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The Value of Changes in Life Expectancy

SHERWIN ROSEN
The University of Chicago

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Abstract

Valuation formulas for age-specific mortality risks are derived from life-cycle allocation theory under uncertainty and related to empirical estimates of the value of life. A change in an age-specific mortality risk affects all subsequent survivor functions and reallocates consumption and labor supply over the entire life cycle. The value of eliminating a risk to life at a specific age is the expected present value of consumer surplus from that age forward. Approximate numerical extrapolations from cross-section estimates imply that values decrease rapidly in current age and in the distance between current age and age at risk.

Professional interest in cost-benefit analysis of safety, illness, and death probabilities had its origins in the environmental concerns and the growth of the medical sector of the 1960's as a practical matter, and in the pioneering work of Schelling (1968) and Mishan (1971) as an intellectual one. Subsequent work has followed two distinct lines. One, beginning with Usher (1973), has analyzed intertemporal risks affecting life expectancy (Conley, 1976; Cropper, 1977; Ehrlich and Chuma, 1984; Arthur, 1981; Shepard and Zeckhauser, 1984; Moore and Viscusi, 1988), where risks are implicitly evaluated at various points in the life cycle. The other uses simpler, atemporal models to guide empirical work (Jones-Lee, 1976; Thaler and Rosen, 1975; Viscusi, 1978). The relationship between the two is developed below.

Section 1 briefly reviews the single-period model, and explores some unusual consequences of state-dependent preferences. The net difference in utility between life and death states is an essential aspect of preferences for life-risk valuation. Paradoxically, risk-averse people can actually prefer more life-risk gambles to less in order to convexify preferences in certain cases. The consumption elasticity of net utility is established as a key determinant of the value of life.

Section 2 examines a deterministic life cycle model and establishes two points. First, intertemporal substitution possibilities in life cycle preferences determine

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the value of life extensions: greater substitution reduces the willingness to pay for life extensions because *quantity* (life-years) and *quality* (consumption per year) of life are better substitutes. Second, the marginal value of life extensions increase with age, a phenomenon that can lead to regret in old age from having voluntarily exposed oneself to irreversible risks when young. However, this is neither intertemporally inconsistent nor irrational.

Section 3 addresses the stochastic life cycle problem and derives valuation formulas for perturbations in age-specific death (hazard) rates. The value of eliminating a risk to life at a specific age is the expected present value of the additional consumer surplus it gives rise to. Using the fact that the value of a current (age-independent) risk is estimated by equalizing wage differences on risky jobs, section 4 imputes middle-of-life-cycle valuations of death hazards based on an approximation of the valuation formula developed in section 3. Valuations of current risks decrease with age in this range of approximation. They also decrease with the future age of risk exposure. These calculations illustrate how to value risk exposures, such at to carcinogenic substances, that involve delays between initial exposure and subsequent risk. Suggestions for future research appear in the concluding section.

1. Some consequences of normalization

Valuing risks to life requires some unusual normalizations of preferences because utility is inherently state-dependent in this problem. The basic issues have not been thoroughly treated in the literature and are best illustrated in a one-period model (e.g., Bailey, 1980; Rosen, 1981).

Consider a person without heirs or altruism toward others. There are two states: If an accident doesn't occur, the risk-averse person survives and enjoys utility $\tilde{u}(c)$, where c is consumption. The person dies if an accident occurs, so it is meaningless to think of consumption. Instead, assign a constant M to utility in this state (with $M \gtreqless 0$). Expected utility is

$$\mathrm{E}\tilde{\mathrm{U}} = p\tilde{u}(c) + (1 - p)M, \tag{1}$$

where p is the probability of survival.

In expected utility theory, preferences are independent of states and the utility function is defined only up to an increasing linear transformation. When preferences are state-dependent, any increasing linear transformation is acceptable so long as the *same* transformation is consistently applied to the utility function of each state. In the case at hand, subtracting M from utility in each state normalizes the utility of nonsurvival to zero:

$$\mathrm{EU} = p[\tilde{u}(c) - M] + (1 - p)[M - M] = pu(c), \tag{2}$$

where

$$u(c) = \tilde{u}(c) - M \tag{3}$$

is the differences in utility between life and death. Only the difference matters, because (1) and (2) order life-and-death gambles in exactly the same way.

