ANALYTICAL EXTENSION OF LUNAR LIBRATION TABLES

NATASHA PETROVA
Department of Astronomy, Kazan State University,
Lenin str. 18, Kazan, 420008, Rep. of Tatarstan, Russia

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Abstract. Tables of lunar physical libration defining the analytical dependence upon the parameters of the lunar gravitational field are presented. The tables are obtained on the framework of the "main problem" in lunar libration by integration of the Hamilton equations reduced to the harmonic oscillator equations.

The variables of physical libration have been obtained in the form of Poisson series. The distinguishing feature of the tables is that these series are the analytical extension of semianalytical solution computed for a number of dynamical parameters LURE2.

A comparison with the Eckhardt's solution is briefly presented. The previously revealed disagreement of the mean inclination of lunar equator to ecliptic with that in Eckhardt's "solution 500" has been maintained.

Key words: Lunar rotation, physical libration

Glossary of Principle Terms

A, B, C	lunar principal moments of inertia.
$a, a_\odot$	mean Earth-Moon and Earth-Sun distances.
$M, M_\odot, M_\oplus$	masses of Moon, Sun and Earth.
$\rho, \rho_\oplus$	mean radius of Moon and equatorial radius of Earth.
λ	correction factor for applying Kepler's third low to lunar orbit.
$n, n_\odot$	mean rates of lunar and solar orbital motion.
$\bar{L}, \bar{L}_\odot$	mean longitude of Moon and of Sun.
$r, \tilde{r}_\odot r_\odot$	Earth-Moon, Earth-Sun and Moon-Sun distances.
l, l', F, D	Delaunay arguments.
L, b, π^*	lunar ecliptical longitude, latitude and sine parallax.
π_0^*	constant part of sine parallax.
$\Lambda = L - \bar{L}$	inequality in lunar ecliptical longitude.
$\Lambda_\odot$	the same for Sun.
f	gravitational constant.
$I, (\bar{\Theta})$	inclination of lunar equator to the ecliptic.
$U, U^\odot, U_2, U_3, U_2^\odot$	gravitational potentials of the Moon and of the Sun, its second and third degree components.
T	kinetic energy.

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μ, ν, π	variables of physical libration, that are the angles defining the position of the principal axes of inertia in the uniformly rotating coordinate system.
P_1, P_2	direction cosines of ecliptic pole in Dynamical System of Coordinate (DSC).
τ	libration in longitude.
u_1, u_2, u_3	direction cosines of lunar radius-vector in ecliptical coordinate system.
$u_{\odot 1}, u_{\odot 2}, u_{\odot 3}$	direction cosines of solar radius-vector in ecliptical coordinate system.
$\Omega, \Omega_x, \Omega_y, \Omega_z$	angular rate of lunar rotation and its components on the DSC-axes.
$\gamma = \frac{B-A}{C}$, $\beta = \frac{C-A}{B}$, $\chi_1 = \frac{C-B}{B}$, $\chi_2 = \frac{C-A}{A}$	ratios of moments or dimensionless moments of inertia.
C_{ij}, S_{ij}	Stockes coefficients of spherical harmonic (degree i, order j) in expansion of lunar gravity potential.

1. Introduction

This article is the continuation of our earlier work (Petrova, 1993) in which the **semianalytical tables** based on the incomplete Schmidt's tables (1980) of lunar motion was presented. These tables are analogous the "solution 500" of Eckhardt(1981). The semianalytical tables of lunar physical libration (LPhL-tables) give the analytical dependence of LPhL-components upon the time t in the form of trigonometric series. Using these tables the physical libration may be computed on the sufficiently large time intervals.

Semianalytical tables are computed with the certain chosen set of characteristics of the gravitational field of the Moon which is known as **the dynamical parameters** or **the dynamical model of lunar gravity field**. The list of these parameters includes among other factors the Stockes coefficients S_{ij} and C_{ij} as well as the dimensionless moments of inertia γ, β. Because their numerical values are obtained from different quality observations it is natural that they possess a certain degree of inaccuracy and will be changed as the observations are perfected.

Practically all modern models of lunar gravitational field are considered in the paper of Kisljuk (1988) where a set of values of dynamical parameters together with their mean square errors are presented (Table I). It is obvious that the scatter in these data is sufficiently large. Hence the semianalytical tables computed for a certain model will not satisfy another model.

In order for this problem to be solved Migus (1980) and later Moons (1982a, 1982b, 1982a) have constructed more universal LPhL-tables which provide the analytical dependence not only on time but on the dynamical parameters as well. These tables were named the **analytical lunar libration tables**.

The availability of different tables which are computed by different mathematical methods and which are based on the different lunar motion tables give

TABLE Ia

Harmonics of selenopotential of the second order obtained from data of different models (in units of 10^{-6}).

No	C_{21}	S_{21}	S_{22}	C_{20}	C_{22}	*Observation type*
1	+15.7±5.9	3.61±3.58	−1.39±1.45	−206.0±2.28	14.0±1.2	*L*
2	−16,6±5.1	0.8±3.9	−3.42±2.5	−205.96±14.1	20.42±2.9	*LO;LA*
3	−8.8±1.31	1.5±1.39	−13.10±3.36	−202.63±1.43	21.91±2.49	*LO;LA*
4	−9.0	−5.56	−3.91	−219.2	32.93	*LO;SA*
5	−0.4	−4.57	+0.21	−207.07	22.42	*LO;SA*
6	−12.8	12.90	+10.14	−195.64	15.87	*LO*
7*	—	—	—	204.8±3.0	22.1±0.5	*LO*
8*	+8.2±2.4	6.33±1.9	+4.54±6.0	−199.6±2.0	23.59±5.3	*LO*
9	+11.0	+13.01	−0.01	−203.79	24.84	*LO;SA*
10	+5.4	6.17	−0.17	−205.60	22.58	*LO*
11	+4.6±2.0	1.95±1.80	+4.85±1.90	−201.24±0.19	20.2±1.9	*E*
12*	—	—	—	202.72±1.48	—	*E*
13*	—	—	—	−203.82	22.4	*LLR−LURE2*
14*	—	—	—	−203.82	22.4	*LLR,VLBI*
15*	—	—	—	203.82	22.3	*LLR*
16*	—	—	—	−202.19±0.91	22.21±1.23	*E*
17	+2.63±2.31	−0.03±2.98	0	−204.64±3.8	21.73±1.16	*LO,SSA*
18	+7.2	−0.77	+2.90	−210.86	22.08	*LO;SSA*
19	+7.7	+3.8	+7.5	−192.7	25.3	*LO;LA*
20	+0.5	+6.9	+3.4	−193.8	27.1	*The same*
21	+7.3	+3.9	+7.2	−200.1	25.0	*LO;LA*
22	+0.2	+6.8	−0.6	−201.0	27.0	*The same*
23	+7.9	+3.8	+8.5	−203.8	22.6	*LO;LA*
24	0	+6.6	+5.8	−203.7	22.5	*The same*
25*	−0.09	0	+0.02	−202.43±1.14	22.26±0.13	*LO;LLR*
26*	+0.10±0.35	0	+0.02±0.01	−202.15±1.2	22.30±0.13	*LO;LLR*
27	−0.9	+6.1	+11.3	−202.6	27.9	*LO;LA+SA*
28	−0.3±0.2	+5.53±1.6	+11.0±7.4	−207.0±6.6	24.3±7.2	*LO;LA+SA*
29	+0.2	+4.6	+11.30	−196.8	27.8	*The same*
30	−3.46±1.91	+6.78±2.56	−1.49±0.54	−204.33±1.54	22.79±0.12	*LOS,PM*
31	+0.48	+5.11	+2.01	−202,6±3.0	21.9±0.4	*LOS*
32	—	—	—	−202,23±0.26	22.27±0.04	*Extend. model*

The Table 1 is the compilation of the tables represented in the book of (Kisljuk, 1988). The type of measurement is denoted by : L-"Luna", LO-"Lunar Orbiter", E-"Explorer",SSA-subsatellites of "Appolo",LLR- Lunar Laser Ranging,VLBI - Very-Long-Base-Interferometry of ALSEP, A-"Appolo", SA-short arches,LA- long arches,LOS- Line-Of-Site acceleration of command modules, PM-point masses.The models used in (Kisljuk, 1988) for determination of extended model of dynamical parameters are denoted by $*$.This model has the number 32.

TABLE Ib

Harmonics of selenopotential of the third order obtained from data of different models (in units of 10^{-6}).

No	C_{30}	C_{31}	C_{32}	C_{33}	S_{31}	S_{32}	S_{33}
1	−36.3±9.9	+56.8±2.6	11.8±4.7	—	+17.8±3.2	−0.7±4.60	—
2	−37.73±18.0	+30.12±2.6	+12.94±2.8	3.17±1.5	+17.62±5.3	−1.47±3.3	−0.43±1.8
3	−22.23±2.62	+36.36±0.25	−2.57±0.58	−2.65±0.79	+7.40±0.32	−2.00±0.63	−4.96±1.14
4	−19.53	+27.18	+6.93	+2.58	+4.07	+4.41	+0.63
5	−6.30	+24.37	+5.02	+1.66	+2.30	+2.03	−0.68
6	−12.99	+34.93	+0.76	+2.84	+9.48	+2.83	−4.47
7	−10.7±9.0	+31.6±0.5	+5.5±1.0	+1.8±0.3	+4.3±12.0	+2.7±1.0	−0.99±0.2
8	−5.88±2.9	+30.01±2.7	+4.70±2.8	+4.85±2.2	+1.42±3.2	+0.57±1.7	−2.92±1.3
9	+28.44	+24.15	+7.63	+1.41	+20.81	+2.27	−0.31
10	+22.16	+35.75	+2.10	+3.01	+8.20	+3.40	+0.55
11	−21.5±1.4	+29.3±1.7	+5.89±0.64	+1.99±0.17	+9.52±1.0	+3.84±0.44	+0.29±0.19
12	-	-	-	-	-	-	-
13	−10.44±4	+28.6	+4.82±0.15	+2.7	+8.8	+1.71±0.15	−1.14±0.7
14	−3±20	+26.0±4	+4.7±0.2	+2.0±2	−1.0±30	+1.8±0.3	−0.3±1
15	−7.6	+28.6	+4.7	+2.9	+8.8	+1.6	−1.3
16	−33.83±25.9	-	-	-	-	-	-
17	−12.95±11.9	+29.21±1.6	+3.93±1.3	+3.48±1.84	+6.48±1.84	+0.40±1.61	−1.26±0.95
18	−2.91	+23.43	+0.89	+0.31	+7.67	+0.89	+1.39
19	−12.2	+24.5	+2.5	−0.5	+8.4	+4.1	−5.6
20	−12.9	+26.4	+2.1	−2.0	+6.9	+4.4	−6.9
21	−2.1	+27.6	+3.3	−1.2	+8.8	+4.1	−4.8
22	−3.1	+29.8	+2.9	−2.4	+7.0	+4.4	−5.8
23	−12.7	+24.6	+3.1	−1.4	+7.5	+4.8	−5.2
24	−13.0	+26.5	+2.6	−2.1	+6.2	+4.6	−6.4
25	−8.89±1.51	+23.72±1.12	+4.83±0.05	+2.21±0.14	+7.16±1.29	+1.63±0.04	−0.34±0.13
26	−12.13±1.8	+30.71±1.9	+4.89±0.05	+1.44±0.17	+5.61±2.59	+1.69±0.04	−0.33±0.17
27	−6.7	+34.6	+1.5	+0.4	+8.2	+1.1	−3.5
28	−4.9±7.1	+28.3±3.2	+3.4±1.8	−0.4±1.74	+7.4±3.1	+3.3±1.8	−3.6±1.7
29	−12.4	+25.5	−1.0	+1.9	+10.7	+2.6	−1.4
30	−7.01±1.4	+24.5±1.14	+4.1±0.3	+5.34±0.11	+5.45±0.79	+2.14±0.26	−1.14±0.06
31	−3.7±4.2	+28.4±1.6	+2.5±0.5	+2.9±0.2	+3.4±1.6	+2.5±0.5	−1.2±0.2
32	−12.17±1.09	+26.84±1.40	+4.853±0.023	+1.92±0.14	+7.97±0.94	+1.661±0.018	−0.474±0.120

a possibility of obtaining a more objective accuracy estimation of physical libration.

This is one of the motives stimulating us to develop one more analytical LPhL-tables based on the lunar motion as computed by the method of Hill, Brown, Eckert under assumption of the "main problem"(Gutzwiller, Schmidt, 1986). Analytical dependence of the lunar coordinates $(\Lambda(t), b(t), \pi^*(t))$ upon time is represented by

TABLE Ic

Dimensionless moments of inertia obtained from data of different models (in units of 10^{-6}).

No	β	γ	Observation type
13*	631.26±0.3	227.37±0.7	$LLR-LURE2$
14*	631.27±0.03	227.7±0.7	$LLR, VLBI$
15*	631.28	227.18	LLR
26*	631.687±0.132	228.022±0.100	$LO; LLR$
32	631.325±0.112	227.735±0.080	*Extend. model*

trigonometrical series whose numerical coefficients and trigonometrical indices of Delaunay arguments (l, l', F, D) are given by the tables called by us HBE-tables.

Furthermore when constructing our tables we have taken into account the fact that semianalytical tables computed for dynamical model LURE2 (e.g. "solution 500" of Ekchardt(1981)) are widely used in modern selenodetic practice. Because of this we have made our tables as the analytical extension of this semianalytical solution.

2. Canonical Form of Rotation Equations

In order to describe lunar rotation the Moon is commonly considered as a rigid body. In this case the frame of reference (xyz) is determined by lunar axes of inertia and it is rigidly bound with lunar body. Let's call this system the **Dynamical System of Coordinates**.

