Journal of Low Temperature Physics, Vol. 32, Nos. 1/2, 1978

Transport and Relaxation Properties of Superfluid ^{3}He. I. Kinetic Equation and Bogoliubov Quasiparticle Relaxation Rate

D. Einzel and P. Wölfle*

Institut für Theoretische Physik, Technische Universität München, Garching, Germany

(Received November 30, 1977)

The kinetic equation for Bogoliubov quasiparticles for both the A and B phases of superfluid ^{3}He is derived from the general matrix kinetic equation. A condensed expression for the exact spin-symmetric collision integral is given. The quasiparticle relaxation rate is calculated for the BW state using the s–p *approximation for the quasiparticle scattering amplitude. By using the results for the quasiparticle relaxation rate, the mean free path of Bogoliubov quasiparticles is calculated for all temperatures.*

1. INTRODUCTION

Since the discovery of the superfluid phases of ^{3}He and their subsequent interpretation as pair-correlated Fermi liquids, considerable effort has been spent in studying the dynamics of this system. One expects that at least for not too high frequencies ω and wave vectors $\mathbf{q}$, such that $\omega < \Delta$ and $q < \Delta / v_F$ (here Δ is the energy gap and v_F the Fermi velocity) a two-fluid description along the lines set forth by Landau and others[1] in the case of ^{4}He should be applicable. Here the role of the superfluid component is played by the condensate of Cooper pairs, while the normal component is represented by the gas of thermal excitations, most importantly the Bogoliubov quasiparticles. In this low-frequency, low-wave vector regime the condensate is characterized by the local structure of the order parameter, which is completely determined by the variables corresponding to the various broken symmetries.

The normal component, on the other hand, is described by a distribution function in momentum space of Bogoliubov quasiparticles (or any

*Also at Max-Planck-Institut für Physik und Astrophysik, München, Germany.

other elementary excitations that happen to be of importance). For frequencies so low that the gas of thermal excitations is in a local equilibrium state, completely characterized by a set of local thermodynamic variables, the motion of the system is described by a set of hydrodynamic equations. The structure of these equations can be derived from rather general arguments involving transformation properties and the symmetry of the ground state.[1,2] These equations contain a number of phenomenological parameters, such as thermodynamic derivatives and transport coefficients.

The purpose of this paper is to derive the scalar kinetic equation for Bogoliubov quasiparticles, which is the starting point for the calculation of the transport coefficients. A matrix kinetic equation was first discussed in detail by Betbeder-Matibet and Nozières[4] for the case of s-wave pairing in the collisionless limit. The effect of quasiparticle collisions on the matrix kinetic equation and its application to the sound propagation problem in superfluid ^{3}He has been extensively studied by Wölfle.[5,6]

The collision integral of the scalar kinetic equation was first derived by Shumeiko[7] for the case of s-wave pairing. Bhattacharyya, Pethick, and Smith[8,9] have given the collision integral for p-wave pairing for temperatures close to the transition and in the low-temperature limit. Ono *et al.*[10] have derived the collision integral for isotropic quasiparticle scattering in the BW state.

The plan of the paper is as follows. In Section 2 the scalar kinetic equation for the distribution function of Bogoliubov quasiparticles is derived from the microscopic theory. The collision integral for the spin-symmetric case is discussed in detail in Section 3. The quasiparticle relaxation time and mean free path are evaluated and discussed in Section 4.

2. SCALAR KINETIC EQUATION

The state of a pair-correlated Fermi liquid is completely described by the quasiparticle (qp) distribution function

$$f_{\mathbf{p}\sigma\sigma'}(\mathbf{r},t)=\int\frac{d^3q}{(2\pi)^3}[\exp(i\mathbf{q}\mathbf{r})]\langle a^\dagger_{\mathbf{p}_-\sigma}(t)a_{\mathbf{p}_+\sigma'}(t)\rangle \tag{1}$$

and the pair amplitude

$$g_{\mathbf{p}\sigma\sigma'}(\mathbf{r},t)=\int\frac{d^3q}{(2\pi)^3}[\exp(i\mathbf{q}\mathbf{r})]\langle a_{-\mathbf{p}_-\sigma}(t)a_{\mathbf{p}_+\sigma'}(t)\rangle \tag{2}$$

Here the subscript on $\mathbf{p}_\pm$ denotes $\mathbf{p}\pm\mathbf{q}/2$. The observable densities and currents are obtained by averaging the respective operators over the distribution function $f_\mathbf{p}$.

It is convenient to combine the two correlation functions in a 4×4 matrix distribution function in particle hole and spin space:

$$\underline{n}_{\mathbf{p}} = \begin{pmatrix} f_{\mathbf{p}} & g_{\mathbf{p}} \\ g_{\mathbf{p}}^{\dagger} & -f_{-\mathbf{p}}^{T} \end{pmatrix} \tag{3}$$

4×4 matrices of this type are here and in the following denoted by an underscore.

In thermal equilibrium, the distribution function is given by the Fermi function

$$\underline{n}_{\mathbf{p}}^{0} = f_{\mathbf{p}}\{\underline{\varepsilon}_{\mathbf{p}}^{0}\} = \tfrac{1}{2}[\underline{1} - (\underline{\varepsilon}_{\mathbf{p}}^{0}/E_{\mathbf{p}})t_{\mathbf{p}}] \tag{4}$$

where $\underline{\varepsilon}_{\mathbf{p}}^{0}$ is the qp energy matrix

$$\underline{\varepsilon}_{\mathbf{p}}^{0} = \begin{pmatrix} \xi_{p} & \Delta_{\mathbf{p}} \\ \Delta_{\mathbf{p}}^{\dagger} & -\xi_{p} \end{pmatrix} \tag{5}$$

with the normal quasiparticle energy measured from the Fermi level, $\xi_p = \mathbf{p}^2/2m^* - \mu$, and the gap parameter $\Delta_{\mathbf{p}}$. The single-particle energy is given by

$$E_{\mathbf{p}} = [\xi_p^2 + \tfrac{1}{2}\,\mathrm{tr}\,\Delta_{\mathbf{p}}\,\Delta_{\mathbf{p}}^{\dagger}]^{1/2} \tag{6}$$

and

$$t_{\mathbf{p}} = \tanh\,(E_{\mathbf{p}}/2T) \tag{7}$$

assuming $\Delta_{\mathbf{p}\sigma\sigma'}$ to be proportional to a unitary matrix. Note that the quantity $E_{\mathbf{p}}$ in general depends on the magnitude and direction of the momentum $\mathbf{p}$.

The linear response of the system to an external disturbance of wave vector $\mathbf{q}$ and frequency ω can be calculated from the linearized distribution function $\delta\underline{n}_{\mathbf{p}} = \underline{n}_{\mathbf{p}} - \underline{n}_{\mathbf{p}}^{0}$. The time evolution of $\delta\underline{n}_{\mathbf{p}}$ is governed by a matrix kinetic equation,[4,5] describing the change in the qp distribution due to streaming, transitions from the normal to the superfluid component and vice versa, and due to collisions between the quasiparticles. It turns out that the collision integral operates on the part of the distribution function characterizing the deviation from local equilibrium $\delta\underline{n}_{\mathbf{p}}' = \delta\underline{n}_{\mathbf{p}} - \delta\underline{n}_{\mathbf{p}}^{l}$, where $\delta\underline{n}_{p}^{l}$ is the local equilibrium distribution function. It is also necessary to distinguish between particle-like and hole-like excitations, by introducing two distribution functions[5] ($\mu = \pm 1$)

$$\delta f_{\mathbf{p}\sigma\sigma'}^{\mu}(\mathbf{q}, t) = \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi}\,\theta(\mu\omega') \times \int_{-\infty}^{\infty} dt'\, e^{i\omega' t'}\,\delta\left\langle \frac{1}{2}\left[a_{\mathbf{p}+\sigma}^{\dagger}\left(t+\frac{t'}{2}\right), a_{\mathbf{p}-\sigma'}\left(t-\frac{t'}{2}\right)\right]_{-}\right\rangle \tag{8}$$

the sum of which is the distribution function $\delta f_{\mathbf{p}}$ defined by Eq. (1), and analogously for $\delta g_{\mathbf{p}}^{\mu}$. In Eq. (8), $\theta(x)$ is the step function. $\delta \underline{n}_{\mathbf{p}}^{\mu}$ and $\delta \underline{n}_{\mathbf{p}}^{\mu l}$ obey the following set of kinetic equations[5]:

$$\begin{aligned} \underline{\Omega}\{\delta \underline{n}_{\mathbf{p}}^{\mu}\} - \delta \underline{Q}_{\mathbf{p}}^{\mu} &= \underline{I}_{\mathbf{p}}^{\mu}\{\delta \underline{n}_{\mathbf{p}}^{\mu}\} \\ \underline{\Omega}\{\delta \underline{n}_{\mathbf{p}}^{\mu l}\} - \delta \underline{Q}_{\mathbf{p}}^{\mu} - \tfrac{1}{2}\omega f_{\mathbf{p}}'[\underline{a}_{\mathbf{p}+}^{\mu}\, \delta \underline{\varepsilon}_{\mathbf{p}} + \delta \underline{\varepsilon}_{\mathbf{p}} \underline{a}_{\mathbf{p}-}^{\mu}] &= 0 \end{aligned} \tag{9}$$

where

$$\underline{\Omega}\{\delta \underline{n}_{\mathbf{p}}^{\mu}\} = \omega\, \delta \underline{n}_{\mathbf{p}}^{\mu} - \underline{\varepsilon}_{\mathbf{p}+}^{0}\, \delta \underline{n}_{\mathbf{p}}^{\mu} + \delta \underline{n}_{\mathbf{p}}^{\mu} \underline{\varepsilon}_{\mathbf{p}-}^{0} \tag{10}$$

$$\delta \underline{Q}_{\mathbf{p}}^{\mu} = \tfrac{1}{2}\mu[t_{\mathbf{p}+}\underline{a}_{\mathbf{p}-}^{\mu}\, \delta \underline{\varepsilon}_{\mathbf{p}} - \delta \underline{\varepsilon}_{\mathbf{p}} \underline{a}_{\mathbf{p}-}^{\mu} t_{\mathbf{p}-}] \tag{11}$$

