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The Capture of Ions by Vortex Lines in Pure ^{4}He and ^{3}He–^{4}He Solutions*

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We present measurements of the capture cross section for the negative ion by vortex lines in pure ^{4}He and a 0.94% solution of ^{3}He in ^{4}He. A measurable capture of positive ions was not observed in either system. In pure ^{4}He the departure of the experimental data from the predictions of stochastic theory at lower temperatures is explained in terms of a simple breakdown in that theory, and lifetime effects are shown to be of only secondary importance. The decrease in cross section with decreasing temperature, due essentially to a reduced number density of rotons, is predicted by the results of a Monte Carlo calculation. The cross section in the solution follows stochastic theory to significantly lower temperatures, but at about 1 K there again occurs a marked discrepancy. Measurements of the ion trapping lifetime at low temperatures indicate that it is the finite lifetime that accounts for this discrepancy. Elimination of lifetime effects results in fair agreement between the cross section data in the solution and the predictions of stochastic theory.

1. INTRODUCTION

In 1962, Careri *et al.*[1] observed an attenuation in a negative ion current propagating in a direction perpendicular to a vortex array in He II. Positive ions were seen, however, to be unaffected. Later experiments by Tanner *et al.*[2] confirmed these results and, in addition, demonstrated that a capture cross section for the trapping of an ion by a vortex line could be defined and measured. Springett *et al.*[3] and Tanner[4] reported measured values for the cross section in the temperature range $1.0 < T < 1.7$ K and observed that the cross section decreased dramatically with increasing temperature above 1.6 K. Below 1.3 K their results indicated a somewhat less rapid decrease with decreasing temperature.

Donnelly[5] explained the trapping mechanism by treating the ion as a Brownian particle immersed in a gas of elementary excitations and moving

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under the influence of an electric field and an effective vortex potential. This potential corresponds approximately to the kinetic energy of the fluid displaced by the ion in the vortex flow. Expanding upon this idea, Donnelly and Roberts[6] obtained fair agreement with the experimental cross section in the temperature range $1.3 < T < 1.6$ K. They were also able to show that the attempt frequency for escape was sufficiently large above 1.6 K to allow escape of the ion from the vortex. This result corresponded precisely to the observed decrease in effective cross section of the negative ion above 1.6 K and indicated why capture of positive ions, which are smaller and therefore experience a shallower potential well, had not been seen. Direct measurements of negative ion trapping lifetimes by Douglass[7] and positive ion lifetimes in vortex rings in high electric fields by Cade[8] confirmed this picture and provided an important measure of the positive and negative ion radii.[9] Under the influence of hydrostatic pressure, measurements of the cross section by Springett and Donnelly[10] and of the trapping lifetime by Pratt and Zimmerman[11] led to a determination of the pressure dependence of the negative ion radius.

The Donnelly and Roberts[6] theory, however, fails to explain the behavior of the capture cross section below 1.3 K. The authors themselves point out that the theory, based on a solution of the Smoluchowski equation, should not be valid at lower temperatures. In fact below 1 K the entire philosophy of the calculation breaks down simply because the ion–roton mean free path becomes comparable to the width of the potential well. In any event one could only speculate about the reasons for the observed decreasing cross section below 1.3 K. One can, for instance, offer an explanation in terms of the reaction of the vortex line to the presence of the ion when the frictional drag on the line decreases. Douglass,[12] on the other hand, suggested that this anomalous decrease in cross section may be a manifestation of the finite trapping lifetimes that he observed in this temperature range.

In an attempt to explain this apparent anomaly, we have extended the measurements of the negative ion capture cross section to temperatures below 1 K and to dilute solutions of ^{3}He in ^{4}He. We find that in pure ^{4}He the cross section does fall off with decreasing temperature much more rapidly than one would expect from the stochastic theory even if the effects of a finite lifetime are included. Because of the expected breakdown in the stochastic theory, we performed a Monte Carlo calculation. Good agreement between the experimental data and the calculation was obtained. This agreement substantiates the idea that the capture cross section anomaly in pure ^{4}He is predominantly a result of the breakdown in the stochastic theory. The effects of the finite lifetime were found to be negligible except at the lowest temperatures.

The addition of ^{3}He impurities enters stochastic theory only in its effect on ion mobility, and reasonable concentrations will keep the diffusion length smaller than other lengths of interest. In this system we should then have a better test of the stochastic theory. We observed that the cross section does indeed follow the calculated values down to somewhat lower temperatures, but that it then decreases rather rapidly. We have found that this decrease, as opposed to that in pure ^{4}He, is due solely to the finite trapping lifetime. If lifetimes are taken into account, fair agreement between stochastic theory and our data is obtained.

2. EXPERIMENTAL PROCEDURE—CROSS SECTIONS

2.1. Apparatus

The refrigerator and low-level electronics used in this experiment were mounted on a rotating platform driven by a variable-speed, 1-hp dc motor. The angular velocity was controlled and regulated by means of a feedback network between a tachometer attached directly to the motor and a Kepco JQE programmable power supply. This arrangement allowed variations in the speed to be kept less than 0.05%.

For temperatures above 1.0 K a ^{4}He bath was used and for lower temperatures a ^{3}He refrigerator was used. In order to minimize mechanical vibrations, the ^{3}He refrigerator incorporated a sorption pump consisting of approximately 100 cm^3 of Linde molecular sieve-type 13X PLTS. This quantity was sufficient to pump, without saturation, approximately 5 cm^3 of ^{3}He liquid up to 12 h or longer. Recovery of the ^{3}He was facilitated by means of a heating coil wrapped around the pump and could be accomplished without heating the experimental cell above about 2.0 K.

2.2 Technique

The procedure used for measuring the cross section was the determination of the current attenuation due to the presence of the vortex array. The change in the current dI over a distance dx perpendicular to the array is given by

$$dI/I = -n\sigma\, dx \tag{1}$$

where $n = 2\omega m_4/h$ is the number of vortex lines per unit area and σ is the capture cross section.

