Journal of Risk and Uncertainty, 4:329-338 (1991)

Paying to Improve Your Chances: Gambling or Insurance?

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Abstract

Will a more risk-averse individual spend more or less to improve probabilities, say on marketing efforts that enhance the chance of a sale? For any two payoffs and starting probabilities, the answer is unfortunately indeterminate. However, interpreting gambling as increasing small chances of good outcomes and insurance as reducing small chances of bad outcomes, the more risk-averse individual will pay less (more) to gamble (insure). We find a critical *switching probability* that depends on the individuals and outcomes involved. If the good outcome is less (more) likely than this critical value, the expenditures represent gambling (insurance).

Key words: gambling, insurance, risk, risk aversion, probability shifting, utility theory

Many a dollar is spent to shift probabilities. Airlines put money into maintenance to reduce the chance of a crash. Businesses court clients to enhance the probability of a sale. A homeowner installs a lightning rod to diminish the likelihood that a fire will destroy his home. A would-be model pays a professional photographer to prepare a lavish portfolio that she hopes will impress agencies.

Do such expenditures represent gambling, or are they a form of insurance? We normally think of gambling as paying a small amount to obtain a small probability of a big prize. Insurance also involves a small expenditure and a low-probability outcome, but its purpose is to ameliorate an otherwise adverse outcome. Under such an interpretation, the lightning rod is insurance; the model's portfolio is a gamble. But there is also a distinction to be made from the traditional literature (which is excellently surveyed by Hirshleifer and Riley, 1979). Gambling and insurance in the standard paradigm are defined as transfers of resources across contingencies *when probabilities are fixed*. In our

*Zeckhauser's research was supported in part by the Bradley Foundation. After this work was essentially complete, we encountered working papers by George Sweeney and T. Randolph Beard of Vanderbilt University, titled "Self-Protection in the State-Independent Expected Utility Model," and "Self-Protection, Risk Aversion, and Caution," which address some of the issues in this article. A referee provided helpful comments.

examples, by contrast, the essence of the transaction is to pay a sum to *change the probabilities* for the better.

In the context of insurance, such expenditures have been described as loss prevention or self-protection (see Ehrlich and Becker, 1972). It has long been noted that the purchase of insurance through markets leads to a reduction in loss-prevention efforts, a process labeled moral hazard (see Arrow, 1963; Pauly, 1968). Our concern in this analysis is the link between risk aversion and the nature and magnitude of loss-prevention expenditures.

We shall focus here on situations in which all expenditures and outcomes are measured in dollar equivalents. With dollars (or any single numeraire) as a metric, the theory of risky choice and risk aversion is well established. A rational individual maximizes his expected utility. A utility function u is defined to be more risk averse than v if the certainty equivalent of every lottery is less for u than for v. Equivalently, at every wealth, the lotteries u would accept are a subset of those acceptable to v. (We adopt the shorthand of referring to an individual by his or her utility function and do not distinguish between "more risk averse than" and "at least as risk averse as," etc.)

How does risk aversion affect choices when the decision maker pays money to improve his chances (i.e., to make favorable outcomes more likely)? Is the influence of risk aversion here the same as when probabilities are fixed? One might think an expenditure to improve chances (such as the business's courting of clients) represents a gamble and as such would be more attractive to less risk-averse decision makers. Alternatively, however, one might regard the expensive marketing effort as a form of insurance against the loss of clients, an expenditure attractive to the more risk-averse business. Without further elaboration, such expenditures apparently cannot be categorized as either insurance or gambling.[1]