Take the right-hand selenocentric ecliptical coordinate system whose axes X and Y are respectively oriented towards the vernal equinox point Υ and the pole P of ecliptic at epoch JD2000.0 as inertial coordinate system (XYZ) (Figure 1).

Introduce an intermediate coordinate system $(\bar{X}\bar{Y}\bar{Z})$ which **uniformly rotates** around the axis directed toward the pole P. The rate of the rotation equal to the mean velocity of lunar orbital motion $n = d\bar{L}/dt$. Then the position of DSC relative to $(\bar{X}\bar{Y}\bar{Z})$-system may be determined by the transformation matrix $\mathbf{d}$:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{pmatrix} \begin{pmatrix} \bar{X} \\ \bar{Y} \\ \bar{Z} \end{pmatrix} \tag{1}$$

Here the matrix $\mathbf{d}$ is obtained as a product of rotation matrices:

$$\mathbf{d} = \mathbf{m}_x(-\pi) \cdot \mathbf{m}_{\bar{Y}}(\nu) \cdot \mathbf{m}_Z(\mu)$$

and $\mathbf{m}_v(\alpha)$ is the rotation matrix around the axis v, the deflection angle α is positive if the rotation is executed in anti-clockwise direction. The matrix elements d_{ij} are

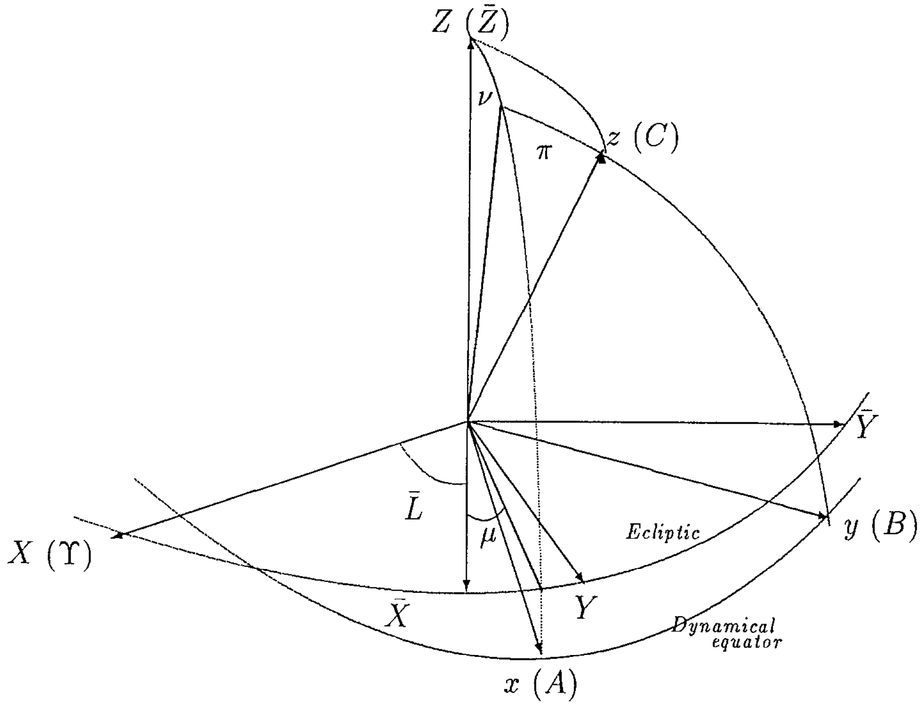

Fig. 1. Selenocentrical coordinate system. XYZ – ecliptical coordinate system, X is directed towards the equinox point, Z - towards the ecliptic north pole ; $\bar{X}\bar{Y}\bar{Z}$ – uniformly rotating coordinate system; xyz – dynamical coordinate system, x is directed towards the minimal moments of inertia A, z - towards the maximal moments C; μ, ν, π – libration angles.

the functions of μ, ν, π-angles (Figure 1):

$$\begin{aligned} d_{11} &= \cos\mu\cos\nu & d_{12} &= \sin\mu\cos\nu \\ d_{21} &= -\cos\mu\sin\nu\sin\pi - \sin\mu\cos\pi & d_{22} &= -\sin\mu\sin\nu\sin\pi + \cos\mu\cos\pi \\ d_{31} &= \cos\mu\sin\nu\cos\pi - \sin\mu\sin\pi & d_{32} &= \sin\mu\sin\nu\cos\pi + \cos\mu\sin\pi \end{aligned}$$

$$\begin{aligned} d_{13} &= -\sin\nu \\ d_{23} &= -\cos\nu\sin\pi \\ d_{33} &= \cos\nu\cos\pi \end{aligned} \qquad (2)$$

These angles describe the deviation of lunar rotation from uniform one or, in other words, the physical libration.

So the final purpose of this problem is the construction of the functional dependence of μ, ν, π-angles upon time t and the dynamical parameters. The Hamilton equations are constructed for handling the problem.

To use the Hamilton-technique it is necessary, firstly, to introduce the **canonical variables** $\mathbf{q} = (q_1, q_2, q_3)^T$ and $\mathbf{p} = (p_1, p_2, p_3)^T$ which are the functions of time t and of libration angles and of its derivatives. And then we can construct the Hamiltonian $H(q, p; t) = T - U$.

Let us write the kinematic equations of lunar rotation. These equations describe the dependence of rotation rate $\boldsymbol{\Omega}$ projections onto the axes of inertial system (XYZ) as a function of libration angles and of its derivatives:

$$\begin{cases} \Omega_x = -\dot{\mathcal{M}} \cdot \sin\nu + \dot{\pi} \\ \Omega_y = -\dot{\mathcal{M}} \cdot \cos\nu \sin\pi + \dot{\nu}\cos\pi \\ \Omega_z = -\dot{\mathcal{M}} \cdot \cos\nu\cos\pi + \dot{\nu}\sin\pi \end{cases}$$

where $\mathcal{M} = \bar{L} + \Lambda$ and $\dot{\mathcal{M}} = n + \dot{\mu}$.

The kinetic energy of lunar rotation may be expressed in the form

$$T = \frac{1}{2}(A\Omega_x^2 + B\Omega_y^2 + C\Omega_z^2) = T(\nu, \pi, \dot{\mathcal{M}}, \dot{\nu}, \dot{\pi})$$

Let us now introduce the canonical variables in the following manner:

$$\bar{q}_1 = \mathcal{M} \qquad \bar{q}_2 = \sin\nu \qquad \bar{q}_3 = \sin\pi$$

The conjugated impulses $\bar{\mathbf{p}} = (\bar{p}_1\bar{p}_2\bar{p}_3)^T$ are respectively determined by the formulae:

$$\bar{p}_1 = \frac{\partial(T)}{\partial(\dot{\bar{q}}_1)} \qquad \bar{p}_2 = \frac{\partial(T)}{\partial(\dot{\bar{q}}_2)} \qquad \bar{p}_3 = \frac{\partial(T)}{\partial(\dot{\bar{q}}_3)}$$

Since the expression for kinetic energy is the polynomial of the second degree in generalized rates then

$$\bar{p}_i = \sum_{s=1}^{3} A_{si} \cdot \dot{\bar{q}}_s + B_i$$

The determinant $\mid A_{si} \mid$ can be shown to be not zero and this fact allows us to perform the Legendre's transformation and to express the kinetic energy as a function of canonical variables:

$$\begin{aligned} T \;=\; & \frac{1}{2C}\{(1+\chi_1)\bar{p}_3^2(1-\bar{q}_3^2) + (1+\chi_2)[-\bar{p}_1\bar{q}_3(1-\bar{q}_2^2)^{-\frac{1}{2}} + \\ & +\bar{p}_2(1-\bar{q}_2^2)^{\frac{1}{2}}(1-\bar{q}_3^2)^{\frac{1}{2}} + \bar{p}_3\bar{q}_2\bar{q}_3(1-\bar{q}_3^2)^{\frac{1}{2}}(1-\bar{q}_2^2)^{-\frac{1}{2}}]^2 + \\ & +[\bar{p}_1(1-\bar{q}_3^2)^{\frac{1}{2}}(1-\bar{q}_2^2)^{-\frac{1}{2}} + \bar{p}_2(1-\bar{q}_2^2)^{\frac{1}{2}}\bar{q}_3 - \\ & -\bar{p}_3\bar{q}_2(1-\bar{q}_2^2)^{-\frac{1}{2}}(1-\bar{q}_3^2)]^2\} \end{aligned} \qquad (3)$$

Let us now go to the second term of Hamiltonian. The lunar potential energy U is represented through its second U_2 and third U_3 degree harmonics for interaction with the Earth and through the second degree harmonic $U_2^{\odot}$ with the Sun:

$$U = U_2 + U_3 + U_2^{\odot} + \ldots$$

These terms are computed through the known expansion over the spherical functions.

$$
\begin{aligned}
U_2 &= \left(\frac{3}{2}\lambda\frac{fM_\oplus}{a^3}\right)\left(\frac{a}{r}\right)^3[(C-B)u_1^2-(C-A)u_3^2]\\
U_3 &= -\left(\frac{3}{2}\lambda\frac{fM_\oplus}{a^3}\right)M\rho^2(\frac{\rho}{a})\left(\frac{a}{r}\right)^4[C_{31}u_1-S_{31}u_2+C_{30}u_3-\\
&\quad -10C_{33}u_1^3+10S_{33}u_2^3-\tfrac{5}{3}C_{30}u_3^3+\\
&\quad +30C_{33}u_1u_2^2-30S_{33}u_1^2u_2-5C_{31}u_1u_3^2-10C_{32}u_1^2u_3-\\
&\quad -5C_{31}u_2u_3^2+10C_{32}u_2^2u_3-20S_{32}u_1u_2u_3]\\
U_2^\odot &= \left(\frac{3}{2}\frac{fM_\odot}{a_\odot^3}\right)\left(\frac{a_\odot}{r_\odot}\right)^3[(C-B)u_{\odot 1}^2-(C-A)u_{\odot 3}^2].
\end{aligned}
\tag{4}
$$

The direction cosines of the Earth's and Sun's radius vectors in the DSC according to (1) may be written in the form:

$$
\begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix} = \frac{1}{r}\begin{pmatrix}d_{11}&d_{12}&d_{13}\\d_{21}&d_{22}&d_{23}\\d_{31}&d_{32}&d_{33}\end{pmatrix}\begin{pmatrix}\bar{X}\\\bar{Y}\\\bar{Z}\end{pmatrix} \quad \begin{pmatrix}u_{\odot 1}\\u_{\odot 2}\\u_{\odot 3}\end{pmatrix} = \frac{1}{r_\odot}\begin{pmatrix}d_{11}&d_{12}&d_{13}\\d_{21}&d_{22}&d_{23}\\d_{31}&d_{32}&d_{33}\end{pmatrix}\begin{pmatrix}\bar{X}_\odot\\\bar{Y}_\odot\\\bar{Z}_\odot\end{pmatrix}
$$

From geometry of spherical triangles (Figure 1) it can be shown that:

$$
\begin{pmatrix}\bar{X}\\\bar{Y}\\\bar{Z}\end{pmatrix} = r\begin{pmatrix}\cos b\cos\Lambda\\\cos b\sin\Lambda\\-\sin b\end{pmatrix}
$$

$$
\begin{pmatrix}\bar{X}_\odot\\\bar{Y}_\odot\\\bar{Z}_\odot\end{pmatrix} = \begin{pmatrix}r\cos b\cos\Lambda-\tilde{r}_\odot\cos(D-\Lambda_\odot)\\r\cos b\sin\Lambda+\tilde{r}_\odot\sin(D-\Lambda_\odot)\\-r\sin b\end{pmatrix}
$$

where

$$
\frac{r}{r_\odot} = (\frac{r}{\tilde{r}_\odot})(\frac{\tilde{r}_\odot}{r_\odot}); \qquad \frac{r}{\tilde{r}_\odot} = (\frac{a}{a_\odot})(\frac{r}{a})(\frac{a_\odot}{\tilde{r}_\odot});
$$

$$
\frac{\tilde{r}_\odot}{r_\odot} = 1-\frac{1}{2}(\frac{r}{\tilde{r}_\odot})^2-(\frac{r}{\tilde{r}_\odot})\cos b\cos(D+\Lambda-\Lambda_\odot)
$$

The trigonometrical functions of b, Λ and also the a/r (or r/a) appeared in the expressions for u_i, $u_{\odot i}$ and for U (4) are calculated as functions of time in the form

TABLE II

Numerical values of constants and parameters of physical libration.

Notation	Value used	Notation	Value used
$\frac{M}{M_\oplus}$	$1.230002 \cdot 10^{-2}$	$\left(\frac{C}{M\rho^2}\right)^0$	0.392
$\frac{M_\odot}{M+M_\oplus}$	328900.5	γ^0	$2.2737 \cdot 10^{-4}$
$\frac{\rho}{a}$	$4.521 \cdot 10^{-3}$	β^0	$6.3126 \cdot 10^{-4}$
$\frac{a}{a_\odot}$	$2.5718814 \cdot 10^{-3}$	C_{20}^0	$-2.027 \cdot 10^{-4}$
n (rad/day)	$2.299708345 \cdot 10^{-1}$	C_{22}^0	$2.23 \cdot 10^{-5}$
$n_\odot$ (rad/day)	$1.7202785 \cdot 10^{-2}$	C_{30}^0	$-1.04 \cdot 10^{-5}$
π_0^* arc. sec	3422.452	C_{31}^0	$2.86 \cdot 10^{-5}$
λ	1.002723	C_{32}^0	$4.8 \cdot 10^{-6}$
$\dot{l}$ (rad/day)	$2.280271372 \cdot 10^{-1}$	C_{33}^0	$2.7 \cdot 10^{-6}$
$\dot{l}'$ (rad/day)	$1.720196956 \cdot 10^{-2}$	S_{31}^0	$8.8 \cdot 10^{-6}$
$\dot{F}$ (rad/day)	$2.308957130 \cdot 10^{-1}$	S_{32}^0	$1.7 \cdot 10^{-6}$
$\dot{D}$ (rad/day)	$2.127087043 \cdot 10^{-1}$	S_{33}^0	$-1.1 \cdot 10^{-6}$

of trigonometrical series whose numerical coefficients and trigonometrical indices are given by HBE-tables. The value $a/a_\odot$ = const (Table II).