In measuring the cross section, the operation of the experimental cell, shown in Fig. 1, is rather simple. A ^{210}Po alpha emitter serves as the ion source. By correctly biasing the source, ions of either sign are injected into

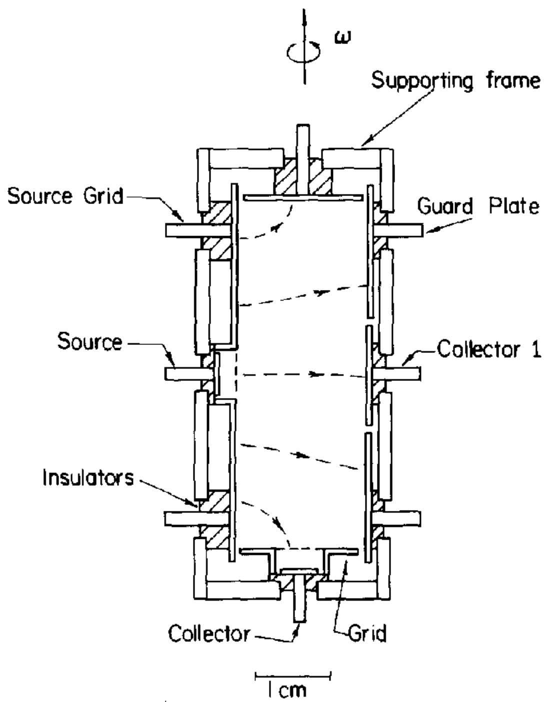

Fig. 1. A schematic of the experimental cell. In some of the cross section measurements the top and bottom electrodes were biased at the same potential as the source grid. This produced a divergent field configuration (as shown by the dashed lines), which helped to minimize space charge difficulties.

the drift region, where a constant electric field drives them in a direction transverse to the vortex array. The collected current is detected by means of a Keithley 640 low-drift vibrating capacitor electrometer mounted on the rotating table. The current is measured for various values of the angular velocity, including $\omega = 0$, and the cross section is obtained by integrating Eq. (1) over the length of the drift region. A typical sample of the raw data showing the actual current attenuation is given in Fig. 2a. As can be seen, it takes the order of minutes to achieve stability with regard to both ion currents and fluid flow. A similar waiting period is required in returning to $\omega = 0$. In some cases current drifts would occur, but it was usually possible to account for this satisfactorily by extrapolating between the initial and final values of the unattenuated current.

A number of precautions had to be taken in order to ensure the accuracy of our results. Of primary importance was the problem of space charge

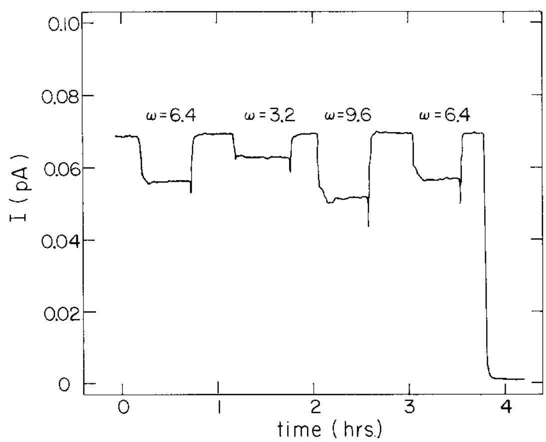

Fig. 2a. A sample of the raw data illustrating the current attenuation in the cross section measurements.

accumulation on vortex lines in the path of the ion beam. This charge could introduce substantial error by modifying the ion–vortex interaction and/or the electric field felt by the ions. To avoid this charge buildup, the top and bottom electrodes were biased at the same potential as the source-grid electrode, resulting in the slightly diverging field configuration as shown by the dashed lines in Fig. 1. This allowed trapped ions to be drawn away from the beam path without seriously effecting the field homogeneity. (Variations in the grid–collector field were no more than about 8%.) In addition, the collected current was always kept less than 0.1 pA by pulsing the source on and off in a manner similar to that described by Tanner.[4]

Since current attenuation occurred in the source grid region as well as the drift region, the measured cross section σ_m consists of two terms,

$$\sigma_m = \sigma_d + \sigma_s(l_s/l_d) \tag{2}$$

where σ_d and σ_s are the effective cross sections in the drift and source regions, respectively, and l_s and l_d are the source–grid and grid–collector separations. The contribution of the second unknown term in Eq. (2) can be minimized by keeping the ratio l_s/l_d small. In our experiment it was about 0.1. Also, noting that σ is approximately inversely proportional to electric field, the contribution of σ_s can be reduced by ensuring that the source field (pulse on) is considerably larger than that in the drift region. (In pure ^{4}He it was necessary, however, to reduce the source field somewhat at lower temperatures so as to avoid the creation of vortex rings.)

Taking the raw data shown in Fig. 2a, we have plotted in Fig. 2b, on a semi-log graph, the collected current I as a function of the angular velocity

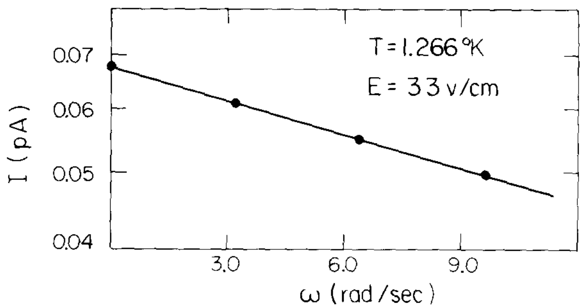

Fig. 2b. A semi-log plot of current versus angular velocity for the data of Fig. 2a.

