Journal of Low Temperature Physics, Vol. 33, Nos. 3/4, 1978

Observations of Quantized Vorticity Generated in Superfluid ^{4}He Flow Through 2-μm-Diameter Orifices*

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(Received April 17, 1978)

A beam of charged vortex rings is used to study the vorticity present downstream from an array of 2-μm-diameter orifices through which a steady flow of superfluid ^{4}He is driven at known velocity and pressure head Δp. From the measured attentuation of the beam at low Δp, we infer a density of vortex line less than predicted by the Feynman critical velocity model. Possible explanations are considered. We find evidence that an interconnected tangle of vortices is formed above a certain value of Δp, which is in reasonable agreement with theory, at least for one orifice plate. Charge transfer and transient attenuation measurements indicate that this tangle decays in part into small vortex rings. These results are compared with an earlier experiment of Gamota.

Nearly all current theories of the critical velocity for the onset of dissipation in the flow of superfluid ^{4}He are based on a threshold for the generation of quantized vortices, a process first suggested by Feynman.[1] A great many experiments over the past forty years have investigated the relation between superfluid velocity v_s and the chemical potential drop $\Delta\mu$ for a variety of flow channel geometries,† but only a very few experiments have provided direct information on the configuration of superfluid vortices in the flow.[3,4]‡ In view of the complexity of critical velocity phenomena and the lack of quantitative agreement with theoretical models, the need for more such information is manifest.

*Research supported in part by the National Science Foundation under grant DMR 72-02971.

†For discussion of some of the experiments, see Ref. 2. A few of the more recent experiments are cited below.

‡There have been a number of studies by ion trapping or second sound attenuation of vortices in counterflow in wide channels, but the turbulent counterflow state is not thought to be sensitive to the vortex nucleation process.[5]

0022-2291/78/1100-0243$05.00/0

A significant step in this direction was an experiment by Gamota,[3] who used charged vortex rings (CVRs) to study the vorticity present a few centimeters downstream from an array of 5-μm-diameter orifices* at low temperatures (0.3–0.5 K), where drag forces on vortices are small. From a combination of CVR attenuation and charge transfer/time-of-flight measurements, he concluded that the flow-generated vorticity consisted of vortex rings of single-quantum circulation, directed downstream, with diameters (of 3.2–1.7 μm) decreasing as the flow drive was increased. The flow was driven by vibrating the membrane containing the orifices at its fundamental resonant frequency of 1.17 kHz. This constitutes the principal limitation of the experiment: Neither v_s nor $\Delta\mu$ could be determined,† and as a result Gamota could not successfully relate his observations to the vortex generation process.

In this paper we report CVR attenuation measurements which complement and clarify the observations of Gamota.‡ Our apparatus also directs CVRs transverse to the presumed path of vortices downstream from an array of orifices. However, we impose a steady superfluid flow through the orifices and are able to measure both $\Delta\mu$ and v_s; thus we know (to logarithmic accuracy) the length of quantized vortex line being generated per second. Using a specific theoretical model, we can predict the configuration of the vortices and can estimate at what magnitude of $\Delta\mu$ close encounters between primary vortices become of dominant importance. For $\Delta\mu$ smaller than this, one may hope to observe the primary vortices, while at larger $\Delta\mu$ one can study superfluid turbulence under conditions of very small viscous drag, complementing extensive recent work in the counterflow regime.[5] Most of our results for large $\Delta\mu$ will be reported elsewhere.

Anderson[8] showed that in steady flow with a chemical potential drop $\Delta\mu$, quantized vortices must cut completely across the flow channel at an average rate ν given by

$$\nu = \Delta\mu/\kappa \tag{1}$$

where $\kappa = h/m$ is the quantum of circulation. Huggins[9] first described the incremental process by which a vortex extracts energy from the general flow as it cuts the streamlines. In order to construct a model of the threshold dissipation process, one must invent a vortex nucleation or regeneration mechanism and compute (or conjecture) the subsequent vortex trajectories. Models can be divided into two classes on the basis of