Here the $\underline{a}_{\mathbf{p}}^{\mu}$ are the single-particle spectral weights

$$\underline{a}_{\mathbf{p}}^{\mu} = \tfrac{1}{2}[\underline{1} + \mu \underline{\varepsilon}_{\mathbf{p}}^{0}/E_{\mathbf{p}}] \tag{12}$$

and $f_{\mathbf{p}}' = -(1/4T)\,\mathrm{sech}^2\,(E_{\mathbf{p}}/2T)$ is the derivative of the Fermi function.

The change in the quasiparticle energy is given by

$$\delta \underline{\varepsilon}_{\mathbf{p}} = \begin{pmatrix} \delta \varepsilon_{\mathbf{p}} & \delta \Delta_{\mathbf{p}} \\ \delta \Delta_{\mathbf{p}}^{\dagger} & -\delta \varepsilon_{-\mathbf{p}}^{T} \end{pmatrix} \tag{13}$$

where $\delta \varepsilon_{\mathbf{p}}$ and $\delta \Delta_{\mathbf{p}}$ are related back to the distribution functions via

$$\delta \varepsilon_{\mathbf{p}} = \sum_{\mathbf{k}} f_{\mathbf{pk}}\, \delta f_{\mathbf{k}} \tag{14}$$

and

$$\delta \Delta_{\mathbf{p}} = \sum_{\mathbf{k}} g_{\mathbf{pk}}\, \delta g_{\mathbf{k}} \tag{15}$$

Here $f_{\mathbf{pk}}$ and $g_{\mathbf{pk}}$ are the Fermi liquid and pair interactions, respectively.

The matrix collision integral appearing in Eq. (9) may be split into an outscattering and an inscattering contribution

$$\underline{I}_{\mathbf{p}}^{\mu} = (\underline{I}_{\mathbf{p}}^{\mu})_{\mathrm{out}} + (\underline{I}_{\mathbf{p}}^{\mu})_{\mathrm{in}} \tag{16}$$

The outscattering term is given by

$$(\underline{I}_{\mathbf{p}}^{\mu})_{\mathrm{out}} = -\tfrac{1}{2} i [\underline{\Gamma}_{\mathbf{p}}^{\mu}, \delta \underline{n}_{\mathbf{p}}^{\mu}]_{+} \tag{17}$$

Here $\underline{\Gamma}_{\mathbf{p}}^{\mu}$ is a generalized matrix quasiparticle relaxation rate, derived from the microscopic theory[5] as

$$\begin{aligned} \{\underline{\Gamma}_{\mathbf{p}}^{\mu}\}_{\sigma\sigma'}^{\rho\rho'} = \frac{1}{(-T) f_{\mathbf{p}}'} \sum_{2,3,4} (2\pi)^4 \delta^3(\mathbf{p} + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4)\, \delta(\mu E + \mu_2 E_2 - \mu_3 E_3 \\ - \mu_4 E_4)\, f_{\mathbf{p}} f_{\mathbf{p}_2} (1 - f_{\mathbf{p}_3})(1 - f_{\mathbf{p}_4}) \cdot \{\underline{G}\}_{\sigma\sigma'}^{\rho\rho'} \end{aligned} \tag{18}$$

where

$$\{\underline{G}\}^{\rho\rho'}_{\sigma\sigma'} = \frac{1}{2} \sum_{\substack{\sigma_2\ldots\sigma_4 \\ \sigma_2'\ldots\sigma_4'}} \{A_1A_1'[\underline{a}^{\mu_2}_{\mathbf{p}_2}]^{\rho'\rho}_{\sigma_2'\sigma_2}[\underline{a}^{\mu_3}_{\mathbf{p}_3}]^{\rho\rho'}_{\sigma_3\sigma_3'}[\underline{a}^{\mu_4}_{\mathbf{p}_4}]^{\rho\rho'}_{\sigma_4\sigma_4'}$$

$$-2A_1A_2'[\underline{a}^{\mu_2}_{\mathbf{p}_2}]^{-\rho',\rho}_{\sigma_2'\sigma_2}[\underline{a}^{\mu_3}_{\mathbf{p}_3}]^{\rho\rho'}_{\sigma_3\sigma_3'}[\underline{a}^{\mu_4}_{\mathbf{p}_4}]^{\rho,-\rho'}_{\sigma_4\sigma_4'}\} \tag{19}$$

The summations in Eq. (18) extend over momenta $\mathbf{p}_i$ and particle–hole indices μ_i for $i=2,3,4$, keeping the particle–hole indices ρ, ρ' fixed, and $f_i = [\exp(E_{\mathbf{p}_i}/T)+1]^{-1}$.

The qp scattering amplitudes A_1 and A_2 are approximated by their normal state values A_N

$$\begin{aligned} A_1 &= A_N(\mathbf{p}_1\sigma_1, \mathbf{p}_2\sigma_2; \mathbf{p}_3\sigma_3, \mathbf{p}_4\sigma_4) \\ A_2' &= A_N(\mathbf{p}_1\sigma_1', -\mathbf{p}_4\sigma_4'; \mathbf{p}_3\sigma_3', -\mathbf{p}_2\sigma_2') \end{aligned} \tag{20}$$

The changes in the scattering amplitude due to pair correlations are confined to a relatively small portion of momentum space and will be neglected. A prime on $A_{1,2}$ indicates spin variables σ_i'.

The inscattering term has the form

$$(\underline{I}^{\mu}_{\mathbf{p}})_{\text{in}} = \tfrac{1}{2}i[\underline{C}^{\mu}_{\mathbf{p}}, \underline{a}^{\mu}_{\mathbf{p}}]_+ \tag{21}$$

where

$$\{\underline{C}^{\mu}_{\mathbf{p}}\}^{\rho\rho'}_{\sigma\sigma'} = \sum_{2,3,4} (2\pi)^4\delta^3(\mathbf{p}+\mathbf{p}_2-\mathbf{p}_3-\mathbf{p}_4)\,\delta(\mu E+\mu_2E_2-\mu_3E_3-\mu_4E_4)$$

$$\times f_{\mathbf{p}}f_{\mathbf{p}_2}(1-f_{\mathbf{p}_3})(1-f_{\mathbf{p}_4})\cdot\{\underline{G}^{\text{in}}\{\delta\underline{n}'\}\}^{\rho\rho'}_{\sigma\sigma'} \tag{22}$$

with

$$\{\underline{G}^{\text{in}}\{\delta\underline{n}'\}\}^{\rho\rho'}_{\sigma\sigma'} = \tfrac{1}{2} \sum_{\substack{\sigma_2\ldots\sigma_4 \\ \sigma_2'\ldots\sigma_4'}} \{A_1A_1'(-[\delta\hat{\underline{n}}^{\mu_2'}_{\mathbf{p}_2}]^{\rho'\rho}_{\sigma_2'\sigma_2}[\underline{a}^{\mu_3}_{\mathbf{p}_3}]^{\rho\rho'}_{\sigma_3\sigma_3'}[\underline{a}^{\mu_4}_{\mathbf{p}_4}]^{\rho\rho'}_{\sigma_4\sigma_4'}$$

$$+[\underline{a}^{\mu_2}_{\mathbf{p}_2}]^{\rho'\rho}_{\sigma_2'\sigma_2}[\delta\hat{\underline{n}}^{\mu_3'}_{\mathbf{p}_3}]^{\rho\rho'}_{\sigma_3\sigma_3'}[\underline{a}^{\mu_4}_{\mathbf{p}_4}]^{\rho\rho'}_{\sigma_4\sigma_4'}$$

$$+[\underline{a}^{\mu_2}_{\mathbf{p}_2}]^{\rho'\rho}_{\sigma_2'\sigma_2}[\underline{a}^{\mu_3}_{\mathbf{p}_3}]^{\rho\rho'}_{\sigma_3\sigma_3'}[\delta\hat{\underline{n}}^{\mu_4'}_{\mathbf{p}_4}]^{\rho\rho'}_{\sigma_4\sigma_4'})$$