The value of life is defined as the willingness to pay for a small increment dp in the survival rate. Budget opportunities obviously enter into this calculation. The person is endowed with nonhuman wealth W. What happens to ownership of these assests if the person dies? Assume the atemporal equivalent of an actuarially fair annuity, a tontine in which all survivors share equally in the unintended bequests of decendents. In a large group of equally endowed persons, a proportion $(1 - p)$ die and their wealth is distributed to p survivors. A survivor's consumption equals initial endowment W plus the tontine share $(1 - p)W/p$, or W/p in all, so the budget constraint is

$$W = pc. \tag{4}$$

Totally differentiate (2) and (4) and eliminate dc:

$$d(\mathrm{EU}) = \frac{\partial \mathrm{EU}}{\partial p} dp + \frac{\partial \mathrm{EU}}{\partial W} dW = \left[u - \left(\frac{W}{p}\right)u'\right]dp + u'dW. \tag{5}$$

The value of life is the marginal rate of substitution between W and p:

$$v = -\frac{dW}{dp} = \frac{u}{u'} - \frac{W}{p} = \left[\frac{u/(W/p)}{u'} - 1\right]\frac{W}{p} = \frac{1-\varepsilon}{\varepsilon}\frac{W}{p}, \tag{6}$$

where $\varepsilon = d \log u/d \log c$ is the ratio of marginal to average utility evaluated at $c = W/p$. Equation (6) shows that a person will pay to reduce death risk if and only if $\varepsilon < 1$. The person will pay to increase risk if $\varepsilon > 1$. The following argument proves that $0 < \varepsilon \leqslant 1$ covers all economically interesting cases (for persons without earnings). Two possible configurations of $u(c) = \tilde{u}(c) - M$ must be considered to show this.

(i) Suppose $u(c) \geqslant 0$ for $c \geqslant 0$; that is, $u(c)$ has a nonnegative intercept (figure 1). Since $u(c)/c$ is the slope of a line from the origin to a point on $u(c)$, it follows from the figure that $u'(c) < u(c)/c$ and $\varepsilon \leqslant 1$ for all c. Furthermore, $\varepsilon > 0$ because u is positive—the utility of survival is at least as large as the utility of death for all c. This, however, need not be true. The second case is more interesting.

(ii) Suppose $u(c) \lesseqgtr 0$ as $c \lesseqgtr \bar{c}$: $u(c)$ has a negative intercept (figure 2) and $\bar{c} > 0$ is *minimum survival* consumption. Since the utility of death has been normalized to zero, death is the preferred state if $c < \bar{c}$ because $M > \tilde{u}(c)$ in that range. Nevertheless, for $\bar{c} < c < c^*$, $\tilde{u}(c) > M$ and the slope of the cord linking the origin with $u(c)$ is less than $u'(c)$. Therefore $\varepsilon > 1$, so equation (6) implies that the person would pay to reduce survival chances even though survival is preferred to nonsurvival. Survival seems to be a bad, not a good, in this range.

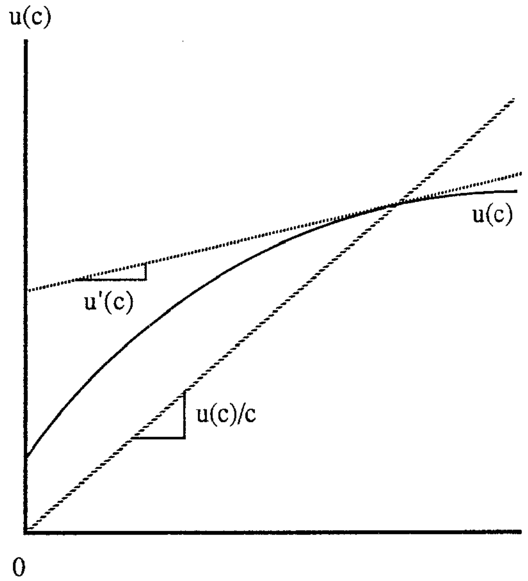

Fig. 1. The utility of survival is at least as large as the utility of death for all c.

