has been given by Galaiko²³ and Shumeiko.²⁴ Their theories are, however, restricted to the *s*-wave BCS superfluid, and no Fermi liquid effects were considered. We shall show that in the presence of Fermi liquid effects one cannot get a correct result for the deviation of the chemical potential from its local equilibrium value by Shumeiko's method.²⁴

The purpose of the present article is to derive in a systematic and transparent manner the two-fluid hydrodynamic equations for ³He-B from the microscopic point of view. We start from the matrix kinetic equations derived by Wölfle²¹ for the matrix Green's functions G^{\approx} .²⁵ We solve them in a power series form in terms of ω and q , by invoking the Enskog-Chapman scheme.²⁹ Apart from Wölfle's method,²¹ we first solve the dynamical equation for the spectral function $A = G^> + G^<$. We show that the spectral function that reflects the structure of the Cooper pair adjusts itself to its local equilibrium value within the time $1/\Delta$. It also will be proved that when the condition

$$\Delta \gg 1/\tau \gg \omega, v_F q \quad (1)$$

(v_F is the Fermi velocity and τ is a typical collision time) is fulfilled, the

Green's functions are expressed in the form

$$G^{\mathbb{N}} = A\phi^{\mathbb{N}} \quad (2)$$

where the scalar functions $\phi^{\mathbb{N}}$ can be interpreted as the distribution functions of the Bogoliubov–Valatin quasiparticles.

Following the usual methods,^{16,24} we eliminate first the global phase of the energy gap by applying the gauge transformation on the kinetic equations. Usually,^{16,24} the number conservation law is introduced a priori as the equation to determine that global phase. We examine that procedure and show that it is a mathematical convenience which can be justified in the macroscopic limit. We also carefully explore the relation between the chemical potential and the phase of the order parameter. The gist of the considerations is that the chemical potential is a quantity defined on a thermodynamic basis whereas the phase of the energy gap is defined completely on a quantum mechanical basis. This observation leads to a clear-cut derivation of the accelerating equation with dissipative terms, i.e., the second viscosity term.

As has been pointed out by Graham and Pleiner,²⁶ the orbital part two-fluid equations for $^3\text{He-B}$ are the same as those for He-II^{31} and the *s*-wave BCS superfluid.²⁴ In this sense, our formulation yields no qualitatively new results. We believe, however, that our results are worth recording because our method provides a most general and transparent language for the microscopic derivation of the hydrodynamic equations, including dissipative effects. Moreover, our method can even handle the nonlinear regime, although we will demonstrate it only in the derivation of the hydrodynamic equations without dissipation. The outline of the present work has been already reported.²⁷

The organization of this paper is as follows. In Section 2, we briefly review Wölfle's method²¹ of deriving the matrix kinetic equation. In Section 3, following Kadanoff and Baym,²⁵ we manipulate the matrix kinetic equation so that it is appropriate to be handled with respect to the macroscopic limit. In Section 4, conservation laws are derived. Special attention is paid to the particle number conservation law. Section 5 is devoted to the study of the spectral function. Local equilibrium (zeroth order in ω and q) forms of the Green's functions are also derived. In Section 6, the thermodynamic relations and the hydrodynamic equations obeyed by the local equilibrium solutions are discussed. In Section 7, the first order (in ω and q) solutions of the kinetic equations are obtained. Section 8 is devoted to the derivation of the hydrodynamic equations, including dissipative terms. Concluding remarks are given in Section 9. In the Appendix, we consider the

relation between the number conservation law and the equation to determine the global phase of the order parameter. Throughout this article, we take the units of $\hbar = k_B = 1$.

2. GENERAL FORMULATION

In this section, we derive a matrix kinetic equation for the matrix Green's function following Kadanoff and Baym²⁵ and Wölfle.²¹ For details, we refer the reader to their work.

For the purpose of dealing with the superfluid Fermi system, it is convenient to define four-dimensional spinor field operators

$$\Psi(1) = \begin{pmatrix} \psi_{\uparrow}(1) \\ \psi_{\downarrow}(1) \\ \psi_{\uparrow}^{\dagger}(1) \\ \psi_{\downarrow}^{\dagger}(1) \end{pmatrix} \quad (3)$$

$$\Psi^{\dagger}(1) = (\psi_{\uparrow}^{\dagger}(1)\psi_{\downarrow}^{\dagger}(1)\psi_{\uparrow}(1)\psi_{\downarrow}(1)) \quad (4)$$

where (1) is the abbreviation for the space-time coordinates (\mathbf{r}_1, t_1) and ψ_{\uparrow} and $\psi_{\uparrow}^{\dagger}$ are the destruction and creation operators, respectively. Nonequilibrium Green's functions in the presence of an external field $U(1, 1')$ are defined as

$$G_{ss'}^>(1, 1'; U) = (-i)\langle \Psi_{U_s}(1)\Psi_{U_{s'}}^{\dagger}(1') \rangle \quad (5)$$

$$G_{ss'}^<(1, 1'; U) = i\langle \Psi_{U_{s'}}^{\dagger}(1')\Psi_{U_s}(1) \rangle \quad (6)$$

where

$$\Psi_U(1) = \mathcal{S}^{-1}(t)\Psi(1)\mathcal{S}(t)$$

with

$$\mathcal{S}(t) = T \exp \left[-i \int_{-\infty}^t d1 d1' \Psi^{\dagger}(1)U(1, 1')\Psi(1') \right]$$

where T is the usual chronological operator. The average is taken over the grandcanonical ensemble at $t = -\infty$:

$$\langle A \rangle = \text{tr } A \exp [-\beta(\mathcal{H} - \mu_0 N)] / \text{tr } \exp [-\beta(\mathcal{H} - \mu_0 N)]$$

The equations of motion for G^{\cong} are given by²¹

$$[D - U - \text{Re } M, G^{\cong}] = I^{\cong} \quad (7)$$

where $[A, B]$ is the commutator,

$$D = \left[i \frac{\partial}{\partial t_1} + \left(\frac{\nabla_1^2}{2m} + \mu_0 \right) \rho_3 \right] \delta(1-1') \tag{8}$$

ρ_3 is the Pauli matrix in the particle-hole space, and $\delta(1-1')$ is the four-dimensional δ -function. The real part of the self-energy $\text{Re } M$ represents the energy shift of the quasiparticles, including the off-diagonal elements which describe the energy gap. The collision terms I^{\cong} are written as

$$I^{\cong} = \pm \frac{1}{2} (\{M_C^{\cong}, G^{\mp}\} - \{M_C^{\mp}, G^{\cong}\}) \tag{9}$$

where $\{A, B\}$ is the anticommutator and M_C^{\cong} denotes the self-energy parts to describe the collision process. For simplicity we take into account only the s -wave part of the scattering amplitude, which is denoted as V , and treat the collision process with the Born approximation. Then, similar to Kadanoff and Baym's result,²⁵ we have

$$M_C^{\cong}(1, 1') = V^2 \int d\bar{2} d\bar{3} \rho_3 G^{\cong}(1, 1') \rho_3 \frac{1}{2} \text{tr} [\rho_3 G^{\mp}(\bar{2}, \bar{3}) \rho_3 G^{\cong}(\bar{3}, \bar{2})] - V^2 \int d\bar{2} d\bar{3} \rho_3 G^{\cong}(1, \bar{2}) \rho_3 G^{\mp}(\bar{2}, \bar{3}) \rho_3 G^{\cong}(\bar{3}, 1') \rho_3 \tag{10}$$

We are interested here in the situation where the disturbances of the system are slowly varying in space and time. In this case, it is convenient to perform beforehand a gauge transformation on the Green's functions so that the fluctuation of the global phase of the energy gap can be eliminated, as has been done in other works.^{14,16,23,24} Let us define the transformed Green's function by

$$\check{G}^{\cong}(1, 1') = e^{-i\rho_3 \Lambda(1)} G^{\cong}(1, 1') e^{i\rho_3 \Lambda(1')} \tag{11}$$

which obeys the equation of motion

$$[\check{D} - \check{U} - \text{Re } \check{M}, \check{G}^{\cong}] = \check{I}^{\cong} \tag{12}$$

where

$$\check{D} = \left\{ i \frac{\partial}{\partial t_1} + \left[\frac{\nabla_1^2}{2m} + \mu_0 - \frac{\partial \Lambda(1)}{\partial t_1} - \frac{1}{2} m \mathbf{v}_s^2(1) \right] \rho_3 \right\} \delta(1-1') \tag{13}$$

and

$$\check{U} = U(1, 1') + \frac{1}{2} i [\nabla_1 \cdot \mathbf{v}_s(1) + \mathbf{v}_s(1) \cdot \nabla_1] \delta(1-1') \tag{14}$$

The superfluid velocity \mathbf{v}_s has been introduced as

$$\mathbf{v}_s(1) = (1/m) \nabla_1 \Lambda(1) \quad (15)$$

where m is the mass of the ^3He atom. The collision terms are invariant under the gauge transformation and so is the diagonal part of $\text{Re } M$. An explicit form of $\text{Re } M$ will be given below. The transformed Green's functions describe the system in the moving frame with the velocity \mathbf{v}_s . In what follows, we omit the tilde for simplicity and also omit the external field U because it was introduced as a convenience for deriving the kinetic equation.

3. DERIVATION OF THE GENERALIZED KINETIC EQUATION

In this section we follow Kadanoff and Baym²⁵ and manipulate the matrix kinetic equation given in the last section to make it appropriate for describing the system near the hydrodynamic limit.

When the disturbance of the system is slowly varying, we can expect that G^{\cong} to be slowly varying functions of the "center-of-mass" coordinates

$$\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}'_1)/2, \quad T = (t_1 + t'_1)/2$$

but to be sharply peaked about zero values of the "relative" coordinates

$$\mathbf{r} = \mathbf{r}_1 - \mathbf{r}'_1, \quad t = t_1 - t'_1$$

We define the Fourier transform of G^{\cong} over \mathbf{r} and t by

$$G^{\cong}(\mathbf{R}, T, \mathbf{p}, \omega') = \pm i \int d\mathbf{r} dt \exp[-i(\mathbf{p} \cdot \mathbf{r} - \omega' t)] G^{\cong}(1, 1') \quad (16)$$

Any physical quantities of interest can be computed from the 4×4 Wigner distribution function N defined by

$$N(\mathbf{R}, T, \mathbf{p}) = \int \frac{d\omega'}{2\pi} G^<(\mathbf{R}, T, \mathbf{p}, \omega') \quad (17)$$

which can be written explicitly as

$$N(\mathbf{R}, T, \mathbf{p}) = \begin{pmatrix} \tilde{n}(\mathbf{R}, T, \mathbf{p}) & -d(\mathbf{R}, T, \mathbf{p}) \\ -d^\dagger(\mathbf{R}, T, \mathbf{p}) & 1 - n(\mathbf{R}, T, -\mathbf{p}) \end{pmatrix} \quad (18)$$

where n and d are 2×2 matrices and \tilde{n} denotes the transpose of n . The matrix n represents the usual Wigner distribution function

$$n_{\alpha\beta}(\mathbf{R}, T, \mathbf{p}) = \int d\mathbf{r} \exp(-i\mathbf{p} \cdot \mathbf{r}) \langle \psi_\alpha^\dagger(\mathbf{R} + \frac{1}{2}\mathbf{r}) \psi_\beta(\mathbf{R} - \frac{1}{2}\mathbf{r}) \rangle \quad (19)$$

whereas the matrix d gives the anomalous correlation function:

$$d_{\alpha\beta}(\mathbf{R}, T, \mathbf{p}) = \int d\mathbf{r} \exp(-i\mathbf{p} \cdot \mathbf{r}) \langle \psi_\alpha(\mathbf{R} + \frac{1}{2}\mathbf{r}) \psi_\beta(\mathbf{R} - \frac{1}{2}\mathbf{r}) \rangle \quad (20)$$