ω. The cross section is determined from the slope of the resulting straight line. The results of our measurements are shown in Figs. 3–6, the specific conditions under which these data were obtained being given in the respective captions. The error bars indicated reflect the reproducibility and scatter obtained for a measurement taken a number of times during a run and on different runs. Other possible errors introduced by, for example, uncertainties in the current readings or inhomogeneities in the electric field were generally smaller.

The data shown pertain to the capture of the negative ion. It was found that the positive-ion capture cross section was never more than about 20 Å, a value corresponding to the resolution of our experiment. This was true at all accessible temperatures in both pure ^{4}He and the ^{3}He–^{4}He solution and for electric fields down to 0.5 V/cm.

3. CROSS SECTIONS IN PURE ^{4}He

In analyzing the capture problem, there are three factors contributing to the ion motion that must be taken into account. The first, and simplest, is the effect of the contant electric field. The second factor is the purely hydrodynamic interaction that exists between a sphere and a vortex line. If the line remains rectilinear, the force is radial and attractive and is given by[13]

$$F_v = -(\rho_s \kappa^2/2\pi)(a/r)^3 + O[(a/r)^5] \tag{3}$$

where ρ_s is the superfluid density and a is the ion radius. This formula is correct only when the ion is rather distant from the vortex. As it approaches within a few ionic radii, the vortex core reacts to the ion's flow pattern, causing it to deviate from rectilinearity. This is turn introduces complicated azimuthal as well as radial forces acting back on the ion. Fortunately, except for capture widths the order of an ion diameter or smaller, these difficulties can probably be neglected and we can assume the validity of Eq. (3).

An ion in He II finds itself continually bombarded by elementary excitations. In the context of the capture problem these ion–excitation interactions can be dealt with, at sufficiently high temperatures, by treating the ion as a Brownian particle moving in the combined electric and vortex fields. The behavior of such a system can, in principle, be determined from the solution of the prescribed Fokker–Planck equation for the steady-state probability density. The validity of this approach is limited. If one considers the ion–roton mean free path, one finds that this length becomes larger than any reasonable characteristic size of the vortex potential well for temperatures not much below 1 K. Thus the Brownian behavior of the ion no longer persists to sufficiently small scale.

At higher temperatures, on the other hand, this approach has proven useful, as shown by Donnelly and Roberts.[6] In their work these authors avoided the mathematical difficulties inherent to the generalized Fokker–Planck equation by assuming, among other things, that the ion velocity distribution is Maxwellian. Under the various assumptions made, they obtained the Smoluchowski equation for the ion distribution in real space; i.e.,

$$\nabla \cdot \{\nabla f(\mathbf{r}) + [f(\mathbf{r})/k_B T]\, \nabla\varphi(\mathbf{r})\} = 0 \tag{4}$$

where $\varphi(\mathbf{r})$ is the potential energy associated with the electric and vortex fields and $f(\mathbf{r})$ is the probability density of finding an ion at position $\mathbf{r}$. If the vortex is aligned along the z axis of a polar coordinate system, the appropriate boundary conditions become

$$\lim_{\mathbf{r}\to 0} f(r, \theta) = 0 \tag{5a}$$

$$\lim_{\mathbf{r}\to \infty} f(r, \theta) = 1 \tag{5b}$$

The ion capture cross section, being defined as the ratio of the ion flux into the vortex divided by the flux at infinity, can then be written as

$$\sigma = \frac{2k_B T}{eE} \lim_{r\to 0} \int_0^\pi d\theta \left(r\frac{\partial f}{\partial r} + \frac{rf}{k_B T}\frac{\partial \varphi}{\partial r}\right) \tag{6}$$

Unfortunately, the solution of Eq. (4) with the subsidiary conditions (5a) and (5b) also presents certain difficulties due to the mixed symmetries of the vortex and electric fields. Donnelly and Roberts get around this problem by neglecting the vortex potential in Eq. (4) beyond a certain distance d and by replacing the boundary condition (5a) with $f(d, \theta) = 1 - P$. This simplification requires that d and P be considered as two related adjustable parameters. The limit in Eq. (6) reverts simply to $r = d$ and the cross section becomes

$$\sigma = \pi r_E P \left[\frac{I_0(d/r_E)}{K_0(d/r_E)} + 2 \sum_{n=1}^{\infty} (-1)^n \frac{I_n(d/r_E)}{K_n(d/r_E)} \right] \tag{7}$$

where $r_E = 2k_B T/eE$. Donnelly and Roberts argue that d should be the order of the radius at which the electric and vortex fields are of comparable magnitude and that P should be about 1/2.

In comparing this result with our data, we retained their value for d,

$$d = (6\rho_s a^3 \kappa^2/eE)^{1/3} \tag{8}$$

and adjusted P to fit theory to experiment at 1.5 K. Here a, the negative ion radius, was taken to be 16 Å. The calculation is shown as curve 1 in Fig. 3, the value of P being 0.48.

McCauley and Onsager[14] performed a more rigorous calculation in which they explicitly included the vortex force. Their result, which has no

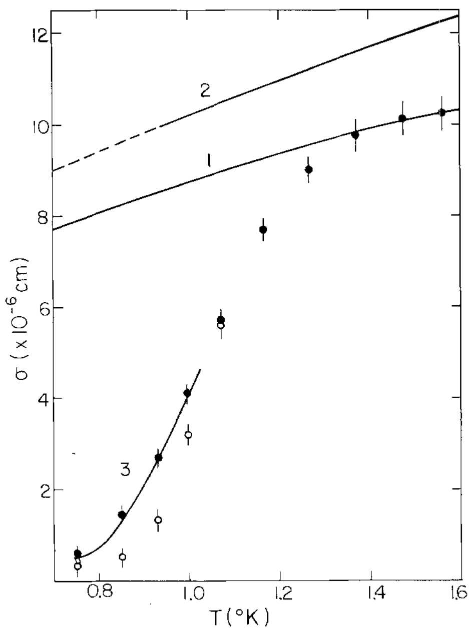

Fig. 3. Negative ion capture cross section in pure ^{4}He as a function of temperature for an electric field $E = 33$ V/cm. The open circles are measurements made with the field configuration shown in Fig. 1 and the solid circles are data obtained with the top and bottom electrodes biased at -50 and $+50$ V, respectively. The curves are discussed in the text.

adjustable parameters, is shown as curve 2 in Fig. 3. It has about the same temperature dependence as the Donnelly and Roberts theory but falls 15–20% higher than the experimental data. In their approach they used a perturbative theory, retaining terms only to first order. Unfortunately, it is not clear to what extent higher order terms might contribute to the cross section.