Using the known expansions for radius-vector and for true anomaly in two bodies problem (Abalakin *et al.*, 1976) we have computed the solar coordinates $\frac{a_\odot}{r_\odot}$ and $\Lambda_\odot$ in the form of trigonometrical series also. We have applied the third Kepler law for determination of the factors:

$$\frac{fM_\oplus}{a^3} = \frac{n^2}{1+\frac{M}{M_\oplus}} \qquad \frac{fM_\odot}{a_\odot^3} = \frac{n_\odot^2}{1+\frac{M+M_\oplus}{M_\odot}}$$

Hence the potential function $U = U(\bar{q}_1 - \bar{L}, \bar{q}_2, \bar{q}_3; t)$ depends of time via the lunar and solar coordinates. Its dependence of the libration angles (or of the canonical variables $\bar{\mathbf{q}}$) is executed via the elements of matrix $\mathbf{d}$:

Let us perform the following canonical transformation:

$$q_1 = \bar{q}_1 - \bar{L}(t) \quad q_2 = \bar{q}_2 \quad q_3 = \bar{q}_3$$

$$p_1 = \bar{p}_1 - Cn^2 \quad p_2 = \bar{p}_2 \quad p_3 = \bar{p}_3$$

The new Hamiltonian H is connected with old Hamiltonian $\bar{H}$ which is written via the new variables $\mathbf{q}$ and $\mathbf{p}$ in the following manner:

$$H = \bar{H}(\bar{q}, \bar{p}; t)_{\bar{q},\bar{p}\to q,p} - p_1\frac{d\bar{L}}{dt} = \bar{H}(q, p; t) - p_1 n$$

According to Kassini laws the Moon is rotated almost uniformly and its equator place has a small inclination ($I \sim 5555.''$) to the ecliptic. Under these conditions the variables q_i are small :

$$q_1 \sim 2 \cdot 10^{-5} \qquad q_2 \sim -2.7 \cdot 10^{-4} \sin F \qquad q_3 \sim -2.7 \cdot 10^{-4} \cos F$$

This fact allows us to develop the fractional and negative degrees in T (3) and the trigonometrical functions in **d** (2) in the power series. To satisfy the required accuracy $\varepsilon = 0.''001$ for libration angles it is sufficient to develop the obtained expression up to the fourth degree relative to **q**. Then we have:

$$\begin{aligned} T &= p_1 n + \frac{1}{2C}[p_1^2 + (1+\chi_1)p_2^2 + (1+\chi_2)p_3^2] \\ &\quad + \frac{1}{2}\chi_1 C n^2 q_3^2 + C n^2 q_2^2 + q_2[-np_3 - \frac{1}{C}p_1 p_3] \\ &\quad - q_3[\frac{\chi_1}{C}p_1 p_2 + \chi_1 n p_2] + \cdots \\ &= p_1 n + C \sum G_{ijklm}(\chi_1, \chi_2) q_2^i q_3^j \frac{p_1}{C}^k \frac{p_2}{C}^l \frac{p_3}{C}^m \end{aligned} \tag{5}$$

We have following expansion for the elements d_{ij} (2):

$$d_{ij} = \sum_{i,j,k=0}^{(i+j+k)\leq 4} S_{ijk}\, q_1^i q_2^j q_3^k$$

where the S_{ijk} are numerical coefficients resulting from the expansion.

Then we have expressed the potential energy in this fashion:

$$U = C \sum_{<U>} Q_{ijk}(\gamma, \beta, S_{ij}, C_{ij}, t) q_1^i q_2^j q_3^k$$

Here the parameters of the dynamical figure of the Moon are maintained in the literal form, and the lunar coordinates are presented in the form of trigonometrical series. The functions Q_{ijk} are derived on the basis of formulae for U (4), for direction cosines $u_i, u_{\odot i}$ and for elements d_{ij} and they may be written in the following common form:

$$Q_{ijk} = \sum_{m=2}^{3} \Psi_m \sum_{r=1}^{\infty} R_r^{ijk} \begin{matrix}\sin\\ \cos\end{matrix}(k_{r1}l + k_{r2}l' + k_{r3}F + k_{r4}D) \tag{6}$$

The summation index m relates to $m-$harmonic of selenopotential. The coefficients $\Psi_m(\gamma, \beta, C_{3n}, S_{3n}, \rho/a, C/M\rho^2)$ in (6) and $G_{ijklm}(\chi_1, \chi_2)$ in (5) have a form of **power polynomial** on the parameters indicated in parentheses. The R_r^{ijk} are numerical coefficients.

Let us take the maximum lunar moment of inertia C as the unit of measurement of principal moments and of impulses p_i. In the subsequent discussion we can use the following relations between the principal moments of inertia and the dimensionless one:

$$A = \frac{1-\gamma\beta}{1+\beta} \sim 1 \qquad B = \frac{1+\gamma}{1+\beta} \sim 1 \tag{7}$$

The Hamiltonian H is finally represented as a power polynomial:

$$H = \sum_{<T>} G_{ijklm}(\chi_1, \chi_2) q_2^i q_3^j p_1^k p_2^l p_3^m + \sum_{<U>} Q_{ijk}(\gamma, \beta, S_{ij}, C_{ij}, t) q_1^i q_2^j q_3^k \tag{8}$$

where summation limits are defined in the following manner

$$< T >= \begin{cases} i, j = 0 \div 4 \\ k, l, m = 0 \div 2 \\ (i+j) \leq 4 \end{cases} \qquad < U >= \begin{cases} i, j, k = 0 \div 4 \\ (i+j+k) \leq 4 \end{cases}$$

Note that the first term in (8) obtained from expression for kinetic energy is not dependent on the q_1 whereas the second term obtained from potential energy is not dependent on the impulses $\mathbf{p}$. This fact is important for solution of Hamilton equations.

On the basis of Hamiltonian (8) we have constructed the equations system associated with it.

$$\begin{cases} \dot{q}_i = +\frac{\partial(H)}{\partial(p_i)} = \tilde{F}_{q_i}(\mathbf{q}, \mathbf{p}) + \tilde{\Phi}_{q_i}(t, \mathbf{q}) \\ \dot{p}_i = -\frac{\partial(H)}{\partial(q_i)} = \tilde{F}_{p_i}(\mathbf{q}, \mathbf{p}) + \tilde{\Phi}_{p_i}(t, \mathbf{q}) \end{cases} \tag{9}$$

On the right-hand side of these equations the functions $\tilde{F}_{q_i} \tilde{F}_{p_i}$ and $\tilde{\Phi}_{q_i} \tilde{\Phi}_{p_i}$ are respectively the partial derivatives from kinetic and potential parts of Hamiltonian (8) with respect to canonical variables.

3. Construction of analytical parameters

Analytical form of right-hand side of Equations (9) is analogous to that of Hamiltonian (8). The Stockes coefficients C_{ij}, S_{ij} and the dimensionless moments of inertia $\gamma, \beta, \chi_1, \chi_2$ enter into the equations as parameters. In addition, they are the power factors at the unknowns $\mathbf{q}$ and $\mathbf{p}$.

For the obtained system to be solved it is necessary to bring these dynamical parameters to the form which for one makes it possible to integrate the equations and for another is convenient for practical use.

When choosing the analytical parameters we are guided by the following two principles:

(1) The solution must be defined as an analytical extension of the semianalytical solution with LURE2-parameters;

(2) The power part of the solution must have a rapid convergence for the most dynamical models of selenopotential represented in Table I.

To satisfy these concepts we have introduced the analytical parameters E_i as the differences between **any potential** value of the dynamical parameter and its value **given by the model** LURE2. Denote the LURE2-values of parameters by the subscript(^{0}).

For the second harmonic of potential we have defined the analytical parameters with the following relationships:

$$E_1 = \frac{A-A^0}{\Delta_1} \quad E_2 = \frac{B-B^0}{\Delta_2} \tag{10}$$

For the third harmonic we have :

$$\begin{aligned} h &= \frac{(C/M\rho^2)^0}{(C/M\rho^2)} = \frac{0.392}{(C/M\rho^2)} & E_3 &= \frac{hC_{30}-C_{30}^0}{\Delta_3} \\ E_4 &= \frac{hC_{31}-C_{31}^0}{\Delta_4} & E_5 &= \frac{hC_{32}-C_{32}^0}{\Delta_5} \\ E_6 &= \frac{hC_{33}-C_{33}^0}{\Delta_6} & E_7 &= \frac{hS_{31}-S_{31}^0}{\Delta_7} \\ E_8 &= \frac{hS_{32}-S_{32}^0}{\Delta_8} & E_9 &= \frac{hS_{33}-S_{33}^0}{\Delta_9} \end{aligned} \tag{11}$$

Here the Δ_i are the coefficients whose numerical values must be chosen on the basis of principles in hand.

With these new notations both the functions Q_{ijk} in(6) and hence the solution for **q** and **p** is represented as the Poisson series (Brumberg, 1980):

$$\sum_{r=1}^{\infty} COEF_r \cdot FACTOR_r \cdot \begin{pmatrix} sin \\ cos \end{pmatrix} (k_{r1}l + k_{r2}l' + k_{r3}F + k_{r4}D) \tag{12}$$

here:
$COEF_r$ is a numerical amplitude in arc seconds,
$FACTOR_r = \prod_{i=1}^{9} E_i^{m_{ri}}$ is a power polynomial,
k_{rj}, m_{ri} are the integer trigonometric and power indices respectively.

The problem is to obtain all numerical values of these quantities by integrating (9) and to represent them in table form.

When substituting the concrete numerical values for the dynamical parameters all analytical parameters must satisfy the following condition:

$$| E_i | \leq 1 \quad \text{where } i = 1 \div 9 \tag{13}$$

This criterion allows us to cut the terms with the high degrees m_i when operating with the Poisson series.

The coefficients Δ_i in (10), (11) must be on the one hand sufficiently large in order for the condition (13) to be satisfied for the most models in Table I. And on the other hand they must be sufficiently small because $COEF_r$ in (12) are directly proportional to the adopted values Δ_i and hence must also be small in itself for the series (12) to have a limited length within the assigned accuracy.

For the optimal choice of Δ_i for E_1 and E_2 we have used "3σ"-criterion where σ_i is the mean square dispersion of γ and β.

Under condition that $\gamma, \beta \sim 10^{-4} << 1$ and using (7) it is easy to obtain

$$E_1 \simeq -\frac{\Delta\beta}{\Delta_1} \qquad E_2 \simeq \frac{\Delta\gamma - \Delta\beta}{\Delta_2}, \qquad \text{where} \quad \Delta\gamma = \gamma - \gamma^0, \quad \Delta\beta = \beta - \beta^0$$

To satisfy (13) we must assume:

$$\begin{aligned} \Delta_1 &= \max \mid \Delta\beta \mid = \mid 3\sigma_{\beta^0} \mid = 1 \cdot 10^{-6} \\ \Delta_2 &= \max \mid \Delta\beta \mid + \max \mid \Delta\gamma \mid = \mid 3\sigma_{\beta^0} \mid + \mid 3\sigma_{\gamma^0} \mid = 3 \cdot 10^{-6} \end{aligned}$$

In the case that only the Stockes coefficients C_{20} and C_{22} are determined from observations the dimensionless moments γ and β may be obtained by following relationships:

$$\gamma = \frac{4C_{22}}{(C/M\rho^2)} \qquad \beta = \frac{\delta + \gamma}{2 + \gamma - \delta}, \qquad \text{where } \delta = -\frac{2C_{20}}{(C/M\rho^2)}$$

We draw attention to the disagreement between the values of γ, β obtained through C_{20}, C_{22} and that obtained directly from observations. This causes the difference in coefficients of semianalytical series calculated with the use of C_{20}, C_{22} and with the γ, β only. So, for instance, both Stockes coefficients and dimensionless moments are given in the model N 26 of Table I (Ferrari *et al.*, 1980). The former were obtained from Doppler tracing data and the latter were obtained from laser ranging data. As result the coefficient in harmonic $\sin F$ for variable ν is equal to $\sim 5540.''$ with the use of C_{20}, C_{22} and to $\sim 5562.''$ with γ, β.

This far exceeds the allowed discrepancy. On account of this great care must be exercised when **different-type observational data** are used in order for the analytical tables to be brought into semianalytical tables.

For the third harmonic the nearest rounding off to the first significant figure of Stockes coefficient value by itself was taken as Δ_i. The formulae **for practical use** may finally be written in the following form:

$$\begin{aligned} E_1 &= -\frac{\beta\cdot 10^4 - 6.3126}{10^{-2}} & E_2 &= -\frac{(\gamma\cdot 10^4 - 2.2737) - (\beta\cdot 10^4 - 6.3126)}{3\cdot 10^{-2}} \\ h &= \frac{0.392}{(C/M\rho^2)} & E_3 &= \frac{hC_{30}\cdot 10^5 + 1.044}{-2} \\ E_4 &= \frac{hC_{31}\cdot 10^5 - 2.86}{3} & E_5 &= \frac{hC_{32}\cdot 10^5 - 0.48}{0.5} \\ E_6 &= \frac{hC_{33}\cdot 10^5 - 0.27}{0.3} & E_7 &= \frac{hC_{31}\cdot 10^5 - 0.88}{1.0} \\ E_8 &= \frac{hC_{32}\cdot 10^5 - 0.17}{0.2} & E_9 &= \frac{hS_{33}\cdot 10^5 + 0.11}{-0.2} \end{aligned} \tag{14}$$

By this means the choice of the analytical parameters in the form(14) satisfies the laid down conditions, namely:
-if the LURE2-model matches our requirements then the powers terms may be eliminated from treatment: the harmonics whose power indexes are equal to zero ($FACTOR = 1$) represent the semianalytical solution analogous to "solution 500" of Eckhardt;

- the terms that have $FACTOR \neq 1$ directly demonstrate the influence of distinction between values of dynamical parameters on physical libration;
- the condition (13) is fulfilled for most of the dynamical models represented in Table I; this provides a rapid convergence of the power part of the libration series.