In this article, we investigate how well our conventional intuition, developed from the fixed-probabilities case, applies to probability-improving outlays. Here, too, it turns out, the more risk-averse decision maker will be more prone to insure, the less risk-averse to gamble, provided that we understand when an expenditure represents a gamble, and when insurance. That is, the more risk-averse individual pays less to secure a small chance of a good outcome, but pays more to avoid a small chance of a bad outcome. We shall demonstrate that this intuition is correct, but defining a "small chance" is crucial. We develop a definition in the form of a critical *switching probability*, which depends on the individuals and outcomes involved. Above this value, odds-improving expenditures reduce the probability of a failure to a low-enough level that they are properly regarded as insurance, and hence more attractive to the more risk averse. Below this critical value, the probability of success is small enough that odds-improving expenditures can be regarded as gambles, and therefore more appealing to less risk-averse individuals. Spending more, it should be noted, worsens the worst possible outcome. In this sense, such spending is risk-taking behavior, and when the more risk-averse individual spends more, he is taking greater risk.

The two extreme cases of the foregoing result say simply that the more risk-averse individual will choose a riskless portfolio if either party does, as the definition of more risk averse requires (see above and theorem 1 below). These cases suffice to show that with u more risk averse than v, it is always possible to construct examples of options to purchase more favorable probabilities at higher cost where u spends more than v, and others where he spends less than v. Riskless options are seldom available or affordable, however. Installing the lightning rod protects against lightning fires, but not against fires caused by poor wiring: some risk remains. Our main concern is what we can say about gambling and insurance behavior when there are residual risks.

1. Formulation and results

For simplicity, we consider situations with but two outcomes, x_1 and x_2. The probability of receiving the greater value x_2 depends on how much one spends, z, according to a function $p(z)$. Thus, the net payoff will be $x_2 - z$ with probability $p(z)$ and $x_1 - z$ otherwise. It is assumed that u is more risk averse than v over an interval including all possible outcomes.

1.1. Who spends more is indeterminate

We show (theorem 2) that, given any outcomes x_1 and x_2, and any nonequivalent utility functions u and v, there exists a probability purchase function $p(z)$ such that u spends less than v, and another function $p^*(z)$ such that u spends more than v. We demonstrate this in cases where the more risk-averse individual chooses a riskless portfolio at one extreme or the other. Obviously, small perturbations away from zero risk could give us the same result without employing riskless portfolios. (Dionne and Eeckhoudt (1985) had previously provided explicit examples of the counterintuitive result that the more risk-averse individual may spend less on risk-reducing activities.)

1.2. Ambiguity of gambling and insurance

Interestingly, for any pair of gross payoffs x_1 and x_2, any cost $z < x_2 - x_1$, and any probability level p_0, $0 < p_0 < 1$, there exist utility functions u and v and a probability purchase function $p(z)$ such that v chooses $p(z) = p_0$, while u is more risk averse and chooses $p(z)$ either larger or smaller than p_0, as desired. (Theorem 4 produces a stronger result, which allows the function $p(z)$ to be given as well.) Evidently, therefore, a particular instance of spending to shift probabilities cannot unambiguously be regarded as gambling or insuring, except in the extreme cases where spending to achieve certainty is insurance and spending that forgoes certainty is gambling.

1.3. Single-switching, critical-probability results

If we relinquish the idea that our exogenous intuition can assess when the probability of success is sufficiently high (low) that expenditures are insurance (gambling), some regularity returns. Basically, the more risk-averse individual will pay more when the likelihood of the good outcome is above an endogenous threshold and less when it is below. We state this proposition more formally as our single-switching result mentioned above. Assume that $p(z)$ is sufficiently well behaved that u's expected utility is a single-peaked function of z. For any two utility functions and any two net outcomes $w_1 = x_1 - z$ and $w_2 = x_2 - z$ with $w_2 > w_1$, there is a critical probability, call it p_s, such that if the optimal purchases for v secure these net outcomes coupled with a probability p_v° of winning w_2 that is greater than p_s, then optimally u spends more than v. Thus $p_v^\circ > p_s$ defines expenditures as insurance, and the more risk-averse person buys more insurance. On the other hand, if $p_v^\circ < p_s$, then such expenditures are gambles and optimally u spends less than v. Even if the functions giving the individuals' expected utilities as a function of expenditure are not single-peaked, u has positive marginal return to z at p_s if $p_v^\circ > p_s$, and u has negative marginal return there if $p_v^\circ < p_s$ (see theorem 3(b)).