*Nuclepore membrane, manufactured by Nuclepore Corporation, Pleasanton, California.
†Also, acceleration effects may have been important.
‡The techniques were developed in part by Schwarz[6] and Gamota and Sanders.[7]

whether the impulse of the vortex ring (or its self-induced velocity, excluding boundary effects) is directed upstream or downstream.* A prototype of the former class is the Iordanskii–Langer–Fisher (ILF) model,[11] in which thermal fluctuations create a vortex ring larger than a threshold size such that its self-induced velocity would just balance the general flow velocity. Such a ring is carried backward by the flow and is expanded by drag forces and by diverging flow at the channel exit. Eventually the flow velocity falls off faster ($\sim r^{-2}$) with distance r downstream than the self-induced velocity of the expanded ring ($\sim r^{-1}$), and the ring moves back "upstream" to annihilate at the wall surrounding the orifice. Such an upstream-directed vortex ring could not be observed at a large distance from the orifice. We remark that the Glaberson-Donnelly[12] "vortex mill" model for the extrinsic critical velocity also produces upstream-directed rings.†

The prototype for downstream-directed rings is the original model of Feynman.[1] The threshold state in this model is a ring just smaller in diameter than the channel or orifice, such that its self-induced velocity plus the general flow are just balanced by the boundary "image" velocity (i.e., the flow induced by the vortex due to its proximity to the wall). A ring smaller than this threshold will proceed downstream, expanding in the diverging flow, but ultimately shrinking due to drag forces, until it either disappears or hits a wall. Feynman did not address the nucleation problem, but argued by analogy to classical flow separation that generation of vorticity at the exit edge of a flow channel was plausible. One would expect thermally nucleated vortices to appear predominantly adjacent to a wall (see, e.g., Hess[13]). Blood[14] has conjectured that a peel-off mechanism might invert such vortices into large downstream-directed rings. That the Feynman model has retained interest is indicated by the number of calculations which have appeared of the resulting vortex trajectories.[9,15]

We will provisionally adopt the Feynman model of downstream-directed vortex rings for the sake of definite predictions and because vortices are in fact observed downstream (whether primary or not remains uncertain). The temperature is assumed to be sufficiently low that normal fluid drag is very small and that $\Delta\mu = \Delta p/\rho$. The asymptotic diameter d of a ring will be large compared to the orifice diameter D if the flow velocity averaged over the orifice v_s is much larger than the threshold velocity found by Feynman,

$$v_F = (\kappa/2\pi D)\eta_F \tag{2}$$

*This corresponds roughly to the outward and inward nucleation of Roberts and Donnelly,[10] but is not restricted to thermal nucleation. At higher temperatures, where normal fluid drag is large, it is also possible to envision a nearly straight vortex moving nearly transversely across the flow and transferring its energy gain immediately to the normal fluid.

†To see this, one can apply the results of Huggins[9] and ask which streamlines remain to be cut after pinch-off.

where $\eta_F = \ln(D/a) \simeq 10$ and a is the vortex core radius. Either by calculating trajectories or from energy considerations, one finds that

$$d = (\eta_F/2\eta)D(v_s/v_F) \tag{3}$$

and the asymptotic self-induced velocity of the ring is

$$v_I = (\kappa/2\pi d)(\eta+1) = [\kappa^2\eta(\eta+1)/2\pi^2 D^2]v_s^{-1} \tag{4}$$

where $\eta = \ln(4d/a) - 3/2 \simeq 11$.

Successive rings from the same orifice can be expected to exchange energy, collide, and tangle if the mean axial separation is small compared to the diameter; that is, if

$$\nu d/v_I \gtrsim 1 \tag{5}$$

with nucleation rate ν given by Eq. (1). While lacking knowledge of the solution of the appropriate stability problem and of the magnitude of fluctuations in nucleation rate, we find it plausible that Eq. (5) will give the onset of tangling within perhaps a factor of 2. With Eqs. (1)–(4),

$$\beta \equiv \nu d/v_I = [2\pi^3/\eta^2(\eta+1)](D^4 v_s^2\, \Delta\mu/\kappa^4) \tag{6}$$

Thus the value of $\Delta\mu$ at which tangling begins ($\beta \simeq 1$) is a rather strong function of D and v_s.

In order to estimate the attenuation α of a transverse CVR beam before the onset of tangling, we make the following assumptions: The neutral rings are large and slow compared to the CVRs and move almost but not exactly perpendicular to them; their trajectories fill a cone or "jet" which, in the plane of the CVR beam, has diameter δ; and $\alpha \ll 1$. Then

$$\alpha \simeq 0.75(8/\pi)(Nd_c/\delta)\beta \tag{7}$$

where N is the number of orifices and d_c is the CVR diameter. The numerical factor incorporates the result of Schwarz[16] for the collision width.