4. Derivation and solution of differential equations

The technique developed in (Petrova, 1993) was used for the solution of system (9). By differentiating both left and right sides of (9) with respect to t the obtained equations are reduced to the form of the equations which describe the harmonic oscillator motion.

$$\begin{cases} \ddot{q}_i + \omega_i^2 q_i = \Phi_{q_i}(t, \mathbf{q}) + F_{q_i}(t, \mathbf{q}, \mathbf{p}) \\ \ddot{p}_i + \omega_i^2 p_i = \Phi_{p_i}(t, \mathbf{q}) + F_{p_i}(t, \mathbf{q}, \mathbf{p}) \end{cases} \tag{15}$$

Here the functions $\Phi_{q/p}$ and $F_{q/p}$ are the linear combinations of derivatives of $\tilde{\Phi}_{q/p}$ and of $\tilde{F}_{q/p}$ with respect to t from which the terms of the form

$$(a_i + b_i\bar{q} + c_i\bar{q}^2 + \cdots) \cdot q_i$$

are previously removed and rearranged on the left side of the equations. The coefficients a_i, b_i, c_i are defined by numerical values of parameters n, γ^0, β_0,-$\chi_1^0, \chi_2^0, C_{3n}^0, S_{3n}^0$, as well as of amplitudes of trigonometrical series L, b, r. The constant terms in series $\mathbf{q}, \mathbf{q}^2$ are denoted by $(^-)$, they have all trigonometrical and power indexes equal to zero. These terms determine, in fact, **the fundamental frequencies(eigenfrequencies)** ω_i of harmonic oscillators described formally by the equation (15):

$$\begin{aligned} \omega_1^2 &= -2\bar{Q}_{200}(\gamma^0, \beta^0, C_{3n}^0, S_{3n}^0) + b_1\bar{q} + c_1\bar{q}^2 + \cdots \\ \omega_2^2 &= n^2 - 2(1 + \chi_1^0)\bar{Q}_{020}(\gamma^0, \beta^0, C_{3n}^0, S_{3n}^0) + b_2\bar{q} + c_2\bar{q}^2 + \cdots \\ \omega_3^2 &= \chi_1^0\chi_2^0 n^2 - 2(1 + \chi_2^0)\bar{Q}_{002}(\gamma^0, \beta^0, C_{3n}^0, S_{3n}^0) + b_3\bar{q} + c_3\bar{q}^2 + \cdots \end{aligned} \tag{16}$$

The system (15) is solved iteratively. The semianalytical series obtained in (Petrova, 1993) was taken as the zero approximation $(\mathbf{q^0}, \mathbf{p^0})$. When substituting them for $\mathbf{q}, \mathbf{p}$ in (15) and(16) the right-hand side of equations become the form of Poisson series. In this case the frequencies ω_i take the concrete numerical values. Every n-approximation $\mathbf{q^n}, \mathbf{p^n}$ is computed through the (n-1)-solution:

$$\left\{\begin{aligned} \ddot{q}_i^{(n)} + \bar{\omega}_i^2 q_i^{(n)} &= \Phi_{q_i}(t, \mathbf{q}^{(n-1)}) + F_{q_i}(t, \mathbf{q}^{(n-1)}, \mathbf{p}^{(n-1)}) = \\ &= \sum_r X_r \prod_s E_s^{m_{sr}} \cdot \binom{sin}{cos}(k_{r1}l + k_{r2}l' + k_{r3}F + k_{r4}D) \\ \ddot{p}_i^{(n)} + \bar{\omega}_i^2 p_i^{(n)} &= \Phi_{p_i}(t, \mathbf{q}^{(n-1)}) + F_{p_i}(t, \mathbf{q}^{(n-1)}, \mathbf{p}^{(n-1)}) = \\ &= \sum_r Y_r \prod_s E_s^{n_{sr}} \cdot \binom{sin}{cos}(k_{r1}l + k_{r2}l' + k_{r3}F + k_{r4}D) \end{aligned}\right. \tag{17}$$

In this form the equations are easily integrated. The solution of respective homogeneous system corresponds to the free libration that may be taken to be zero when data of observations are considered (Calame, 1976).

The solution **for the forced libration** is written in the following manner :

$$\left\{\begin{aligned} q_i &= \sum_r X'_r \prod_s E_s^{m_{sr}} \cdot \binom{sin}{cos}(k_{r1}l + k_{r2}l' + k_{r3}F + k_{r4}D) \\ p_i &= \sum_r Y'_r \prod_s E_s^{n_{sr}} \cdot \binom{sin}{cos}(k_{r1}l + k_{r2}l' + k_{r3}F + k_{r4}D) \end{aligned}\right. \tag{18}$$

where the numerical coefficients X'_r and Y'_r are deduced from the expressions:

$$\left\{\begin{aligned} X'_r &= \frac{X_r}{\bar{\omega}_i^2 - (k_{r1}\dot{l} + k_{r2}\dot{l}' + k_{r3}\dot{F} + k_{r4}\dot{D})^2} \\ Y'_r &= \frac{Y_r}{\bar{\omega}_i^2 - (k_{r1}\dot{l} + k_{r2}\dot{l}' + k_{r3}\dot{F} + k_{r4}\dot{D})^2} \end{aligned}\right. \tag{19}$$

The derivatives $\dot{l}, \dot{l}', \dot{F}, \dot{D}$ in (19) are constant, because we have taken only the linear dependence on the time t of Delaunay arguments. The iterative process continues until the difference in numerical coefficients of $n-$iteration $(X')^n, (Y')^n$ and $(n-1)-$iteration $(X')^{n-1}, (Y')^{n-1}$ do not exceed the preassigned value $\varepsilon = 0.''001$, that is until the following condition

$$\begin{aligned} &| (X'_i)^n - (X'_i)^{(n-1)} | \leq 0.''001 \\ &| (Y'_i)^n - (Y'_i)^{(n-1)} | \leq 0.''001 \end{aligned} \tag{20}$$

is fulfilled for each $i-$harmonic of series(18).

The angles ν and π were obtained by the expansion of arcsin q_i in the power series up to the q_i^5:

$$\arcsin q = q + \frac{q^3}{2 \cdot 3} + \frac{1 \cdot 3q^5}{2 \cdot 4 \cdot 6 \cdot 7} + \cdots$$

5. Computer Realization of the Problem and Analysis of Obtained Solution

The performance of great number operations with Poisson series is required for the solution of problem in hand. For this purpose Universal Poisson Processor (UPP)

(Tarasevich, 1979) was used. This subroutine package was adapted for FORTRAN-IV of ES-electronic computer and was favorably placed at our disposal by Dr.Titov V.B.(St-Petersburg University, Russia). A part of the UPP-program was rewritten by us in Assembler with the view to speeding the performance. Nevertheless the calculation process still remained very laborious, processor time and memory consuming. This is explained by the fact that in the calculations the length of intermediate worked series may be related up to two thousand terms.

After 2-3 iterations the condition (20) was fulfilled for the most of harmonics in series (18). But it has taken 6-7 iterations before the required accuracy was reached for the resonant terms and for the harmonics having small denominators.

The final result for μ, ν, π and for P_1, P_2, τ is represented in Table III. As it shown in (Petrova, 1993) the former are convenient to use for the transformation of observed coordinates of lunar craters to DSC. But the variables P_1, P_2, τ are widely used for the description of physical libration: the position of ecliptic pole in DSC and libration in longitude are given by these variables. They are little different from μ, ν, π:

$$\begin{aligned} P_1 &= -\sin\nu = -\nu + o(\nu^2) \\ P_2 &= -\sin\pi\cos\nu = -\pi + o(\pi^2) \\ \tau &= \mu + \tan\tfrac{\Theta}{2}^2 \sin[2(F+\mu-\sigma)] + o(\tfrac{\Theta}{2}^2) \end{aligned} \tag{21}$$

The angular variable $\Theta = \arcsin(P_1^2 + P_2^2)$ defines the inclination of the lunar equator to the ecliptic and its value averaged over time $I = \bar{\Theta}$ is the **mean inclination.**

The terms that have the power indexes equal to zero give the semianalytical solution corresponding to the dynamical model LURE2. These terms are emphasized in Table III by bold type.

The terms having the non-zero power indexes produce a correction to the semi-analytical solution. This correction is caused by the deviation of chosen dynamical parameter values from those of the LURE2-model.

Let us now turn to the discussion of the obtained results. In this work we have all over again deduced U_3-expansion (4) with the use of analytical system REDUCE. Consequently two coefficients were refined. This fact and also the use of the HBE-tables which are more complete and accurate than the tables of Schmidt (1980) have yielded a better agreement (as compared with (Petrova, 1993)) between the present results and the results of Moons (1982b) and Ekchardt (1981). The harmonics which give the difference of amplitudes exceeding $0.''01$ on the comparison are presented in Table IV. It is the harmonics that have small denominators.

But it should be noticed that the discrepancy between the value of the mean inclination of the lunar equator obtained in our calculations I^P and the value

TABLE III.
Physical libration tables

TRIG	FACTOR		COEF			COEF		
l l' F D	$E_1 E_2$	$E_3 E_4 E_5 E_6 E_7 E_8 E_9$	sin	cos		sin	cos	
0 0 0 0	**0 0**	**0 0 0 0 0 0 0**		**214.352**	τ		**214.352**	μ
	0 0	0 0 0 0 0 0 1		307.822	τ		307.822	μ
	0 0	0 0 0 0 0 1 0		-.081	τ		-.081	μ
	0 0	0 0 0 0 1 0 0		51.281	τ		51.281	μ
	0 0	0 0 0 1 0 0 0		-1.441	τ		-1.441	μ
	0 0	0 0 1 0 0 0 0		-.059	τ		-.059	μ
	0 0	0 1 0 0 0 0 0		.162	τ		.162	μ
	0 1	0 0 0 0 0 0 0		-2.813	τ		-2.813	μ
	1 0	0 0 0 0 0 0 0		.938	τ		.938	μ
	0 0	0 0 0 1 0 0 1		-2.069	τ		-2.069	μ
	0 0	0 0 0 1 1 0 0		-.345	τ		-.345	μ
	0 1	0 0 0 0 0 0 1		-4.040	τ		-4.040	μ
	0 1	0 0 0 0 1 0 0		-.673	τ		-.673	μ
0 0 0 1	**0 0**	**0 0 0 0 0 0 0**	**.098**		τ	**.098**		μ
	0 1	0 0 0 0 0 0 0	.001		τ	.001		μ
0 0 0 2	**0 0**	**0 0 0 0 0 0 0**	**-.487**		τ	**-.447**		μ
	0 0	0 0 0 1 0 0 0	-.004		τ	-.004		μ
	0 1	0 0 0 0 0 0 0	-.006		τ	-.006		μ
	1 0	0 0 0 0 0 0 0	.002		τ	.002		μ
0 0 0 4	**0 0**	**0 0 0 0 0 0 0**	**-.004**		τ	**-.004**		μ
0 0 1 -2	**0 0**	**0 0 0 0 0 0 0**			τ		**-.001**	μ
0 0 1 -1	**0 0**	**0 0 0 0 0 0 0**		**-.003**	τ		**-.003**	μ
	0 0	0 0 1 0 0 0 0		-.004	τ		-.004	μ
0 0 1 0	**0 0**	**0 0 0 0 0 0 0**	**-.010**	**1.087**	τ	**-.016**	**2.171**	μ
	0 0	0 0 0 0 0 0 1	-.001		τ	-.003		μ
	0 0	0 0 0 0 0 1 0	-.016		τ	-.016		μ
	0 0	0 0 0 1 0 0 0		.001	τ		.002	μ
	0 0	0 0 1 0 0 0 0	-.002	1.394	τ	-.002	1.860	μ
	0 0	0 1 0 0 0 0 0		-.001	τ		-.008	μ
	0 0	1 0 0 0 0 0 0		.558	τ		.744	μ
	0 1	0 0 0 0 0 0 0		-.003	τ		-.003	μ
	1 0	0 0 0 0 0 0 0			τ		-.004	μ
0 0 2 -2	**0 0**	**0 0 0 0 0 0 0**	**1.669**	**.001**	τ	**1.670**	**.001**	μ
	0 0	0 0 0 0 1 0 0		.002	τ		.002	μ
	0 0	0 0 0 1 0 0 0	.013		τ	.013		μ
	0 0	0 1 0 0 0 0 0	-.002		τ	-.002		μ
	0 1	0 0 0 0 0 0 0	.022		τ	.022		μ
	1 0	0 0 0 0 0 0 0	-.006		τ	-.006		μ
0 0 2 -1	**0 0**	**0 0 0 0 0 0 0**	**-.001**		τ	**-.002**		μ
0 0 2 0	**0 0**	**0 0 0 0 0 0 0**	**-.027**	**.000**	τ	**-37.380**	**-.078**	μ
	0 0	0 0 0 0 0 0 1		-.112	τ		-.112	μ
	0 0	0 0 0 0 1 0 0		-.019	τ		-.019	μ
	0 0	0 0 0 1 0 0 0			τ	-.079		μ
	0 0	0 0 1 0 0 0 0	.014		τ			μ
	0 0	0 1 0 0 0 0 0			τ	.290		μ
	0 0	1 0 0 0 0 0 0	.005		τ			μ
	0 1	0 0 0 0 0 0 0	.001		τ	.001		μ
	1 0	0 0 0 0 0 0 0			τ	.153		μ
0 0 2 2	**0 0**	**0 0 0 0 0 0 0**			τ	**-.001**		μ
0 0 3 0	**0 0**	**0 0 0 0 0 0 0**		**.002**	τ			μ
	0 0	0 0 0 0 0 0 1	.002		τ			μ
	0 0	0 0 0 1 0 0 0		-.001	τ			μ
	0 0	0 0 1 0 0 0 0		.466	τ			μ
	0 0	0 1 0 0 0 0 0		.002	τ			μ