Moreover, we can compare two arbitrary levels of expenditure without an assumption of single-peakedness or a restriction to marginal changes. We obtain a similar result (theorem 3(a)), now utilizing a pair of critical probabilities. Specifically, for any two utility functions, and any two expenditure levels y and z with $y < z$, there exist two critical probabilities p_y and p_z such that if u prefers z to y and v prefers y to z, then the probabilities $p(y)$ and $p(z)$ must exceed the respective critical probabilities, while the reverse preferences are only possible for $p(y) < p_y$ and $p(z) < p_z$. Figure 1 illustrates this relationship between the preferences of u and v. In the figure, the indifference line for v must be steeper than that for u. This fact leads to theorem 3(a).

1.4. Multiple local optima

The above critical probability results are simplest when expected utility has only one local optimum as a function of the expenditure z (the single-peaked case). Some intuitive insight into the possibility of multiple local optima and the effect of risk aversion thereon can be obtained by considering marginal benefits and marginal costs separately. The expected utility maximizer may be regarded as comparing

$$\text{Marginal Benefits (MB)} = p'(z)(u(w_2) - u(w_1))$$

and

$$\text{Marginal Costs (MC)} = p(z)u'(w_2) + (1 - p(z))u'(w_1),$$

where both margins are measured in utility terms. (Equating MB and MC yields equation (28) of Ehrlich and Becker (1972), p. 639.) Standardizing (dividing) by $u(w_2) - u(w_1)$ does not change their relative magnitudes, even though $w_i = x_i - z$ depends on z, and gives

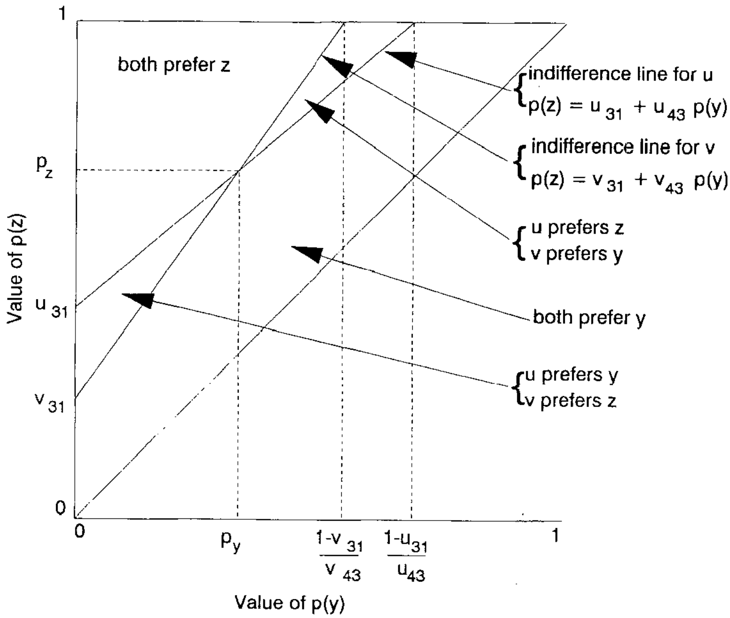

Figure 1. Possible preferences between two expenditure levels z and y.

$$\text{Standardized Marginal Benefit (SMB)} = p'(z)$$

and

$$\text{Standardized Marginal Cost (SMC)} = p(z)s_2(z) + (1 - p(z))s_1(z),$$

where $s_i(z) = u'(w_i)/(u(w_2) - u(w_1))$, the ratio of the slope of u at w_i to the change in u on the interval $(w_1, w_2), i = 1, 2$. If u is risk averse, then $s_1(z) > 1/(x_2 - x_1) > s_2(z) > 0$. The more risk averse u is, the more the $s_i(z)$ differ from $1/(x_2 - x_1)$. After rescaling, $s_1(z)$ and $s_2(z)$ can be interpreted as (nonlocal) measures of risk aversion, relating specifically to two-point gambles on w_1 and w_2, with infinitesimal probability on w_2 and w_1, respectively.