We consider briefly what may be expected if, on the other hand, upstream-directed vortex rings are nucleated. The energy extracted is the same as in the other case, so the diameter of a ring after it passes the axial turning point and approaches the wall (not too closely) is still given approximately by Eq. (3), and Eq. (4) gives the correct order of magnitude for the vortex velocity in this region. Consequently the condition for onset of tangling should remain essentially the same. It is conceivable that an individual ring, particularly if it is nucleated off axis, may develop an instability on approaching the wall, which could lead to the pinching off

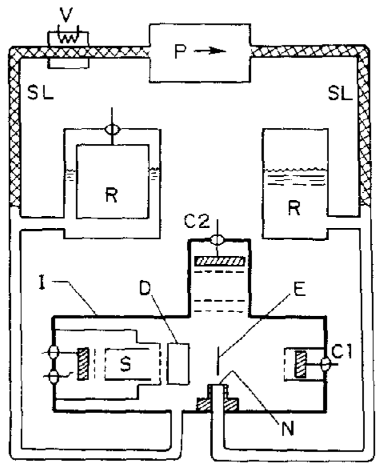

Fig. 1. Schematic diagram of the apparatus. The lettered components are identified in the text. Components of the interaction cell I are drawn approximately to scale.

and escape of a smaller free ring. After tangling begins, it seems probable that ring fragments will be released. Thus some downstream vorticity might be observed.

Figure 1 is a schematic drawing of our apparatus. A superfluid pump* P at $T = 1.6$ K, connected through long superleaks SL, drives the flow of superfluid upward through orifices in plate N at the bottom of the interaction cell I at $T = 0.4$ K. The liquid ^{4}He sample has free surfaces in reservoirs R, also at 0.4 K, one of which contains a capacitive level sensor. This permits us to monitor the pressure difference Δp across the orifice plate and connecting tubing. The CVR gun[18] S focuses a beam of positively charged vortex rings to a neck about 1.5 mm in diameter at a point 4.5 mm above the orifice plate. (The focusing voltage was adjusted and the neck diameter determined by using horizontal deflection plates D to sweep the beam across the gap between knife edges E, which were present above the orifice plate in a few runs.) Primary collector C1 is used with a vibrating capacitor electrometer to measure the transmitted CVR current. Secondary collector C2 is available for charge transfer measurements.

We have used two orifice plates, which are samples of Nuclepore membrane containing 1600 (N-1) or 1300 (N-2) holes of 2μm diameter,

*The operation of our mutual friction pump, which is essentially a shunted counterflow channel, will be described elsewhere. Similar devices have been employed by others.[17]

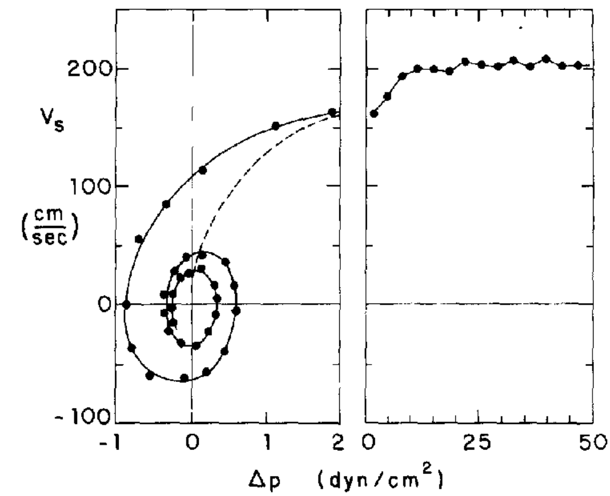

Fig. 2. Superfluid velocity vs. pressure head in gravitational flow through membrane N-2 at $T = 0.4$ K. U-tube oscillations at the end of the pass are shown on an expanded pressure scale on the left. The period of oscillation is 2.9 sec. The measured pressure head in this region is largely due to the inertia of the liquid in the connecting tubes. The dashed line is an estimate of the dissipative pressure head across the membrane.

randomly distributed over a 1.7-mm-diameter disk.† The holes are cylindrical, smooth, and sharp-edged at the downstream side, although the membrane surface is very rough on the upstream side.