It is clear from Fig. 3 that there exists fair agreement between theory and experiment above about 1.3 K. However, one notes a rapidly increasing discrepancy with decreasing temperature. This is also evident from the data of Fig. 4, which indicates that the discrepancy persists over a wide range of electric fields. A number of explanations for this anomalous behavior have been suggested, the three most prominent being: (a) a breakdown in stochastic theory, (b) a lifetime effect, and (c) a modification in the ion–vortex interaction. We shall consider each of these possibilities in turn.

Stochastic theory in its most general form would be valid only above about 1.0 K at best, and it is to be expected that the condition for the validity of the Smoluchowski equation is even more restrictive. That this is the case can be deduced by a careful consideration of the assumptions involved in the derivation of the Smoluchowski equation.* The two basic assumptions are that (1) the ion velocity distribution is Maxwellian and (2) the external force acting on the ion does not change appreciably over a diffusion length L_D. In the case of the ion–vortex interaction the latter condition holds true as long as the ion remains a distance r,

$$r \gg L_D = (M_i\mu/e)(2k_B T/M_i)^{1/2} \tag{9}$$

from the vortex core. Here M_i and μ are the ion mass and mobility, respectively. Now if the ion is thermalized (i.e., condition 1 is satisfied) and if it finds itself within a distance r_T of the vortex, where r_T is the radius within which the vortex potential is less than $-k_B T$, the ion will then be effectively trapped. Thus the validity of the Smoluchowski equation is ensured by having

$$r_T = (\rho_s a^3 \kappa^2/4\pi k_B T)^{1/2} \gg L_D \tag{10}$$

or $\mu T \ll 1.5\ \text{cm}^2$ K/V-sec. Inequality (10) begins to break down at temperatures in the vicinity of 1.2–1.3 K. This is, in fact, coincident with the onset of the anomalous behavior seen in Fig. 3.

Because of the inapplicability of stochastic theory at lower temperatures, we have attempted to clarify the cross-section behavior by performing a Monte Carlo calculation. In this calculation a fixed, straight vortex line is located along the z axis of a three-dimensional coordinate system and an electric field is imposed in the positive x direction. Ions are started at some

*See Refs. 15 for a detailed discussion of this problem.

initial position $(-x_i, y_i)$ to the left of the origin for various values of y_i and are allowed to move in the combined electric and vortex fields. Individual ion–roton scattering events are taken into account by assuming isotropic scattering and the frequency of collision is adjusted to give the correct mobility. The calculated cross section remained invariant to the choice of x_i as long as it was an order of magnitude greater than d [Eq. (8)] or the diffusion length, whichever was larger. The initial ordinate y_i was varied incrementally from 0 until capture events no longer occurred, and if the ion propagated from $-x_i$ to $+x_i$, it was assumed to have escaped capture. The effect of phonon scattering was determined by putting an appropriate drag term into the ion equation of motion, and was found to be negligible.

In doing the calculation, we considered two possible criteria for capture. In the first (stochastic capture), capture was said to occur when the ion remained within a distance r_T of the vortex during the course of 100 roton collisions. As the second criterion (direct capture), we defined the ion as being trapped when it approached to within an ionic radius of the vortex core. This criterion enhanced the cross section somewhat at lower temperatures and leads to curve 3 in Fig. 3. In Fig. 4 we compare the field dependences and note that at 0.996 K the two calculations are in reasonable agreement with the data, but at 0.752 K we found that "stochastic capture" gave cross sections that were smaller than our experimental resolution for all values of electric field. This seems to indicate that "direct capture" is of primary importance in the actual trapping process.

As can be seen from Figs. 3 and 4, we obtain fair quantitative agreement between our experimental data and the Monte Carlo calculation using no adjustable parameters. There were, however, a number of assumptions

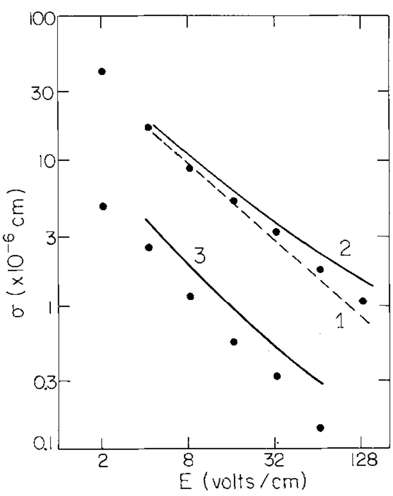

Fig. 4. Negative ion capture cross section in pure ^{4}He as a function of electric field. The upper set of points corresponds to $T = 0.996$ K and the lower set to $T = 0.752$ K. These data were obtained using the field configuration of Fig. 1.

made in this calculation. It is almost certainly true that isotropic scattering is a gross oversimplification of the ion–roton interaction. We found that modifications in the scattering dynamics did not significantly affect the calculated results, and therefore it appears that the detailed nature of this interaction is not very important in understanding the capture process.

A possibly more serious difficulty, especially at lower temperatures where the capture width is small, is the complex hydrodynamics inherent to the ion–vortex interaction. Schwarz[16] has dealt with this difficult problem and has shown that in the hydrodynamic limit the cross section is approximately geometric. Furthermore, Schwarz's results make it unlikely that convective motion of the vortex plays any major role in the anomalous cross-section decrease. Excess drag that the ion experiences in the vicinity of the vortex core, associated with a larger excitation density there,[17] also leads to a geometric contribution to the capture. These notions support the validity of the "direct capture" criterion.