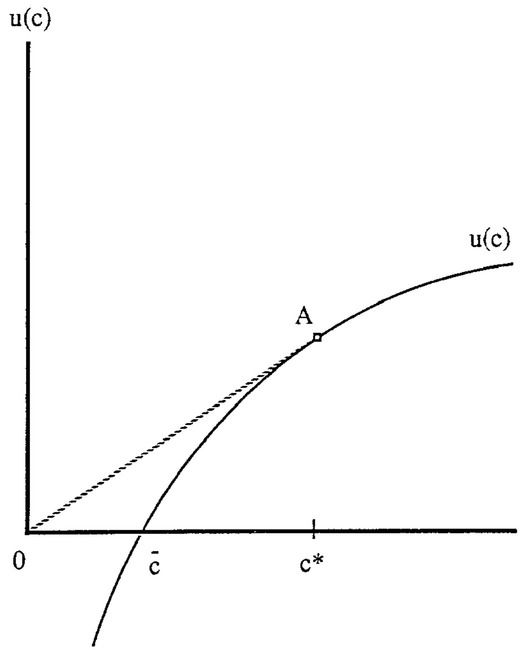

Fig. 2. The utility of death is preferred to the utility of survival for $c < \bar{c}$.

It is incorrect to apply equation (6) in case (ii) because of a nonconvexity in the utility function. The effective utility function in figure 2 is the abscissa for c in the range $[0,\bar{c}]$ connecting to $u(c)$ for $c \geqslant \bar{c}$. It exhibits an increasing return because the outcome is indivisibile—one either lives or dies. However, the indivisibility is smoothed (or *convexified*) by randomizing between death (zero utility) and life utility at consumption c^* if one survives. From figure 2, if W/p falls in the interval $(0,c^*)$ there exists $p^* = W/c^*$, with $0 < p^* < p < 1$ such that survivors enjoy $u(c^*) = u(W/p^*)$ and nonsurvivors receive zero utility. Ex ante expected utility is on the cord connecting 0 and $u(c^*)$ and is larger than the sure thing for $0 < c \leqslant c^*$. The smoothed utility function is the envelope 0A for $0 < c \leqslant c^*$ and the original function AB thereafter. $\varepsilon = 1.0$ on the straight line segment and $\varepsilon < 1.0$ for $c > c^*$.

Put differently, calculations leading to (6) imply $\partial \text{EU}/\partial p = \lambda[(1 - \varepsilon)/\varepsilon]W/p$, where λ is the marginal utility of money. If $c < c^*$ then $(1 - \varepsilon) < 0$ and $\partial \text{EU}/\partial p < 0$. Think of a choice problem in which p^* is chosen to maximize EU in (2) subject to (4) and to the constraint $p^* \leqslant p$. The solution $p^* = W/c^*$ is the optimum value of p^* when $\varepsilon > 1$ in the utility function $u(c)$. $p = p^*$ is chosen when $\varepsilon \leqslant 1$.

Convexification is achieved by adopting modes of behavior that increase the risk sufficiently to enable survivors to attain consumption standard c^*. Applying equation (6) to the convexified utility function shows that the person will not pay anything to extend life chances in the straight line section of figure 2 because excess risks are already being taken to smooth out preferences before the experiment is presented to the person.[1] Whatever risk is offered will be undone by randomization.[2]

For a working person, specify utility in the first state as $u(c,l)$, where l is leisure, and the budget equation as

$$c = w(1 - l) + \frac{W}{P}, \tag{7}$$

where w is the wage rate, and earnings (but not assets) are assumed to be at risk. If expected utility maximization implies that the person does not work, the analysis remains as above. If some labor is supplied to the market, the envelope theorem yields

$$v = \frac{\partial \text{EU}}{\partial p} \bigg/ \frac{\partial \text{EU}}{\partial W} = -\frac{dW}{dp} = \frac{1 - \varepsilon}{\varepsilon} c + w(1 - l), \tag{8}$$

where c and l in (8) are their optimal values and $\varepsilon = cu_c/u$ is evaluated at those values. Foregone earnings are now included in v.

In this case, the derivative $\partial \text{EU}/\partial p$ is proportional to the right-hand side of (8). There is no scope for randomization to increase expected utility if that sum is positive. In particular, $\varepsilon > 1$ is consistent with utility maximization and no randomization for a working person so long as $w(1 - l) > (1 - \varepsilon)c/\varepsilon$. Consequently, the value of life can either exceed or fall short of earnings according as $\varepsilon \lesseqgtr 1$, and no theoretical bounds connecting the two can be established a priori. The early human capital estimates of Weisbrod (1971) and Rice (1966) remain unjustified from utility theory, unless other evidence suggests that $\varepsilon \approx 1$. If ε is so large that $\upsilon < 0$ in (8) then randomization reduces υ to zero, whatever earnings and consumption happen to be.