$$-2A_1A_2'(-[\delta\hat{\underline{n}}^{\mu_2'}_{\mathbf{p}_2}]^{-\rho',\rho}_{\sigma_2'\sigma_2}[\underline{a}^{\mu_3}_{\mathbf{p}_3}]^{\rho\rho'}_{\sigma_3\sigma_3'}[\underline{a}^{\mu_4}_{\mathbf{p}_4}]^{\rho,-\rho'}_{\sigma_4\sigma_4'}$$

$$+[\underline{a}^{\mu_2}_{\mathbf{p}_2}]^{-\rho'\rho}_{\sigma_2'\sigma_2}[\delta\hat{\underline{n}}^{\mu_3'}_{\mathbf{p}_3}]^{\rho\rho'}_{\sigma_3\sigma_3'}[\underline{a}^{\mu_4}_{\mathbf{p}_4}]^{\rho,-\rho'}_{\sigma_4\sigma_4'}$$

$$+[\underline{a}^{\mu_2}_{\mathbf{p}_2}]^{-\rho'\rho}_{\sigma_2'\sigma_2}[\underline{a}^{\mu_3}_{\mathbf{p}_3}]^{\rho\rho'}_{\sigma_3\sigma_3'}[\delta\hat{\underline{n}}^{\mu_4'}_{\mathbf{p}_4}]^{\rho,-\rho'}_{\sigma_4\sigma_4'})\} \tag{23}$$

and the same summations as in Eq. (18) are implied. In Eq. (23) we have introduced the abbreviation

$$\delta\hat{\underline{n}}^{\mu}_{\mathbf{p}} = \frac{1}{(-T)f'_{\mathbf{p}}}\,\delta\underline{n}^{\mu}_{\mathbf{p}} = \frac{1}{f_{\mathbf{p}}(1-f_{\mathbf{p}})}\,\delta\underline{n}^{\mu}_{\mathbf{p}}$$

Note that the whole matrix structure with respect to particle–hole and spin variables is contained in the quantities $\underline{G}^{\text{in}}$ and $\underline{G}$.

We now perform a Bogoliubov transformation $\underline{U}$ on the kinetic equation. $\underline{U}$ diagonalizes the equilibrium energy matrix

$$\underline{U}_{\mathbf{p}}\underline{\varepsilon}^0_{\mathbf{p}}\underline{U}^{\dagger}_{\mathbf{p}} = E_{\mathbf{p}}\underline{\rho}_3 \tag{24}$$

(here $\underline{\rho}_3 = \tau^3 \otimes \tau^0$ and $\{\tau^i\}$, $i = 0, x, y, z$, is the system of Pauli matrices, including the unit matrix) and is found as

$$\underline{U}_{\mathbf{p}} = \begin{pmatrix} u_{\mathbf{p}} & v_{\mathbf{p}} \\ -v^{+}_{\mathbf{p}} & u_{\mathbf{p}} \end{pmatrix} \tag{25}$$

with

$$u_{\mathbf{p}} = \left[\frac{1}{2}\left(1 + \frac{\xi_p}{E_{\mathbf{p}}}\right)\right]^{1/2}, \qquad v_{\mathbf{p}} = \frac{\Delta_{\mathbf{p}}}{|\Delta_{\mathbf{p}}|}\left[\frac{1}{2}\left(1 - \frac{\xi_p}{E_{\mathbf{p}}}\right)\right]^{1/2} \tag{26}$$

Note that in the case of triplet pairing $v_{\mathbf{p}}$ is a nontrivial matrix in spin space, depending on both the magnitude and direction of the momentum $\mathbf{p}$.

The transform of the spectral weights $\underline{a}^{\mu}_{\mathbf{p}}$ is given by

$$\tilde{\underline{a}}^{\mu}_{p} = \underline{U}_{\mathbf{p}}\underline{a}^{\mu}_{\mathbf{p}}\underline{U}^{\dagger}_{\mathbf{p}} = \tfrac{1}{2}[\underline{1} + \mu\underline{\rho}_3] = \begin{cases} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, & \mu = 1 \\ \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, & \mu = -1 \end{cases} \tag{27}$$

Let us define Bogoliubov-transformed distibution functions by

$$\delta\tilde{\underline{n}}^{\mu}_{\mathbf{p}} = \underline{U}_{\mathbf{p}}\,\delta\underline{n}^{\mu}_{\mathbf{p}}\underline{U}^{\dagger}_{\mathbf{p}} = \begin{pmatrix} (\delta\tilde{\underline{n}}^{\mu}_{\mathbf{p}})_{++} & (\delta\tilde{\underline{n}}^{\mu}_{\mathbf{p}})_{+-} \\ (\delta\tilde{\underline{n}}^{\mu}_{\mathbf{p}})_{-+} & (\delta\tilde{\underline{n}}^{\mu}_{\mathbf{p}})_{--} \end{pmatrix} \tag{28}$$

and correspondingly

$$\delta\tilde{\underline{\varepsilon}}_{\mathbf{p}} = \underline{U}_{\mathbf{p}}\,\delta\underline{\varepsilon}_{\mathbf{p}}\underline{U}^{\dagger}_{\mathbf{p}} \tag{29}$$

Multiplying the kinetic equation (9) by $\underline{U}_{p+}$ from the left and by $\underline{U}^{\dagger}_{p-}$ from the right and expanding in $v_{\mathrm{F}}q/|\Delta|$, one first observes that

$$(\delta\tilde{\underline{n}}^{\mu=-1}_{\mathbf{p}})_{++} = (\delta\tilde{\underline{n}}^{\mu=+1}_{\mathbf{p}})_{--} = 0 \tag{30}$$

as a consequence of the projection property of $\tilde{\underline{a}}^{\mu}_{p}$.

$(\delta\tilde{\underline{n}}^{\mu}_{p})_{+-}$ is found to be independent of μ and decoupled from $(\delta\tilde{\underline{n}}^{\mu}_{p})_{++}$,

$$(\delta\tilde{\underline{n}}^{\mu}_{\mathbf{p}})_{+-} = -\frac{t_{\mathbf{p}}}{2E_{\mathbf{p}}}(\delta\tilde{\underline{\varepsilon}}_{\mathbf{p}})_{+-} + O\left(\frac{\omega}{\Delta}, \frac{v_{\mathrm{F}}q}{\Delta}, \frac{1}{\tau\Delta}\right) \tag{31}$$

where the collision integral has been characterized by the relaxation rate

$1/\tau$. In this limit $(\delta \tilde{\underline{n}}_p^\mu)_{+-}$ is equal to the local equilibrium distribution $(\delta \tilde{\underline{n}}_{\mathbf{p}}^{\mu'})_{+-}$ and consequently drops out of the collision integral. This corresponds to the fact that the superfluid component is in a local equilibrium state provided the conditions ω, $v_F q$, $1/\tau \ll \Delta$ are satisfied.

There is only one nontrivial component of the matrix distribution function left, namely

$$\delta \nu_{\mathbf{p}} \equiv (\delta \tilde{\underline{n}}_{\mathbf{p}}^{\mu=1})_{++} \tag{32}$$

$\delta \nu_{\mathbf{p}}$ characterizes the occupation of single-qp eigenstates of energy $E_{\mathbf{p}}$.

The scalar kinetic equation for $\delta \nu_{\mathbf{p}}$ reads

$$\omega\, \delta \nu_{\mathbf{p}} - \mathbf{q} \nabla_{\mathbf{p}} E_{\mathbf{p}}\, \delta \nu'_{\mathbf{p}} = -(i/\tau_{\mathbf{p}})\, \delta \nu'_{\mathbf{p}} + i \tilde{C}_{\mathbf{p}}^{\mu=1} \{\delta \nu'\} \tag{33}$$

where

$$\delta \nu'_{\mathbf{p}} = \delta \nu_{\mathbf{p}} - \delta \nu^{l}_{\mathbf{p}} = \delta \nu_{\mathbf{p}} - f'_{\mathbf{p}} \delta E_{\mathbf{p}} \tag{34}$$

is the deviation of $\delta \nu_p$ from local equilibrium, and

$$\delta E_{\mathbf{p}} \equiv (\delta \tilde{\underline{\varepsilon}}_{\mathbf{p}})_{++} \tag{35}$$

Here $1/\tau_p$ is the quasiparticle relaxation rate,

$$1/\tau_{\mathbf{p}} = \tfrac{1}{2}\, \mathrm{tr}_\sigma \left[(\underline{U}_{\mathbf{p}} \underline{\Gamma}_{\mathbf{p}}^{\mu=1} \underline{U}_{\mathbf{p}}^{+})_{++} \right] \tag{36}$$

and $\tilde{C}_{\mathbf{p}}$ is the inscattering collision integral,

$$\tilde{C}_{\mathbf{p}} = (\underline{U}_{\mathbf{p}} \underline{C}_{\mathbf{p}}^{\mu=1} \underline{U}_{\mathbf{p}}^{+})_{++} \tag{37}$$

It should be emphasized that the kinetic equation (33) is quite general in that it is valid for both spin-symmetric and spin-antisymmetric distribution functions $\delta \nu_{\mathbf{p}}$.