Lifetime effects have been considered as a possible cause of the cross-section anomaly. This was first suggested by Douglass[12] as a result of his observations of a finite trapping lifetime in this temperature range. The effect of a lifetime τ would be to reduce the measured cross section σ_m such that[10]

$$\sigma_m = \sigma_0 e^{-t/\tau} \tag{11}$$

where σ_0 is the true cross section and t is some average time it takes an ion to drift along a line outside the field of view of the collector. The value of t can be decreased by imposing an electric field parallel to the vortex array. A sufficiently large field should therefore reduce lifetime effects to a negligible level. This was indeed found to be the case, in that a clearing field shifted the high-temperature lifetime edge to higher temperatures.

In Fig. 3 we have plotted two sets of data. The open circles are the results obtained with the cell operated in the mode indicated in Fig. 1, i.e., the top and bottom electrodes grounded. In this mode there existed some parallel clearing field (off the symmetry plane) but it was insufficient to remove lifetime effects from the measurements. By biasing the top and bottom electrodes at opposite polarities, we observed an increase in the cross section to some limiting values. These values at the various temperatures are shown as the solid circles in Fig. 3. They correspond to the true cross section in which the effects of the finite trapping lifetime have been eliminated.

A number of important points should be noted. First, it is clear that lifetimes modify the measured cross section only for temperatures at and below 1.0 K. This is consistent with the temperature dependence of the trapping lifetime (see Section 5). Second, the data in Fig. 4 were taken without any clearing field, but the results of Fig. 3 indicate that the actual cross section should be somewhat higher and therefore in better agreement with the predictions of the Monte Carlo calculation using the direct capture criterion. This is borne out by a comparison of the temperature dependence of this calculation, namely curve 3 in Fig. 3, with the data. The good agreement strongly supports the claim that the cross section anomaly is indeed due to a simple breakdown in the stochastic theory: the cross section drops at lower temperatures because the ion, as it passes through the vortex well, suffers too few roton collisions to remain in thermal equilibrium and "fall" to the bottom of the well.

In concluding this section, we briefly discuss our failure to see positive ion capture. It should first be noted that the validity criterion, (10), for the Smoluchowski equation is about the same for the positive ion as for the negative ion. Because of its smaller size and therefore smaller effective vortex potential, however, the corresponding high-temperature lifetime edge will be shifted to a lower temperature than that for the negative ions. If one does a reasonable extrapolation, this temperature turns out to be about 0.5 K, well below the temperatures at which the stochastic theory is valid. If one extrapolates the expected behavior for the positive ion cross section from what is observed experimentally for the negative ions at higher temperatures, it should be very small indeed. Pursuing this one step further, we did a number of Monte Carlo calculations scanning the full experimentally accessible range of temperatures and electric fields. The resulting cross section was found, in all cases, to be less than our experimental resolution.

4. CROSS SECTIONS IN A ^3He–^4He SOLUTION (0.94%)

The Smoluchowski equation is not valid at lower temperatures in pure ^4He basically because of the reduced number density of rotons. In order to extend its validity and thereby provide a further test of stochastic theory, we have measured the capture cross section in a 0.94% solution of ^3He in ^4He. This concentration was sufficient to ensure the satisfaction of the validity condition, (10), over the entire temperature range of our experiment.

The results of our measurements in the solution are shown in Figs. 5 and 6. In Fig. 5 we have also plotted, as curves 1 and 2, the theoretical calculations of Donnelly and Roberts[6] and McCauley and Onsager.[14] Here again we have adjusted P to fit the Donnelly–Roberts theory to experiment at 1.4 K, P in this case being 0.47. By comparing the data with that in pure ^4He, it is clear that the cross section does follow theory down to significantly lower temperatures in the solution. A pronounced departure from theory, however, does occur at about 1.0 K.

One might suppose that an explanation for this anomaly could be found in a possible breakdown in the theory. The perturbative approach of

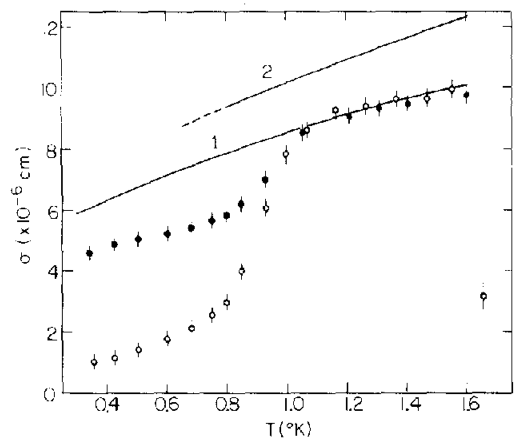

Fig. 5. Negative ion capture cross section in a 0.94% ^{3}He–^{4}He solution as a function of temperature for an electric field $E = 33$ V/cm. The open circles are measurements made with the field configuration of Fig. 1 and the solid circles are data obtained with the top and bottom electrodes biased at -50 and $+50$ V, respectively.

McCauley and Onsager, for instance, fails at lower temperatures because the expansion parameter in their calculation no longer remains small and higher order terms should be included. In the case of the Donnelly–Roberts theory it is not clear what might be wrong, though their approximations with regard to the vortex potential could be open to question.

The true reason, however, lies elsewhere. In Fig. 5 we have plotted two sets of data. The open circles correspond to the measured cross section for the electric field configuration shown in Fig. 1. When we increased the parallel clearing field we obtained a substantial increase in the low-temperature cross section, as indicated by the solid circles. The field dependence of the cross section at two different temperatures in the presence of the parallel clearing field is shown in Fig. 6 along with the predictions of the Donnelly–Roberts theory. It appears, then, that lifetime effects are the primary, if not sole, cause of the anomalous decrease in cross section. These results are consistent with the measured temperature behavior of the trapping lifetime in this solution, as discussed in Section 5.