TABLE III.
Continued

TRIG	FACTOR		COEF			COEF		
$l\ l'\ F\ D$	E_1E_2	$E_3E_4E_5E_6E_7E_8E_9$	sin	cos		sin	cos	
	0 0	1 0 0 0 0 0 0		.187	τ			μ
	1 0	0 0 0 0 0 0 0		.001	τ			μ
0 0 4 0	0 0	0 0 0 0 0 0 0	-.003		τ	-.003		μ
	0 0	0 0 1 0 0 0 0	.007		τ			μ
	0 0	1 0 0 0 0 0 0	.003		τ			μ
0 1 -4 1	0 0	0 0 0 0 0 0 0	-.022		τ			μ
0 1 -2 0	0 0	0 0 0 0 0 0 0	.001		τ	-.013		μ
0 1 -2 1	0 0	0 0 0 0 0 0 0	-.004		τ	-.004		μ
0 1 -2 2	0 0	0 0 0 0 0 0 0	.145		τ	.145		μ
0 1 -1 1	0 0	0 0 0 0 0 0 0		-.004	τ		-.004	μ
0 1 0 -4	0 0	0 0 0 0 0 0 0	.001		τ	.001		μ
0 1 0 -2	0 0	0 0 0 0 0 0 0	.038		τ	.037		μ
0 1 0 -1	0 0	0 0 0 0 0 0 0	-.001		τ	-.001		μ
0 1 0 0	0 0	0 0 0 0 0 0 0	90.704	.005	τ	90.705	.005	μ
	0 0	0 0 0 0 0 0 1		.009	τ		.009	μ
	0 0	0 0 0 1 0 0 0	.694		τ	.694		μ
	0 0	0 1 0 0 0 0 0	-.078		τ	-.078		μ
	0 1	0 0 0 0 0 0 0	1.355		τ	1.355		μ
	1 0	0 0 0 0 0 0 0	-.452		τ	-.452		μ
0 1 0 1	0 0	0 0 0 0 0 0 0	-.017		τ	-.016		μ
0 1 0 2	0 0	0 0 0 0 0 0 0	.007		τ	.006		μ
0 1 1 0	0 0	0 0 0 0 0 0 0		.001	τ		.001	μ
0 1 2 -2	0 0	0 0 0 0 0 0 0	.032		τ	.032		μ
0 1 2 0	0 0	0 0 0 0 0 0 0	.001		τ	-.017		μ
0 1 2 1	0 0	0 0 0 0 0 0 0	-.002		τ			μ
0 2 -2 2	0 0	0 0 0 0 0 0 0	-.025		τ	-.025		μ
0 2 0 -2	0 0	0 0 0 0 0 0 0	.002		τ	.002		μ
0 2 0 0	0 0	0 0 0 0 0 0 0	.225		τ	.225		μ
	0 0	0 0 0 1 0 0 0	.002		τ	.002		μ
	0 1	0 0 0 0 0 0 0	.003		τ	.003		μ
0 2 2 -2	0 0	0 0 0 0 0 0 0	.001		τ	.001		μ
0 3 0 0	0 0	0 0 0 0 0 0 0	.001		τ	.001		μ
1 -2 0 -2	0 0	0 0 0 0 0 0 0	-.001		τ	-.001		μ
1 -2 0 0	0 0	0 0 0 0 0 0 0	-.003		τ	-.003		μ
1 -2 2 -2	0 0	0 0 0 0 0 0 0			τ	.001		μ
1 -1 -2 0	0 0	0 0 0 0 0 0 0	-.001		τ	-.003		μ
1 -1 -1 0	0 0	0 0 0 0 0 0 0	.001	.004	τ	.001	.004	μ
	0 0	0 0 1 0 0 0 0		.006	τ		.006	μ
1 -1 0 -2	0 0	0 0 0 0 0 0 0	-.031		τ	-.031		μ
1 -1 0 -1	0 0	0 0 0 0 0 0 0	-1.148	-.005	τ	-1.148	-.005	μ
1 -1 0 0	0 0	0 0 0 0 0 0 0	-.164		τ	-.164		μ
	0 0	0 0 0 1 0 0 0	-.001		τ	-.001		μ
	0 1	0 0 0 0 0 0 0	-.002		τ	-.002		μ
1 -1 0 2	0 0	0 0 0 0 0 0 0	-.005		τ	-.005		μ
1 -1 1 -2	0 0	0 0 0 0 0 0 0		-.002	τ		-.002	μ
1 -1 2 -2	0 0	0 0 0 0 0 0 0			τ	-.001		μ
1 0 -5 0	0 0	0 0 1 0 0 0 0		-.002	τ		-.002	μ
1 0 -4 0	0 0	0 0 0 0 0 0 0	.000		τ			μ
1 0 -4 0	0 0	0 0 0 1 0 0 0	.002		τ			μ
	0 0	0 1 0 0 0 0 0	.001		τ			μ
	0 1	0 0 0 0 0 0 0	.004		τ			μ
1 0 -3 0	0 0	0 0 0 0 0 0 0			τ		-.001	μ
	0 0	0 0 0 0 0 1 0	.005		τ			μ
	0 0	0 0 1 0 0 0 0		-.013	τ			μ

TABLE III.
Continued

TRIG	FACTOR		COEF			COEF		
l l' F D	$E_1 E_2$	$E_3 E_4 E_5 E_6 E_7 E_8 E_9$	sin	cos		sin	cos	
	0 0	1 0 0 0 0 0 0		-.004	τ			μ
1 0-2-2	**0 0**	**0 0 0 0 0 0 0**	**.001**		τ	**.004**		μ
1 0-2-1	**0 0**	**0 0 0 0 0 0 0**			τ	**.001**		μ
1 0-2 0	**0 0**	**0 0 0 0 0 0 0**	**-.426**	**.002**	τ	**-1.771**	**.003**	μ
	0 0	0 0 0 0 0 0 1		.005	τ		.005	μ
	0 0	0 0 0 1 0 0 0	.003		τ	.001		μ
	0 0	0 1 0 0 0 0 0	.004		τ	.015		μ
	0 1	0 0 0 0 0 0 0	.009		τ	.011		μ
	1 0	0 0 0 0 0 0 0			τ	.005		μ
1 0-2 1	**0 0**	**0 0 0 0 0 0 0**	**.008**		τ	**.008**		μ
1 0-2 2	**0 0**	**0 0 0 0 0 0 0**	**.005**		τ	**.006**		μ
1 0-1 0	**0 0**	**0 0 0 0 0 0 0**	**-1.397**	**-6.620**	τ	**-1.397**	**-6.635**	μ
	0 0	0 0 0 0 0 1 0	-1.619		τ	-1.619		μ
	0 0	0 0 0 1 0 0 0		.058	τ		.058	μ
	0 0	0 0 1 0 0 0 0		-7.186	τ		-7.192	μ
	0 0	1 0 0 0 0 0 0		.585	τ		.583	μ
	0 1	0 0 0 0 0 0 0		.113	τ		.113	μ
1 0 0-4	**0 0**	**0 0 0 0 0 0 0**	**.014**		τ	**.014**		μ
1 0 0-3	**0 0**	**0 0 0 0 0 0 0**	**-.002**		τ	**-.002**		μ
1 0 0-2	**0 0**	**0 0 0 0 0 0 0**	**4.130**	**-.002**	τ	**4.140**	**-.002**	μ
	0 0	0 0 0 0 0 0 1		-.002	τ		-.002	μ
	0 0	0 0 0 1 0 0 0	.027		τ	.027		μ
	0 0	0 1 0 0 0 0 0	-.003		τ	-.003		μ
	0 1	0 0 0 0 0 0 0	.054		τ	.054		μ
	1 0	0 0 0 0 0 0 0	-.018		τ	-.018		μ
1 0 0-1	**0 0**	**0 0 0 0 0 0 0**	**-3.453**	**-.003**	τ	**-3.453**	**-.003**	μ
	0 0	0 0 0 0 0 0 1		-.008	τ		-.008	μ
	0 0	0 0 0 1 0 0 0	-.024		τ	-.024		μ
	0 1	0 0 0 0 0 0 0	-.054		τ	-.054		μ
1 0 0 0	**0 0**	**0 0 0 0 0 0 0**	**-16.795**	**-.009**	τ	**-17.131**	**-.008**	μ
	0 0	0 0 0 0 0 0 1		.012	τ		.012	μ
	0 0	0 0 0 0 1 0 0		-.002	τ		-.002	μ
	0 0	0 0 0 1 0 0 0	-.104		τ	-.102		μ
	0 0	0 1 0 0 0 0 0	.025		τ	.027		μ
	0 1	0 0 0 0 0 0 0	-.194		τ	-.190		μ
	1 0	0 0 0 0 0 0 0	.072		τ	.072		μ
1 0 0 1	**0 0**	**0 0 0 0 0 0 0**	**.005**		τ	**.005**		μ
1 0 0 2	**0 0**	**0 0 0 0 0 0 0**	**-.063**		τ	**-.063**		μ
1 0 0 4	**0 0**	**0 0 0 0 0 0 0**	**-.001**		τ	**-.001**		μ
1 0 1-2	**0 0**	**0 0 0 0 0 0 0**	**-.006**	**.038**	τ	**-.006**	**.037**	μ
	0 0	0 0 0 0 0 1 0	-.007		τ	-.007		μ
	0 0	0 0 1 0 0 0 0		.040	τ		.040	μ
	0 0	1 0 0 0 0 0 0		-.004	τ		-.004	μ
1 0 1 0	**0 0**	**0 0 0 0 0 0 0**	**-.001**	**-.001**	τ	**.009**	**.010**	μ
	0 0	0 0 0 0 0 1 0	.005		τ	.010		μ
	0 0	0 0 1 0 0 0 0		.007	τ		.007	μ
	0 0	1 0 0 0 0 0 0		.003	τ		.003	μ
1 0 2-4	**0 0**	**0 0 0 0 0 0 0**	**.001**		τ	**.001**		μ
1 0 2-3	**0 0**	**0 0 0 0 0 0 0**	**-.001**		τ	**-.001**		μ
1 0 2-2	**0 0**	**0 0 0 0 0 0 0**	**.022**		τ	**.051**		μ
1 0 2-1	**0 0**	**0 0 0 0 0 0 0**			τ	**.001**		μ
1 0 2 0	**0 0**	**0 0 0 0 0 0 0**	**-.005**		τ	**-.016**		μ
	0 0	0 0 0 1 0 0 0	.001		τ			μ
	0 0	0 1 0 0 0 0 0	.001		τ			μ