Let us turn to the kinetic equation given in the last section. Since the $G^{\cong}(\mathbf{R}, T, \mathbf{r}, t)$ are sharply peaked about $\mathbf{r} = 0$ and $t = 0$, we can expand the quantities \mathbf{v}_s and $\text{Re } M$ in powers of \mathbf{r} and t and retain only the lowest order terms. Then we have

$$\begin{aligned} \frac{\partial}{\partial T} G^{\cong} + i[\mathcal{E}, G^{\cong}] + \frac{1}{2} \left\{ \frac{\partial \mathcal{E}}{\partial \mathbf{p}}, \frac{\partial G^{\cong}}{\partial \mathbf{R}} \right\} \\ - \frac{1}{2} \left\{ \frac{\partial \mathcal{E}}{\partial \mathbf{R}}, \frac{\partial G^{\cong}}{\partial \mathbf{p}} \right\} + \frac{1}{2} \left\{ \frac{\partial \mathcal{E}}{\partial T}, \frac{\partial G^{\cong}}{\partial \omega'} \right\} = I^{\cong} \end{aligned} \quad (21)$$

where the energy matrix \mathcal{E} is given by

$$\mathcal{E} = \left(\frac{\mathbf{p}^2}{2m} - \mu_0 + \dot{\Lambda} + \frac{1}{2} m \mathbf{v}_s^2 \right) \rho_3 + \mathbf{p} \cdot \mathbf{v}_s + \text{Re } M(\mathbf{R}, T, \mathbf{p}) \quad (22)$$

We have neglected the ω' dependence of $\text{Re } M$ because the correction is of order T/E_F .²¹ We write \mathcal{E} as a sum of the equilibrium energy matrix \mathcal{E}_0 and the fluctuating part $\delta\mathcal{E}$:

$$\mathcal{E} = \mathcal{E}_0 + \delta\mathcal{E} \quad (23)$$

In the BW state²⁸ \mathcal{E}_0 is given as

$$\mathcal{E}_0 = \begin{pmatrix} \xi & \Delta_{\mathbf{p}} \\ \Delta_{\mathbf{p}}^{\dagger} & -\xi \end{pmatrix} \quad (24)$$

where $\xi = \mathbf{p}^2/2m^* - \mu_0$ (m^* the effective mass) and the 2×2 energy gap matrix $\Delta_{\mathbf{p}}$ is given²⁸ by

$$\Delta_{\mathbf{p}} = i\Delta \sum_{\mu, \nu=1,2,3} R_{\mu\nu} \hat{\mathbf{p}}_{\nu} \sigma_{\nu} \sigma_2 \equiv \Delta \hat{\Lambda}(\hat{\mathbf{p}}) \quad (25)$$

where Δ is the absolute magnitude of the energy gap, $\hat{\mathbf{p}}$ is the unit vector parallel to \mathbf{P} , $R_{\mu\nu}$ is some orthogonal matrix, and the σ are Pauli matrices in spin space. It is convenient for the BW state²⁸ to introduce a new set of Pauli matrices in particle-hole space as

$$\begin{aligned} \rho'_0(\hat{\mathbf{p}}) &= \rho_0 = 1 \\ \rho'_1(\hat{\mathbf{p}}) &= \hat{\Delta}'(\hat{\mathbf{p}})\rho_1 - \hat{\Delta}''(\hat{\mathbf{p}})\rho_2 \\ \rho'_2(\hat{\mathbf{p}}) &= \hat{\Delta}''(\hat{\mathbf{p}})\rho_1 + \hat{\Delta}'(\hat{\mathbf{p}})\rho_2 \\ \rho'_3(\hat{\mathbf{p}}) &= \rho_3 \end{aligned} \quad (26)$$

where $\hat{\Delta}' = (\hat{\Delta} + \hat{\Delta}^{\dagger})/2$ and $\hat{\Delta}'' = (\hat{\Delta} - \hat{\Delta}^{\dagger})/2i$. These new Pauli matrices obey the same commutation relations as the original Pauli matrices. By use of the

new Pauli matrices we can express \mathcal{E}_0 simply as

$$\mathcal{E}_0 = \xi \rho'_3 + \Delta \rho'_1 \quad (27)$$

As for the fluctuating part $\delta\mathcal{E}$, we can write

$$\delta\mathcal{E} = (\dot{\Lambda} + \frac{1}{2} m \mathbf{v}_s^2) \rho'_3 + \mathbf{p} \cdot \mathbf{v}_s + \Sigma \quad (28)$$

where Σ is expressed self-consistently in terms of the Wigner distribution functions via the Fermi liquid interaction and the pairing interaction.^{16,21} We can neglect the antisymmetric part of the Fermi liquid interaction because we are dealing with the spin-independent problem. Further, we retain conventionally the Fermi liquid interactions up to the p -wave part to write

$$f_{\mathbf{p}\mathbf{p}'} = f_0^S + f_1^S P_1(\hat{\mathbf{p}} \cdot \hat{\mathbf{p}}') \quad (29)$$

where P_1 is the first-order Legendre polynomial. The pairing interaction is given in our problem as

$$g_{\mathbf{p}\mathbf{p}'} = -3g_1 P_1(\hat{\mathbf{p}} \cdot \hat{\mathbf{p}}') \quad (30)$$

Now let us expand Σ and N in terms of new Pauli matrices:

$$\Sigma = \sum_{i=0}^3 \Sigma^{(i)} \rho'_i, \quad N = \sum_{i=0}^3 N^{(i)} \rho'_i \quad (31)$$

Then the self-consistent relations are given by

$$\Sigma^{(0)} = 2 \sum_{\mathbf{p}'} f_1^S P_1(\hat{\mathbf{p}} \cdot \hat{\mathbf{p}}') [N^{(0)}(\mathbf{p}') - N_0^{(0)}(\mathbf{p}')] \quad (32)$$

$$\Sigma^{(3)} = 2 \sum_{\mathbf{p}'} f_0^S [N^{(3)}(\mathbf{p}') - N_0^{(3)}(\mathbf{p}')] \quad (33)$$

$$\Sigma^{(1)} \rho'_1(\hat{\mathbf{p}}) = \sum_{\mathbf{p}'} g_{\mathbf{p}\mathbf{p}'} [N^{(1)}(\mathbf{p}') - N_0^{(1)}(\mathbf{p}')] \rho'_1(\hat{\mathbf{p}}) \quad (34)$$

$$\Sigma^{(2)} \rho'_2(\hat{\mathbf{p}}) = \sum_{\mathbf{p}'} g_{\mathbf{p}\mathbf{p}'} [N^{(2)}(\mathbf{p}') - N_0^{(2)}(\mathbf{p}')] \rho'_2(\hat{\mathbf{p}}) \quad (35)$$

with N_0 the equilibrium Wigner distribution function.

It will be shown a posteriori that in the macroscopic limit the pair correlation function depends on the angle $\hat{\mathbf{p}}$ in the same way as the equilibrium one; therefore, the angle dependence of the off-diagonal elements of N is absorbed in $\rho'_1(\hat{\mathbf{p}})$ and $\rho'_2(\hat{\mathbf{p}})$. Then the last two of the above

equations are simplified to be

$$\Sigma^{(1)} \equiv \delta\Delta = -g_1 \sum_{\mathbf{p}} [N^{(1)}(\mathbf{p}') - N_0^{(1)}(\mathbf{p}')] \quad (34')$$

$$\Sigma^{(2)} = -g_1 \sum_{\mathbf{p}} [N^{(2)}(\mathbf{p}') - N_0^{(2)}(\mathbf{p}')] \quad (35')$$

where $\Sigma^{(1)}$ or $\delta\Delta$ represents the fluctuation of the magnitude of the order parameter. Since we have already eliminated the fluctuation of the phase of the energy gap, we have

$$\Sigma^{(2)} = \sum_{\mathbf{p}} N^{(2)}(\mathbf{p}) = 0 \quad (36)$$

which will play an important role in the derivation of the number conservation law.

Finally, we write down the collision term:

$$I^{\cong}(\mathbf{R}, T, \mathbf{p}, \omega') = \pm \frac{1}{2} [\{M^>(\mathbf{R}, T, \mathbf{p}, \omega'), G^<(\mathbf{R}, T, \mathbf{p}, \omega')\} - \{M^<, G^>\}] \quad (37a)$$

where

$$\begin{aligned} M^{\cong}(\mathbf{R}, T, \mathbf{p}, \omega') &= V^2 \int \frac{d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}_4 d\omega_2 d\omega_3 d\omega_4}{(2\pi)^{12}} \\ &\times (2\pi)^4 \delta(\omega' + \omega_2 - \omega_3 - \omega_4) \delta(\mathbf{p} + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4) \\ &\times \{\rho_3 G^{\cong}(\mathbf{R}, T, \mathbf{p}_3, \omega_3) \rho_3 \frac{1}{2} \text{tr} [\rho_3 G^{\cong}(\mathbf{R}, T, \mathbf{p}_2, \omega_2) \rho_3 G^{\cong}(\mathbf{R}, T, \mathbf{p}_4, \omega_4)] \\ &- \rho_3 G^{\cong}(\mathbf{R}, T, \mathbf{p}_3, \omega_3) \rho_3 G^{\cong}(\mathbf{R}, T, \mathbf{p}_2, \omega_2) \rho_3 G^{\cong}(\mathbf{R}, T, \mathbf{p}_4, \omega_4) \rho_3\} \end{aligned} \quad (37b)$$

The above equations compose a complete set to describe a superfluid Fermi system. We collect them here for convenience:

$$\begin{aligned} \frac{\partial}{\partial T} G^{\cong} + i[\mathcal{E}, G^{\cong}] + \frac{1}{2} \left\{ \frac{\partial \mathcal{E}}{\partial \mathbf{p}}, \frac{\partial G^{\cong}}{\partial \mathbf{R}} \right\} \\ - \frac{1}{2} \left\{ \frac{\partial \mathcal{E}}{\partial \mathbf{R}}, \frac{\partial G^{\cong}}{\partial \mathbf{p}} \right\} \\ + \frac{1}{2} \left\{ \frac{\partial \mathcal{E}}{\partial T}, \frac{\partial G^{\cong}}{\partial \omega'} \right\} = I^{\cong} \end{aligned} \quad (38)$$

$$I^{\infty} = \pm \frac{1}{2} (\{M^{\>}, G^{\<}\} - \{M^{\<}, G^{\>}\}) \quad (39)$$

$$\sum_{\mathbf{p}} N^{(2)}(\mathbf{p}) = 0 \quad (40)$$

It is to be noted that, if one neglects the off-diagonal elements in the above equations, one immediately has the kinetic equation for the normal system given by Kadanoff and Baym.²⁵

4. CONSERVATION LAWS

We are interested in the derivation of the hydrodynamic equations where the frequency ω as well as the wave number q is very small. We can solve the kinetic equation in a power series form in terms of ω and q : the zeroth-order solutions give the local equilibrium solution, while the higher order solutions describe the deviation from the local equilibrium. This is essentially the same as the Enskog–Chapman method²⁹ of solving the Boltzmann equation for a dilute gaseous system. In fact, we shall see that the kinetic equations of our problem in the macroscopic limit also reduce to the Boltzmann-type equations for weakly interacting Bogoliubov–Valatin quasiparticles. In the Enskog–Chapman method, at each stage of the approximation, the hydrodynamic equations, which have the form of the conservation laws, are given by the solubility condition for the next order distribution function.²⁹ This procedure can also be followed in our problem for the momentum and the energy conservation laws as was done by Shumeiko.²⁴ However, we can alternatively obtain the complete conservation laws for both the momentum and the energy densities by taking appropriate moments of the kinetic equation (21). The hydrodynamic equations at each stage of the approximation are obtained by substituting the solved distribution function into the conservation laws.