Unfortunately, we were unable to impose a sufficiently strong clearing field to completely eliminate the effects of lifetime. If the field were made much larger than that used, the transverse field becomes seriously distorted. It appears, though, that the data are converging to a reasonable limit. In Figs. 7a and 7b we have taken our data and plotted them as $\sigma(T/\rho_s)^{1/2}$ vs. $E(\rho_s/T^3)^{1/2}$. The reason for doing this follows from an analysis of the

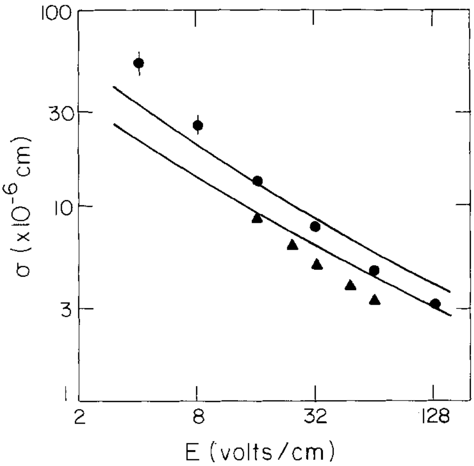

Fig. 6. Negative ion capture cross section in a 0.94% ^3He–^4He solution as a function of electric field. The circles correspond to $T = 0.996$ K and the triangles to $T = 0.426$ K. These data were obtained with the top and bottom electrodes biased at -50 and $+50$ V, respectively. The upper and lower curves are the Donnelly–Roberts theory for the two temperatures.

mathematical problem defined by Eqs. (4)–(6), using Eq. (3) as the vortex force. The results of this analysis is a general scaling law for the cross section

$$\sigma(T, E) = (\rho_s/T)^{1/2} g(E(\rho_s/T^3)^{1/2}) \tag{12}$$

where g is a function determined only by explicit solution of Eqs. (4) and (5). Figure 7a shows the cross section measured without a clearing field. There is no indication whatsoever of any scaling. In Fig. 7b, on the other hand, the data, taken with a substantial clearing field, have essentially converged to a single curve and the effects of lifetime have been reduced dramatically.

The theoretical calculations also obey Eq. (12). We have plotted, as curve 1 in Fig. 7b, the prediction of Donnelly and Roberts fitted to experiment at $T = 1.4$ K and $E = 33$ V/cm. One finds reasonable agreement between theory and experiment, though there do exist some discrepancies. At higher values of $E(\rho_s/T^3)^{1/2}$ these are probably due to residual lifetime effects. The McCauley–Onsager calculation, shown as curve 2, is seen to break down in the regime where their expansion parameter, proportional to $E(\rho_s/T^3)^{1/2}$, becomes large.

A search for positive ion capture in the solution proved unsuccessful. Unlike the situation in pure ^4He, one would expect measurable capture to

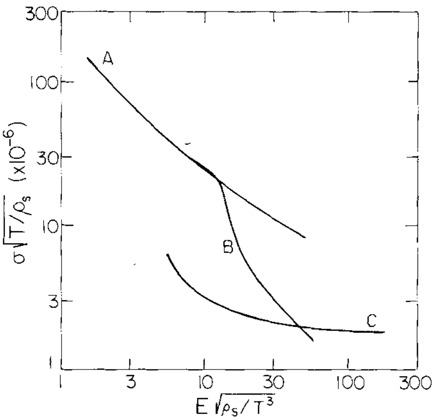

Fig. 7a. A plot of $\sigma(T/\rho_s)^{1/2}$ versus $E(\rho_s/T^3)^{1/2}$ for the 0.94% ^{3}He–^{4}He solution using cross section data obtained with the field configuration of Fig. 1. The curves A and C are electric field scans at $T = 0.996$ and 0.426 K and curve B is a temperature scan at $E = 33$ V/cm.

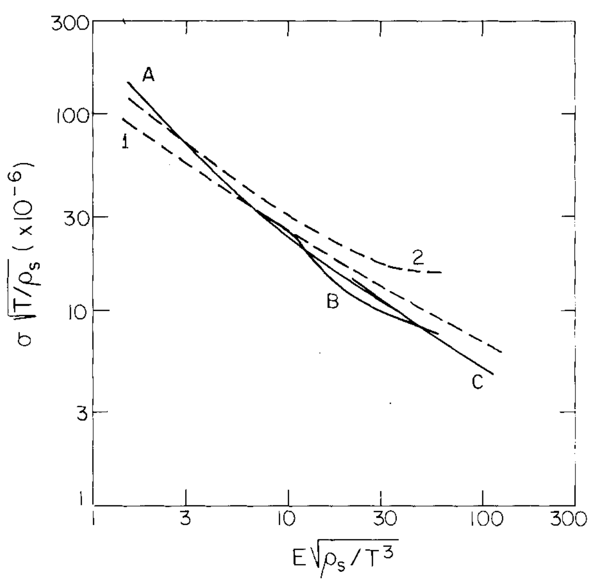

Fig. 7b. A plot of $\sigma(T/\rho_s)^{1/2}$ versus $E(\rho_s/T^3)^{1/2}$ for the 0.94% ^{3}He–^{4}He solution using the cross section data obtained with the top and bottom electrodes biased at −50 and +50 V. Curve 1 is the Donnelly–Roberts theory fit to experiment at $T = 1.4$ K and $E = 33$ V/cm. Curve 2 is the theory of McCauley and Onsager.

occur, at least below the expected lifetime edge. That this should be so is also indicated by Monte Carlo calculations incorporating hard sphere ^{3}He-positive ion scattering. Why capture does not occur is not clear. It may be related to a possible change in the vortex core radius due to the solute ^{3}He atoms. Ohmi *et al.*[18] have predicted the condensation of ^{3}He onto the core and an associated increase in its radius to about 10 Å for a 1% solution. This value is probably too high since Ohmi *et al.* neglected the effects of surface tension at the phase boundary, but a core parameter* as small as 2 Å might be sufficient to lower the positive ion lifetime edge beyond our experimental temperature range.