TABLE III.
Continued

TRIG	FACTOR		COEF			COEF		
$l\ l'\ F\ D$	$E_1 E_2$	$E_3 E_4 E_5 E_6 E_7 E_8 E_9$	sin	cos		sin	cos	
	0 1	0 0 0 0 0 0 0	.003		τ			μ
	1 0	0 0 0 0 0 0 0	-.001		τ			μ
1 0 3 0	0 0	0 0 1 0 0 0 0		.009	τ		.009	μ
	0 0	1 0 0 0 0 0 0		.003	τ		.003	μ
1 1-2 0	0 0	0 0 0 0 0 0 0	.001		τ	-.001		μ
1 1-2 1	0 0	0 0 0 0 0 0 0	.008		τ	.008		μ
1 1-1 0	0 0	0 0 0 0 0 0 0	-.002	-.007	τ	-.002	-.007	μ
	0 0	0 0 1 0 0 0 0		-.010	τ		-.010	μ
1 1 0-4	0 0	0 0 0 0 0 0 0	.002		τ	.002		μ
1 1 0-2	0 0	0 0 0 0 0 0 0	.231		τ	.231		μ
	0 0	0 0 0 1 0 0 0	.002		τ	.002		μ
	0 1	0 0 0 0 0 0 0	.003		τ	.003		μ
	1 0	0 0 0 0 0 0 0	-.001		τ	-.001		μ
1 1 0-1	0 0	0 0 0 0 0 0 0	-.002		τ	-.002		μ
1 1 0 0	0 0	0 0 0 0 0 0 0	.102		τ	.103		μ
	0 1	0 0 0 0 0 0 0	.001		τ	.001		μ
1 1 0 1	0 0	0 0 0 0 0 0 0	-.001		τ	-.001		μ
1 1 0 2	0 0	0 0 0 0 0 0 0	.001		τ	.001		μ
1 1 1-2	0 0	0 0 0 0 0 0 0		.001	τ		.001	μ
	0 0	0 0 1 0 0 0 0		.001	τ		.001	μ
1 1 2-2	0 0	0 0 0 0 0 0 0	.001		τ	.002		μ
1 2 0-2	0 0	0 0 0 0 0 0 0	.010		τ	.010		μ
1 2 0 0	0 0	0 0 0 0 0 0 0	.001		τ	.001		μ
2-2 0-2	0 0	0 0 0 0 0 0 0	.405		τ	.405		μ
2-1-2 0	0 0	0 0 0 0 0 0 0	-.005		τ	-.005		μ
2-1 0-2	0 0	0 0 0 0 0 0 0	.950	.001	τ	.950	.001	μ
	0 0	0 0 0 1 0 0 0	.009		τ	.009		μ
	0 1	0 0 0 0 0 0 0	.012		τ	.012		μ
2-1 0 0	0 0	0 0 0 0 0 0 0	-.008		τ	-.008		μ
2 0-4 0	0 0	0 0 0 0 0 0 0	.001		τ	-.003		μ
2 0-2-2	0 0	0 0 0 0 0 0 0	-.001		τ	-.004		μ
2 0-2 0	0 0	0 0 0 0 0 0 0	16.858	.530	τ	16.870	.530	μ
	0 0	0 0 0 0 0 0 1		1.049	τ		1.049	μ
	0 0	0 0 0 1 0 0 0	-1.017		τ	-1.017		μ
	0 1	0 0 0 0 0 0 0	-3.047		τ	-3.047		μ
2 0 0-4	0 0	0 0 0 0 0 0 0	.022		τ	.022		μ
2 0 0-3	0 0	0 0 0 0 0 0 0	-.001		τ	-.001		μ
2 0 0-2	0 0	0 0 0 0 0 0 0	9.940	.022	τ	9.940	.022	μ
	0 0	0 0 0 0 0 0 1		.057	τ		.057	μ
	0 0	0 0 0 0 1 0 0		-.011	τ		-.011	μ
	0 0	0 0 0 1 0 0 0	.075		τ	.075		μ
	0 0	0 1 0 0 0 0 0	-.007		τ	-.007		μ
	0 1	0 0 0 0 0 0 0	.136		τ	.136		μ
	1 0	0 0 0 0 0 0 0	-.044		τ	-.044		μ
2 0 0-1	0 0	0 0 0 0 0 0 0	-.003		τ	-.003		μ
2 0 0 0	0 0	0 0 0 0 0 0 0	-.445	-.001	τ	-.451	-.001	μ
	0 0	0 0 0 0 0 0 1		.001	τ		.001	μ
	0 0	0 0 0 1 0 0 0	-.004		τ	-.004		μ
	0 1	0 0 0 0 0 0 0	-.006		τ	-.006		μ
	1 0	0 0 0 0 0 0 0	.002		τ	.002		μ
2 0 0 2	0 0	0 0 0 0 0 0 0	-.006		τ	-.006		μ
2 0 2-2	0 0	0 0 0 0 0 0 0	.001		τ	-.001		μ
2 0 2 0	0 0	0 0 0 0 0 0 0			τ	-.001		μ
2 1-2 0	0 0	0 0 0 0 0 0 0	.018		τ	.018		μ

TABLE III.
Continued

TRIG	FACTOR		COEF			COEF		
$l\ l'\ F\ D$	E_1E_2	$E_3E_4E_5E_6E_7E_8E_9$	sin	cos		sin	cos	
2 1 0-4	0 0	0 0 0 0 0 0 0	.002		τ	.002		μ
2 1 0-2	0 0	0 0 0 0 0 0 0	.154		τ	.154		μ
	0 0	0 0 0 1 0 0 0	.001		τ	.001		μ
	0 1	0 0 0 0 0 0 0	.002		τ	.002		μ
2 1 0 0	0 0	0 0 0 0 0 0 0	.006		τ	.006		μ
2 2 0-2	0 0	0 0 0 0 0 0 0	.003		τ	.003		μ
3-1 0-2	0 0	0 0 0 0 0 0 0	.001		τ	.001		μ
3-1 0 0	0 0	0 0 0 0 0 0 0	-.001		τ	-.001		μ
3 0-2 0	0 0	0 0 0 0 0 0 0	.000		τ	-.001		μ
3 0 0-4	0 0	0 0 0 0 0 0 0	-.001		τ	-.001		μ
3 0 0-2	0 0	0 0 0 0 0 0 0	.034		τ	.034		μ
3 0 0 0	0 0	0 0 0 0 0 0 0	-.022		τ	-.022		μ
3 1 0-2	0 0	0 0 0 0 0 0 0	.001		τ	.001		μ
4 0-2-2	0 0	0 0 0 0 0 0 0	-.002		τ	-.002		μ
4 0 0-4	0 0	0 0 0 0 0 0 0	-.004		τ	-.004		μ
4 0 0-2	0 0	0 0 0 0 0 0 0	.002		τ	.002		μ
4 0 0 0	0 0	0 0 0 0 0 0 0	-.001		τ	-.001		μ
0 0 0 0	0 0	0 0 0 0 0 0 0		-80.803	P_1		80.818	ν
	0 0	0 0 1 0 0 0 0		-69.220	P_1		69.233	ν
	0 0	0 1 0 0 0 0 0		-.029	P_1		.029	ν
	0 0	1 0 0 0 0 0 0		-27.698	P_1		27.703	ν
	0 1	0 0 0 0 0 0 0		.105	P_1		-.105	ν
	1 0	0 0 0 0 0 0 0		-.006	P_1		.006	ν
	0 1	0 0 1 0 0 0 0		.020	P_1		-.020	ν
	0 1	1 0 0 0 0 0 0		.032	P_1		-.032	ν
0 0 0 1	0 0	0 0 0 0 0 0 0		.001	P_1		-.001	ν
0 0 0 2	0 0	0 0 0 0 0 0 0	.001	.002	P_1	-.001	-.002	ν
	0 0	0 0 1 0 0 0 0		.002	P_1		-.002	ν
0 0 1-4	0 0	0 0 0 0 0 0 0	-.003		P_1	.003		ν
0 0 1-3	0 0	0 0 0 0 0 0 0	.001		P_1	-.001		ν
0 0 1-2	0 0	0 0 0 0 0 0 0	2.910		P_1	-2.911		ν
	0 0	0 1 0 0 0 0 0	-.003		P_1	.003		ν
	1 0	0 0 0 0 0 0 0	-.003		P_1	.003		ν
0 0 1-1	0 0	0 0 0 0 0 0 0	.116		P_1	-.116		ν
0 0 1 0	0 0	0 0 0 0 0 0 0	5561.491	5.754	P_1	-5561.997	-5.754	ν
	0 0	0 0 0 0 0 0 1	.014	8.270	P_1	-.014	-8.270	ν
	0 0	0 0 0 0 1 0 0	.010	1.378	P_1	-.010	-1.378	ν
	0 0	0 0 0 1 0 0 0	5.880	.014	P_1	-5.880	-.014	ν
	0 0	0 0 1 0 0 0 0	.020	.001	P_1	-.020	-.001	ν
	0 0	0 1 0 0 0 0 0	-21.509	-.001	P_1	21.515	.001	ν
	0 0	1 0 0 0 0 0 0	.011		P_1	-.011		ν
	0 1	0 0 0 0 0 0 0	.034	.028	P_1	-.034	-.028	ν
	1 0	0 0 0 0 0 0 0	-11.417	-.006	P_1	11.419	.006	ν
	0 0	0 1 0 0 0 0 1		-.010	P_1		.010	ν
	0 0	0 1 0 1 0 0 0	-.027		P_1	.027		ν
	0 0	0 2 0 0 0 0 0	.065		P_1	-.065		ν
	0 0	2 0 0 0 0 0 0	-.022		P_1	.022		ν
	1 0	0 1 0 0 0 0 0	.069		P_1	-.069		ν
	2 0	0 0 0 0 0 0 0	.014		P_1	-.014		ν
0 0 1 1	0 0	0 0 0 0 0 0 0	-.009		P_1	.009		ν
0 0 1 2	0 0	0 0 0 0 0 0 0	.119		P_1	-.119		ν
0 0 1 4	0 0	0 0 0 0 0 0 0	.001		P_1	-.001		ν
0 0 2 0	0 0	0 0 0 0 0 0 0		.001	P_1		-.016	ν
	0 0	0 0 1 0 0 0 0			P_1		-.013	ν

TABLE III.
Continued

TRIG	FACTOR		COEF			COEF		
l l' F D	$E_1 E_2$	$E_3 E_4 E_5 E_6 E_7 E_8 E_9$	sin	cos		sin	cos	
	0 0	1 0 0 0 0 0 0			P_1		-.006	ν
0 0 3-2	0 0	0 0 0 0 0 0 0	-.018		P_1	.019		ν
0 0 3 0	0 0	0 0 0 0 0 0 0	.013		P_1	.156	.001	ν
	0 0	0 1 0 0 0 0 0			P_1	-.002		ν
0 1-1-2	0 0	0 0 0 0 0 0 0	-.008		P_1	.008		ν
0 1-1 0	0 0	0 0 0 0 0 0 0	1.021	-.002	P_1	-1.021	.002	ν
	0 0	0 0 0 0 0 0 1		-.002	P_1		.002	ν
	0 0	0 0 0 1 0 0 0	.011		P_1	-.011		ν
	0 0	0 1 0 0 0 0 0	-.006		P_1	.006		ν
	0 1	0 0 0 0 0 0 0	.019		P_1	-.019		ν
	1 0	0 0 0 0 0 0 0	-.009		P_1	.009		ν
0 1-1 1	0 0	0 0 0 0 0 0 0	.308		P_1	-.308		ν
0 1-1 2	0 0	0 0 0 0 0 0 0	.113		P_1	-.113		ν
0 1 0 0	0 0	0 0 0 0 0 0 0		-.003	P_1		.003	ν
	0 0	0 0 1 0 0 0 0		-.002	P_1		.002	ν
0 1 0 1	0 0	0 0 0 0 0 0 0	-.004	-.007	P_1	.004	.007	ν
	0 0	0 0 1 0 0 0 0		.003	P_1		-.003	ν
	0 0	1 0 0 0 0 0 0		.001	P_1		-.001	ν
0 1 1-2	0 0	0 0 0 0 0 0 0	.081		P_1	-.081		ν
0 1 1 0	0 0	0 0 0 0 0 0 0	1.244	.001	P_1	-1.244	-.001	ν
	0 0	0 0 0 0 0 0 1		.002	P_1		-.002	ν
	0 0	0 0 0 1 0 0 0	.010		P_1	-.010		ν
	0 0	0 1 0 0 0 0 0	-.004		P_1	.004		ν
	0 1	0 0 0 0 0 0 0	.017		P_1	-.017		ν
	1 0	0 0 0 0 0 0 0	-.008		P_1	.008		ν
0 1 1 1	0 0	0 0 0 0 0 0 0	.002		P_1	-.002		ν
0 1 1 2	0 0	0 0 0 0 0 0 0	-.002		P_1	.002		ν
0 1 3-2	0 0	0 0 0 0 0 0 0	-.001		P_1	.001		ν
0 2-3 2	0 0	0 0 0 0 0 0 0	-.001		P_1	.001		ν
0 2-1 0	0 0	0 0 0 0 0 0 0	.003		P_1	-.003		ν
0 2-1 2	0 0	0 0 0 0 0 0 0	.019		P_1	-.019		ν
0 2 1-2	0 0	0 0 0 0 0 0 0	.001		P_1	-.001		ν
0 2 1 0	0 0	0 0 0 0 0 0 0	.002		P_1	-.002		ν
1-2 1-2	0 0	0 0 0 0 0 0 0	-.048		P_1	.048		ν
1-1-1-2	0 0	0 0 0 0 0 0 0	.003		P_1	-.003		ν
1-1-1-1	0 0	0 0 0 0 0 0 0	.004		P_1	-.004		ν
1-1-1 0	0 0	0 0 0 0 0 0 0	.128		P_1	-.128		ν
1-1-1 2	0 0	0 0 0 0 0 0 0	.002		P_1	-.002		ν
1-1 0 0	0 0	0 0 0 0 0 0 0	-.001	-.002	P_1	.001	.002	ν
	0 0	0 0 1 0 0 0 0		-.002	P_1		.002	ν
1-1 1-2	0 0	0 0 0 0 0 0 0	.038		P_1	-.038		ν
1-1 1-1	0 0	0 0 0 0 0 0 0	.001		P_1	-.001		ν
1-1 1 0	0 0	0 0 0 0 0 0 0	.011		P_1	-.011		ν
1-1 1 2	0 0	0 0 0 0 0 0 0	.001		P_1	-.001		ν
1 0-3 0	0 0	0 0 0 0 0 0 0	.007		P_1	.004		ν
1 0-3 2	0 0	0 0 0 0 0 0 0	-.007		P_1	.007		ν
1 0-2 0	0 0	0 0 0 0 0 0 0		-.053	P_1		.053	ν
	0 0	0 0 0 0 0 1 0	.026		P_1	-.026		ν
	0 0	0 0 1 0 0 0 0		.031	P_1		-.031	ν
	0 0	1 0 0 0 0 0 0		.009	P_1		-.009	ν
1 0-1-4	0 0	0 0 0 0 0 0 0	-.003		P_1	.003		ν
1 0-1-2	0 0	0 0 0 0 0 0 0	-.350		P_1	.349		ν
	0 0	0 1 0 0 0 0 0	.002		P_1	-.002		ν
1 0-1-1	0 0	0 0 0 0 0 0 0	-.050		P_1	.050		ν