2. Deterministic life-cycle model

Consider the following problem. A person with time-separable preferences and access to a perfect capital market lives for certain until age T. How much wealth will the person give up to extend life years by a small increment dT? The concept of risk aversion in the one-period problem is replaced with the concept of intertemporal substitution in a life-cycle problem. Furthermore, discounting of future risks implies that the value of life extension systematically changes with age.

2.1 The decision problem

Preferences remain state-dependent in this problem. Assume the person enjoys a flow of utility $\tilde{u}(c(t))$ at age t. Assigning a constant M to instantaneous "utility" beyond the age of death T,

$$\tilde{U} = \int_0^T (\tilde{u}(c(t)))e^{-\rho t}dt + \int_T^\infty Me^{-\rho t}dt$$

$$= \int_0^T (\tilde{u}(c(t) - M)e^{-\rho t}dt + M/\rho, \tag{9}$$

where ρ is the rate of time preference. However, the constant M/ρ may be dropped because any monotone transformation of $\tilde{U}$ preserves orderings, resulting in

$$U = \int_0^T (\tilde{u}(c(t)) - M)e^{-\rho t}dt = \int_0^T u(c)e^{-\rho t}dt, \tag{10}$$

where $u(c)$ has exactly the same form as equation (3).

A person endowed with wealth W confronts a pure-consumption-loans market at interest rate r and cannot die in debt. Then all capital is consumed in the lifetime, so the choice of consumption path $c(t)$ is constrained by

$$W = \int_0^T c(t)e^{-rt}dt. \tag{11}$$

The Lagrangian expression for this problem is

$$\int_0^T u(c(t))e^{-\rho t}dt + \lambda\left[W - \int_0^T c(t)^{-rt}dt\right], \tag{12}$$

and the marginal condition is familiar:

$$u'(c(t))e^{-\rho t} = \lambda e^{-rt} \qquad \text{for } 0 \leqslant t \leqslant T. \tag{13}$$

2.2. Valuation of longevity

Indirect utility is a function of W and T (and r and ρ),

$$U(W,T;r,\rho) = \max_{c(t)} \int_0^T u(c(t))e^{-\rho t}dt \quad \text{s.t. (11)}, \tag{14}$$

and defines (W,T) indifference curves from which the marginal rate of substitution $-dW/dT = \upsilon$ follows. Applying the envelope theorem to (12),

$$\begin{aligned} U_W &= \lambda, \\ U_T &= u(c(T))e^{-\rho T} - \lambda c(T)e^{-rt} = [u(c(T) - c(T)u'(c(T))]e^{-\rho T}, \end{aligned} \tag{15}$$

after exploiting (13). The marginal utility of life-years has two components. Extending life has a direct effect which adds a term $u(c(T))$ to the lifetime sum of utilities on the one hand, but it also requires reallocating consumption away from other points in the life cycle, given W, on the other. These indirect increments are valued at *marginal cost* $u'(c(T))$. The term in square brackets in (15) is the utility surplus at $t = T$, and is discounted by the rate of time preference, ρ, since it occurs T years in the future.

Collecting results,

$$\begin{aligned} \upsilon &= -\frac{dW}{dT} = \frac{\partial U/\partial T}{\partial U/\partial W} = u(c(T))e^{-\rho t}/\lambda - c(T)e^{-rT} \\ &= \left[\frac{u(c(T))}{u'(c(T))} - c(T)\right]e^{-rT} = \frac{1-\varepsilon(T)}{\varepsilon(T)}c(T)e^{-rT}. \end{aligned} \tag{16}$$

2.3. Discussion

υ is decreasing in ε in both the one-period and intertemporal problems, but now ε is related to the concept of intertemporal substitution in preferences. This is illustrated by examining some extreme cases.

For $|M|$ sufficiently small, $u(c)$ approaches a linear function of c as ε goes to unity, and from (10), U is essentially summable in $c(t)$. How that total is distributed over time hardly matters, because consumption at any one time is a very good substitute for consumption at any other time. Equation (16) shows that v goes to zero in this case. A person won't pay for an increment dT because the increased horizon is completely offset by lower per-period consumption. This is the sense in which large intertemporal substitution implies large substitution between the *quantity* (T) and the *quality* (c) of life.