The flow velocity v_s, unlike Δp, cannot be monitored during CVR measurements. However, the reservoirs can be used to measure v_s as a function of Δp in gravitational flow. During these measurements the thermal valve V serves to block flow through the pump. We first attempt to characterize the dissipative mode in our samples. Data for a gravitational flow pass through orifice plate N-2 are shown in Fig. 2. The spiral at the left shows, on an expanded scale, the damped U-tube oscillation at the end of the pass. The calculated inertance of the parallel orifices is only 2.5% of the inertance of the connecting tubes. The inertial contribution to Δp of the tubes can be subtracted off, leaving the dissipative pressure drop across the orifices, given by the dashed line in Fig. 2.

Some information on the dissipation at velocities less than 70 cm/sec can be extracted from the damping of the U-tube oscillation. The decrement in amplitude is reproducibly 0.33 per half-cycle on each of the first

†This is a nonstandard material with nominal pore diameter 3 μm and pore density 9×10^4 per cm^2. The actual pore diameter, determined by scanning electron microscope measurements and room temperature gas flow, is 2.0 μm.

two half-cycles following the first velocity zero crossing. It then decreases abruptly to 0.08 for the next several half-cycles and to 0.04 for the 20th through 40th half-cycles (not shown). This damping might be interpreted as due to the tail end of a common $\Delta p(v_s)$ characteristic, or as due to a few channels of low critical velocity. In view of the CVR measurements reported below, it is of interest to consider the bimodal hypothesis that a minority of the orifices carry most of the flow (perhaps with an intrinsic critical velocity of order 1000 cm/sec) and are subcritical during the U-tube oscillation, while the remaining orifices have much lower critical velocities and account for the damping. The magnitude of the damping makes this hypothesis untenable: It is necessary that most of the flow be through channels in which the dissipative force greatly exceeds the inertia, at least during the first several cycles of the oscillation. Consequently the "soft" behavior of $\Delta p(v_s)$, which falls off only gradually with decreasing v_s, must be characteristic of individual orifices. Under the assumption of equal pore velocities, the observed damping yields the estimates $\Delta p \simeq 0.13\ \mathrm{dyn/cm^2}$ at $v_s = 50$ cm/sec; $0.02\ \mathrm{dyn/cm^2}$ at 30 cm/sec; and $0.003\ \mathrm{dyn/cm^2}$ at 10 cm/sec. There is no substantial directional asymmetry, at least in the range of 25–50 cm/sec. Observations of the U-tube oscillation with the orifice plate not installed confirm that the dissipative pressure drop appears across the orifices and not along the connecting tubes.

This soft behavior is not consistent with the intrinsic dissipative process,[11] and the magnitude of the velocity (at 1 $\mathrm{dyn/cm^2}$) is at least six times less than has been observed in some other experiments at 0.4 K.* On the other hand, the observed velocity is 20 times the Feynman critical velocity and about five times the extrinsic critical velocity most often observed in thin micrometer-diameter orifices.[21] It is possible that the flat velocity beyond 10 $\mathrm{dyn/cm^2}$ results from a different vortex generation mechanism. We have not studied the temperature dependence of the velocity in the neighborhood of 0.4 K. A temperature-dependent critical velocity is observed in this material over the temperature range 1.36–2.16 K.[22] Sample N-1 yielded gravitational flow data very similar to those in Fig. 2, except the velocity was 17% larger.

Attenuation measurements are made by establishing a properly focused CVR beam, then recording simultaneously the CVR current I reaching C1 and the pressure head Δp as functions of the power supplied to the superfluid pump. Such plots exhibit a subcritical regime, which establishes the unattenuated current I_0. Typical 15-eV CVR transmission data are shown in Fig. 3. It is evident that the attenuation is very nonlinear.

*Schofield[19] reported a velocity of 950 cm/sec at 0.4 K in a 2.2-μm-diameter, electroformed channel. Similar results in a 6-μm orifice were reported by Hulin.[20]