TABLE III.
Continued

TRIG	FACTOR		COEF			COEF		
l l' F D	$E_1 E_2$	$E_3 E_4 E_5 E_6 E_7 E_8 E_9$	sin	cos		sin	cos	
1 0-1 0	**0 0**	**0 0 0 0 0 0 0**	**124.477**	**-.123**	P_1	**-124.499**	**.123**	ν
	0 0	0 0 0 0 0 0 1		-.172	P_1		.172	ν
	0 0	0 0 0 0 1 0 0		-.032	P_1		.032	ν
	0 0	0 0 0 1 0 0 0	-.301		P_1	.301		ν
	0 0	0 1 0 0 0 0 0	-.640		P_1	.640		ν
	0 1	0 0 0 0 0 0 0	-.933		P_1	.933		ν
	1 0	0 0 0 0 0 0 0	-.054		P_1	.054		ν
1 0-1 1	**0 0**	**0 0 0 0 0 0 0**	**-.004**		P_1	**.004**		ν
1 0-1 2	**0 0**	**0 0 0 0 0 0 0**	**.036**		P_1	**-.036**		ν
1 0 0-2	**0 0**	**0 0 0 0 0 0 0**	**.010**	**-.008**	P_1	**-.010**	**.008**	ν
	0 0	0 0 0 0 0 1 0	.011		P_1	-.011		ν
	0 0	0 0 1 0 0 0 0		-.006	P_1		.006	ν
	0 0	1 0 0 0 0 0 0		-.007	P_1		.007	ν
1 0 0 0	**0 0**	**0 0 0 0 0 0 0**	**-.704**	**-.822**	P_1	**.704**	**.822**	ν
	0 0	0 0 0 0 0 0 1	-.008	.002	P_1	.008	-.002	ν
	0 0	0 0 0 0 0 1 0	-.778	.002	P_1	.778	-.002	ν
	0 0	0 0 0 0 1 0 0	-.001		P_1	.001		ν
	0 0	0 0 1 0 0 0 0	-.004	-.573	P_1	.004	.573	ν
	0 0	0 1 0 0 0 0 0		-.005	P_1		.005	ν
	0 0	1 0 0 0 0 0 0		-.246	P_1		.246	ν
	0 1	0 0 0 0 0 0 0		-.005	P_1		.005	ν
	1 0	0 0 0 0 0 0 0		.001	P_1		-.001	ν
1 0 1-4	**0 0**	**0 0 0 0 0 0 0**	**-.008**		P_1	**.008**		ν
1 0 1-2	**0 0**	**0 0 0 0 0 0 0**	**-2.678**		P_1	**2.678**		ν
	0 0	0 0 0 0 0 0 1		-.003	P_1		.003	ν
	0 0	0 0 0 1 0 0 0	.006		P_1	-.006		ν
	0 0	0 1 0 0 0 0 0	.014		P_1	-.014		ν
	0 1	0 0 0 0 0 0 0	.020		P_1	-.020		ν
1 0 1-1	**0 0**	**0 0 0 0 0 0 0**	**-.052**		P_1	**.052**		ν
1 0 1 0	**0 0**	**0 0 0 0 0 0 0**	**1.575**	**.001**	P_1	**-1.564**	**-.001**	ν
	0 0	0 1 0 0 0 0 0	-.007		P_1	.007		ν
	1 0	0 0 0 0 0 0 0	-.003		P_1	.003		ν
1 0 1 1	**0 0**	**0 0 0 0 0 0 0**	**-.001**		P_1	**.001**		ν
1 0 1 2	**0 0**	**0 0 0 0 0 0 0**	**.013**		P_1	**-.013**		ν
1 0 3-2	**0 0**	**0 0 0 0 0 0 0**	**-.001**		P_1	**.001**		ν
1 0 3 0	**0 0**	**0 0 0 0 0 0 0**	**.001**		P_1	**-.001**		ν
1 1-1-2	**0 0**	**0 0 0 0 0 0 0**	**-.017**		P_1	**.017**		ν
1 1-1 0	**0 0**	**0 0 0 0 0 0 0**	**.116**		P_1	**-.116**		ν
1 1-1 1	**0 0**	**0 0 0 0 0 0 0**	**.008**		P_1	**-.008**		ν
1 1-1 2	**0 0**	**0 0 0 0 0 0 0**	**-.002**		P_1	**.002**		ν
1 1 0-2	**0 0**	**0 0 0 0 0 0 0**		**-.001**	P_1		**.001**	ν
1 1 0 0	**0 0**	**0 0 0 0 0 0 0**	**-.001**	**-.002**	P_1	**.001**	**.002**	ν
	0 0	0 0 0 0 0 1 0	-.001		P_1	.001		ν
	0 0	0 0 1 0 0 0 0		-.002	P_1		.002	ν
1 1 1-4	**0 0**	**0 0 0 0 0 0 0**	**-.001**		P_1	**.001**		ν
1 1 1-2	**0 0**	**0 0 0 0 0 0 0**	**-.083**		P_1	**.083**		ν
1 1 1 0	**0 0**	**0 0 0 0 0 0 0**	**-.008**		P_1	**.008**		ν
1 2-1-2	**0 0**	**0 0 0 0 0 0 0**	**-.001**		P_1	**.001**		ν
2-1-1-2	**0 0**	**0 0 0 0 0 0 0**	**.016**		P_1	**-.016**		ν
2-1-1 0	**0 0**	**0 0 0 0 0 0 0**	**.009**		P_1	**-.009**		ν
2-1 1-2	**0 0**	**0 0 0 0 0 0 0**	**.009**		P_1	**-.009**		ν
2-1 1 0	**0 0**	**0 0 0 0 0 0 0**	**.001**		P_1	**-.001**		ν
2 0-3 0	**0 0**	**0 0 0 0 0 0 0**	**.200**		P_1	**-.200**		ν
2 0-1-4	**0 0**	**0 0 0 0 0 0 0**	**-.003**		P_1	**.003**		ν

TABLE III.
Continued

TRIG $l\ l'\ F\ D$	FACTOR $E_1 E_2$	$E_3 E_4 E_5 E_6 E_7 E_8 E_9$	COEF sin	cos		COEF sin	cos	
2 0-1-2	0 0	0 0 0 0 0 0 0	.231		P_1	-.231		ν
	0 0	0 0 0 1 0 0 0	.001		P_1	-.001		ν
	0 1	0 0 0 0 0 0 0	.002		P_1	-.002		ν
2 0-1 0	0 0	0 0 0 0 0 0 0	.338	.005	P_1	-.338	-.005	ν
	0 0	0 0 0 0 0 0 1		.001	P_1		-.001	ν
	0 0	0 0 0 0 1 0 0		.003	P_1		-.003	ν
	0 0	0 0 0 1 0 0 0	.010		P_1	-.010		ν
	0 0	0 1 0 0 0 0 0	-.019		P_1	.019		ν
	0 1	0 0 0 0 0 0 0	.002		P_1	-.002		ν
	1 0	0 0 0 0 0 0 0	-.004		P_1	.004		ν
2 0-1 2	0 0	0 0 0 0 0 0 0	.002		P_1	-.002		ν
2 0 0-2	0 0	0 0 0 0 0 0 0	-.002	.001	P_1	.002	-.001	ν
	0 0	0 0 0 0 0 1 0	-.002		P_1	.002		ν
2 0 0 0	0 0	0 0 0 0 0 0 0	.001	.001	P_1	-.001	-.001	ν
	0 0	0 0 1 0 0 0 0		.001	P_1		-.001	ν
2 0 1-4	0 0	0 0 0 0 0 0 0	.002		P_1	-.002		ν
2 0 1-2	0 0	0 0 0 0 0 0 0	.152		P_1	-.152		ν
	0 0	0 0 0 0 0 0 1		-.001	P_1		.001	ν
	0 1	0 0 0 0 0 0 0	.002		P_1	-.002		ν
2 0 1 0	0 0	0 0 0 0 0 0 0	.073		P_1	-.073		ν
2 0 1 2	0 0	0 0 0 0 0 0 0	.001		P_1	-.001		ν
2 1-1-2	0 0	0 0 0 0 0 0 0	.002		P_1	-.002		ν
2 1-1 0	0 0	0 0 0 0 0 0 0	.011		P_1	-.011		ν
2 1 1-4	0 0	0 0 0 0 0 0 0	-.001		P_1	.001		ν
2 1 1-2	0 0	0 0 0 0 0 0 0	-.002		P_1	.002		ν
2 1 1 0	0 0	0 0 0 0 0 0 0	-.001		P_1	.001		ν
3 0-1-2	0 0	0 0 0 0 0 0 0	-.004		P_1	.004		ν
3 0-1 0	0 0	0 0 0 0 0 0 0	-.001		P_1	.001		ν
3 0 1-4	0 0	0 0 0 0 0 0 0	-.001		P_1	.001		ν
3 0 1-2	0 0	0 0 0 0 0 0 0	-.002		P_1	.002		ν
3 0 1 0	0 0	0 0 0 0 0 0 0	.004		P_1	-.004		ν
0 0 0 0	0 0	0 0 0 0 0 0 0		.373	P_2		-.373	π
	0 0	0 0 0 0 0 0 1		.417	P_2		-.417	π
	0 0	0 0 0 0 0 1 0		.014	P_2		-.014	π
	0 0	0 0 0 0 1 0 0		.115	P_2		-.115	π
	0 0	0 0 1 0 0 0 0		-.074	P_2		.074	π
	0 0	1 0 0 0 0 0 0		.123	P_2		-.123	π
	0 0	0 0 1 0 0 0 1		.623	P_2		-.623	π
	0 1	0 0 0 0 0 0 1		-.165	P_2		.165	π
	0 1	0 0 1 0 0 0 0		-.137	P_2		.137	π
0 0 1-4	0 0	0 0 0 0 0 0 0		.001	P_2		-.001	π
0 0 1-2	0 0	0 0 0 0 0 0 0	.001	-3.198	P_2	-.001	3.200	π
	0 0	0 0 0 0 0 0 1	-.003		P_2	.003		π
	0 0	0 1 0 0 0 0 0		.007	P_2		-.007	π
	0 1	0 0 0 0 0 0 0		.004	P_2		-.004	π
	1 0	0 0 0 0 0 0 0		.003	P_2		-.003	π
0 0 1-1	0 0	0 0 0 0 0 0 0	.002	.097	P_2	-.002	-.097	π
	0 0	0 1 0 0 0 0 0		.005	P_2		-.005	π
	0 1	0 0 0 0 0 0 0		.011	P_2		-.011	π
0 0 1 0	0 0	0 0 0 0 0 0 0	-5.777	5539.366	P_2	5.778	-5540.370	π
	0 0	0 0 0 0 0 0 1	-8.304	.017	P_2	8.304	-.017	π
	0 0	0 0 0 0 1 0 0	-1.384	.010	P_2	1.384	-.010	π
	0 0	0 0 0 1 0 0 0	.027	5.793	P_2	-.027	-5.795	π
	0 0	0 0 1 0 0 0 0		.022	P_2		-.022	π

TABLE III.
Continued

TRIG	FACTOR		COEF			COEF		
$l\ l'\ F\ D$	$E_1 E_2$	$E_3 E_4 E_5 E_6 E_7 E_8 E_9$	sin	cos		sin	cos	
	0 0	0 1 0 0 0 0 0	.021	-21.542	P_2	-.021	21.554	π
	0 0	1 0 0 0 0 0 0		.011	P_2		-.011	π
	0 1	0 0 0 0 0 0 0	.070	-.150	P_2	-.070	.150	π
	1 0	0 0 0 0 0 0 0	-.010	-11.373	P_2	.010	11.378	π
	0 0	0 0 0 1 0 0 1	.033		P_2	-.033		π
	0 0	0 1 0 0 0 0 1	.010		P_2	-.010		π
	0 0	0 1 0 1 0 0 0		-.027	P_2		.027	π
	0 0	0 2 0 0 0 0 0		.065	P_2		-.065	π
	0 0	2 0 0 0 0 0 0		-.022	P_2		.022	π
	0 1	0 0 0 0 0 0 1	.099		P_2	-.099		π
	1 0	0 1 0 0 0 0 0		.069	P_2		-.069	π
	2 0	0 0 0 0 0 0 0		.014	P_2		-.014	π
0 0 1 1	**0 0**	**0 0 0 0 0 0 0**		**-.002**	P_2		**.002**	π
0 0 1 2	**0 0**	**0 0 0 0 0 0 0**		**.008**	P_2		**-.008**	π
0 0 2-2	**0 0**	**0 0 0 0 0 0 0**	**-.001**	**-.002**	P_2	**.001**	**.002**	π
	0 0	0 0 0 0 0 1 0		-.002	P_2		.002	π
	0 0	1 0 0 0 0 0 0	-.001		P_2	.001		π
0 0 2 0	**0 0**	**0 0 0 0 0 0 0**	**.000**		P_2	**.030**		π
	0 0	0 0 1 0 0 0 0			P_2	.025		π
	0 0	1 0 0 0 0 0 0			P_2	.010		π
0 0 3-2	**0 0**	**0 0 0 0 0 0 0**		**-.006**	P_2		**.007**	π
0 0 3 0	**0 0**	**0 0 0 0 0 0 0**		**.011**	P_2	**-.001**	**.326**	π
	0 0	0 0 0 0 0 0 1			P_2	-.002		π
	0 0	0 0 0 1 0 0 0			P_2		.002	π
	0 0	0 1 0 0 0 0 0			P_2		-.004	π
	1 0	0 0 0 0 0 0 0			P_2		-.003	π
0 1-1-2	**0 0**	**0 0 0 0 0 0 0**		**.001**	P_2		**-.001**	π
0 1-1 0	**0 0**	**0 0 0 0 0 0 0**		**-1.059**	P_2		**1.059**	π
	0 0	0 0 0 1 0 0 0		-.012	P_2		.012	π
	0 0	0 1 0 0 0 0 0		.004	P_2		-.004	π
	0 1	0 0 0 0 0 0 0		-.021	P_2		.021	π
	1 0	0 0 0 0 0 0 0		.009	P_2		-.009	π
0 1-1 1	**0 0**	**0 0 0 0 0 0 0**		**-.029**	P_2		**.029**	π
	0 0	0 0 1 0 0 0 0		-.003	P_2		.003	π
	0 0	1 0 0 0 0 0 0		-.007	P_2		.007	π
0 1-1 2	**0 0**	**0 0 0 0 0 0 0**		**.128**	P_2		**-.128**	π
0 1 0 0	**0 0**	**0 0 0 0 0 0 0**	**-.034**	**.001**	P_2	**.034**	**-.001**	π
	0 0	0 0 1 0 0 0 0	-.029		P_2	.029		π
	0 0	1 0 0 0 0 0 0	-.015		P_2	.015		π
0 1 0 1	**0 0**	**0 0 0 0 0 0 0**		**-.004**	P_2		**.004**	π
	0 0	0 0 1 0 0 0 0	-.003		P_2	.003		π
0 1 1-2	**0 0**	**0 0 0 0 0 0 0**		**-.086**	P_2		**.086**	π
0 1 1 0	**0 0**	**0 0 0 0 0 0 0**		**1.284**	P_2		**-1.284**	π
	0 0	0 0 0 1 0 0 0		.010	P_2		-.010	π
	0 0	0 1 0 0 0 0 0		-.005	P_2		.005	π
	0 1	0 0 0 0 0 0 0		.016	P_2		-.016	π
	1 0	0 0 0 0 0 0 0		-.008	P_2		.008	π
0 2-1 2	**0 0**	**0 0 0 0 0 0 0**		**.022**	P_2		**-.022**	π
0 2 1-2	**0 0**	**0 0 0 0 0 0 0**		**.001**	P_2		**-.001**	π
0 2 1 0	**0 0**	**0 0 0 0 0 0 0**		**.004**	P_2		**-.004**	π
1-2-1 0	**0 0**	**0 0 0 0 0 0 0**		**.007**	P_2		**-.007**	π
1-2 1-2	**0 0**	**0 0 0 0 0 0 0**		**-.066**	P_2		**.066**	π
1-1-3 0	**0 0**	**0 0 0 0 0 0 0**		**-.001**	P_2		**.001**	π
1-1-1 0	**0 0**	**0 0 0 0 0 0 0**	**.001**	**-.136**	P_2	**-.001**	**.136**	π