5. TRAPPING LIFETIMES

Because of the low-temperature behavior observed for the cross section, especially in the ^{3}He–^{4}He solution, it was necessary to measure the ion trapping lifetime. In order to avoid cell-dependent effects,[20] we used the same experimental cell as used in the cross section measurements. Its operation in measuring the lifetime consisted of a three-phase cycle as indicated schematically in Fig. 8. During the first or charging phase, which generally lasted anywhere from 10 to 60 sec for a given measurement, the source was

*The energy well depends on the energy per unit length of a vortex line, which is only indirectly related to the core radius; see, e.g., Ref. 19.

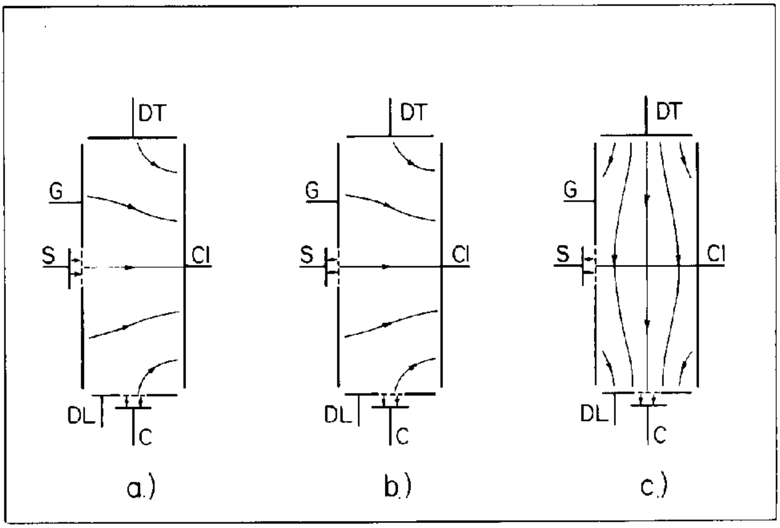

Fig. 8. A schematic showing the three phases of the cell operation used in the lifetime measurements. The arrows indicate the electric field lines.

biased on and the electric fields were as shown in Fig. 8a. This configuration prevented ions, once trapped, from leaking off the lines in the vertical direction.

During the clearing phase the source was biased off. This allowed free ions and ions escaping from the vortex lines to be swept away to the right-hand electrode. This phase lasted for a varying amount of delay time t_D. In the final or flushing phase the electrodes were biased in such a way (Fig. 8c) as to drive the remaining trapped charge downward to the collector at the base of the cell. The entire cycle was repeated a number of times, the collected current being transmitted to the Keithley vibrating capacitor electrometer and thence to a Fabri-Tek model 1062 instrument computer. The latter signal averaged and ultimately integrated the accumulated signal.

A typical set of data for various delay times is given in Fig. 9. The amplitude of each of the curves is approximately proportional to the charge collected as a function of time. The proportionality is not precise because the collector does experience a certain amount of capacitive pickup when the electric field is switched at the transition between the clearing and flushing phases. This pickup, amounting to anywhere from 1 to 20% of the total signal, can be determined separately and subtracted from the raw data. Taking the corrected data of Fig. 9, we have plotted the total accumulated charge as a function of the delay time on a semi-log graph as shown in Fig. 10. For this case it is clear that the charge decay is exponential, i.e.,

$$Q = Q_0 e^{-t/\tau} \tag{13}$$

where τ is defined to be the trapping lifetime. In fact we have found that

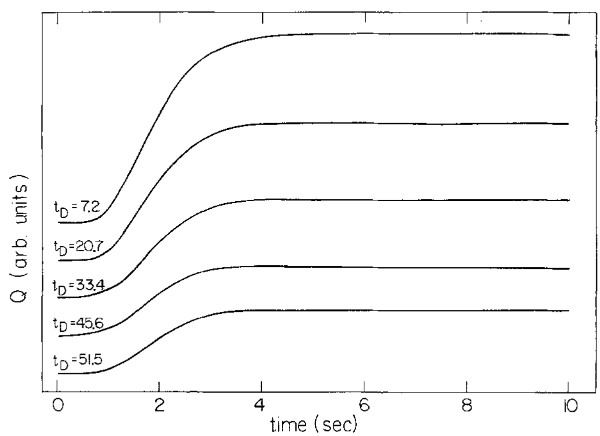

Fig. 9. A sample of the raw data showing the integrated current for various delay times t_D. The ordinate corresponds to the time at the beginning of phase C in the cell operation.

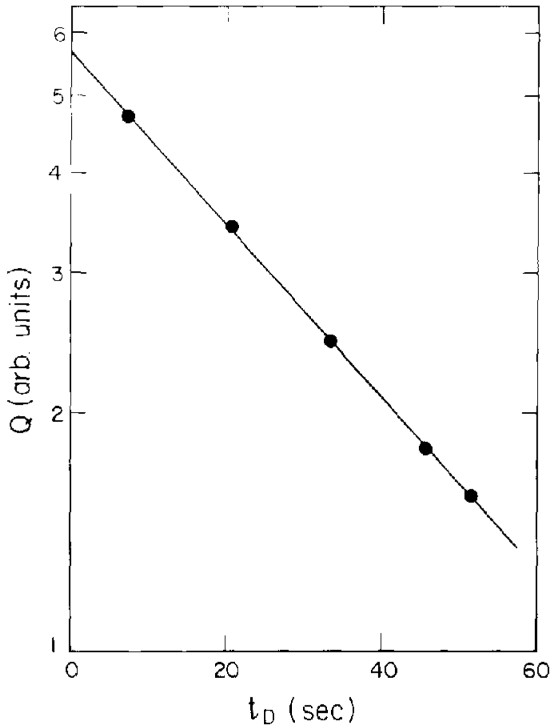

Fig. 10. The total collected charge as a function of delay time using the data of Fig. 9. These data were obtained in pure ^{4}He at $T = 0.930$ K.