The number density ρ , the momentum density \mathbf{J} in the moving frame and the energy density U are computed from the Wigner distribution function N as

$$\rho = \sum_{\mathbf{p}} \frac{1}{2} \text{tr} \{ \rho_3 [N + \frac{1}{2}(\rho_3 - 1)] \} \quad (41)$$

$$\mathbf{J} = \sum_{\mathbf{p}} \frac{1}{2} \mathbf{p} \text{tr} [N + \frac{1}{2}(\rho_3 - 1)] \quad (42)$$

$$\begin{aligned} U = & \frac{1}{2} \sum_{\mathbf{p}} \{ \frac{1}{2} \text{tr} \{ \mathcal{E} [N + \frac{1}{2}(\rho_3 - 1)] \} \\ & + \frac{1}{2} \text{tr} \{ [\mathbf{p} \cdot \mathbf{v}_s + (\xi + \dot{\Lambda} + \frac{1}{2} m \mathbf{v}_s^2) \rho_3] [N + \frac{1}{2}(\rho_3 - 1)] \} \\ & + (\mu_0 - \dot{\Lambda}) \text{tr} \{ \rho_3 [N + \frac{1}{2}(\rho_3 - 1)] \} \} \end{aligned} \quad (43)$$

The expression for the energy density is rather complicated for two reasons. One is that we should avoid the double counting of the mutual two-body interactions. The other is that we must discard the contribution from the gauge field $\dot{\Lambda}$, because it should have no contribution to the total energy although it plays an important role in determining the dynamics of the system.

We first consider the momentum and energy conservation laws. Noting that the collision term in the kinetic equation (21) conserves momentum and energy, we have two conservation laws:

$$\begin{aligned} \frac{\partial}{\partial T} J'_i + \frac{\partial}{\partial R_j} \sum_{\mathbf{p}} \frac{1}{4} \text{tr} \left[p_i \left\{ \frac{\partial \mathcal{E}}{\partial p_j}, N \right\} + \delta_{ij} \{ \mathcal{E}, N \} \right] \\ - \sum_{\mathbf{p}} \frac{1}{4} \text{tr} \left[\left\{ \mathcal{E}, \frac{\partial N}{\partial R_i} \right\} \right] = 0 \end{aligned} \quad (44)$$

$$\frac{\partial U}{\partial T} = (\mu_0 - \dot{\Lambda}) \dot{\rho} + m \dot{\mathbf{v}}_s \cdot (\mathbf{J}' + \rho \mathbf{v}_s) - \frac{\partial}{\partial R_j} \sum_{\mathbf{p}} \frac{1}{4} \text{tr} \left[\mathcal{E} \left\{ \frac{\partial \mathcal{E}}{\partial p_j}, N \right\} \right] \quad (45)$$

Let us turn to the particle number conservation law. From the definition of ρ given by Eq. (41) it follows that

$$\frac{\partial \rho}{\partial T} + \sum_{\mathbf{p}} \frac{1}{2} \text{tr} [\rho_3 i [\mathcal{E}, N]] + \frac{\partial}{\partial R_j} \sum_{\mathbf{p}} \frac{1}{4} \text{tr} \left[\rho_3 \left\{ \frac{\partial \mathcal{E}}{\partial p_j}, N \right\} \right] = 0 \quad (46)$$

where the third term is interpreted to be the divergence of the current density, while the second term can be rewritten by use of Eqs. (34') and (35') to be

$$\sum_{\mathbf{p}} \frac{1}{2} \text{tr} [\rho_3 i [\mathcal{E}, N]] = -4(\Delta + \delta\Delta) \sum_{\mathbf{p}} N^{(2)}(\mathbf{p})$$

Thus we have

$$\frac{\partial \rho}{\partial T} + \frac{\partial}{\partial R_j} \sum_{\mathbf{p}} \frac{1}{4} \text{tr} \left[\rho_3 \left\{ \frac{\partial \mathcal{E}}{\partial p_j}, N \right\} \right] - 4(\Delta + \delta\Delta) \sum_{\mathbf{p}} N^{(2)}(\mathbf{p}) = 0 \quad (47)$$

We recall that the kinetic equations should be solved under the constraint of (36),

$$\sum_{\mathbf{p}} N^{(2)}(\mathbf{p}) = 0$$

The solutions of the kinetic equations automatically satisfy the number conservation law

$$\frac{\partial \rho}{\partial T} + \frac{\partial}{\partial R_j} \sum_{\mathbf{p}} \frac{1}{4} \text{tr} \left[\rho_3 \left\{ \frac{\partial \mathcal{E}}{\partial p_j}, N \right\} \right] = 0 \quad (48)$$

On the other hand, we can adopt instead of Eq. (36) the conservation law (48) itself as the constraint on the solutions of the kinetic equation, as was done by Betbeder-Matibet and Nozières¹⁶ and Shumeiko.²⁴ As will be shown in the Appendix, this is a mathematical convenience which can be justified in the macroscopic limit where the structure of the order parameter does not change and the distribution functions are described in terms of the conserved quantities or their space derivatives.

In fact, the distribution functions will be written in terms of the following parameters: the local temperature $1/\beta$, the normal velocity \mathbf{v}_n , and the gauge parameter Λ . The set of conservation laws obtained in this section is sufficient to determine the behaviors of these parameters.

From the requirement of Galilei invariance, the current density given above can be related to the momentum density \mathbf{J} in the fixed frame as

$$\mathbf{J} = \mathbf{J}' + m\rho\mathbf{v}_s = m \sum_{\mathbf{p}} \frac{1}{4} \text{tr} [\rho_3 \{ \partial \mathcal{E} / \partial \mathbf{p}, N \}] \quad (49)$$

This yields the well-known relation between the effective mass and the real mass of the ^3He atom:

$$m^*/m = 1 + \frac{1}{3}F_1 \quad (50)$$

where $F_1 = N_F f_1^S$ (N_F is the density of states at the Fermi surface for both spin projections).

5. SPECTRAL FUNCTION AND THE LOCAL EQUILIBRIUM SOLUTION

In the rest of this paper, we solve the kinetic equations in a power series in terms of ω and q and substitute the results into the conservation laws given in the last section to obtain the hydrodynamic equations. However, the kinetic equations are still complicated because they still contain the internal frequency ω' . Wölfle²¹ has eliminated ω' by directly integrating the kinetic equation to show that two types of matrix distribution functions (δn and δr in his notation) are necessary to describe the deviation of the system from local equilibrium. We instead take another approach very similar to that of Kadanoff and Baym²⁵; that is, first we solve the kinetic equation for the spectral function. In our method we can have a more transparent understanding of the local equilibrium solution.

The spectral function is defined by

$$A(\mathbf{R}, T, \mathbf{p}, \omega') = G^>(\mathbf{R}, T, \mathbf{p}, \omega') + G^<(\mathbf{R}, T, \mathbf{p}, \omega') \quad (51)$$

Noting that the collision terms $I^>$ and $I^<$ have different signs, we have for A

the kinetic equation

$$\frac{\partial A}{\partial T} + i[\mathcal{E}, A] + \frac{1}{2} \left\{ \frac{\partial \mathcal{E}}{\partial \mathbf{p}}, \frac{\partial A}{\partial \mathbf{R}} \right\} - \frac{1}{2} \left\{ \frac{\partial \mathcal{E}}{\partial \mathbf{R}}, \frac{\partial A}{\partial \mathbf{p}} \right\} + \frac{1}{2} \left\{ \frac{\partial \mathcal{E}}{\partial T}, \frac{\partial A}{\partial \omega'} \right\} = 0 \quad (52)$$

We solve it in a power series in terms of ω and q . For that purpose, it is convenient to expand the energy matrices \mathcal{E} and $\delta\mathcal{E}$ in terms of the Pauli matrices ρ'_i :

$$\begin{aligned} \mathcal{E} &= \mathcal{E}^{(0)} + \mathcal{E}^{(1)} \rho'_1(\hat{\mathbf{p}}) + \mathcal{E}^{(3)} \rho'_3(\hat{\mathbf{p}}), \\ \delta\mathcal{E} &= \delta\mathcal{E}^{(0)} + \delta\mathcal{E}^{(1)} \rho'_1(\hat{\mathbf{p}}) + \delta\mathcal{E}^{(3)} \rho'_3(\hat{\mathbf{p}}) \end{aligned} \quad (53)$$

The zeroth-order solution is given by

$$A_0 = \sum_{\nu=\pm 1} \frac{1}{2} (1 + \nu \bar{\mathcal{E}}_{\mathbf{p}} / \bar{E}_{\mathbf{p}}) 2\pi\delta(\omega' - \nu E_{\nu\mathbf{p}}) \quad (54)$$

where ν is the particle-hole index and $\bar{\mathcal{E}}_{\mathbf{p}}$, $\bar{E}_{\mathbf{p}}$, and $E_{\nu\mathbf{p}}$ are defined by

$$\bar{\mathcal{E}}_{\mathbf{p}} = \mathcal{E}^{(1)} \rho'_1(\hat{\mathbf{p}}) + \mathcal{E}^{(3)} \rho'_3(\hat{\mathbf{p}}) \quad (55)$$

$$\bar{E}_{\mathbf{p}} = [(\mathcal{E}^{(1)})^2 + (\mathcal{E}^{(3)})^2]^{1/2} = [(\xi + \dot{\lambda} + \frac{1}{2} m v_s^2 + \Sigma^{(3)})^2 + (\Delta + \delta\Delta)^2]^{1/2} \quad (56)$$

and

$$E_{\nu\mathbf{p}} = \bar{E}_{\mathbf{p}} + \nu\delta\mathcal{E}^{(0)} = \bar{E}_{\mathbf{p}} + \nu(\mathbf{p} \cdot \mathbf{v}_s + \Sigma^{(0)}) \quad (57)$$

The first-order solution can be obtained iteratively from Eq. (52):

$$A_1 = \sum_{\nu=\pm 1} [2\pi\delta(\omega' - \nu E_{\nu\mathbf{p}}) g_{\nu} + 2\pi\delta'(\omega' - \nu E_{\nu\mathbf{p}}) h_{\nu}] \quad (58)$$

with

$$g_{\nu} = \frac{i\nu}{8\bar{E}^3} \left(\left[\bar{\mathcal{E}}, \frac{\partial \bar{\mathcal{E}}}{\partial T} \right] + \left[\bar{\mathcal{E}}, \frac{\partial \bar{\mathcal{E}}}{\partial \mathbf{R}} \right] \frac{\partial \mathcal{E}^{(0)}}{\partial \mathbf{p}} - \left[\bar{\mathcal{E}}, \frac{\partial \bar{\mathcal{E}}}{\partial \mathbf{p}} \right] \frac{\partial \mathcal{E}^{(0)}}{\partial \mathbf{R}} \right) \quad (59)$$

and

$$\begin{aligned} h_{\nu} &= \frac{i}{8\bar{E}^2} \left(\left[\bar{\mathcal{E}}, \frac{\partial \bar{\mathcal{E}}}{\partial T} \right] + \left[\bar{\mathcal{E}}, \frac{\partial \bar{\mathcal{E}}}{\partial \mathbf{R}} \right] \frac{\partial \mathcal{E}^{(0)}}{\partial \mathbf{p}} - \left[\bar{\mathcal{E}}, \frac{\partial \bar{\mathcal{E}}}{\partial \mathbf{p}} \right] \frac{\partial \mathcal{E}^{(0)}}{\partial \mathbf{R}} \right. \\ &\quad \left. + \nu \bar{E} \left[\frac{\partial \bar{\mathcal{E}}}{\partial \mathbf{p}}, \frac{\partial \bar{\mathcal{E}}}{\partial \mathbf{R}} \right] \right) \end{aligned} \quad (60)$$

The first-order correction A_1 to the spectral function is smaller than A_0 by the factor of ω/Δ or qv_F/Δ . Physically this means that the spectral function adjusts itself to A_0 on the time scale $1/\Delta$ or within the space range of the coherence length ξ by virtue of quantum mechanical interference effects.