If $p(z)$ exhibits diminishing returns to expenditure, then SMB is positive and decreasing, and it is as smooth (or rough) as p' is. SMC is a weighted average of $s_1(z)$ and $s_2(z)$. As z increases, the weight on the smaller, $s_2(z)$, increases. This tends to make SMC decreasing in z. The more risk averse u is, the greater are $s_1 - s_2$ and this tendency. On the other hand, if u has decreasing risk aversion, then s_1 is an increasing function and s_2 is a decreasing function. This tends to make SMC increasing in z where $p(z)$ is small. The more rapidly risk aversion decreases, the greater this tendency. Thus it appears that multiple local optima can occur more easily the larger risk aversion is, the more rapidly it decreases, and the more $p(z)$ varies for $0 < z < x_2 - x_1$. (Local irregularities can multiply optima as well, of course.)

For $u(x) = -e^{-cx}$ (constant risk aversion), we obtain

$$\text{SMC} = c(K - p(z)),$$

where $K = 1/(1 - e^{cx_1 - cx_2} > 1$. Comparing this SMC to SMB, we see that multiple local optima are clearly possible even when risk aversion is constant and $p(z)$ exhibits diminishing returns.

2. Conclusions

Our four theorems taken together tell us a great deal about the relationship between risk aversion and behavior in the common situations in which one can pay to shift probabilities. First, the more risk-averse individual may purchase a riskless portfolio when the less risk-averse person does not, but not vice versa. Second, when some risks persist, as in most situations, merely knowing that one individual spent more than another to improve probabilities does not tell us whether such behavior is properly thought of as gambling or insuring. Third, if a good outcome is likely enough, so that we are essentially in an insurance situation, the more risk-averse individual spends the larger amount to improve his or her chances. By contrast, if the likelihood of the bad outcome is sufficiently large, expenditures essentially represent gambles and the less risk-averse individual will spend more. Fourth, one cannot diagnose any behavior in a vacuum as constituting either gambling or insurance. How likely is likely enough to make an expenditure insurance will depend on the degree of risk aversion of the individual observed and the reference group. Sharper demarcations would not seem to be available.

These results, we believe, accord with intuition. In a more general vein, they suggest that examining how individuals behave in the types of real situations in which expenditures shift probabilities may provide an insightful way to study behavior toward risk. It is reassuring that the classic concept of risk aversion bears a natural relationship to such behavior.

Appendix: Theorems and proofs

Assume in theorems 1 through 3 that u is strictly more risk averse than v on an interval including all possible outcomes.

Theorem 1. Given any choice set with all outcomes in (x_1, x_2), if v chooses a riskless portfolio, then u does also.

This is an immediate consequence of the fact that if u is more risk averse than v, then u prefers a certainty to a lottery whenever v does. (In the nonstrict case, this is an equivalence, essentially the definition, and theorem 1 still holds up to indifference in the conclusion.)

Assume hereafter that the choice to be made is a value of z, which yields outcome $x_2 - z$ with probability $p(z)$ and $x_1 - z$ otherwise, where p is an increasing function with $p(0) = 0$.

Theorem 2.

(a) There exists a p such that $p(z) = 0$ is optimum for u but $p(z) > 0$ at the optimum for v.

(b) There exists a p such that $p(z) = 1$ is optimum for u but $p(z) < 1$ at the optimum for v.