Eq. (13) is satisfied to within experimental error for all the measurements that were made.

During our preliminary runs we made a number of measurements at the high-temperature lifetime edge. We found our data to be in good agreement with those of Douglass[7] and Pratt and Zimmerman.[11] Of particular interest to us was the anomalous low-temperature behavior of the trapping lifetime first observed by Douglass.[21] We therefore repeated the experiment at lower temperatures, obtaining the results shown in Fig. 11 for pure ^{4}He and the 0.94 % ^{3}He–^{4}He solution. Within the estimated error of about ± 15 % we found the lifetimes to be independent of reasonable variations in the charging time or flushing field or to variations up to 100 μW in the heater power. The data were taken with the apparatus rotating at 10.64 rad/sec, the clearing field between electrodes G and Cl during the first and second phases being 15 V/cm. These parameters were about the same as those used by Douglass in his measurements. We found that our measured lifetimes are qualitatively in agreement with his but are generally longer. DeConde *et*

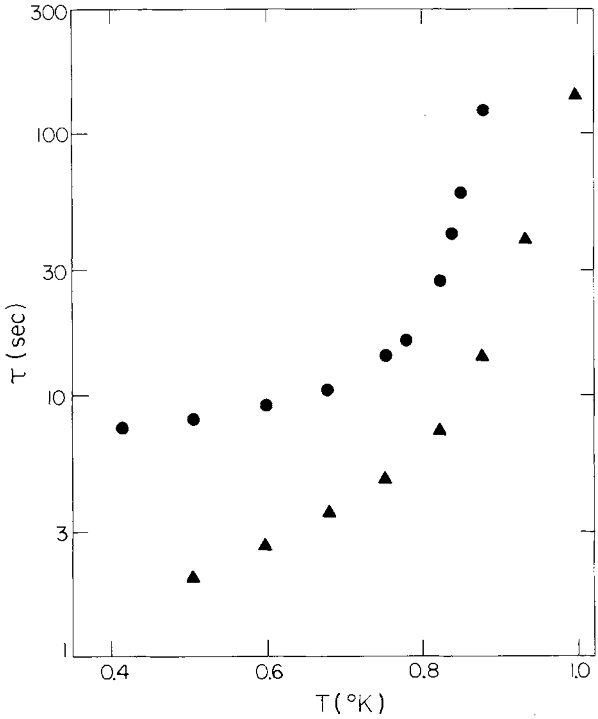

Fig. 11. The negative ion trapping lifetime as a function of temperature. The triangles are data obtained in pure ^{4}He and the circles are data obtained in the 0.94% ^{3}He–^{4}He solution.

al.[20] have observed variations in lifetime apparently due to the size of the experimental cell. The difference we observed between our results and Douglass' might be of this nature.

The effect of lifetime on capture depends critically on the length of time t in Eq. (11). If τ is of the order of or less than t, lifetime effects will be significant in a cross section measurement. A rough approximation for t can be given as

$$t \simeq d_c/\mu_{\parallel} E_{\parallel} \tag{14}$$

where d_c is a characteristic dimension of the collector and $\mu_{\parallel}$ and $E_{\parallel}$ are the ion mobility and electric field along a vortex line. In our experiment, using Eq. (14), we have estimated t to be of the order of 10–20 sec or less, depending on the solution, the magnitude of $E_{\parallel}$, and, to a lesser extent, the temperature. The temperature dependence due to the mobility $\mu_{\parallel}$ is relatively weak, certainly much weaker than that of the lifetime as seen in Fig. 11. As the temperature increases, therefore, the ratio of t to τ decreases until, at about 1 K, the effect of lifetime on the measured cross section becomes negligible.

6. SUMMARY

The main purpose of this research has been to understand the capture of ions by vortices in rotating He II. In the case of pure ^{4}He we found that predictions based on stochastic theory[6,14] were in fair agreement with experiment above about 1.3 K. At lower temperatures the cross section decreased much more rapidly than predicted. The results of a Monte Carlo calculation, using a simple model for the ion–roton interaction, showed that this decrease is due mainly to the reduced number of rotons at lower temperatures.[22] The effect of a finite trapping lifetime was shown to play only a minor role in the gross behavior of the measured cross section and only for temperatures below 1.1 K. Our failure to observe positive ion trapping was shown to be consistent with theoretical predictions.

In order to provide a more detailed test of stochastic theory, we measured the capture cross section in a 0.94 % solution of ^{3}He in ^{4}He. The data taken in this solution followed theory to significantly lower temperatures than those for pure ^{4}He, but there did occur, at about 1.0 K, a marked departure from the theory. We found that a clearing electric field imposed parallel to the vortex array substantially decreased this discrepancy. This was interpreted as signifying the presence of lifetime effects at low temperatures. When these effects were eliminated by the clearing field, considerable improvement in the agreement between stochastic theory and experiment was obtained.

An absence of a measurable cross section for the positive ion in the solution was noted. This result is contrary to both the predictions of stochastic theory and Monte Carlo calculations. A possible explanation may be a decrease in the effective vortex potential due to the condensation of ^{3}He atoms onto the core.

Finally, our indirect observations of a finite trapping lifetime were verified by a direct measurement. In this experiment we used the same cell as used in the cross section measurements so as to avoid cell-dependent effects.[20] Our results for the lifetime were seen to be in qualitative agreement with those of Douglass,[21] though somewhat longer. The rapid decrease in lifetime at about 1.0 K was coincident and consistent with the onset of the observed discrepancies between our cross section data and the theoretical predictions.

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