This fact has been pointed out by Galaiko,²³ although he did not directly treat the spectral function. In the recent theory of the spin dynamics of superfluid ³He, Leggett and Takagi³⁰ introduced the concepts of superspin and normal spin. The superspin polarization comes from the deformation of the Cooper pair wave function due to the applied magnetic field and it adjusts itself to its local equilibrium value quantum mechanically within the time $1/\Delta$. On the other hand, the change in the configuration of the Bogoliubov–Valatin quasiparticles gives rise to the normal spin polarization. It is worth noting that the spectral function in our formalism just describes the behavior of the Cooper pair structure.

From the definition of the spectral function, we see that the matrix elements of G^{\cong} are written as

$$G_{ss'}^< = A_{ss'} f_{ss'}(\mathbf{R}, T, \mathbf{p}, \omega') \quad (61)$$

$$G_{ss'}^> = A_{ss'} [1 - f_{ss'}(\mathbf{R}, T, \mathbf{p}, \omega')] \quad (62)$$

We also expand $f_{ss'}$ in terms of ω and q and retain up to the first-order terms:

$$f_{ss'} = f_{0ss'} + f_{1ss'} \quad (63)$$

Then we can write $G^<$ up to the first order as

$$\begin{aligned} G_{ss'}^< &= (A_{0ss'} + A_{1ss'}) (f_{0ss'} + f_{1ss'}) \\ &\cong (A_0 + A_1)_{ss'} f_{0ss'} + A_{0ss'} f_{1ss'} \end{aligned} \quad (64)$$

We expect the zeroth-order solution to give the local equilibrium solution which makes the collision terms I^{\cong} vanish. Then we can choose the most general form of $f_{0ss'}$ to be

$$f_{0ss'} = \delta_{ss'} f^< \{ \beta(\mathbf{R}, T) [\omega' - \mathbf{p} \cdot \mathbf{v}_n(\mathbf{R}, T)] \} \quad (65)$$

with the equilibrium Fermi distribution function $f^<[x] = 1/(e^x + 1)$. It will be shown later that β and \mathbf{v}_n are the inverse local temperature and the normal velocity, respectively. It should be emphasized that, contrary to the normal case,²⁵ we cannot include the local chemical potential in the argument of the Fermi distribution function, because it would violate the particle–hole symmetry relation to be obeyed by the matrix Green's functions:

$$G^>(\mathbf{R}, T, \mathbf{p}, \omega') = \rho_1 \tilde{G}^<(\mathbf{R}, T, -\mathbf{p}, -\omega') \rho_1 \quad (66)$$

From Eqs. (54), (58), (64), and (65), we have for G^{\cong} up to the first order

$$G^< = A f^< [\beta(\omega' - \mathbf{p} \cdot \mathbf{v}_n)] + \sum_{\nu} 2\pi \delta(\omega' - \nu E_{\nu\mathbf{p}}) \delta F_{\nu}(\mathbf{p}) \quad (67)$$

and

$$G^> = Af^>[\beta(\omega' - \mathbf{p} \cdot \mathbf{v}_n)] - \sum_{\nu} 2\pi\delta(\omega' - \nu E_{\nu\mathbf{p}}) \delta F_{\nu}(\mathbf{p}) \tag{68}$$

where $f^> = 1 - f^<$.

In this section, we have derived the local equilibrium solution by a rather intuitive method. It can be shown, however, that in the linear regime our solution coincides with Wölfle's result. Moreover, we can show that the two types of distribution functions δn and δr introduced by Wölfle²¹ to describe the deviation of the system from local equilibrium are related to our δF_{ν} by the relationships

$$\delta n = \sum_{\nu} \delta F_{\nu}(\mathbf{p}) \tag{69}$$

$$\delta r = \sum_{\nu} \nu \delta F_{\nu}(\mathbf{p}) \tag{70}$$

6. THERMODYNAMIC RELATIONS AND THE HYDRODYNAMIC EQUATIONS IN LOCAL EQUILIBRIUM

In this section, we compute the local equilibrium values of the conserved quantities and derive the thermodynamic relations as well as the hydrodynamic equations.

The Wigner distribution function in local equilibrium is

$$N_l(\mathbf{R}, T, \mathbf{p}) = \int \frac{d\omega'}{2\pi} (A_0 + A_1) f^<[\beta(\omega' - \mathbf{p} \cdot \mathbf{v}_n)] \tag{71}$$

We can neglect the contribution from A_1 because it is small by the factor of ω/Δ or qv_F/Δ . Then we have

$$N_{\pm} = \frac{1}{2} (f_{\pm}^< - f_{\pm}^> + 1) + \frac{\mathcal{E}_{\mathbf{p}}}{2\bar{E}_{\mathbf{p}}} (f_{\pm}^< + f_{\pm}^> - 1) \tag{72}$$

with

$$f_{\pm}^< = f^<[\beta(E_{\mp\mathbf{p}} \mp \mathbf{p} \cdot \mathbf{v}_n)] \tag{73}$$

Substituting this into Eqs. (41) and (42), we have

$$\rho = \sum_{\mathbf{p}} \left[1 + \frac{\mathcal{E}^{(3)}}{\bar{E}_{\mathbf{p}}} (f_{+}^< + f_{-}^< - 1) \right] \tag{74}$$

and

$$\mathbf{J} = \sum_{\mathbf{p}} \mathbf{p} (f_{+}^< - f_{-}^<) \tag{75}$$

By inspection of the rhs of Eq. (75), we note that \mathbf{J}' is proportional to $\mathbf{v}_n - \mathbf{v}_s$. Then, we define the normal density ρ_n by

$$\mathbf{J}' \equiv m\rho_n(\mathbf{v}_n - \mathbf{v}_s) \quad (76)$$

which together with Eq. (49) leads to

$$\mathbf{J} = m(\rho_n\mathbf{v}_n + \rho_s\mathbf{v}_s) \quad (77)$$

where $\rho_s = \rho - \rho_n$. Thus we can interpret \mathbf{v}_n to be the normal velocity.³¹

As for the energy density, it is more convenient to compute it in a variational form. From Eq. (43), we have for δU

$$\delta U = \sum_{\mathbf{p}} \frac{1}{2} \text{tr} (\mathcal{E} \delta N) + \mathbf{J}' \cdot \delta \mathbf{v}_s + \rho \delta \left(\frac{1}{2} m \mathbf{v}_s^2 \right) + (\mu_0 - \dot{\Lambda}) \delta \rho \quad (78)$$

Substituting Eq. (72) into Eq. (78), we obtain

$$\begin{aligned} \delta U = & \delta (\mathbf{J}' \cdot \mathbf{v}_s + \frac{1}{2} m \rho \mathbf{v}_s^2) + (1/\beta) \delta S + (\mu_0 - \dot{\Lambda} - \frac{1}{2} m v_s^2) \delta \rho \\ & + (\mathbf{v}_n - \mathbf{v}_s) \cdot \delta \mathbf{J}' \end{aligned} \quad (79)$$

where we have defined the entropy density S by

$$S = -2 \sum_{\mathbf{p}} (f_+^< \ln f_+^< + f_+^> \ln f_+^>) \quad (80)$$

Comparing this result with the Landau–Khalatnikov phenomenological theory,³¹ we can interpret $1/\beta$ as the local temperature. Further, the chemical potential in local equilibrium can be defined by

$$\mu = \mu_0 - \dot{\Lambda} - \frac{1}{2} m v_s^2 \quad (81)$$

from which we have the well-known acceleration relation³¹

$$m \partial \mathbf{v}_s / \partial T = -\nabla (\mu + \frac{1}{2} m \mathbf{v}_s^2) \quad (82)$$

Other hydrodynamic equations are given by substituting Eq. (72) into the conservation laws derived in Section 4.

The number conservation is trivial:

$$m \partial \rho / \partial T + \text{div } \mathbf{J} = 0 \quad (83)$$

The momentum conservation law leads to

$$\frac{\partial}{\partial T} J'_i + (\mathbf{v}_n \cdot \nabla) J'_i + J'_i \text{div } \mathbf{v}_n + \mathbf{J}' \cdot \frac{\partial \mathbf{v}_n}{\partial \mathbf{R}_i} + S \nabla_i (1/\beta) = 0 \quad (84)$$

Combining Eq. (84) with the acceleration equation (82), we have

$$\frac{\partial \mathbf{J}}{\partial T} + \mathbf{v}_s \text{div } \mathbf{J} + (\mathbf{J} \cdot \nabla) \mathbf{v}_s + (\mathbf{v}_n \cdot \nabla) \mathbf{J}' + \mathbf{J}' \text{div } \mathbf{v}_n + \nabla p = 0 \quad (85)$$

where the local pressure p is defined via the Gibbs–Duhem relation³¹

$$\delta p = \rho \delta \mu + S \delta(1/\beta) + \mathbf{J}' \cdot \delta(\mathbf{v}_n - \mathbf{v}_s) \quad (86)$$

Finally, the energy conservation law has the form

$$\frac{\partial U}{\partial T} = -\operatorname{div} \left[\left(\mu + \frac{1}{2} m v_s^2 \right) \mathbf{J} + (\mathbf{J}' \cdot \mathbf{v}_n) \mathbf{J}' + \frac{S}{\beta} \mathbf{v}_n \right] \quad (87)$$

The local equilibrium distribution functions are completely determined by solving the hydrodynamic equations (83), (85), and (87) under some appropriate boundary conditions.

It is convenient for later use to write down the linear version of the above results:

$$\partial m \rho / \partial T + \operatorname{div} \mathbf{J} = 0 \quad (88)$$

$$\partial \mathbf{J} / \partial T + \nabla p = 0 \quad (89)$$

$$\partial U / \partial T + \operatorname{div} [\mu_0 \mathbf{J} + (S_0 / \beta_0) \mathbf{v}_n] = 0 \quad (90)$$

$$\mathbf{J} = m(\rho_{s0} \mathbf{v}_s + \rho_{n0} \mathbf{v}_n) \quad (91)$$

$$\delta p = \rho_0 \delta \mu + S_0 \delta(1/\beta) \quad (92)$$

$$\rho = [N_F / (1 + F_0)] \delta \mu = [N_F / (1 + F_0)] (-\dot{\Lambda}) \quad (93)$$

$$\delta U = C_V \delta(1/\beta) + \mu_0 \delta \rho \quad (94)$$

where $F_0 = N_F f_0^s$ is the s -wave Fermi liquid parameter, C_V the equilibrium specific heat per volume and ρ_{n0} is expressed in terms of the Yosida function Y ,³²

$$\rho_{n0} = \rho_0 - \rho_{s0} = \rho_0 Y(1 + \frac{1}{3} F_1) / (1 + \frac{1}{3} F_1 Y) \quad (95)$$

We also give the Wigner distribution function in the linear regime:

$$N_l = N_0 + \delta N_l \quad (96)$$

with

$$\begin{aligned} \delta N_l = & \frac{\delta \mathcal{E}^{(3)} \Delta - \delta \mathcal{E}^{(1)} \xi}{E_0^3} [\Delta \rho'_3(\hat{\mathbf{p}}) - \xi \rho'_1(\hat{\mathbf{p}})] \theta(E_0) \\ & + \left\{ \delta \mathcal{E}^{(0)} - \mathbf{p} \cdot \mathbf{v}_n + \frac{\mathcal{E}_0}{E_0} \left[\frac{\xi}{E_0} \delta \mathcal{E}^{(3)} + \frac{\Delta}{E_0} \delta \mathcal{E}^{(1)} - \beta E_0 \delta \left(\frac{1}{\beta} \right) \right] \right\} f^{<'}(E_0) \end{aligned} \quad (97)$$

where $\theta = f^{<} - \frac{1}{2}$; the fluctuation part of the energy matrix $\delta \mathcal{E}$ is determined

by

$$\delta \mathcal{E}^{(0)} = \frac{\frac{1}{3} F_1 Y}{1 + \frac{1}{3} F_1 Y} \mathbf{p} \cdot (\mathbf{v}_n - \mathbf{v}_s) + \mathbf{p} \cdot \mathbf{v}_s \quad (98)$$

$$\delta \mathcal{E}^{(1)} = \frac{\partial \Delta}{\partial (1/\beta)} \delta \left(\frac{1}{\beta} \right) = \frac{Y}{Y-1} \beta \Delta \delta \left(\frac{1}{\beta} \right) \quad (99)$$

$$\delta \mathcal{E}^{(3)} = -\dot{\Lambda} - f_0^S \delta \rho = \frac{-1}{1+F_0} \delta \mu \quad (100)$$

7. FIRST-ORDER SOLUTION OF KINETIC EQUATIONS

As has been shown in Section 5, the deviation of the system from local equilibrium is described by δF_ν ($\nu = \pm 1$). The equations to determine the first-order solutions of δF_ν are obtained from Eq. (21) by an iterative method. For simplicity, we confine ourselves in the rest of this paper to the linear regime; then the Green's functions G^{\approx} are written as [see Eqs. (67) and (68)]

$$G^{\approx} = A f^{\approx} \mp \sum_{\nu} \delta F_{\nu} 2\pi \delta(\omega' - \nu E_{0\nu}) \quad (101)$$

In what follows we drop the index zero to indicate the equilibrium value except for special cases.