Proof of theorem 2(a). Let $U(z)$ be the expected utility if u spends z, and similarly for $V(z)$. Then

$$U(z) = p(z)u(x_2 - z) + (1 - p(z))u(x_1 - z) \tag{1}$$

$$U'(0) = p'(0)[u(x_2) - u(x_1)] - u'(x_1)$$

$$U'(0) > (<)\, 0 \text{ iff } p'(0) > (<) \frac{u'(x_1)}{u(x_2)-u(x_1)}$$

$$V'(0) > 0 > U'(0) \text{ if } \frac{v'(x_1)}{v(x_2)-v(x_1)} < p'(0) < \frac{u'(x_1)}{u(x_2)-u(x_1)}.$$

Choose $p(z)$ to satisfy this condition. $U(z) < U(0)$ and $V(z) > V(0)$ for small z. Let $p(z)$ increase so slowly that $U(z) < U(0)$ for all z. The result follows. The condition is possible by Pratt's (1964) theorem 1(e) or equation (21) or Pratt (1988), section 4.1; alternatively,

$$\frac{v(x_2)-v(x_1)}{v'(x_1)} - \frac{u(x_2)-u(x_1)}{u'(x_1)} = (x_2 - x_1)\left[\frac{v'(t)}{v'(x_1)} - \frac{u'(t)}{u'(x_1)}\right]$$

for some $t \in (x_1, x_2)$, and this is strictly positive since $\frac{u'(x)}{v'(x)}$ is decreasing because u is more risk averse than v. Q.E.D.

Proof of theorem 2(b). Suppose $p(y) = 1$. Let $x_i' = x_i - y$ for $i = 1, 2$. Then

$$U'(y) = p'(y)[u(x_2') - u(x_1')] - u'(x_2'),$$

$$U'(y) > (<)\, 0 \text{ iff } p'(y) > (<) \frac{u'(x_2')}{u(x_2')-u(x_1')},$$

$$V'(y) < 0 < U'(y) \text{ if } \frac{v'(x_2')}{v(x_2')-v(x_1')} < p'(y) < \frac{u'(x_2')}{u(x_2')-u(x_1')}.$$

Choose p to satisfy this condition, but to increase so slowly that $U(z) < U(y)$ for all z. The condition is possible for sufficiently small y, since

$$\frac{v(x_2)-v(x_1)}{v'(x_2)} > \frac{u(x_2)-u(x_1)}{u'(x_2)}$$

by Pratt (1964), equation (22). Q.E.D.

Corollary. Given any nonequivalent utility functions u and v, there exist x_1, x_2 and functions p_1 and p_2 such that u will spend less than v for p_1 and more for p_2.

Proof. If u and v are not equivalent, then there exists some interval $[x_1,x_2]$ on which one is strictly more risk averse than the other, so the two functions of theorem 2 serve. (We assume u and v are smooth—say thrice continuously differentiable—although this may not be necessary.) Q.E.D.

Theorem 3.
(a) Suppose $0 < y < z < x_2 - x_1$ and let $w_1 = x_1 - z$, $w_2 = x_2 - z$, $w_3 = x_1 - y$, $w_4 = x_2 - y$, $u_{ij} = u(w_i) - u(w_j)$, $v_{ij} = v(w_i) - v(w_j)$, and

$$p_y = \frac{u_{31}/u_{21} - v_{31}/v_{21}}{v_{43}/v_{21} - u_{43}/u_{21}},$$

$$p_z = \frac{u_{31}/u_{43} - v_{31}/v_{43}}{u_{21}/u_{43} - v_{21}/v_{43}}.$$

Then $0 < p_y < p_z < 1$. If u prefers z to y, and v prefers y to z, then $p(y) > p_y$ and $p(z) > p_z$. The opposite preferences imply $p(y) < p_y$ and $p(z) < p_z$.

(b) Let $p_s = \dfrac{u'(w_1)/u_{21} - v'(w_1)/v_{21}}{v'(w_2)/v_{21} - u'(w_2)/u_{21}}$.

If $p(z) > (<)\, p_s$ and the marginal return to spending is positive (negative) for v, then so it is for u.