3. COLLISION INTEGRAL FOR THE SPIN-SYMMETRIC CASE

Performing the Bogoliubov transformation on the particle–hole matrix $\underline{\Gamma}_p^\mu$, one obtains

$$(\underline{U}_{\mathbf{p}} \underline{\Gamma}_{\mathbf{p}}^{\mu} \underline{U}_{\mathbf{p}}^{+})_{++} = \frac{1}{2} \left\{ \left(1 + \frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}\right) (\underline{\Gamma}_{\mathbf{p}}^{\mu})_{++} + \left(1 - \frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}\right) \frac{\Delta_{\mathbf{p}} (\underline{\Gamma}_{\mathbf{p}}^{\mu})_{--} \Delta_{\mathbf{p}}^{+}}{|\Delta_{\mathbf{p}}|^2} + (\underline{\Gamma}_{\mathbf{p}}^{\mu})_{+-} \frac{\Delta_{\mathbf{p}}}{E_{\mathbf{p}}} + \frac{\Delta_{\mathbf{p}}^{+}}{E_{\mathbf{p}}} (\underline{\Gamma}_{\mathbf{p}}^{\mu})_{-+} \right\} \tag{38}$$

$\tilde{C}_{\mathbf{p}}$ is given by an analogous expression.

As seen from Eqs. (18) and (22) for Γ and C, the transformation involves only manipulations with the quantities $\underline{G}$ and $\underline{G}^{\mathrm{in}}\{\delta \underline{\hat{n}}'\}$. We first eliminate $\delta \underline{\hat{n}}'$ in favor of $\delta \nu'$ by performing the reverse transformation on the transformed matrix distribution function

$$\delta n_{\mathbf{p}}^{\mu'} = \begin{pmatrix} \frac{1}{2}(1+\mu)\,\delta\nu'_{\mathbf{p}} & 0 \\ 0 & -\frac{1}{2}(1-\mu)\,\delta\nu'^{T}_{\mathbf{p}} \end{pmatrix} \tag{39}$$

Assuming that $\delta\nu'_{\mathbf{p}}$ is either even or odd under the sign reversal of $\mathbf{p}$ (which is the case in all later applications), we can write

$$\delta\nu'_{-\mathbf{p}} = s\,\delta\nu'_{\mathbf{p}}; \qquad s = \pm 1 \tag{40}$$

Then $\delta \underline{n}_{\mathbf{p}}^{\mu'}$ can be written in the compact form

$$\delta \underline{n}_{\mathbf{p}}^{\mu'} = \underline{U}_{\mathbf{p}}^{\dagger}\,\delta\underline{\nu}_{\mathbf{p}}^{\mu'}\,\underline{U}_{\mathbf{p}} = p_{\mu}^{s}\,\underline{a}_{\mathbf{p}}^{\mu}\,\delta\nu'_{\mathbf{p}} \tag{41}$$

where

$$p_{\mu}^{s} = (1+\mu)/2 - s(1-\mu)/2 \tag{42}$$

In deriving (41) we have specialized the distribution function $\delta\nu'_{\mathbf{p}}$ to be spin-symmetric.

Substituting Eq. (41) into Eq. (22), one immediately finds that the matrix structure of $\underline{C}$ reduces to that of $\underline{\Gamma}$, viz.,

$$\{\underline{C}_{\mathbf{p}}^{\mu}\}_{\sigma\sigma'}^{\rho\rho'} = \sum_{2,3,4} (2\pi)^4 \delta^3(\mathbf{p}+\mathbf{p}_2-\mathbf{p}_3-\mathbf{p}_4)\,\delta(\mu E+\mu_2 E_2-\mu_3 E_3-\mu_4 E_4)$$
$$\times f_{\mathbf{p}} f_{\mathbf{p}_2}(1-f_{\mathbf{p}_3})(1-f_{\mathbf{p}_4})\{\underline{G}\}_{\sigma\sigma'}^{\rho\rho'} \sum_{i=2}^{4} (-1)^{\delta_{i,2}} p_{\mu_i}^{s}\,\delta\hat{\nu}'_{p_i} \tag{43}$$

with $\delta\hat{\nu}$ defined by

$$\delta\hat{\nu}'_{\mathbf{p}} = \frac{1}{(-T)f'_{\mathbf{p}}}\delta\nu'_{\mathbf{p}} = \frac{1}{f_{\mathbf{p}}(1-f_{\mathbf{p}})}\delta\nu'_{\mathbf{p}}$$

In order to evaluate Eq. (43) further, it is convenient to introduce the spin-rotation invariant representation of the scattering amplitude in terms of the dimensionless singlet and triplet components A_s and A_t, respectively,

$$N_F A_N(\mathbf{p}_1\sigma_1, \mathbf{p}_2\sigma_2; \mathbf{p}_3\sigma_3, \mathbf{p}_4\sigma_4)$$
$$= \tfrac{1}{4}(A_s+3A_t)\tau^0_{\sigma_1\sigma_3}\tau^0_{\sigma_2\sigma_4} + \tfrac{1}{4}(A_t-A_s)\boldsymbol{\tau}_{\sigma_1\sigma_3}\cdot\boldsymbol{\tau}_{\sigma_2\sigma_4} \tag{44}$$

where $\boldsymbol{\tau} = \{\tau^x, \tau^y, \tau^z\}$ is the vector of Pauli matrices, τ^0 is the unit matrix, and

$$N_F = m^* p_F/\pi^2$$

is the density of states at the Fermi level for both spin projections.

Also, the following abbreviations for the spectral weights are introduced:

$$\underline{a}_p^{\mu} = \frac{1}{2}\begin{pmatrix} \alpha_{\mathbf{p}}^{\mu} & D_{\mathbf{p}}^{\mu} \\ (D_{\mathbf{p}}^{\mu})^{\dagger} & \alpha_{\mathbf{p}}^{-\mu} \end{pmatrix} \tag{45}$$

where

$$\alpha_{\mathbf{p}}^{\mu} = 1 + \mu\xi_{\mathbf{p}}/E_{\mathbf{p}}, \qquad D_{\mathbf{p}}^{\mu} = \mu\ \Delta_{\mathbf{p}}/E_{\mathbf{p}}$$

Substituting Eqs. (44) and (45) into the expression for $\underset{\sim}{G}$, Eq. (19), and performing the spin sums, one obtains

$$\{\underset{\sim}{G}\}_{\sigma\sigma'}^{++} = (1/8\pi N_{\mathrm{F}})\{W\alpha_2\alpha_3\alpha_4 - W_{12}\alpha_3\tfrac{1}{2}\,\mathrm{tr}\,[D_2^{+}D_4]\}\tau_{\sigma\sigma'}^{0} \tag{46a}$$

$$\{\underset{\sim}{G}\}_{\sigma\sigma'}^{+-} = (1/8\pi N_{\mathrm{F}})\{W_{+}\tfrac{1}{2}\,\mathrm{tr}\,[D_2^{+}D_4]D_{3\sigma\sigma'} + W_{-}\tfrac{1}{2}\,\mathrm{tr}\,[D_2^{+}D_3]D_{4\sigma\sigma'} \tag{46b}$$
$$+ W_{+-}[D_3D_2^{+}D_4]_{\sigma\sigma'} - W_{12}\alpha_2\alpha_4D_{3\sigma\sigma'}\}$$

$$\{\underset{\sim}{G}\}_{\sigma\sigma'}^{-+} = (\{\underset{\sim}{G}\}_{\sigma\sigma'}^{+-})^{\dagger}[\alpha_{\mathbf{p}_i}^{\mu_i} \to \alpha_{\mathbf{p}_i}^{-\mu_i}] \tag{46c}$$

$$\{\underset{\sim}{G}\}_{\sigma\sigma'}^{--} = (\{\underset{\sim}{G}\}^{++})^{\dagger}[\alpha_{\mathbf{p}_i}^{\mu_i} \to \alpha_{\mathbf{p}_i}^{-\mu_i}] \tag{46d}$$

where several traceless terms have not been written down explicitly. The functions W abbreviate various combinations of scattering amplitudes

$$W = \tfrac{1}{4}\pi[3A_t^2 + A_s^2] \tag{47a}$$

$$W_{12} = \tfrac{1}{4}\pi[(A_s + A_t)(A_{2s} + A_{2t}) + 4A_tA_{2t}] \tag{47b}$$

$$W_{(-)}^{+} = \tfrac{1}{4}\pi[A_t \underset{(-)}{+} A_s]^2 \tag{47c}$$

$$W_{+-} = \tfrac{1}{4}\pi[A_t^2 - A_s^2] \tag{47d}$$

By a well-known argument,[11] the dependence of the qp scattering amplitude on the magnitudes of momenta can be neglected and A_s and A_t are only functions of θ and ϕ, where θ is the angle between $\hat{\mathbf{p}}_1$ and $\hat{\mathbf{p}}_2$, and ϕ is the angle enclosed by the planes containing $(\hat{\mathbf{p}}_1, \hat{\mathbf{p}}_2)$ and $(\hat{\mathbf{p}}_3, \hat{\mathbf{p}}_4)$, respectively. The amplitude A_2 can be expressed in terms of θ and ϕ by

$$N_{\mathrm{F}}A_2 = \tfrac{1}{4}(3A_{2t} + A_{2s})\tau_{\sigma_1\sigma_3}^{0}\tau_{\sigma_2\sigma_4}^{0} + \tfrac{1}{4}(A_{2t} - A_{2s})\boldsymbol{\tau}_{\sigma_1\sigma_3}\cdot\boldsymbol{\tau}_{\sigma_2\sigma_4} \tag{48}$$

where

$$A_{2;s,t}(\theta, \phi) = A_{s,t}(\vartheta, \varphi)$$

and

$$\cos\vartheta = -\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}\cos\phi, \qquad \cos\varphi = \frac{\cot^2\frac{1}{2}\theta - \sin^2\frac{1}{2}\phi}{\cot^2\frac{1}{2}\theta + \sin^2\frac{1}{2}\phi} \tag{49}$$