If M is small but ε goes to zero, the indifference curves between $c(t)$ and $c(t')$ increasingly resemble those of fixed proportions, and v grows very large. A person with such preferences is willing to pay a great deal for dT because each year of life becomes essential, and quantity and quality of life are poor substitutes. Intertemporal substitution plays a similar role in life-cycle theory as risk aversion does in atemporal models.[3]

The presence of the discount factor in (16) implies that v systematically increases with age. It is known that consumption plans are intertemporally consistent when preferences are time-separable. No matter what the person's age, planned consumption at T, $c(T)$, remains unchanged and so does $\varepsilon(T)$. Nonetheless, v changes because the horizon is shortened. If $v(t)$ is the value at age t and $v(t')$ is the value at some later age t', equation (16) implies

$$v(t') = v(t)e^{r(t'-t)}. \tag{17}$$

A person close to the end is willing to pay more to extend life than a person whose horizon is longer. One implication is that risky personal actions that have long latency periods have smaller value to younger people than to older people. The young may appear reckless on this account, but such recklessness may pass a personal cost-benefit test. Moreover, there is a natural tendency for participation in the risky activity to fall as the person ages.

There is nothing irrational about this. Suppose a person trades $v(t)$ dollars for a reversible decrement $-dT$ at age t. Investing the money makes it grow to $v(t') = v(t)e^{r(t'-t)}$ at age t'. At that point, the person would be willing and able to pay $v(t')$ to re-extend life by dT. However, if the earlier action is irreversible, the willingness to pay $v(t') - v(r)$ at t' in excess of compensation at t suggests a sense in which earlier actions are regretted later, even when regrets are fully foreseen (if information is perfect) when the initial action is taken. Regret is not irrational. It is similar to a gambler regretting having played a game ex post even though the prospect of losing was fully weighed in the decision to gamble in the first place.

Extending the analysis to include labor supply is straightforward. Earnings at T discounted back to the present are added to (16). Again $v = 0$ if randomization is optimal.

3. Valuation of risks over the life cycle

The more general problem of a stochastic horizon is analyzed in this section.

3.1. Tastes, opportunities, and optimization

A person who lives exactly t years enjoys utility (as in (10)) of

$$U(t) = \int_0^t u(c(\tau), l(\tau))e^{-\rho\tau}d\tau. \quad (18)$$

Let $f(t)$ be the probability density of living for t years, and let $F(t)$ be the *cdf*, the probability of dying on or before age t. Then expected utility is

$$\text{EU} = \int_0^\infty f(t)U(t)dt = \int_0^\infty \int_0^t f(t)u(c(\tau), l(\tau))e^{-\rho\tau}d\tau dt$$

$$= \int_0^\infty (1 - F(t))u(c(t), l(t))e^{-\rho\tau}dt, \quad (19)$$

where the third equality follows from changing the order of integration. $(1 - F(t))$ is the survivor function (from birth), hereafter written $S(t)$, and is itself a function of the pattern of age-specific death rates:

$$S(t) = (1 - F(t)) = e^{-\int_0^t h(\tau)d\tau} \quad (20)$$

where $h(t) = f(t)/(1 - F(t))$ is the death rate at age t. Substituting (20) in (19) shows that the force of mortality increases the effective rate of time preference. The future is discounted more heavily because a person may not live to see it.

Analysis is confined to the stochastic equivalent of a perfect capital market in which actuarially fair life-assured annuities (Yaari, 1965) are available.[4] In effect a person assigns all current and future claims to income to an insurance company in exchange for a contract that guarantees consumption $c(t)$ until death. The consumption risk of death is insured because those who die earlier than the average leave enough wealth behind to finance the consumption claims of those who live longer than the average.

A person who lives for exactly t years imposes a capital liability on the insurance company of

$$\int_0^t [c(\tau) - w(\tau)(1 - l(\tau))]e^{-r\tau}d\tau,$$

where $w(1 - l)$ is earned income. Budget balance requires that the expected liability over all claimants equals endowed wealth, or

$$W = \int_0^\infty \int_0^t f(t)[c - w(1 - l)]e^{-r\tau}d\tau dt$$

$$= \int_0^\infty S(t)[c(t) - w(t)(1 - l(t))e^{-rt}dt \quad (21)$$

after changing the order of integration. Mortality increases the net interest rate because net credits are earned on those who die early.

Optimal choices of $c(t)$ and $l(t)$ maximize EU subject to constraint (21). The marginal conditions

$$u_c(c(t),l(t))e^{-\rho t} = \lambda e^{-rt}, \quad u_l(c(t),l(t))e^{-\rho t} = w(t)\lambda e^{-rt}, \tag{22}$$

are identical to a deterministic problem because financial uncertainty is fully insured by annuities.