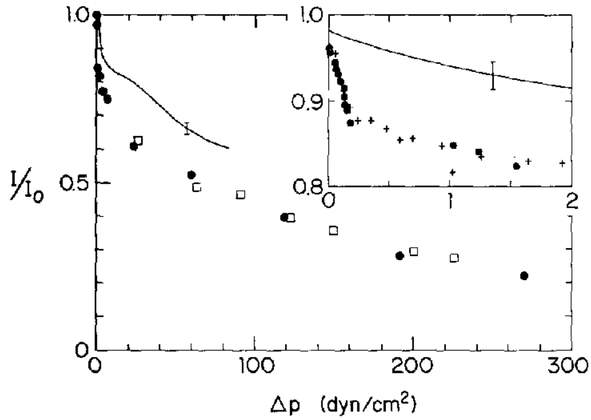

Fig. 3. Transmission of CVR current to collector Cl as a function of pressure head across the orifices, in quasisteady flow. Different symbols indicate different passes with orifice plate N-1, while the solid line represents smoothed data for several passes with orifice plate N-2. The inset gives data at small Δp on an expanded scale. The unattenuated current I_0 is on the order of 1 pA.

There is an initial regime, extending to about 0.2 dyn/cm^2 (for N-1), which might plausibly be identified with attenuation by independent primary vortex rings. For larger Δp the attenuation increases much less rapidly, suggesting that vortices either are being blocked from reaching the interaction region or are being broken into smaller, faster fragments. The predicted onset of tangling, obtained from Eqs. (5) and (6) with velocities from gravitational flow, is at $\Delta p = 0.3$ dyn/cm^2, in reasonable agreement with the observed break.

To predict the magnitude of attenuation at the break with Eq. (7), it is necessary to estimate the width δ of the vortex jet. A lower limit is the diameter of the sample: $\delta > 1.7$ mm. There is some indirect evidence that the vorticity is in fact concentrated in the direction normal to the sample, but we can set an upper limit by calculating the attenuation under the assumption that vortex rings are emitted isotropically into the half-space; this is numerically equivalent to $\delta = 11$ mm. In the spirit of the Feynman model, we will assume that vortex rings are emitted in the direction of the individual pore axes, which, under microscopic examination, are found to be inclined up to 10° to the sample normal. This yields $\delta \simeq 3.4$ mm, at least up to the onset of tangling. With this value and with $\beta = 1$, we find $\alpha \simeq 0.72$ (not corrected for finite opacity) for N-1, about a factor of six larger than is observed experimentally. Consequently, the concentration of vorticity before tangling is about one-fourth that predicted by the Feynman model.

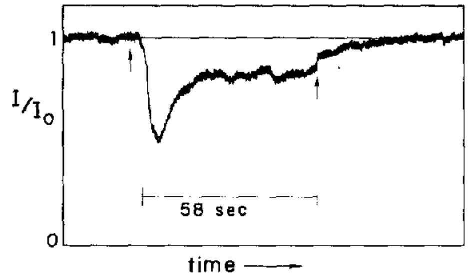

Fig. 4. Time-dependent CVR current at collector C1 when the flow is switched on for a period indicated by the bar. During the flow $\Delta p =$ 6.1 dyn/cm^2. The arrows indicate the times at which heater current to the thermal valve was switched off and on.

With orifice plate N-2 the CVR attenuation (solid line in Fig. 3) is substantially smaller out to about 4 dyn/cm^2, and there is no well-defined break. (The differences at larger Δp may be attributed in part to the smaller number of orifices in N-2 and its smaller critical velocity.) The discrepancy with the Feynman model in this case is roughly a factor of 40.

In order to confirm the existence of a vortex tangle, we have made some transient measurements of the CVR current when the flow through N-1 is suddenly started or stopped, of which Fig. 4 is an example. If only primary vortex rings were present, the transmitted current should change suddenly after a transit time of 0.2 sec. Unfortunately we have no measurements in this regime. When the flow is started at either 2.4 or 6.1 dyn/cm^2, there appears a transient attenuation of two to three times the steady state value and about 10 sec duration.* On stopping, with an initial pressure head $\Delta p = 2.4$ dyn/cm^2, the current recovers approximately exponentially with a time constant of 6 sec. At $\Delta p = 6.1$ dyn/cm^2, about one-fourth of the attenuated current is restored promptly (in less than the response time of the electronics, about 0.3 sec) and the rest with a time constant of about 8 sec. At $\Delta p \simeq 260$ dyn/cm^2 the prompt fraction is 55 ± 15% and the time constant for the remainder is about 14 sec.

The slow component of the recovery strongly suggests the presence of connected vortices of large spatial extent. The fast component at larger Δp suggests that this tangle is increasingly permeated by fragments small enough to propagate through it (i.e., not much larger than the CVRs). The starting transient attenuation is evidently due to vortices penetrating to the

*During these measurements the reservoirs R were completely filled to allow rapid changes of Δp. The pressure head was determined from prior calibration of the pump.