The spin traces in Eqs. (46) may be performed after introducing the vector representation for triplet pairing[3]

$$\Delta_{\mathbf{p}\sigma\sigma'} = \mathbf{d}(\hat{p})\cdot[\boldsymbol{\tau}i\tau^2]_{\sigma\sigma'} \tag{50}$$

As can be seen from Eqs. (45), (46), and (50), the vector $\mathbf{d}(\hat{p})$ appears exclusively in the combination

$$\mathbf{D}_{\mathbf{p}}^{\mu} = \mu \mathbf{d}(\hat{p})/E_{\mathbf{p}} \tag{51}$$

Also, the sums on momenta are decomposed into energy and angle integrations according to

$$\sum_{\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4} (2\pi)^3\delta^3(\mathbf{p}+\mathbf{p}_2-\mathbf{p}_3-\mathbf{p}_4)\cdots = \frac{N_F^2}{(4\pi)^2\varepsilon_F}\int_{-1}^{1}\frac{d\cos\theta}{2\cos(\theta/2)}\int_0^{2\pi}\frac{d\phi}{2\pi}\int_0^{2\pi}\frac{d\phi_2}{2\pi}\frac{1}{8}\int_{-\infty}^{\infty}d\xi_2\int_{-\infty}^{\infty}d\xi_3\int_{-\infty}^{\infty}d\xi_4\cdots \tag{52}$$

The Bogoliubov quasiparticle relaxation rate for arbitrary (unitary) triplet pairing is finally obtained as

$$\frac{1}{\tau_{\mathbf{p}}} = \frac{1}{\tau_N(0,T)}\frac{2}{\pi^2T^2}\frac{1}{(-T)f'_{\mathbf{p}}}\int_0^{\infty}d\xi_2\,d\xi_3\,d\xi_4\sum_{\mu_2\mu_3\mu_4} \times\langle\delta(E+\mu_2E_2-\mu_3E_3-\mu_4E_4) \times f_{\mathbf{p}}f_{\mathbf{p}_2}(1-f_{\mathbf{p}_3})(1-f_{\mathbf{p}_4})\Omega(\mathbf{p},\mathbf{p}_2,\ldots,\mathbf{p}_4)\rangle\frac{1}{\langle W\rangle} \tag{53}$$

Here the angular brackets $\langle\cdots\rangle$ denote averaging over angles according to

$$\langle A\rangle = \frac{1}{2}\int_{-1}^{1}\frac{d\cos\theta}{\cos(\theta/2)}\int_0^{2\pi}\frac{d\phi}{2\pi}\int_0^{2\pi}\frac{d\phi_2}{2\pi}A(\theta,\phi,\phi_2) \tag{54}$$

where ϕ_2 is the azimuthal angle of $\hat{\mathbf{p}}_2$ about the axis $\hat{\mathbf{p}}_1$. The function Ω contains angular dependences via the qp scattering amplitudes and the energy gap and is defined by

$$\Omega(\mathbf{p}_1\cdots\mathbf{p}_4) = W - W_{12}[\mathbf{D}_{\mathbf{p}_1}^{*}\cdot\mathbf{D}_{\mathbf{p}_3}^{\mu_3}+\mathbf{D}_{\mathbf{p}_2}^{\mu_2 *}\cdot\mathbf{D}_{\mathbf{p}_4}^{\mu_4}] + W_{+-}\mathbf{D}_{\mathbf{p}_1}^{*}\cdot\mathbf{D}_{\mathbf{p}_2}^{\mu_2 *}\mathbf{D}_{\mathbf{p}_3}^{\mu_3}\cdot\mathbf{D}_{\mathbf{p}_4}^{\mu_4} + 2W_a\mathbf{D}_{\mathbf{p}_1}^{*}\cdot\mathbf{D}_{\mathbf{p}_3}^{\mu_3}\mathbf{D}_{\mathbf{p}_2}^{\mu_2 *}\cdot\mathbf{D}_{\mathbf{p}_4}^{\mu_4} + 2W_b\mathbf{D}_{\mathbf{p}_1}^{*}\cdot\mathbf{D}_{\mathbf{p}_4}^{\mu_4}\mathbf{D}_{\mathbf{p}_2}^{\mu_2 *}\cdot\mathbf{D}_{\mathbf{p}_3}^{\mu_3} \tag{55}$$

where

$$W_a = \tfrac{1}{4}\pi A_t(A_t+A_s), \qquad W_b = \tfrac{1}{4}\pi A_t(A_t-A_s) \tag{56}$$

Ω thus depends on $\hat{\mathbf{p}}_1$, θ, ϕ, and ϕ_2 as well as on ξ_1, ξ_2, ξ_3, and ξ_4. In Eq. (53), $\tau_N(0,T)$ is the quasiparticle lifetime at the Fermi surface in the normal state

$$\tau_N(0,T) = \frac{32\varepsilon_F}{\pi^2\langle W\rangle}\frac{1}{T^2} = \tau_N(0,T_c)\left(\frac{T_c}{T}\right)^2 \tag{57}$$

The inscattering term of the collision integral in Eq. (33) is evaluated in complete analogy. It is convenient to assume definite parity of $\delta\nu$ with respect to ξ. For $\delta\nu$ even in ξ one finds

$$I_{\mathbf{p}}^{\text{in}}=\frac{i}{2}\operatorname{tr}[\tilde{C}_{\mathbf{p}}]=\frac{i}{\tau_N(0,T)}\frac{2}{\pi^2T^2}\sum_{\mu_2\cdots\mu_4}\int_0^{\infty}d\xi_2\,d\xi_3\,d\xi_4$$
$$\times\frac{1}{\langle W\rangle}\Big\langle\delta(E+\mu_2E_2-\mu_3E_3-\mu_4E_4)f_{\mathbf{p}}f_{\mathbf{p}_2}(1-f_{\mathbf{p}_3})$$
$$\times(1-f_{\mathbf{p}_4})\Big[\Omega(\mathbf{p},\mathbf{p}_2,\ldots,\mathbf{p}_4)\sum_{i=2}^{4}(-1)^{\delta_{i,2}}p_{\mu_i}^{s}\delta\hat{v}'_{\mathbf{p}_i}\Big]\Big\rangle \tag{58}$$

For $\delta\nu$ odd in ξ the expression in square brackets in Eq. (58) has to be replaced by

$$[\cdots]\to\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}\Big[\sum_{i=2}^{4}p_{\mu_i}^{-s}\frac{\xi_{\mathbf{p}_i}}{E_{\mathbf{p}_i}}W\delta\hat{v}'_{\mathbf{p}_i}-p_{\mu_3}^{-s}\mathbf{D}_{\mathbf{p}_2}^{\mu_2 *}\cdot\mathbf{D}_{\mathbf{p}_4}^{\mu_4}\frac{\xi_{\mathbf{p}_3}}{E_{\mathbf{p}_3}}\,\mathrm{W}_{+-}\,\delta\hat{v}'_{\mathbf{p}_3}\Big] \tag{59}$$

The expressions for the collision integral can be further simplified for definite pairing states, i.e., given $\mathbf{d}(\hat{\mathbf{p}})$. In this paper we only consider the pseudo-isotropic or Balian–Werthamer state, for which the simplification is particularly pronounced. In this case the order parameter is given by

$$\mathbf{d}(\hat{\mathbf{p}})=\Delta(T)\mathbf{R}\cdot\hat{\mathbf{p}} \tag{60}$$

where $\mathbf{R}$ is a rotation matrix and $\Delta(T)$ is the temperature-dependent magnitude of the gap. Since in Eq. (55) only scalar products of $\mathbf{d}$ enter, the rotation matrix $\mathbf{R}$ drops out. Indeed, spin-symmetric properties should not depend on the relative orientations of spin and orbital variables.