Let us first consider the collision term. Substituting Eq. (101) into Eq. (37a), we have

$$I_{\nu}^{<} = \sum_{\nu=\pm 1} I_{\nu} 2\pi \delta(\omega' - \nu E) \quad (102)$$

The expression for I_{ν} is rather complicated:

$$\begin{aligned} I_{\nu} = & - \int \frac{d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}_4}{(2\pi)^5} V^2 \sum_{\nu_2, \nu_3, \nu_4} \delta_{\mathbf{p}} \delta_{\nu E} f_1^< f_2^< f_3^> f_4^> \\ & \times \left(\frac{1}{2} \{ \rho_3 A_{\nu_4} \rho_3, \delta \bar{F}_{\nu} \} \frac{1}{2} \text{tr} [\rho_3 A_{\nu_2} \rho_3 A_{\nu_3}] \right. \\ & - \frac{1}{2} \{ \rho_3 \delta \bar{F}_{\nu} \rho_3, A_{\nu_4} \} \frac{1}{2} \text{tr} [\rho_3 A_{\nu_2} \rho_3 A_{\nu_3}] \\ & + \frac{1}{2} \{ \rho_3 A_{\nu_4} \rho_3, A_{\nu} \} \frac{1}{2} \text{tr} [\rho_3 \delta \bar{F}_{\nu_2} \rho_3 A_{\nu_3}] \\ & - \frac{1}{2} \{ \rho_3 A_{\nu_4} \rho_3, A_{\nu} \} \frac{1}{2} \text{tr} [\rho_3 A_{\nu_2} \rho_3 \delta \bar{F}_{\nu_3}] \\ & - \frac{1}{2} \{ \rho_3 A_{\nu_4} \rho_3 A_{\nu_3} \rho_3 A_{\nu_2} \rho_3, \delta \bar{F}_{\nu} \} \\ & + \frac{1}{2} \{ \rho_3 \delta \bar{F}_{\nu_4} \rho_3 A_{\nu_3} \rho_3 A_{\nu_2} \rho_3, A_{\nu} \} \\ & - \frac{1}{2} \{ \rho_3 A_{\nu_4} \rho_3 \delta \bar{F}_{\nu_3} \rho_3 A_{\nu_2} \rho_3, A_{\nu} \} \\ & \left. + \frac{1}{2} \{ \rho_3 A_{\nu_4} \rho_3 A_{\nu_3} \rho_3 \delta \bar{F}_{\nu_2} \rho_3, A_{\nu} \} \right) \quad (103) \end{aligned}$$

where $\delta_{\mathbf{p}} = \delta(\mathbf{p} + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4)$, $\delta_{\nu E} = \delta(\nu E + \nu_2 E_2 - \nu_3 E_3 - \nu_4 E_4)$, $f_i^{\approx} = f_i^{\approx}(\nu_i E_i)$,

$$A_{\nu} = \frac{1}{2}(1 + \nu \mathcal{E}/E) \tag{104}$$

and

$$\delta \bar{F}_{\nu} = \delta F_{\nu}(\mathbf{p})/f^>(E)f^<(E) \tag{105}$$

We restrict ourselves to the neighborhood of the transition temperature and consider superfluid effects up to linear order in $\beta_c \Delta$. For that purpose it suffices to replace all the terms in the collision term by its normal value except for the terms that yield $(\Delta/E)^2$. Then, the collision term is simplified to

$$\begin{aligned} I_{\nu} = & -\frac{1}{\tau_{\mathbf{p}}} \delta F_{\nu}(\mathbf{p}) + \frac{1}{\tau(0)} \frac{f^> f^<}{\langle V^2 / \cos \frac{1}{2} \theta \rangle} \\ & \times \sum_{\nu'} \int dx' \int \frac{\sin \theta d\theta d\phi d\tilde{\phi}'}{4\pi \cos(\theta/2) 2\pi} V^2 \\ & \times B(\nu'|x'| + \nu|x|) \frac{\text{ch } \frac{1}{2} x}{\text{ch } \frac{1}{2} x'} \frac{1}{2} \{ \delta \bar{F}_{\nu'}(\mathbf{p}') - 4 \delta \bar{F}_{\nu'}^{(0)}(\mathbf{p}'), A_{\nu} \} \end{aligned} \tag{106}$$

where $\tau_{\mathbf{p}}$ is the normal relaxation time: $\tau(0)$ is its value at the Fermi surface; $x = \beta \xi$; θ and ϕ are the angle variables used by Abrikosov and Khalatnikov³³; and $\tilde{\phi}'$ is the azimuthal angle of \mathbf{p}' around \mathbf{p} . The function B is given by⁶

$$B(x_1 + x_2) = \frac{1}{\pi^2} \frac{x_1 + x_2}{\text{sh} [(x_1 + x_2)/2]} \tag{107}$$

Let us turn to the drift term. Substituting Eq. (101) into the lhs of Eq. (21) and retaining the first-order terms in ω and q , we find the drift terms:

$$\begin{aligned} & \sum_{\nu} 2\pi \delta(\omega' - \nu E) \\ & \times \left(i[\mathcal{E}, \delta F_{\nu}] + f' A_{\nu} \left(-\nu \beta E \frac{\partial(1/\beta)}{\partial T} - \mathbf{p} \cdot \frac{\partial \mathbf{v}_n}{\partial T} \right) \right. \\ & \left. + \frac{1}{2} f' \left\{ \frac{\partial \mathcal{E}}{\partial \mathbf{p}}, A_{\nu} \right\} \left(-\nu \beta E \frac{\partial(1/\beta)}{\partial \mathbf{R}} - \mathbf{p} \cdot \frac{\partial \mathbf{v}_n}{\partial \mathbf{R}} \right) + \frac{1}{2} f' \left\{ \frac{\partial \delta \mathcal{E}}{\partial T}, A_{\nu} \right\} \right) \end{aligned} \tag{108}$$

where $f' = (\partial f^</\partial E) = -\beta f^>(E) f^<(E)$. The time derivative of $\delta \mathcal{E}$ is

computed by use of the zeroth-order solutions as

$$\begin{aligned} \frac{\partial \delta \mathcal{E}}{\partial T} = & \frac{\frac{1}{3} F_1 Y}{1 + \frac{1}{3} F_1 Y} \mathbf{p} \cdot \frac{\partial}{\partial T} (\mathbf{v}_n - \mathbf{v}_s) + \mathbf{p} \cdot \frac{\partial \mathbf{v}_s}{\partial T} \\ & + \frac{\partial \Delta}{\partial(1/\beta)} \frac{\partial(1/\beta)}{\partial T} \rho'_1(\hat{\mathbf{p}}) - \frac{1}{N_F} \frac{\partial \rho}{\partial T} \rho'_3(\hat{\mathbf{p}}) \end{aligned} \quad (109)$$

Following the Enskog–Chapman scheme,^{24,29} we replace the time derivatives by the space derivatives via the hydrodynamic equations derived in Section 6. Finally we have for δF_ν the equation

$$\begin{aligned} i[\mathcal{E}, \delta F_\nu] + \frac{1}{2} f' \left(\left\{ \frac{2}{1 + \frac{1}{3} F_1 Y} \frac{S}{m \rho_n \beta} A_\nu \mathbf{p} - E \left[\frac{\xi}{E} + \nu \rho'_3(\hat{\mathbf{p}}) \right] \frac{\partial \xi}{\partial \mathbf{p}} \right\} \cdot \beta \nabla \left(\frac{1}{\beta} \right) \right. \\ + \left[\nu \frac{\xi}{E} + \rho'_3(\hat{\mathbf{p}}) \right] \frac{1}{N_F} \operatorname{div} (\mathbf{J} - \rho \mathbf{v}_n) \\ + \left[\nu \frac{\xi}{E} + \rho'_3(\hat{\mathbf{p}}) \right] \left(\frac{\rho}{N_F} - \frac{1}{3} \mathbf{p} \cdot \frac{\partial \xi}{\partial \mathbf{p}} \right) \operatorname{div} \mathbf{v}_n \\ + \left\{ 2 \nu E A_\nu - \frac{\partial \Delta}{\partial(1/\beta)} \frac{1}{\beta} \left[\frac{\nu \Delta}{E} + \rho'_1(\hat{\mathbf{p}}) \right] \right\} \frac{S}{C_V} \operatorname{div} \mathbf{v}_n \\ - \left. \left[\nu \frac{\xi}{E} + \rho'_3(\hat{\mathbf{p}}) \right] \frac{1}{2} p_i \frac{\partial \xi}{\partial p_j} \left(\frac{\partial v_{nj}}{\partial R_i} + \frac{\partial v_{ni}}{\partial R_j} - \frac{2}{3} \delta_{ij} \operatorname{div} \mathbf{v}_n \right) \right) \\ = I_\nu \end{aligned} \quad (110)$$

The general solutions of Eq. (110) are given by the sum of the particular solutions and the trivial solutions of the homogeneous equation

$$I_\nu - i[\mathcal{E}, \delta F_\nu] = 0 \quad (111)$$

By inspection of Eq. (103), we find that the trivial solutions that satisfy the particle–hole symmetry are written as

$$\delta F_\nu = f' A_\nu (\mathbf{p} \cdot \mathbf{B}(\mathbf{R}, T) + \nu E C(\mathbf{R}, T)) \quad (112)$$

where \mathbf{B} and C are arbitrary functions of \mathbf{R} and T .