3.2. Valuation of life risks

Valuation formulas follow, as usual, from the indirect utility function. However, there are two technical complications. First, expected utility in the optimal program varies with attained age because the probability of surviving to any given age depends on age itself. The conditional probability of attaining some future age must be continually renormalized as the person ages. If $S(t)$ in (20) is the survival probability at birth, the conditional probability of surviving until age t given that one has survived until age a is

$$S_a(t) = S(t)/S(a) = e^{-\int_a^t h(\tau)d\tau}. \tag{23}$$

Writing EU_a for discounted expected utility given that the person has survived until age a, (19) becomes

$$EU_a = \int_a^\infty S_a(t)u(c(t),l(t))e^{-\rho(t-a)}dt. \tag{24}$$

Second, expected utility at a depends on the *entire function* $S_a(t)$, and calculating marginal rates of substitution requires extending the concept of differentiation to a perturbation or variation in the function $S(t)$. This is technically a Frechet derivative, as pointed out by Arthur (1981). Willingness to pay for any pattern of (small) changes in death probabilities can be calculated by examining how variations in $h(t)$ affects $S_a(t)$.

Using notation δ to indicate this kind of differentiation, from (24),

$$\frac{\delta EU_a}{\delta S_a} = \int_a^\infty \left(u + S_a u_c \frac{\delta c}{\delta S_a} + S_a u_l \frac{\delta l}{\delta S_a}\right)(\delta S_a)e^{-\rho(t-a)}dt,$$

where δS_a is the variation is $S_a(t)$ and δc and δl are the equilibrium variations in $c(t)$ and $l(t)$ that are caused by it. Exploiting the time-consistent nature of the solution, reconditioning and differentiation of constraint (21) yields

$$\int_a^\infty S_a\left(\frac{\delta c}{\delta S_a} + w\,\frac{\delta l}{\delta S_a}\right)(\delta S_a)e^{-r(t-a)}dt = -\int_a^\infty (c - w(1-l))(\delta S_a)e^{-r(t-a)}dt.$$

These two expressions and the marginal conditions give the envelope result

$$\frac{\delta \mathrm{EU}_a}{\delta S_a} = \int_a^\infty [u - u_c \cdot (c - w(1-l))](\delta S_a)e^{-\rho(t-a)}dt. \tag{25}$$

Similarly, $\delta \mathrm{EU}_a/\delta W^a = \partial \mathrm{EU}_a/\partial W^a = u_c e^{(r-\rho)(t-a)}$, where W^a is wealth remaining at age a, so the appropriate marginal rate of substitution reduces to

$$-\frac{\delta W^a}{\delta S_a} = \int_a^\infty \left[\left(\frac{1-\varepsilon}{\varepsilon}\right)c + w(1-l)\right](\delta S_a)e^{-rt}dt, \tag{26}$$

which generalizes equation (16) above.[5] The value of a perturbation in $S_a(t)$ is the change in expected discounted consumer surplus it gives rise to along the optimum (c,l) path. Further calculations reveal that $\delta \mathrm{EU}_a/\delta w = \lambda\int_0^\infty(1 - l(t))e^{-rt}dt$, implying that

$$\frac{\delta w}{\delta S_a} = \frac{\delta W^a}{\delta S_a} \Big/ \int_0^\infty (1 - l(t))e^{-rt}dt \tag{27}$$

is the intertemporal version of Slutsky compensation for a change in the intertemporal pattern of wage rates.

3.3. *The value of saving a life*

$S_a(t)$ is related to $h(t)$ through (23). Taking logs and differentiating,

$$\delta S_a = -S_a(t)\int_a^t \delta h(\tau)d\tau. \tag{28}$$

Substituting (28) into (26) gives the valuation formula for changes in death rates, the natural primitives of the problem.

Consider the canonical experiment where δh is the Dirac-delta function taking a point-mass jump of size Δ at age α and otherwise remaining unchanged. Then the perturbation δS in (26) is zero for $t < \alpha$, because $\delta h = 0$ for $t < \alpha$, and also for $a > \alpha$ because the person has survived the risk. However, for $a < \alpha$ and $t > \alpha$ there is a persistent effect of $S_a(t)$ because $-\log S_a(t)$ is the sum of all previous hazard rates, from (23).

$$\begin{aligned} \delta S_a &= 0 \quad \text{for } a > \alpha, \\ \delta S_a &= 0 \quad \text{for } t < \alpha \text{ and } a < \alpha, \\ \delta S_a &= -\Delta[S(t)/S(a)] = -\Delta \cdot S_a(t) \quad t \geqslant \alpha \text{ and } a < \alpha. \end{aligned} \tag{29}$$