Since the scalar products $\hat{\mathbf{p}}_i\cdot\hat{\mathbf{p}}_j$ do not depend on the angle ϕ_2, the ϕ_2-integration in Eq. (53) is trivial. The angular average of Ω is given by

$$\langle\Omega(\mathbf{p}_1\cdots\mathbf{p}_4)\rangle=\langle W\rangle-[D_1D_3+D_2D_4]\langle W_I\rangle+D_1D_2D_3D_4\langle W_D\rangle \tag{61}$$

where we have defined

$$D_i=\mu_i\,\Delta(T)/E_{\mathbf{p}_i} \tag{62}$$

and

$$W_I=\tfrac{1}{4}\pi[(A_s+A_t)(A_{2s}+A_{2t})+4A_tA_{2t}][\cos^2\tfrac{1}{2}\theta+\sin^2\tfrac{1}{2}\theta\cos\phi] \tag{63a}$$

$$W_D=\tfrac{1}{4}\pi[A_t^2(4\cos^4\tfrac{1}{2}\theta+4\sin^4\tfrac{1}{2}\theta\cos^2\phi-\cos^2\theta)$$
$$+2A_sA_t\sin^2\theta\cos\phi+\mathrm{A}_s^2\cos^2\theta] \tag{63b}$$

Introducing dimensionless isotropic energy integrals

$$I_n(E)=\frac{2}{\pi^2T^2}\frac{1}{(-T)f'_1}\sum_{\mu_2\cdots\mu_4}\int_0^{\infty}d\xi_2\,d\xi_3\,d\xi_4\,\delta(E+\mu_2E_2-\mu_3E_3-\mu_4E_4)$$
$$\times f_{\mathbf{p}}f_{\mathbf{p}_2}(1-f_{\mathbf{p}_3})(1-f_{\mathbf{p}_4})K_n \tag{64}$$

with

$$K_n = \delta_{n,0} + \mu_2 \frac{\Delta_0^2}{E_{\mathbf{p}_1} E_{\mathbf{p}_2}} \delta_{n,1} - \mu_2\mu_3 \frac{\Delta_0^2}{E_{\mathbf{p}_2} E_{\mathbf{p}_3}} \delta_{n,2} + \mu_2\mu_3\mu_4 \frac{\Delta_0^4}{E_{\mathbf{p}_1} E_{\mathbf{p}_2} E_{\mathbf{p}_3} E_{\mathbf{p}_4}} \delta_{n,3}$$

we can write the quasiparticle relaxation rate in the BW state as

$$\frac{1}{\tau_p} = \frac{1}{\tau_N(0, T)} \left[I_0 + \gamma_0 (I_1 + I_2) \frac{\Delta^2(T)}{\Delta_0^2} + \delta_0 I_3 \frac{\Delta^4(T)}{\Delta_0^4} \right] \tag{65}$$

Here we have defined

$$\gamma_0 = \langle W_I \rangle / \langle W \rangle \tag{66}$$

and

$$\delta_0 = \langle W_D \rangle / \langle W \rangle \tag{67}$$

and $\Delta_0 = \Delta(T=0)$.

The inscattering term of the collision integral in the BW state is obtained analogously from the general expressions (58) and (59) by inserting (61). The resulting expressions may be further simplified by interchanging variables such that $I_{\mathbf{p}}^{\text{in}}$ operates on $\delta\nu'_{\hat{p}_2}(\xi_2)$ only. Defining energy integrals

$$B_n^s(\xi_1, \xi_2) = \frac{1}{2\pi^2 T} \sum_{\mu_2} p^s_{-\mu_2} \sum_{\mu_3\mu_4} \int_0^\infty d\xi_3 \, d\xi_4$$

$$\times \delta(E_1 + \mu_2 E_2 - \mu_3 E_3 - \mu_4 E_4) \prod_{j=3}^{4} \left(\operatorname{sech} \frac{E_{\mathbf{p}_i}}{2T} \right) b_n \tag{68}$$

where

$$b_n = \delta_{n,0} + \frac{\Delta_0}{\Delta(T)} D_3 \, \delta_{n,1} + \frac{\Delta_0^2}{\Delta^2(T)} D_3 D_4 \, \delta_{n,2}$$

and expanding Ω in terms of spherical harmonics, one finally obtains

$$I_{\mathbf{p}_1}^{\text{in}} = \frac{i}{\tau_N(0, T)} \frac{1}{T N_F} \sum_{\mathbf{p}_2} \frac{\cosh(E_2/2T)}{\cosh(E_1/2T)}$$

$$\times \sum_{l=0}^{\infty} (2l+1) P_l(\hat{\mathbf{p}}_1 \cdot \hat{\mathbf{p}}_2) S_l(\xi_1, \xi_2) \, \delta\hat{\nu}'_{\mathbf{p}_2} \tag{69}$$

where

$$S_l(\xi_1, \xi_2) = \lambda_l^{-s} B_0^s + \alpha_l \left[\frac{\Delta}{E_1} B_1^s - \frac{\Delta}{E_2} B_1^{-s} \right] \frac{\Delta}{\Delta_0}$$

$$+ \gamma_l \left[\frac{\Delta^2}{\Delta_0^2} B_2^s - \frac{\Delta^2}{E_1 E_2} B_0^{-s} \right] - \delta_l^{-s} \frac{\Delta^2}{E_1 E_2} \frac{\Delta^2}{\Delta_0^2} B_2^{-s} \tag{70a}$$

for $\delta\nu'(\xi)$ even and

$$S_l(\xi_1, \xi_2) = \frac{\xi_1}{E_1}\frac{\xi_2}{E_2}\left(\lambda_l^s B_0^{-s} + \gamma_l \frac{\Delta^2}{\Delta_0^2} B_2^{-s}\right) \tag{70b}$$

for $\delta\nu'(\xi)$ odd in ξ. In all later applications $\delta\nu'$ will be odd in ξ.

The angular dependence of the collision operator is contained in the parameters λ_l^s, α_l^s, γ_l, and δ_l^s, defined by

$$\lambda_l^s = (1/\langle W\rangle)\langle W(\theta, \phi)[-sP_l(\hat{\mathbf{p}}_1 \cdot \hat{\mathbf{p}}_2) + P_l(\hat{\mathbf{p}}_1 \cdot \hat{\mathbf{p}}_3) + P_l(\hat{\mathbf{p}}_1 \cdot \hat{\mathbf{p}}_4)]\rangle \tag{71a}$$

$$\alpha_l^s = (1/\langle W\rangle)\langle W_I(\theta, \phi)[-sP_l(\hat{\mathbf{p}}_1 \cdot \hat{\mathbf{p}}_2) + P_l(\hat{\mathbf{p}}_1 \cdot \hat{\mathbf{p}}_4)]\rangle \tag{71b}$$

$$\gamma_l = (1/\langle W\rangle)\langle W_I(\theta, \phi)P_l(\hat{\mathbf{p}}_1 \cdot \hat{\mathbf{p}}_3)\rangle \tag{71c}$$

$$\delta_l^s = (1/\langle W\rangle)\langle W_D(\theta, \phi)[-sP_l(\hat{\mathbf{p}}_1 \cdot \hat{\mathbf{p}}_2) + P_l(\hat{\mathbf{p}}_1 \cdot \hat{\mathbf{p}}_3) + P_l(\hat{\mathbf{p}}_1 \cdot \hat{\mathbf{p}}_4)]\rangle \tag{71d}$$

For $s = +1$ we shall omit the superscript + on λ_l^+, α_l^+, and δ_l^+ in the following.

The parameters λ_l and λ_l^- are identical with the ones characterizing the normal-state collision integral, which is obtained as the limit as $\Delta \to 0$ of Eqs. (65) and (69). This limit is equivalent to the exact low-temperature result of Sykes and Brooker.[12] The component even in ξ is, for instance, given by

$$I_{\mathbf{p}_1} = -\frac{i}{\tau_N(\xi_{p_1})}\delta f'_{\mathbf{p}_1} + \frac{i}{\tau_N(0)}\frac{1}{TN_F}\sum_{\mathbf{p}_2}\frac{\cosh(\xi_2/2T)}{\cosh(\xi_1/2T)} \times \sum_{l=0}^{\infty}(2l+1)P_l(\hat{\mathbf{p}}_1 \cdot \hat{\mathbf{p}}_2)B\left(\frac{\xi_1 - \xi_2}{2T}\right)\lambda_l\,\delta f'_{\mathbf{p}_2} \tag{72}$$

where

$$B(x) = \frac{x}{\sinh(x/2)}$$

$\delta f'_{\mathbf{p}_2}$ is the deviation of the normal-state distribution function from local equilibrium and $\tau_N^{-1}(\xi_p)$ is the limit as $\Delta \to 0$ of (65):

$$\frac{1}{\tau_N(\xi_p)} = \frac{1}{\tau_N(0)}\left[1 + \left(\frac{\xi_p}{\pi T}\right)^2\right] \tag{73}$$

The first few parameters are fixed by the conservation properties of normal quasiparticle collisions,

$$\lambda_0 = \lambda_1 = 1, \qquad \lambda_0^- = 3$$

The parameters λ_2, γ_2, and λ_1^-, γ_1 entering in the cases of shear viscosity and thermal conductivity are given in a more explicit form by

$$\begin{aligned}
\lambda_1^- &= 1+(2/\langle W\rangle)\langle W(\theta,\phi)\cos\theta\rangle \\
\lambda_2 &= 1-(3/\langle W\rangle)\langle W(\theta,\phi)\sin^4\tfrac{1}{2}\theta\,\sin^2\phi\rangle \\
\gamma_1 &= (1/\langle W\rangle)\langle W_I(\theta,\phi)[\cos^2\tfrac{1}{2}\theta+\sin^2\tfrac{1}{2}\theta\cos\phi]\rangle \\
\gamma_2 &= (1/\langle W\rangle)\langle W_I(\theta,\phi)[\tfrac{3}{2}(\cos^2\tfrac{1}{2}\theta+\sin^2\tfrac{1}{2}\theta\cos\phi)^2-\tfrac{1}{2}]\rangle
\end{aligned} \tag{74}$$

The two parameters λ_2 and λ_1^- can be determined from measurements of these transport properties in normal ^{3}He, provided the quasiparticle relaxation time on the Fermi surface in the normal state $\tau_N(0)$ is known independently.