We proceed to the particular solutions. Let us first consider the shear viscosity term for illustration. The equation to be solved is

$$i[\mathcal{E}, \delta F_\nu] + \frac{1}{2} f' \left(\nu \frac{\xi}{E} + \rho'_3 \right) p_i \frac{\partial \xi}{\partial p_j} X_{ij} = I_\nu \quad (113)$$

with

$$X_{ij} = -\frac{1}{2} \left(\frac{\partial v_{nj}}{\partial R_i} + \frac{\partial v_{ni}}{\partial R_j} - \frac{2}{3} \delta_{ij} \operatorname{div} \mathbf{v}_n \right)$$

We can easily find the solution within the relaxation time approximation $I_\nu = -\delta F_\nu/\tau_{\mathbf{p}}$. Making use of that solution, we can find a solution for Eq. (113) of the form

$$\delta F_\nu = -\tau_1(\xi) f' p_i \frac{\partial \xi}{\partial p_i} X_{ij} \left[\nu \frac{\xi}{E} A_\nu - \frac{1}{\tau_{\mathbf{p}}} \frac{\Delta}{4E^2} \rho'_2(\hat{\mathbf{p}}) \right] \quad (114)$$

where we have assumed $\tau_{\mathbf{p}}\Delta \gg 1$ and also the particle-hole symmetry of the density of states for the ξ integral. The effective relaxation time $\tau_1(\xi)$ obeys the relation

$$1 = \frac{\tau_1}{\tau_{\mathbf{p}}} - \frac{\alpha_1}{\tau(0)} \int dx' B(x-x') \frac{\text{ch}(x/2)}{\text{ch}(x'/2)} \left(\frac{\xi'}{E'} \right)^2 \tau_1(\xi') \quad (115)$$

where α_1 is the angle-averaged quantity

$$\alpha_1 = \langle V^2(\cos^2 \theta - \frac{1}{2} \sin^2 \theta) / \cos \frac{1}{2}\theta \rangle / \langle V^2 / \cos \frac{1}{2}\theta \rangle \quad (116)$$

The result (114) is composed of the sum of the term proportional to the spectral function A_ν and the correction term. The correction term is necessary to solve the matrix equation within an accuracy of order $1/\tau_{\mathbf{p}}\Delta$; however, we can neglect it in the following calculation because not only is it smaller than the main term by $1/\tau_{\mathbf{p}}\Delta$, but also it makes no contribution to the physical quantities of interest. A similar situation holds for all the other drift terms. Then, we write the set of particular solutions of Eq. (110) omitting the correction terms:

$$\begin{aligned} \delta F_{p\nu} = & -f' A_\nu \\ & \times \left[p_i \frac{\partial \xi}{\partial p_i} X_{ij} \nu \frac{\xi}{E} \tau_1(\xi) - \beta \frac{\partial \xi}{\partial \mathbf{p}} \cdot \nabla \left(\frac{1}{\beta} \right) \xi \tau_2(\xi) \right. \\ & + \frac{1}{N_F} \text{div}(\mathbf{J} - \rho \mathbf{v}_n) \nu \frac{\xi}{E} \tau_3(\xi) \\ & + \left(\frac{\rho}{N_F} - \frac{1}{3} \mathbf{p} \cdot \frac{\partial \xi}{\partial \mathbf{p}} \right) \text{div} \mathbf{v}_n \nu \frac{\xi}{E} \tau_3(\xi) \\ & \left. + \frac{S}{C_V} \text{div} \mathbf{v}_n \nu E \tau_4(\xi) - \frac{S}{\beta C_V} \frac{\partial \Delta}{\partial(1/\beta)} \text{div} \mathbf{v}_n \nu \frac{\Delta}{E} \tau_5(\xi) \right] \quad (117) \end{aligned}$$

where the effective relaxation times τ_2, τ_3, τ_4 , and τ_5 obey

$$1 = \frac{\tau_2}{\tau_{\mathbf{p}}} - \frac{\alpha_2}{\tau(0)} \int dx' B(x'-|x|) \frac{\text{ch}(x/2)}{\text{ch}(x'/2)} \left(\frac{\xi'}{E'} \right)^2 \frac{E'}{E} \tau_2(\xi') \quad (118)$$

with

$$\alpha_2 = \langle V^2 \cos \theta / \cos \frac{1}{2}\theta \rangle / \langle V^2 / \cos \frac{1}{2}\theta \rangle \quad (119)$$

$$1 = \frac{\tau_3}{\tau_p} - \frac{1}{\tau(0)} \int dx' B(x-x') \frac{\text{ch}(x/2)}{\text{ch}(x'/2)} \left(\frac{\xi'}{E'}\right)^2 \tau_3(\xi') \quad (120)$$

$$1 = \frac{\tau_4}{\tau_p} + \frac{3}{\tau(0)} \int dx' B(x'-|x|) \frac{\text{ch}(x/2)}{\text{ch}(x'/2)} \frac{E'}{E} \tau_4(\xi') \\ - \frac{\alpha_2}{\tau(0)} \int dx' B(x-x') \frac{\text{ch}(x/2)}{\text{ch}(x'/2)} \frac{\Delta^2}{EE'} \tau_4(\xi') \quad (121)$$

and

$$1 = \frac{\tau_5}{\tau_p} + \frac{3}{\tau(0)} \int dx' B(x'-|x|) \frac{\text{ch}(x/2)}{\text{ch}(x'/2)} \frac{E}{E'} \tau_5(\xi') \\ - \frac{\alpha_2}{\tau(0)} \int dx' B(x-x') \frac{\text{ch}(x/2)}{\text{ch}(x'/2)} \left(\frac{\Delta}{E'}\right)^2 \tau_5(\xi') \quad (122)$$

where the integral $\int dx'$ means

$$\int dx' = \int_0^\infty dx' - \int_{-\infty}^0 dx' \quad (123)$$

In the above results, we have neglected one of the two terms that are proportional to $\nabla(1/\beta)$. This can be justified by the following argument: these force terms smoothly reduce to the normal version when $\beta \rightarrow \beta_c$ (β_c the inverse transition temperature), whereas in the normal state, as has been shown by Sykes and Brooker,³⁴ the neglected term gives rise to a smaller contribution to the thermal conductivity than the other one by a factor of $1/\beta E_F$ (E_F is the Fermi energy).

We have shown that all the first-order solutions have the form of

$$\delta F_\nu = f' A_\nu \phi_\nu(\mathbf{p}) \quad (124)$$

as long as the condition $\tau_p \Delta \gg 1$ is fulfilled. We can easily diagonalize Eq. (124) by the Bogoliubov transformation as

$$U^\dagger \delta F_\nu U = \frac{1}{2} f' [1 + \nu \rho'_3(\hat{\mathbf{p}})] \phi_\nu(\mathbf{p}) \quad (125)$$

This indicates that, when $\tau_p \Delta \gg 1$, the first-order solutions are all described by the Bogoliubov–Valatin quasiparticle distribution functions. This fact has been already used by many authors to compute the various transport coefficients^{2–11}. Before concluding this section, let us estimate the superfluid effects on the various effective relaxation times within the order of $\beta_c \Delta$.

If we define $q_1(x)$ by

$$\tau_1(\xi) = \tau(0)[\text{ch}(x/2)]q_1(x) \tag{126}$$

it satisfies the following equation:

$$\frac{1}{\text{ch}(x/2)} = \left[1 + \left(\frac{x}{\pi} \right)^2 \right] q_1 - \alpha_1 \int dx' B(x-x')q_1(x') \left(\frac{\xi'}{E'} \right)^2 \tag{127}$$

which has been treated by Pethick *et al.*⁴ to investigate the shear viscosity near the transition temperature. By making use of the relation

$$\lim_{\Delta \rightarrow 0} \frac{\xi^2}{E^2} = 1 - \frac{\Delta^2}{E^2} = 1 - \pi \Delta \delta(\xi) \tag{128}$$

we can estimate the deviation of q_1 from its normal value q_{n1} to be

$$\delta q_1 = \frac{-\alpha_1}{1 + (x/\pi)^2} \frac{\beta \Delta}{\pi} q_n(0) \frac{x}{\text{sh}(x/2)} \tag{129}$$

For $\tau_2(\xi)$, we find no correction to the normal result apart from the correction of order $(\beta_c \Delta)^2$. If we define q_2 by

$$\tau_2(\xi) = \tau(0) [\text{ch}(x/2)]q_2(x)/x \tag{130}$$

then q_2 satisfies

$$\frac{x}{\text{ch}(x/2)} = \left[1 + \left(\frac{x}{\pi} \right)^2 \right] q_2 - \alpha_2 \int dx' B(x-x')q_2(x') \tag{131}$$

which is nothing but the equation to be solved for the thermal conductivity in the normal Fermi liquid.^{33,34} A very interesting situation occurs with regard to τ_3 . If we define q_3 by

$$\tau_3(\xi) = \tau(0)[\text{ch}(x/2)]q_3(x) \tag{132}$$

we have

$$\frac{1}{\text{ch}(x/2)} = \left[1 + \left(\frac{x}{\pi} \right)^2 \right] q_3 - \int dx' B(x-x') \left(\frac{\xi'}{E'} \right)^2 q_3(x') \tag{133}$$

As has been shown by Wölfle,⁶ one finds a singular solution of the form

$$q_3(x) = \frac{1}{\text{ch}(x/2)} \frac{4}{\pi \beta_c \Delta} \tag{134}$$

It is worth noting that Eq. (133) is the same as that considered by Bhattacharrya *et al.*² for the spin relaxation effects.³⁰ The mechanism of the spin relaxation³⁰ is the same as that in the deviation of the chemical potential from its local equilibrium.

As for τ_4 and τ_5 , it suffices for our purpose only to note that they have no such singular behavior as τ_3 .

8. CONSTRUCTION OF THE HYDRODYNAMIC EQUATIONS WITH DISSIPATIVE TERMS

In this section, we shall derive the linearized version of the two-fluid hydrodynamic equations with dissipative terms for $^3\text{He-B}$. This can be achieved by substituting the Wigner distribution function N up to the first order into the conservation laws given in Section 4. The Wigner distribution function can be expressed in terms of β , \mathbf{v}_n , and Λ (not μ).

Before going into details, we make some remarks on the phenomenological derivation of the two-fluid hydrodynamic equations.³¹ In the phenomenological theories^{26,31} it is assumed that the thermodynamic relations which hold in local equilibrium are also valid even if the system slightly deviates from local equilibrium. Then, in order to derive the phenomenological theory from the microscopic point of view, one should postulate that the thermodynamic relations [see Eqs. (91)–(94)].

$$\delta\rho = [N_F/(1 + F_0)] \delta\mu \quad (135)$$

$$\delta\mathbf{J} = m\rho_{n0}(\mathbf{v}_n - \mathbf{v}_s) \quad (136)$$

$$\delta U = C_V \delta(1/\beta) + \mu_0 \delta\rho \quad (137)$$

as well as the Gibbs–Duhem relation

$$\delta p = \rho_0 \delta\mu + S_0 \delta(1/\beta) \quad (138)$$

hold even if the system deviates from local equilibrium. This means that the variables \mathbf{v}_n and $1/\beta$ still have the meaning of the normal velocity and the local temperature, respectively. The quantities $\delta\rho$, $\delta\mathbf{J}$, δU , and \mathbf{v}_s are completely defined on a microscopic basis [see Eqs. (41)–(43) and also Eq. (15)]. Then it becomes necessary to redefine the chemical potential and the local pressure.

We can make the thermodynamic relations (136) and (137) hold by choosing appropriately the quantities \mathbf{B} and C in the trivial solution (112) so that the first-order corrections to $\delta\mathbf{J}$ and δU vanish. Contrary to the cases of $\delta\mathbf{J}$ and δU , we cannot find any trivial solution which would contribute to $\delta\rho$, because of the particle–hole symmetry relation imposed on the Green's functions [see eq. (66)]. Thus, it is inevitable to redefine the chemical potential microscopically in order to keep the thermodynamic relation (135).