Proof of theorem 3(a). Scale u and v so that $u_{21} = v_{21} = 1$. By equation (1), u prefers y to z iff $p(z) < u_{31} + u_{43}p(y)$. Similarly, v prefers y to z iff $p(z) < v_{31} + v_{43}p(y)$. Since u is more risk averse than v, we have $u_{31} > v_{31}$, $u_{43} < v_{43}$, and $(1 - u_{31})/u_{43} > (1 - v_{31})/v_{43}$. Hence, the relationships shown in figure 1 hold. The formulas for p_y, p_z where the indifference lines intersect are easily obtained. The result follows. Q.E.D.

Proof of theorem 3(b). This follows from 3(a) by taking limits as $y \to z$ and as $z \to y$. For a direct proof, note that with the scaling $u_{21} = v_{21} = 1$.

$$U'(z) = p'(z) - pu'(w_2) - (1 - p)u'(w_1),$$
$$U'(z) > (<)\, 0 \text{ iff } p'(z) > (<)\, pu'(w_2) + (1 - p)u'(w_1),$$

and similarly for v, where $p = p(z)$. Hence,

$$U'(z) > (<)\, 0 > (<)\, V'(z)$$

is impossible if

$$pu'(w_2) + (1 - p)u'(w_1) - pv'(w_2) - (1 - p)v'(w_1) > (<)\, 0. \qquad (2)$$

Since $u'(w_1) > v'(w_1)$ and $u'(w_2) < v'(w_2)$, the left-hand side of inequality (2) is decreasing as a function of p, positive for $p = 0$, and negative for $p = 1$. Let p_s be the value at which it is 0. Then $U'(z) > (<) 0 > (<) V'(z)$ is impossible for $p > (<) p_s$. Q.E.D.

Theorem 4. Given any x_1, x_2, z, and p, there exists a v for which z is optimum if and only if $z < x_2 - x_1$ and for all $t < z$,

$$p(t) + (1 - p(t))(z - t)/(x_2 - x_1) < p(z).$$

In regular cases, v can be chosen strictly monotone, strictly concave, and twice (indeed infinitely often) continuously differentiable. Then there exists a more risk-averse u for which z is locally and globally too small, and another for which z is locally and globally too big.

Proof. Let $v_0(w) = -\infty$ for $w < x_1 - z$ and $v_0(w) = \min(w, x_2 - z)$ for $w \geq x_1 - z$. Given the utility values at $x_1 - z$ and $x_2 - z$, which can be chosen arbitrarily by scaling, v_0 has the minimum possible value everywhere. It follows that if z is optimal for any utility function, then it is optimal for v_0. By straightforward algebra, z is optimal for v_0 if and only if the conditions given in the first sentence of the theorem hold. The first sentence of the theorem follows.

In regular cases, if v_0 is replaced by a sufficiently close, smooth approximation v, the optimum will be close to z and can be made equal to z by reducing $v(w)$ for $w > (<) x_1 - z$ if the optimum is larger (smaller) than z. The regularity condition needed is that for all $\epsilon > 0$, the leeway in the inequality is bounded away from 0 for $t \leq z - \epsilon$. For this it suffices that the inequality hold and p be continuous, or that the derivative of the left-hand side be positive for $t \leq z$, that is, $1 - p(t) < p'(t)(t + x_2 - x_1 - z)$.

The last sentence of the theorem can be proved as follows. (We omit details.) Let $u(w) = v(w)$ for $w \leq x_2 - z - \epsilon$ and elsewhere let u be slightly more risk averse than v. Then, for sufficiently small ϵ, z is locally and globally too small. One can make u strictly more risk averse than v everywhere without losing this property by adding to u a sufficiently small multiple of any more risk-averse function. If $w \leq x_2 - z - \epsilon$ here is replaced by $w \geq x_1 - z + \epsilon$, the effect on the optimum is reversed. Q.E.D.

Note

1. Ordinarily, insurance is actuarially unfavorable (reduces expected monetary value) but reduces risk, while gambling increases risk and hence must be actuarially favorable to be desirable for a risk-averse decision maker. For probability-improving expenditures, however, actuarial favorability is not key, and we make no assumption about it.

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