The inscattering term of the collision integral satisfies a number of exact relations. Substituting $\delta\nu_p' = f_p' Y_{lm}(\hat{p})\phi_l(\xi_p)$ into Eq. (69), one obtains after some rearrangement

$$I_{\mathbf{p}}^{\text{in}}\{f_{\mathbf{p}}' Y_{lm}(\hat{p})\phi_l(\xi_p)\} = \frac{i}{\tau_N(0)} f_{\mathbf{p}}' Y_{lm}(\hat{p})\psi_l(\xi_p); \qquad l=0,1,2,\ldots \tag{75}$$

with

$$\psi_l(\xi_p) = \begin{cases} J_l^+(E_{\mathbf{p}}) \\ \tfrac{1}{3}E_{\mathbf{p}}J_l^-(E_{\mathbf{p}}) \\ (\xi_p/E_{\mathbf{p}})K_l^+(E_{\mathbf{p}}) \\ \tfrac{1}{3}\xi_p K_l^-(E_p) \end{cases} \quad \text{for} \quad \phi_l(\xi_p) = \begin{cases} 1, & l \text{ odd} \\ E_{\mathbf{p}}, & l \text{ even} \\ E_p/\xi_p, & l \text{ even} \\ E_p^2/\xi_p, & l \text{ odd} \end{cases}$$

where

$$\begin{aligned}
J_l^s(E) &= \lambda_l^s I_0 + [\alpha_l^s+\gamma_l][I_1+I_2](\Delta/\Delta_0)^2 + \delta_l^s I_3(\Delta/\Delta_0)^4 \\
K_l^s(E) &= \lambda_l^s I_0 + \gamma_l I_2(\Delta/\Delta_0)^2
\end{aligned}$$

and $I_n(E)$, $n=0,1,2,3$, is defined by Eq. (64). Conversely, due to the symmetry of the collision operator, the following sum rules complement the above relations:

$$\sum_{\mathbf{p}} Y_{lm}(\hat{\mathbf{p}})\phi_l(\xi_p)I_{\mathbf{p}}^{\text{in}} = \frac{i}{\tau_N(0)}\sum_{\mathbf{p}} \psi_l(\xi_p)Y_{lm}(\hat{p})\,\delta\nu_{\mathbf{p}}'; \qquad l=0,1,2,\ldots \tag{76}$$

4. QUASIPARTICLE RELAXATION RATE

The qp relaxation rate is a measure of the effect of qp collisions on the damping of nonequilibrium processes. In the normal state, the dependence of $1/\tau_p$ on temperature and energy ξ_p is given by (73) and thus is rather simple. $\tau_{\mathbf{p}}$ cannot be measured directly in the normal state. However, it has been possible recently to determine $\tau_N(0)$ from the various relaxation effects in the superfluid state near T_c. The values for the temperature-independent quantity $\tau_N(0)T^2$ obtained in this way range from 0.31 to 0.26 $\mu\text{sec}\cdot\text{mK}^2$ for pressures from about 20 bar to the melting pressure.[13] The corresponding value for $1/\tau_N(0)$ for 20 bar at $T_c = 2.4$ mK expressed in temperature units is $1/\tau_N(0) = 0.15$ mK. The condition $1/\tau \ll T$ for the validity of the quasiparticle description is seen to be amply satisfied.

In the superfluid state the dependence of $1/\tau_{\mathbf{p}}$ on temperature and momentum $\mathbf{p}$ is much more complex. In the BW state the relaxation rate is isotropic, as in the normal state. It is seen from Eq. (63) that the temperature dependence of the qp relaxation rate at the Fermi surface ($\xi_p = 0$) is given by a universal function of $\Delta(T)/T$. In general, the expression (65) for $1/\tau_{\mathbf{p}}$ has to be evaluated numerically.

However, in the limit of low temperatures the energy integrations may be done analytically. Expanding the qp energy as

$$E_{\mathbf{p}} = \Delta[1 + (\xi_p^2/2\Delta^2) + \cdots] \tag{77}$$

and inserting this into Eq. (64), it is seen that the lowest order contribution in $e^{-\Delta/T}$ is given by those terms in the sum on μ_2, μ_3, μ_4 for which $\mu_2 + \mu_3 + \mu_4 = -1$, guaranteeing a cancellation of the zeroth-order energies in the energy delta function. The energy integrals are easily done in lowest order in T/Δ and $e^{-\Delta/T}$, with the result

$$I_0 = 3I_1 = 3I_2 = I_3 = \frac{3}{2\pi}\frac{\Delta}{T} Y_0(T) \tag{78}$$

where

$$Y_0(T) = \lim_{T\to 0} Y(T) = \lim_{T\to 0} \int_{-\infty}^{\infty} d\xi_p[-f'_{\mathbf{p}}] = [2\pi\Delta/T]^{1/2} e^{-\Delta/T}$$

is Yoshida's function for $T \to 0$. The function $Y(T)$ is a measure of the number of thermally excited quasiparticles.

Substituting Eq. (78) into Eq. (65), we obtain the qp relaxation rate in the BW state at low temperatures

$$\frac{1}{\tau(E)} = \frac{1}{\tau_N(0, T)} \frac{3}{(2\pi)^{1/2}} \left(\frac{\Delta}{T}\right)^{3/2} e^{-\Delta/T} w_0 \tag{79}$$

where

$$w_0 = 1 - \tfrac{2}{3}\gamma_0 + \delta_0$$

with δ_0 and γ_0 defined by Eqs. (66) and (67). Thus $1/\tau_p$ tends to zero exponentially fast for $T \to 0$, as opposed to the T^2 behavior in the normal state.

In the opposite limit $T \to T_c$ an expansion in lowest order in Δ^2/T^2 leads to the following result for the qp relaxation rate[6]:

$$\frac{1}{\tau_p} = \frac{1}{\tau_N(0)}\left[1 + \left(\frac{E_p}{\pi T}\right)^2 - \left(\frac{\Delta}{\pi T}\right)^2\left(6 \ln 2 + \frac{\pi^2}{18} + 3.3\left(\frac{E_p}{\pi T}\right)^2\right)\right] \quad (80)$$

The averaged cross sections $\langle W \rangle$, $\langle W_l \rangle$, and $\langle W_D \rangle$ are of importance for the quantitative evaluation of Eq. (65). We have estimated these quantities in the so-called s–p approximation, in which only the $l = 0$ and $l = 1$ components of the scattering amplitudes are kept, viz.,

$$A_s(\theta, \phi) = S_0 + S_1 \cos\theta, \qquad A_t(\theta, \phi) = (T_0 + T_1 \cos\theta)\cos\phi \quad (81)$$

In terms of the four parameters S_0, S_1, T_0, T_1 the averages are found as

$$\langle W \rangle = \frac{\pi}{2}\left(S_0^2 - \frac{2}{3}S_0 S_1 + \frac{7}{15}S_1^2 + \frac{3}{2}T_0^2 - T_0 T_1 + \frac{7}{10}T_1^2\right)$$

$$\langle W_l \rangle = \frac{\pi}{2}\left[\frac{1}{3}S_0^2 + \frac{2}{15}S_0 S_1 - \frac{29}{105}S_1^2 + \frac{2}{3}S_0 T_0 - \frac{2}{5}S_0 T_1 \right.$$
$$\left. + \frac{5}{3}(25 - 36 \ln 2)T_0^2 + (84 - 120 \ln 2)T_0 T_1 + \frac{5}{21}(173 - 252 \ln 2)T_1^2\right] \quad (82)$$

$$\langle W_D \rangle = \frac{\pi}{2}\left[\frac{7}{15}S_0^2 - \frac{18}{35}S_0 S_1 + \frac{107}{315}S_1^2 + \frac{8}{15}S_0 T_0 - \frac{8}{105}(S_0 T_1 + S_1 T_0) \right.$$
$$\left. + \frac{8}{63}S_1 T_1 + \frac{29}{30}T_0^2 - \frac{19}{35}T_0 T_1 + \frac{33}{70}T_1^2\right]$$