The Wigner distribution function N up to first order (in q) is given by

$$N = N_0 + \delta N_l + \sum_{\nu} (\delta F_{p\nu} + \delta F_{l\nu}) \tag{139}$$

The second term δN_l has the same form as Eq. (97), although the energy matrix $\delta \mathcal{E}$ should be determined self-consistently by use of Eq. (139). Let us decompose formally $\delta \mathcal{E}$ into the zeroth-order (in q) term $\delta \mathcal{E}'_l$ and the first-order (in q) term $\delta \mathcal{E}_1$ as

$$\delta \mathcal{E} = \delta \mathcal{E}'_l + \delta \mathcal{E}_1 \tag{140}$$

The zeroth-order term $\delta \mathcal{E}'_l$ is given by the same form as Eqs. (98)–(100) as

$$\delta \mathcal{E}'_l{}^{(0)} = \frac{\frac{1}{3}F_1 Y}{1 + \frac{1}{3}F_1 Y} \mathbf{p} \cdot (\mathbf{v}_n - \mathbf{v}_s) + \mathbf{p} \cdot \mathbf{v}_s \tag{141}$$

$$\delta \mathcal{E}'_l{}^{(1)} = \frac{\partial \Delta}{\partial (1/\beta)} \delta \left(\frac{1}{\beta} \right) \tag{142}$$

$$\delta \mathcal{E}'_l{}^{(3)} = -\dot{\Lambda} - f_0^S \delta \rho \tag{143}$$

although the actual values of β , \mathbf{v}_n , and Λ are different from the local equilibrium case. The contribution to $\delta \mathcal{E}_1$ comes only from the first-order correction to the magnitude of the order parameter, which we denote by $\delta \Delta_1$; therefore

$$\delta \mathcal{E}_1 = \delta \Delta_1 \rho'_1(\hat{\mathbf{p}}) \tag{144}$$

Let us also formally decompose the Wigner distribution function given by Eq. (139) into the zeroth-order term $\delta N'_l$ and the first-order term δN_1 as

$$N = N_0 + \delta N'_l + \delta N_1 \tag{145}$$

where $\delta N'_l$ is obtained from Eq. (97) by replacing $\delta \mathcal{E}$ by $\delta \mathcal{E}'_l$ given by Eqs. (141)–(143). The first-order term δN_1 is expressed as

$$\begin{aligned} \delta N_1 = & \sum_{\nu} (\delta F_{p\nu} + \delta F_{l\nu}) - \frac{\theta}{E^3} \xi \delta \Delta_1 (\Delta \rho'_3 - \xi \rho'_1) \\ & + \frac{f' \Delta}{E^2} \delta \Delta_1 \mathcal{E} \end{aligned} \tag{146}$$

The number density $\delta\rho$ can be computed by substituting Eq. (145) into Eq. (41):

$$\begin{aligned} \delta\rho = & N_F(-\dot{\Lambda} - f_0^S \delta\rho) \\ & - 2 \sum_{\mathbf{p}} \tau_3 f' \left(\frac{\xi}{E} \right)^2 \left[\frac{1}{N_F} \operatorname{div} (\mathbf{J} - \rho \mathbf{v}_n) \right. \\ & \left. + \left(\frac{\rho}{N_F} - \frac{1}{3} \mathbf{p} \cdot \frac{\partial \xi}{\partial \mathbf{p}} \right) \operatorname{div} \mathbf{v}_n \right] \end{aligned} \quad (147)$$

Solving Eq. (147) in terms of $\delta\rho$, we obtain

$$\begin{aligned} \delta\rho = & \frac{1}{1+F_0} \left\{ N_F(-\dot{\Lambda}) - 2 \sum_{\mathbf{p}} \tau_3 f' \left(\frac{\xi}{E} \right)^2 \left[\frac{1}{N_F} \operatorname{div} (\mathbf{J} - \rho \mathbf{v}_n) \right. \right. \\ & \left. \left. + \left(\frac{\rho}{N_F} - \frac{1}{3} \mathbf{p} \cdot \frac{\partial \xi}{\partial \mathbf{p}} \right) \operatorname{div} \mathbf{v}_n \right] \right\} \end{aligned} \quad (148)$$

Equating Eq. (148) with Eq. (135), we have a new microscopic definition of the chemical potential:

$$\delta\mu = -\dot{\Lambda} + \zeta_3 \operatorname{div} (\mathbf{J} - \rho \mathbf{v}_n) + \zeta_4 \operatorname{div} \mathbf{v}_n \quad (149)$$

where

$$\zeta_3 = -\frac{1}{N_F} \int d\xi \tau_3(\xi) \left(\frac{\xi}{E} \right)^2 f' \quad (150)$$

and

$$\zeta_4 = - \int d\xi \tau_3(\xi) \left(\frac{\xi}{E} \right)^2 f' \left(\frac{\rho}{N_F} - \frac{1}{3} \mathbf{p} \cdot \frac{\partial \xi}{\partial \mathbf{p}} \right) \quad (151)$$

Recalling the definition of \mathbf{v}_s [Eq. (15)], we have the acceleration equation with dissipative terms:

$$m \partial \mathbf{v}_s / \partial T = -\nabla[\mu - \zeta_3 \operatorname{div} (\mathbf{J} - \rho \mathbf{v}_n) - \zeta_4 \operatorname{div} \mathbf{v}_n] \quad (152)$$

It is worthwhile to give some comments on the treatment of this problem by Shumeiko²⁴ and Wölfle.⁶ They tried to determine the chemical potential shift essentially through the relation

$$\delta\mu = \left(\frac{\partial\mu}{\partial\rho} \right) \delta\rho_1 \equiv \left(\frac{\partial\mu}{\partial\rho} \right) \sum_{\mathbf{p}} \frac{1}{2} \operatorname{tr} [\rho_3 \delta N_1] \quad (153)$$

This method gives a result $(1+F_0)$ times larger than the correct result, although one could obtain the correct result from Eq. (153) if one put

$(\partial\mu/\partial\rho) = 1/N_F$ by imposing a somewhat artificial restriction⁷ upon the partial derivative $(\partial\mu/\partial\rho)$.

Now let us determine $\delta\Delta_1$. The magnitude of the order parameter $\delta\mathcal{E}^{(1)}$ is computed from Eqs. (34') and (145) self-consistently as

$$\delta\mathcal{E}^{(1)} = \frac{\partial\Delta}{\partial(1/\theta)} \delta\left(\frac{1}{\beta}\right) - \frac{Y\Delta}{Y-1}C + \frac{\Delta}{Y-1} \ll \tau_4 - \frac{\Delta}{E^2} \frac{1}{\beta} \frac{\partial\Delta}{\partial(1/\beta)} \tau_5 \gg \frac{S}{C_V} \text{div } \mathbf{v}_n \quad (154)$$

where $\langle\langle A \rangle\rangle$ denotes the energy average:

$$\langle\langle A \rangle\rangle = \int d\xi (-A)^f \quad (155)$$

Then we can write

$$\delta\Delta_1 = \frac{\Delta}{Y-1} \left(\left\langle\left\langle \tau_4 - \frac{\Delta}{E^2} \frac{1}{\beta} \frac{\partial\Delta}{\partial(1/\beta)} \tau_5 \right\rangle\right\rangle \frac{S}{C_V} \text{div } \mathbf{v}_n - YC \right) \quad (156)$$

From the requirement that the first-order corrections to $\delta\mathbf{J}'$ and δU vanish, we have

$$\begin{aligned} \frac{1}{2} \sum_{\mathbf{p}} p_i \text{tr } \delta N_1 = -N_F \int \frac{d\Omega}{4\pi} \langle\langle \beta \nabla_i \left(\frac{1}{\beta}\right) \xi \tau_2 p_i \frac{\partial\xi}{\partial p_i} \\ + B_i p_i p_i \rangle\rangle = 0 \end{aligned} \quad (157)$$

and

$$\begin{aligned} \frac{1}{2} \sum_{\mathbf{p}} \text{tr } [\mathcal{E} \delta N_1] \\ = -N_F \left\langle\left\langle \Delta \delta\Delta_1 - \left(\tau_4 E^2 - \frac{\partial\Delta}{\partial(1/\beta)} \frac{\Delta}{\beta} \tau_5 \right) \frac{S}{C_V} \text{div } \mathbf{v}_n + CE^2 \right\rangle\right\rangle \\ = 0 \end{aligned} \quad (158)$$

from which it follows that

$$\mathbf{B} = -\frac{\langle\langle \tau_2 \xi^2 \rangle\rangle}{m^* E_F Y} \beta \nabla \left(\frac{1}{\beta}\right) \quad (159)$$

and

$$\begin{aligned} C = \left\{ \frac{Y\Delta^2}{Y-1} \left\langle\left\langle \tau_4 - \frac{\Delta}{E^2} \frac{1}{\beta} \frac{\partial\Delta}{\partial(1/\beta)} \tau_5 \right\rangle\right\rangle - \left\langle\left\langle \tau_4 E^2 - \frac{\Delta}{\beta} \frac{\partial\Delta}{\partial(1/\beta)} \tau_5 \right\rangle\right\rangle \right\} \\ \times \left\{ \frac{Y^2 \Delta^2}{Y-1} - \langle\langle E^2 \rangle\rangle \right\}^{-1} \frac{S}{C_V} \text{div } \mathbf{v}_n \end{aligned} \quad (160)$$

By choosing \mathbf{B} and C as Eqs. (159) and (160), we can recover the thermodynamic relations (136) and (137).

Let us turn to the momentum flux. Within the linear regime, the momentum flux Π_{ij} in the moving frame is obtained from Eq. (44) as

$$\Pi_{ij} = \sum_{\mathbf{p}} \frac{1}{4} \text{tr} \left[p_i \left\{ \frac{\partial \mathcal{E}}{\partial p_j}, \delta N \right\} + p_i \left\{ \frac{\partial \delta \mathcal{E}}{\partial p_j}, N_0 \right\} + \delta_{ij} \{ \delta \mathcal{E}, N_0 \} \right] \quad (161)$$

where

$$\delta N = \delta N'_i + \delta N_1 \quad \text{and} \quad \delta \mathcal{E} = \delta \mathcal{E}'_i + \delta \mathcal{E}_1.$$

The zeroth-order contribution to Π in the fixed frame is written in the same form as in the local equilibrium case:

$$(\Pi')_{ij} = \delta_{ij} [\rho(-\dot{\Lambda}) + S \delta(1/\beta)] \quad (162)$$

The first-order correction to Π is given by

$$(\Pi_1)_{ij} = \sum_{\mathbf{p}} \frac{1}{4} \text{tr} \left[p_i \left\{ \frac{\partial \mathcal{E}_0}{\partial p_j}, \delta N_1 \right\} \right] \quad (163)$$

Let us first treat the off-diagonal part:

$$\begin{aligned} (\Pi_1)_{i \neq j} &= \sum_{\mathbf{p}} \frac{1}{4} \text{tr} \left[p_i \left\{ \frac{\partial \mathcal{E}_0}{\partial p_j}, \delta N_1 \right\} \right] \\ &= -4 \sum_{\mathbf{p}} p_i^2 \left(\frac{\partial \xi}{\partial p_j} \right)^2 \tau_1(\xi) \left(\frac{\xi}{E} \right)^2 f' X_{ij} \\ &\equiv -\eta \left(\frac{\partial v_{nj}}{\partial R_i} + \frac{\partial v_{ni}}{\partial R_j} - \delta_{ij} \frac{2}{3} \text{div} \mathbf{v}_n \right) \end{aligned} \quad (164)$$

where

$$\eta = \frac{N_F}{15} \frac{p_f^4}{m^{*2}} \left\langle \left\langle \tau_1 \left(\frac{\xi}{E} \right)^2 \right\rangle \right\rangle \quad (165)$$

The diagonal part is computed from

$$\begin{aligned} (\Pi_1)_{ii} &= \sum_{\mathbf{p}} \frac{1}{4} \text{tr} \left[p_i \left\{ \frac{\partial \mathcal{E}_0}{\partial p_i}, \delta N_1 \right\} \right] \\ &= \rho \left\langle \left\langle \tau_3 \left(\frac{\xi}{E} \right)^2 \left[\frac{1}{N_F} \text{div} (\mathbf{J} - \rho \mathbf{v}_n) + \left(\frac{\rho}{N_F} - \frac{1}{3} \mathbf{p} \cdot \frac{\partial \xi}{\partial \mathbf{p}} \right) \text{div} \mathbf{v}_n \right] \right\rangle \right\rangle \\ &\quad - N_F \left\langle \left\langle \tau_3 \left(\frac{\xi}{E} \right)^2 \left(\frac{\rho}{N_F} - \frac{1}{3} \mathbf{p} \cdot \frac{\partial \xi}{\partial \mathbf{p}} \right) \left[\frac{1}{N_F} \text{div} (\mathbf{J} - \rho \mathbf{v}_n) \right. \right. \right. \\ &\quad \left. \left. \left. + \left(\frac{\rho}{N_F} - \frac{1}{3} \mathbf{p} \cdot \frac{\partial \xi}{\partial \mathbf{p}} \right) \text{div} \mathbf{v}_n \right] \right\rangle \right\rangle \end{aligned} \quad (166)$$