CVR interaction region which later would be intercepted by the fully developed tangle.

Much of the charge that is lost from the CVR beam, at least at large Δp, appears as free positive ions and can be collected on any exposed electrode that is made a few volts negative. A small current reaches C2 even in the presence of a small retarding potential, which indicates that this current is carried by vortex rings moving perpendicular to the CVR beam. Some measurements of this charge-transfer current are shown in Fig. 5. It is evident that the rings involved here are much smaller than the primary rings ($d \simeq 10\,\mu$m), which would require a stopping potential of about 500 V, and also somewhat smaller than the rings seen by Gamota.

In summary, a substantial density of quantized vortices is detected downstream from an array of orifices when the superfluid flow is supercritical; the vortices appear to produce an interconnected tangle if the pressure head exceeds a threshold given approximately by inequality (5); and there is evidence of an efficient mechanism for removing vorticity from the tangle at larger Δp, even though dissipative forces are very small, by pinching off small, fast vortex rings.

There are at least two possible explanations for the observation at the onset of tangling of a lower density of vortices than is predicted by the Feynman model (roughly by a factor of four for N-1 and 40 for N-2). One explanation is that most of the orifices, especially in N-2, are producing upstream- rather than downstream-directed vortex rings. A second possibility is that vortices from a fraction of the orifices with somewhat higher

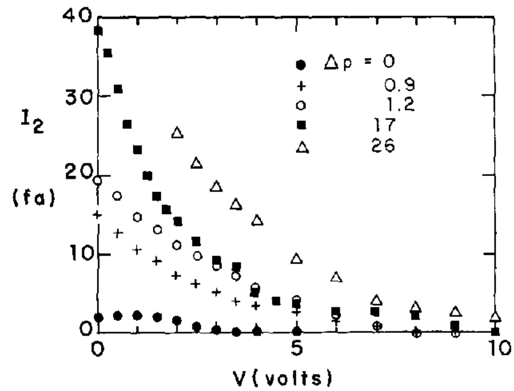

Fig. 5. Charge-transfer current collected at C2 vs. retarding potential, applied to the second grid, for several values of pressure head across the orifices. The current collected with $\Delta p = 0$ is due to a halo of the primary CVR beam. The primary CVR current is approximately 1.5 pA.

than average critical velocities form a tangle at a lower Δp, which intercepts the primary vortices from the other channels as well. With our limited resolution in determining α and v_s much below 0.2 dyn/cm^2, it is difficult to rule this out in the case of N-1. For N-2 it appears untenable on account of limits on the velocity maldistribution from U-tube oscillation damping, and because an attenuation of only 2 or 3% for 0.8-μm CVRs suggests that the tangle must be reasonably transparent to individual 10 μm or smaller primary vortex rings.

We have seen nothing resembling the nearly monoenergetic vortex rings which Gamota observed. On the basis of currently accepted theory, it is very difficult to understand how primary rings could be significantly smaller in diameter than the orifice in which they are generated (as Gamota's were) in steady flow. These rings may have been collision products or primary rings modified by the time-varying flow. As collision products would not be monoenergetic, we consider the second possibility. If the pore critical velocity in Gamota's experiment were much larger than $v_F = 3$ cm/sec, then tangling should occur on increasing drive soon after the critical velocity was reached. (This estimate depends on the cube root of the unreported Q of the membrane resonance.) Assuming that primary rings were seen even at the larger drive voltage, the flow velocity and hence also the expansion factor (for steady flow) given by Eq. (3) must have been small. A downstream-directed vortex ring nucleated at the peak flow velocity would have moved only on the order of one diameter downstream before the flow reversed, and would therefore make a significant portion of its escape against a converging flow. This might well produce a net contraction.

In retrospect, the use of even slightly smaller orifices would make an important increase in the threshold for tangling and would greatly improve conditions for studying primary vortex rings. We hope to do this in the future. It would also be desirable to try to make such studies with a better understood or at least better characterized dissipative mode. The "intrinsic" critical velocity is the most clear-cut example, but there appear also to be several recurring extrinsic syndromes.*

ACKNOWLEDGMENT

We wish to thank Prof. A. J. Dahm for helpful comments on the manuscript.

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