The money value of an excess risk incurred at age α from the prespective of a person currently of age $a < \alpha$ is, from (26) and (29),

$$v(\alpha,a) = \int_{\alpha}^{\infty} Z(t)S_a(t)e^{-r(t-a)}dt, \tag{30}$$

where

$$Z(t) = \frac{1-\varepsilon}{\varepsilon}c + w(1-l) \tag{31}$$

is consumer surplus at t. Of course, the value is zero for a person older than α. Define $V(\alpha) = v(\alpha,\alpha)$ as the value of eliminating a current risk. Then

$$V(\alpha) = \int_{\alpha}^{\infty} Z(t)S_{\alpha}(t)e^{-r(t-\alpha)}dt, \tag{32}$$

and after simple manipulations,

$$v(\alpha,a) = \frac{S(\alpha)}{S(a)}e^{-r(\alpha-a)}V(\alpha). \tag{33}$$

$V(\alpha)$ in (32) is the value of saving a current life. It is the expected present value of consumer surplus at age α. Since $h(\alpha)$ is a probability, the jump Δ lies in the unit interval. For example, suppose $\Delta = 1/1000$. Then $\Delta \cdot V(\alpha)$ in (32) is the amount of money an age α person would pay to eliminate the extra risk, and $1/\Delta = 1000$ such people would collectively pay $(V \cdot \Delta)/\Delta = V$ to eliminate a risk that on average takes one life among them. The value of a prospective risk in (30) or (33) is smaller than the value of a current risk for two reasons. First, $Z(t)$ is discounted by $e^{-r(\alpha-a)}$ because it occurs in the future, as in the deterministic model; and second, not all people of age a will survive until age α to enjoy the benefit. The term $S(\alpha)/S(a)$ in (33) reflects this latter fact. In that sense $v(\alpha,a)$ is the *fractional value of a life* with fraction $S(\alpha)/S(a)$. It is a *whole life* value when $a = \alpha$. Prospective risks have smaller value than current risks at a given age because they are discounted by interviewing mortality as well as by the rate of interest.

Since $V(\alpha)$ is the value of eliminating exposure to a current risk, its derivative indicates whether older people would pay more or less than younger people to eliminate age-independent risks. We have

$$\begin{aligned}\frac{dV(\alpha)}{d\alpha} &= -Z(\alpha) + (r + h(\alpha))V(\alpha). \\ &= \int_0^{\infty} [Z'(\tau + \alpha) - (h(\tau + \alpha) - h(\alpha))Z(\tau + \alpha)]S_{\alpha}(\tau + \alpha)e^{-r\tau}d\tau.\end{aligned} \tag{34}$$

The first form of (34) follows the usual relationship between flow and stock values, taking care to gross-up the interest rate by the current mortality rate. Value is rising with age if current surplus is small relative to discounted future surpluses. The second form of (34) shows that $Z(t)$ must be increasing for $V(\alpha)$ to be increasing, which is likely to be true in the interval between youth and middle age. [6] Surplus $Z(t)$ would be constant in the case where $r = \rho$ and $w(t)$ is constant. Then V falls with age because $h(t)$ is strictly increasing. The latter fact must make V fall in very old age in any case, because there is little surplus left to discount. This point would be reinforced if age-dependence had been specified in utility to reflect deteriorating health and quality of life with age as well as greater mortality per se.

Older persons may nonetheless put greater value on some risks than younger people because the risk is more immediate, as in the difference between (32) and (33). As longevity increases, it is natural to expect more resources to be devoted to curing specific diseases of older age, such as cancer and Alzeimer's disease, because in earlier eras people did not live long enough to be exposed to them. However, $V(\alpha)$ declining for a large α is paradoxical for the incidence of voluntary exposure to immediate risks. To account for why such risks are most often borne by younger people requires an auxilary physiological hypothesis that younger people produce less real risk per unit of exposure than older people do.

4. Estimates

Applying (30) – (33) requires estimates of surplus Z and the elasticity ε. It is important to understand that ε cannot be inferred from consumption or labor supply behavior because marginal conditions (22) do not involve the parameter M or other aspects of mortality, and ε depends on M, the curvature of $\tilde{u}$, and on c and l. In principle ε could be identified by repeated observations on risky choices over the life cycle. The estimate below is based on cross-section wage premiums observed on risky jobs. Since panel data are not available, only average values of Z and ε can be estimated.