Recalling the definition of ζ_3 and ζ_4 in Eqs. (150) and (151), we can write Eq. (166) as

$$(\Pi_1)_{ii} = \rho[\zeta_3 \operatorname{div}(\mathbf{J} - \rho \mathbf{v}_n) + \zeta_4 \operatorname{div} \mathbf{v}_n] - \zeta_1 \operatorname{div}(\mathbf{J} - \rho \mathbf{v}_n) - \zeta_2 \operatorname{div} \mathbf{v}_n \quad (167)$$

where

$$\zeta_1 = \left\langle \left\langle \left(\frac{\rho}{N_F} - \frac{1}{3} \mathbf{p} \cdot \frac{\partial \xi}{\partial \mathbf{p}} \right) \tau_3 \left(\frac{\xi}{E} \right)^2 \right\rangle \right\rangle \quad (168)$$

and

$$\zeta_2 = N_F \left\langle \left\langle \left(\frac{\rho}{N_F} - \frac{1}{3} \mathbf{p} \cdot \frac{\partial \xi}{\partial \mathbf{p}} \right)^2 \tau_3 \left(\frac{\xi}{E} \right)^2 \right\rangle \right\rangle \quad (169)$$

We note that ζ_1 and ζ_4 satisfy the Onsager reciprocal relation³¹

$$\zeta_1 = \zeta_4 \quad (170)$$

although they are small compared with $\rho \zeta_3$ because they include in the integrand a small factor $(\rho/N_F) - \frac{1}{3} \mathbf{p} \cdot \partial \xi / \partial \mathbf{p}$. Collecting the results (162), (164), and (167) we have

$$\begin{aligned} \Pi_{ij} = & \delta_{ij} [\rho(-\dot{\Lambda} + \zeta_3 \operatorname{div}(\mathbf{J} - \rho \mathbf{v}_n) + \zeta_4 \operatorname{div} \mathbf{v}_n) + S \delta(1/\beta)] \\ & - \delta_{ij} [\zeta_1 \operatorname{div}(\mathbf{J} - \rho \mathbf{v}_n) + \zeta_2 \operatorname{div} \mathbf{v}_n] \\ & - \eta \left(\frac{\partial v_{nj}}{\partial R_i} + \frac{\partial v_{ni}}{\partial R_j} - \frac{2}{3} \delta_{ij} \operatorname{div} \mathbf{v}_n \right) \end{aligned} \quad (171)$$

If we redefine the pressure by

$$\delta p = \rho[-\dot{\Lambda} + \zeta_3 \operatorname{div}(\mathbf{J} - \rho \mathbf{v}_n) + \zeta_4 \operatorname{div} \mathbf{v}_n] + S \delta(1/\beta) \quad (172)$$

we find that the Gibbs–Duhem relation (138) is recovered [see Eq. (149)] and also that the momentum flux has the anticipated form³¹

$$\begin{aligned} \Pi_{ij} = & \delta p \delta_{ij} - \eta \left(\frac{\partial v_{nj}}{\partial R_i} + \frac{\partial v_{ni}}{\partial R_j} - \frac{2}{3} \delta_{ij} \operatorname{div} \mathbf{v}_n \right) \\ & - \delta_{ij} [\zeta_1 \operatorname{div}(\mathbf{J} - \rho \mathbf{v}_n) + \zeta_2 \operatorname{div} \mathbf{v}_n] \end{aligned} \quad (173)$$

Let us turn to the energy flux. In the same way as for the momentum flux, we obtain

$$\begin{aligned} \frac{\partial U}{\partial T} = & -\operatorname{div} \left(\mu_0 \mathbf{J} + \frac{S}{\beta} \mathbf{v}_n \right) \\ & - \frac{\partial}{\partial R_j} \sum_{\mathbf{p}} \frac{1}{4} \operatorname{tr} \left[\mathcal{E}_0 \left\{ \frac{\partial \mathcal{E}_0}{\partial p_j}, \delta N_1 \right\} \right] \end{aligned} \quad (174)$$

where the zeroth-order term has been already included in the first term. Substituting Eq. (146) in to Eq. (174), we finally obtain

$$\frac{\partial U}{\partial T} + \text{div} \left[\mu_0 \mathbf{J} + \frac{S}{\beta} \mathbf{v}_n - \kappa \nabla \left(\frac{1}{\beta} \right) \right] = 0 \quad (175)$$

where the thermal conductivity κ is given by

$$\kappa = \frac{2N_F E_F \beta}{3m^*} \langle \xi^2 \tau_2 \rangle \left(1 - \frac{\langle \xi^2 / E_F^2 \rangle}{\langle 1 \rangle} \right) \quad (176)$$

We can neglect the second term, which comes from the trivial solution, because it is smaller than the first by a factor of $(1/\beta E_F)^2$. Then it follows that

$$\kappa = (\beta \rho / m^*) \langle \xi^2 \tau_2 \rangle \quad (177)$$

Thus we have derived the complete set of hydrodynamic equations as well as the thermodynamic relations from the microscopic point of view.

For completeness, we estimate the first order correction to the magnitude of the order parameter given by Eqs. (156) and (160). We have seen in Section 7 that τ_4 and τ_5 have no singular behavior at the transition temperature; therefore it suffices to put $\tau_4 = \tau_5 = \tau(0)$. Then we have

$$\delta \Delta_1 = - \left[1 - \frac{\chi}{1 - Y} \left(1 + \frac{N_F}{C_V} Y \Delta \frac{\partial \Delta}{\partial (1/\beta)} \right) \right] \frac{1}{\beta} \frac{\partial \Delta}{\partial (1/\beta)} \frac{S}{C_V} \text{div} \mathbf{v}_n \tau(0) \quad (178)$$

where

$$\chi = 1 - \langle \xi^2 / E^2 \rangle \quad (179)$$

The leading term near T_c diverges as $1/\Delta^2$ which is in agreement with Shumeiko.²⁴

Finally, we investigate briefly the behavior of the transport coefficients near the transition temperature. The shear viscosity coefficient η was first investigated by Pethick *et al.*⁴ It decreases below the transition temperature proportionally to $\beta_c \Delta$, although such a behavior can be actually observed in the narrow temperature range of $|1 - \beta/\beta_c| \leq 10^{-3}$.^{5,35} To the thermal conductivity κ , there is no correction of order $\beta_c \Delta$. The second-viscosity coefficient ζ_3 was investigated by Wölfle⁶ and Wölfle and Einzel.⁷ It diverges near the transition temperature proportionally to $1/\beta_c \Delta$. From the expressions for $\zeta_1 = \zeta_4$ [Eqs. (151) and (168)], we can easily see that $\zeta_1 = \zeta_4$ is of order $\rho \zeta_3 / (\beta_c E_F)^2$ or of order $\eta / \rho \beta_c \Delta (\beta_c E_F)^2$. Thus, $\zeta_1 = \zeta_4$ can be neglected compared with $\rho \zeta_3$ and η / ρ in the temperature range where $\tau_p \Delta \gg 1$ is satisfied. Another second viscosity coefficient ζ_2 is estimated from Eq. (169) to be $\rho^2 \zeta_3 / (\beta_c E_F)^2$ which is also small except for the immediate vicinity of the transition temperature. All the results agree with previous calculations.^{6,24}

9. CONCLUDING REMARKS

In the preceding sections, we have derived the two-fluid hydrodynamic equations for $^3\text{He-B}$ by applying the generalized Enskog–Chapman scheme to the matrix kinetic equation. Our method is a natural extension of Kadanoff and Baym’s method²⁵ for normal systems to superfluid systems.

We have derived the Boltzmann equation for the Bogoliubov–Valatin quasiparticle distribution function only in the neighborhood of the transition temperature. It can be also done throughout the temperature range $0 \leq 1/\beta \leq 1/\beta_c$ as long as the condition $\tau_p \Delta \gg 1$ is fulfilled. Moreover, it is possible to incorporate a more realistic scattering amplitude, including the higher partial wave contribution.^{7,8} The numerical evaluation of the transport coefficients by use of the variational method⁵ will be reported elsewhere.

We have calculated the scattering processes in the Born approximation. However, it is not satisfactory in the low-temperature limit because the group velocities of the Bogoliubov–Valatin quasiparticles in the BW state vanish at $E = \Delta$. Therefore, as was shown in the case of rotons in He II,³⁶ we can expect that the transport coefficient, for example η , will diverge logarithmically at the low-temperature limit, contrary to the prediction by Pethick *et al.*³ It is still not clear whether this divergence can be observed or not. This problem will be considered separately.

Our method for deriving the hydrodynamic equations also can be applied to the spin dynamics and the orbital dynamics for both the A and B phases of superfluid ^3He . Investigations in this direction are now in progress.

APPENDIX

We show in this appendix in detail that the condition

$$\sum_{\mathbf{p}} N^{(2)}(\mathbf{p}) = 0$$

is equivalent to requiring the number conservation law. For simplicity, we consider the linear regime within the first order in ω and q . From Eq. (71) and (145), we note that the first-order term of $N^{(2)}$ is written as

$$N^{(2)} = N_A^{(2)} + N_1^{(2)} \quad (\text{A1})$$

with

$$N_A^{(2)} = \int \frac{d\omega'}{2\pi} A_1^{(2)} f^<(\omega') \quad (\text{A2})$$

and

$$N_1^{(2)} = \sum_{\nu} \delta F_{\nu}^{(2)} \quad (\text{A3})$$

The first term $N_A^{(2)}$ is obtained from Eq. (58) to be

$$N_A^{(2)} = \left(\frac{\theta}{2E^3} - \frac{f'}{2E^2} \right) \left(-\xi \frac{\partial \delta \mathcal{E}^{(1)}}{\partial T} + \Delta \frac{\partial \delta \mathcal{E}^{(3)}}{\partial T} - \Delta \frac{\partial \xi}{\partial \mathbf{p}} \cdot \frac{\partial \delta \mathcal{E}^{(0)}}{\partial \mathbf{R}} \right) \quad (\text{A4})$$

Then, by use of Eqs. (98)–(100) we have

$$\sum_{\mathbf{p}} N_A^{(2)} = \frac{Y-1}{4\Delta} \left[-\frac{\partial \rho}{\partial T} - \text{div } \mathbf{J} + \frac{Y}{Y-1} \text{div} (\mathbf{J} - \rho \mathbf{v}_n) \right] \quad (\text{A5})$$

where Y is the Yosida function and we have neglected the odd term in ξ during the integration over ξ .

On the other hand, the second contribution $\sum_{\mathbf{p}} N_1^{(2)}$ can be directly evaluated from Eq. (110). Multiplying by ρ_3 on both sides of Eq. (110) and also taking the trace, we sum over \mathbf{p} to yield

$$\sum_{\mathbf{p}} N_1^{(2)} = \frac{Y}{4\Delta} \left(\frac{\partial \rho}{\partial T} + \rho \text{div } \mathbf{v}_n \right) \quad (\text{A6})$$

Finally, we obtain the anticipated result:

$$\sum_{\mathbf{p}} N^{(2)} = \frac{1}{4\Delta} \left(\frac{\partial \rho}{\partial T} + \text{div } \mathbf{J} \right)$$

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