TABLE III.
Continued

TRIG	FACTOR		COEF			COEF		
l l' F D	E_1E_2	$E_3E_4E_5E_6E_7E_8E_9$	sin	cos		sin	cos	
	0 0	0 0 0 1 0 0 0		-.003	P_2		.003	π
	0 0	0 1 0 0 0 0 0		-.009	P_2		.009	π
	0 1	0 0 0 0 0 0 0		-.012	P_2		.012	π
1-1-1 2	**0 0**	**0 0 0 0 0 0 0**		**.001**	P_2		**-.001**	π
1-1 1-2	**0 0**	**0 0 0 0 0 0 0**		**.029**	P_2		**-.029**	π
1-1 1-1	**0 0**	**0 0 0 0 0 0 0**		**-.012**	P_2		**.012**	π
1 0-3 0	**0 0**	**0 0 0 0 0 0 0**		**-.002**	P_2		**-.021**	π
1 0-3 2	**0 0**	**0 0 0 0 0 0 0**		**-.038**	P_2		**.038**	π
1 0-2 0	**0 0**	**0 0 0 0 0 0 0**	**-.079**		P_2	**.080**		π
	0 0	0 0 0 0 0 1 0		-.024	P_2		.024	π
	0 0	0 0 1 0 0 0 0	.003		P_2	-.003		π
	0 0	1 0 0 0 0 0 0	.010		P_2	-.010		π
1 0-1-2	**0 0**	**0 0 0 0 0 0 0**		**.078**	P_2		**-.078**	π
1 0-1-1	**0 0**	**0 0 0 0 0 0 0**		**.047**	P_2		**-.047**	π
1 0-1 0	**0 0**	**0 0 0 0 0 0 0**	**.092**	**-75.449**	P_2	**-.092**	**75.476**	π
	0 0	0 0 0 0 0 0 1	-.059		P_2	.059		π
	0 0	0 0 0 0 1 0 0	.138		P_2	-.138		π
	0 0	0 0 0 1 0 0 0		-.226	P_2		.226	π
	0 0	0 0 1 0 0 0 0		-.004	P_2		.004	π
	0 0	0 1 0 0 0 0 0		.504	P_2		-.504	π
	0 0	1 0 0 0 0 0 0	-.001	-.003	P_2	.001	.003	π
	0 1	0 0 0 0 0 0 0	.004	-.184	P_2	-.004	.184	π
	1 0	0 0 0 0 0 0 0		.152	P_2		-.152	π
1 0-1 1	**0 0**	**0 0 0 0 0 0 0**		**.002**	P_2		**-.002**	π
1 0-1 2	**0 0**	**0 0 0 0 0 0 0**		**.019**	P_2		**-.019**	π
1 0 0-2	**0 0**	**0 0 0 0 0 0 0**	**-.011**	**-.012**	P_2	**.011**	**.012**	π
	0 0	0 0 0 0 0 1 0		-.013	P_2		.013	π
	0 0	0 0 1 0 0 0 0	-.009		P_2	.009		π
	0 0	1 0 0 0 0 0 0	-.004		P_2	.004		π
1 0 0-1	**0 0**	**0 0 0 0 0 0 0**	**.002**	**-.002**	P_2	**-.002**	**.002**	π
	0 0	0 0 1 0 0 0 0	-.001		P_2	.001		π
1 0 0 0	**0 0**	**0 0 0 0 0 0 0**	**.834**	**-.703**	P_2	**-.834**	**.703**	π
	0 0	0 0 0 0 0 0 1		-.005	P_2		.005	π
	0 0	0 0 0 0 0 1 0		-.779	P_2		.779	π
	0 0	0 0 1 0 0 0 0	.583		P_2	-.583		π
	0 0	0 1 0 0 0 0 0	.003		P_2	-.003		π
	0 0	1 0 0 0 0 0 0	.250		P_2	-.250		π
1 0 1-4	**0 0**	**0 0 0 0 0 0 0**		**.005**	P_2		**-.005**	π
1 0 1-3	**0 0**	**0 0 0 0 0 0 0**		**-.001**	P_2		**.001**	π
1 0 1-2	**0 0**	**0 0 0 0 0 0 0**	**-.014**	**-1.612**	P_2	**.014**	**1.613**	π
	0 0	0 0 0 0 0 0 1	-.018		P_2	.018		π
	0 0	0 0 0 0 1 0 0	-.004		P_2	.004		π
	0 0	0 0 0 1 0 0 0		-.043	P_2		.043	π
	0 0	0 1 0 0 0 0 0		-.086	P_2		.086	π
	0 1	0 0 0 0 0 0 0		.003	P_2		-.003	π
	1 0	0 0 0 0 0 0 0		-.005	P_2		.005	π
1 0 1-1	**0 0**	**0 0 0 0 0 0 0**		**-.054**	P_2		**.054**	π
1 0 1 0	**0 0**	**0 0 0 0 0 0 0**	**-.002**	**.299**	P_2	**.002**	**-.276**	π
	0 0	0 0 0 0 0 0 1	-.002		P_2	.002		π
	0 1	0 0 0 0 0 0 0		.002	P_2		-.002	π
1 0 1 2	**0 0**	**0 0 0 0 0 0 0**		**-.002**	P_2		**.002**	π
1 0 3-4	**0 0**	**0 0 0 0 0 0 0**		**-.001**	P_2		**.001**	π
1 0 3-2	**0 0**	**0 0 0 0 0 0 0**			P_2		**-.001**	π
1 0 3 0	**0 0**	**0 0 0 0 0 0 0**		**.001**	P_2		**-.001**	π

TABLE III.
Continued

TRIG	FACTOR		COEF			COEF		
l l' F D	$E_1 E_2$	$E_3 E_4 E_5 E_6 E_7 E_8 E_9$	sin	cos		sin	cos	
1 1-3 2	0 0	0 0 0 0 0 0 0		.003	P_2		-.003	π
1 1-1-2	0 0	0 0 0 0 0 0 0		.004	P_2		-.004	π
1 1-1 0	0 0	0 0 0 0 0 0 0	-.003	-.161	P_2	.003	.161	π
	0 0	0 0 0 1 0 0 0		.005	P_2		-.005	π
	0 0	0 1 0 0 0 0 0		.014	P_2		-.014	π
	0 1	0 0 0 0 0 0 0		.018	P_2		-.018	π
1 1-1 1	0 0	0 0 0 0 0 0 0		.002	P_2		-.002	π
1 1-1 2	0 0	0 0 0 0 0 0 0		-.001	P_2		.001	π
1 1 1-2	0 0	0 0 0 0 0 0 0		-.053	P_2		.053	π
	0 0	0 1 0 0 0 0 0		-.002	P_2		.002	π
	0 1	0 0 0 0 0 0 0		-.003	P_2		.003	π
1 2-1 0	0 0	0 0 0 0 0 0 0		-.004	P_2		.004	π
1 2 1-2	0 0	0 0 0 0 0 0 0		.006	P_2		-.006	π
2-1-1-2	0 0	0 0 0 0 0 0 0		-.013	P_2		.013	π
2-1-1-1	0 0	0 0 0 0 0 0 0		-.057	P_2		.057	π
2-1-1 0	0 0	0 0 0 0 0 0 0		-.002	P_2		.002	π
2-1 0-2	0 0	0 0 0 0 0 0 0	-.001		P_2	.001		π
2-1 1-2	0 0	0 0 0 0 0 0 0		.011	P_2		-.011	π
2 0-3 0	0 0	0 0 0 0 0 0 0		-.201	P_2		.201	π
	0 1	0 0 0 0 0 0 0		.003	P_2		-.003	π
2 0-2 0	0 0	0 0 0 0 0 0 0	.002	-.001	P_2	-.002	.001	π
	0 0	0 0 1 0 0 0 0	.005		P_2	-.005		π
2 0-1-2	0 0	0 0 0 0 0 0 0		-.212	P_2		.212	π
2 0-1-1	0 0	0 0 0 0 0 0 0		.030	P_2		-.030	π
2 0-1 0	0 0	0 0 0 0 0 0 0	-.006	.441	P_2	.006	-.441	π
	0 0	0 0 0 0 0 0 1	-.007		P_2	.007		π
	0 0	0 0 0 1 0 0 0		.018	P_2		-.018	π
	0 0	0 1 0 0 0 0 0		-.006	P_2		.006	π
	0 1	0 0 0 0 0 0 0		.022	P_2		-.022	π
	1 0	0 0 0 0 0 0 0		-.003	P_2		.003	π
2 0 0-2	0 0	0 0 0 0 0 0 0	-.005	.017	P_2	.005	-.017	π
	0 0	0 0 0 0 0 1 0		.020	P_2		-.020	π
	0 0	0 0 1 0 0 0 0	-.004		P_2	.004		π
	0 0	1 0 0 0 0 0 0	-.001		P_2	.001		π
2 0 1 4	0 0	0 0 0 0 0 0 0		.009	P_2		-.009	π
2 0 1-3	0 0	0 0 0 0 0 0 0		-.003	P_2		.003	π
2 0 1-2	0 0	0 0 0 0 0 0 0		.192	P_2		-.192	π
2 0 1 0	0 0	0 0 0 0 0 0 0		-.009	P_2		.009	π
2 1-1-2	0 0	0 0 0 0 0 0 0		-.004	P_2		.004	π

obtained by Eckhardt I^E or Moons $I^{\hat{i}}$ takes place again, as well as it was in (Petrova, 1993):

$$| I^P - I^E |= 0.''965 \qquad | I^P - I^M |= 0.''966$$

We have used the same set of parameters and of constants (Table II) as Eckhardt and Moons.The comparison of HBE-tables and ELP2000 (Chapront, Chapront-Toze,

TABLE IV

Comparison of obtained results with Moon's and Eckhardt's data: the harmonics whose amplitudes are different by more than 0.″01 are only presented.

TRIG	$COEF * FACTOR$(sin)			$COEF * FACTOR$(cos)			
l l' F D	*Eckhardt*	*Moons*	*Petrova*	*Eckhardt*	*Moons*	*Petrova*	
0 0 0 0				214.170	214.187	214.352	τ
0 0 2-2	1.647	1.646	1.669				τ
1 0-1 0	-1.394	-1.394	-1.397	-6.594	-6.597	-6.620	τ
1 0 0-1	-3.460	-3.463	-3.453				τ
2 0-2 0	17.014	17.020	16.858	0.495	0.494	0530	τ
0 0 0 0				-80.724	-80.644	-80.803	p_1
1 0-2 0	-0.011	-0.011		-0.077	-0.078	-0.053	p_1
0 0 1 0	**5562.462**	**5562.459**	**5561.491**	**5.746**	**5.752**	**5.754**	p_1
1 0-1 0	124.492	124.483	124.477				p_1
2 0-3 0	0.231	0.232	0.200				p_1
2 0-1 0	0.379	0.379	0.338	0.014	0.013	0.005	p_1
0 0 0 0				0.392	0.390	0.373	p_2
0 0 1 0	**-5.769**	**-5.775**	**-5.777**	**5540.334**	**5540.330**	**5539.366**	p_2
0 1-1 1				-0.62	-0.60	-0.029	p_2
1 0-1 0	0.082	0.082	0.092	-75.458	-75.433	-75.449	p_2
1 0 0 0	0.836	0.835	0.833	-0.723	-0.721	-0.703	p_2
2 0-3 0				-0.232	-0.233	-0.201	p_2
2 0-1 0	-0.014	-0.013	-0.006	0.482	0.483	0.441	p_2

1983) does not show some essential deviations. The reality of the revealed effect may be validated by the high-accuracy observations.

As a whole the analysis of the Table IV leads one to believe that within the assigned accuracy 0.″01 our solution may be used in practice.

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