S_l and T_l are related to the spin-symmetric (antisymmetric) Landau parameters F_l^s (F_l^a) by $S_l = A_l^s - 3A_l^a$ and $T_l = A_l^s + A_l^a$, with

$$A_l^{s,a} = \frac{F_l^{s,a}}{1 + [F_l^{s,a}/(2l+1)]}$$

TABLE I

Components and Angular Averages of the Quasiparticle Scattering Amplitude in ^{3}He at Two Different Pressures As Defined in the Text

	S_0	S_1	T_0	T_1	γ_0	δ_0	w_0	$\langle W \rangle$
20 bar	9.5	5.1	−1.8	1.5	0.12	0.29	1.21	$79\pi/2$
34 bar	11.0	5.2	−2.34	1.54	0.16	0.39	1.18	$109\pi/2$

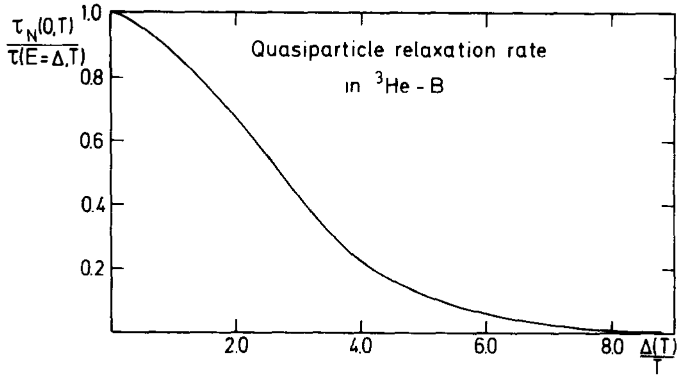

Fig. 1. Quasiparticle relaxation rate $1/\tau$ at the Fermi surface in the BW state normalized to the normal-state value $1/\tau_N$ vs. $\Delta(T)/T$.

Taking the values of $F_l^{s,a}$ appropriate for ^{3}He at 20 and 34 bar, respectively one obtains the results listed in Table I.

The terms involving $\langle W_I \rangle$ and $\langle W_D \rangle$ cancel partially in this approximation. Thus we can conclude that the term involving I_0 in Eq. (65) dominates.

In Fig. 1 the result of a numerical evaluation of the qp relaxation rate normalized to the normal-state value at the Fermi surface ($\xi = 0$) using the parameters appropriate for $P = 20$ bar is plotted as a function of $\Delta(T)/T$. The dependence of the normalized qp relaxation rate on pressure is weak.

The relaxation rate of the superfluid state is seen to fall off rapidly with decreasing temperature relative to the normal-state value. This is due to the appearance of the gap in the single-particle excitation spectrum.

We have used the following interpolation formula for $\Delta(T)$ to extract values for $1/\tau$ as a funtion of temperature from the result of Fig. 1:

$$\Delta(T) = 1.76 T_c \tanh\left[\frac{\pi}{1.76}\left(\frac{2}{3}\frac{\Delta C}{C_N}\right)^{1/2}\left(\frac{T_c}{T} - 1\right)^{1/2}\right] \tag{83}$$

Equation (83) is consistent with a relative specific heat discontinuity $\Delta C/C_N$ and tends to the weak coupling value for $T \to 0$.

Given the qp lifetime τ_p, we may also calculate the mean free path l of Bogoliubov quasiparticles by taking the root mean square of the average distance traveled by a quasiparticle of momentum $\mathbf{p}$, which is $\mathbf{l}_p = \tau_p \nabla_p E_p = \tau_p \mathbf{v}_p \xi_p / E_p$. This is given by

$$l(T) = \left[\frac{\sum_{\mathbf{p}} \mathbf{l}_{\mathbf{p}}^2 f_{\mathbf{p}}}{\sum_{\mathbf{p}} f_{\mathbf{p}}}\right]^{1/2} = v_F \tau_N(0)\left[\frac{\int_0^\infty d\xi_p\, (\tau_p^2/\tau_N^2)(\xi_p^2/E_p^2) f_p}{\int_0^\infty d\xi_p\, f_p}\right]^{1/2} \tag{84}$$

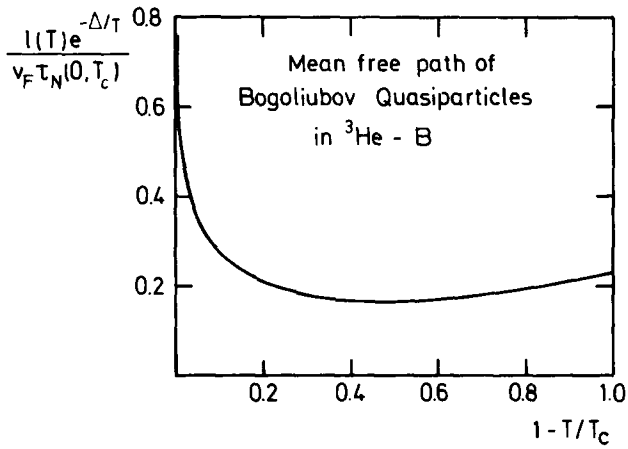

Fig. 2. Mean free path $l(T)$ as defined by Eq. (84) normalized to $v_F\tau_N(0, T_c)\exp(\Delta/T)$ vs. reduced temperature $1-T/T_c$.

where f_p is the Fermi function. In the limit of low temperatures l diverges as

$$\lim_{T\to 0} l(T) = v_F\tau_N(0, T_c)\frac{(2\pi)^{1/2}}{3}\left(\frac{T_c}{\Delta}\right)^2 e^{\Delta/T}\frac{1}{w_0} \tag{85}$$

The result of a numerical evaluation of Eq. (84) is plotted in Fig. 2.

In order to obtain values for the mean free path $l(T)$ itself, we need an estimate of the quantity $v_F\tau_N(T_c)$. We take v_F from Wheatley's table,[13] but with an effective mass ratio scaled down by 5% for 21 bar and 11% for melting pressure. $\tau_N(T_c)$ is taken from Ref. 14. The results are listed in Table II.

One immediately recognizes that for reduced temperatures T/T_c lower than 0.2, the mean free path of Bogoliubov quasiparticles becomes comparable with a typical size of the experimental apparatus, in essential

TABLE II

Mean Free Path of Bogoliubov Quasiparticles in the B Phase of ^{3}He at 21 bar for Different Values of the Reduced Temperature T/T_c

T/T_c	l, cm	T/T_c	l, cm
1.0	1.70×10^{-4}	0.3	1.17×10^{-2}
0.6	4.68×10^{-4}	0.25	3.99×10^{-2}
0.5	9.13×10^{-4}	0.2	2.45×10^{-1}
0.4	2.42×10^{-3}	0.15	4.87
0.35	4.78×10^{-3}	0.1	1.82×10^{3}

agreement with the result of Ono's estimate.[15] Generally speaking, the temperature range for which a transport theory accounting for only quasiparticle collisions is valid is bounded from below in a way which depends upon geometry and size of the sample container. In order to get correct answers for lower temperatures, one has to account for quasiparticle scattering processes at the wall of the container.

REFERENCES

1. I. M. Khalatnikov, *Introduction to the Theory of Superfluidity* (Benjamin, New York, 1965).
2. R. Graham, *Phys. Rev. Lett.* **33**, 1431 (1974); R. Graham and H. Pleiner, *J. Phys. C* **9**, 279 (1976).
3. A. J. Leggett, *Rev. Mod. Phys.* **47**, 331 (1975).
4. O. Betbeder-Matibet and P. Nozières, *Ann. Phys.* (*N.Y.*) **51**, 392 (1969).
5. P. Wölfle, *J. Low Temp. Phys.* **22**, 157 (1976).
6. P. Wölfle, *Phys. Rev. B* **14**, 89 (1976).
7. V. S. Shumeiko, *Zh. Eksp. Teor. Fiz.* **61**, 382 (1971) [*Sov. Phys.—JETP* **34**, 203 (1972)].
8. P. Bhattacharyya, C. J. Pethick, and H. Smith, *Phys. Rev. B* **15**, 3367 (1977).
9. C. J. Pethick, H. Smith, and P. Bhattacharyya, *Phys. Rev. B* **15**, 3384 (1977).
10. Y. A. Ono, J. Hara, K. Nagai, and K. Kawamura, *J. Low Temp. Phys.* **27**, 513 (1977).
11. A. A. Abrikosov and I. M. Khalatnikov, *Rep. Prog. Phys.* **22**, 329 (1959).
12. J. Sykes and G. A. Brooker, *Ann. Phys.* **56**, 1 (1970).
13. J. C. Wheatley, *Rev. Mod. Phys.* **47**, 415 (1976).
14. J. C. Wheatley, in *Progress in Low Temperature Physics*, Vol. 7, D. Brewer, ed. (North-Holland, Amsterdam, 1978).
15. Y. A. Ono, *Prog. Theor. Phys.* **58**, 1068 (1977).