The idea of the method is to interpret observed wage–risk premiums as an estimate of $\upsilon(a,a) = V(a)$ in (30) or (32). Then assume a factorization of (32) into its Z and discount components and use data on consumption and earnings to infer average value of ε and Z.

The risks that are priced in labor market studies generally refer to immediate risks to life from fatal accidents at the work site. This is not strictly true in all cases, but is a reasonable assumption for most of the risky occupations used by Thaler and Rosen (1975; T–R hereafter), in which case that study estimates $V(a)$ in (32). Now $\int S_a(t)dt$ is remaining life expectancy for a person who has attained a years of age; and if the interest rate were zero, $V(a)$ in (32) is approximately the remaining average annual surplus times average remaining life-years. Discounting requires a simple actuarial adjustment because $\int_a^\infty S_a(t)e^{-r(t-a)}dt = A(a,r)$ is the present value of a unit annuity at age a when the interest rate is r, tabulated (as a_x) in actuarial tables.

To proceed, assume a factorization of (32):

$$V(a) = Z \cdot A(a,r). \tag{35}$$

Then average surplus Z is approximately $V(a)/A(a,r)$. Finally, use consumption and earnings data to infer ε from (31) interpreted as a relationship about averages.

This procedure is *exact* if $\rho = r$ and w does not change over the remaining horizon (though the implied estimate of ε is valid only in the neighborhood of the constant equilibrium values of c and l). If $\rho \neq r$ and w changes with age, the approximation may be less useful. Still, the average worker in the T–R sample is observed in the middle of work life, when relative age-earnings growth has mostly disappeared and subsequent living standards are largely set. Insofar as earnings and consumption growth are correlated over time due to the common factor of economic growth, the growth rate is netted out of the real interest rate r in discounting. When all is said and done, however, the quality of the approximations is unknown. A warning of *caveat emptor* hardly seems necessary, but a crude estimate may be better than none at all.

Using (27) to transform the risk-earnings estimate of T–R to a wealth estimate (with annual hours worked at the sample mean in the denominator because the estimate refers to one year each of risk and wage rates) implies a value for $V(a)$ of \$630,000, converted to dollars of 1986 purchasing power.[7] The mean age of workers in that sample is 41.8, so $a = 42$. Table 1 reports $A(42,r)$ based on mortality experience of white males. The third column reports corresponding values of Z in 1986 dollars, dividing $A(42,r)$ into $V(a) = 630{,}000$ from (35). The estimate of Z is sensitive to the rate of interest, rising by about \$6000 for each percentage point in r for small r and by \$11,000 for large values of r.

Table 1. Estimated elasticities and average consumer surplus, 42-year-old white males, by interest rate

r(%)	$A(42,r)$	Z (1986\$)	ε
0	32.1	\$19,660	1.06
2	24.4	25,820	.81
4	16.9	37,280	.56
6	13.2	47,730	.44
8	10.8	58,330	.36
10	9.0	70,000	.30
12	7.7	81,820	.25

Notes: $A(42,r)$ from U.S. Social Security Administration, Office of the Actuary, "Actuarial Tables Based on U.S. Life Tables, 1979-81." Actuarial Study No. 96, August 1986. Z in 1986 dollars based on $v(42,42) = 630{,}000$, from T–R. Elasticity calculated assuming $c = w(1 - l) = 20{,}800$ in 1986 dollars, from T–R.

The estimate of ε in column 4 is based on mean earnings of \$20,800 (1986 dollars) in the sample. This was a relatively low-income population whose full-time earnings averaged 25% below the mean of all full-time wage and salary earners, so their consumption expenditure must have been well approximated by earnings. The estimate in column 4 assumes $c = w(1 - l) = 20{,}800$, implying $\varepsilon = w(1 - l)/Z$ from (31). The sensitivity of Z to r in the third column carries over to ε, with the estimate declining geometrically at about 10% per percentage point increase in r. The estimate also is sensitive to the estimate of $V(a)$, which had a sampling error alone of 25% of the estimated level value.[8]

There are several additional possible sources of bias in these numbers.

i) Savings. Assume that $c = \beta w(1 - l)$ with $\beta < 1$. If β is .9, the implied saving rate is 10% and the estimates in column 4 fall by less than 5%. A 10% savings rate surely is an upper bound for this population.

ii) Costs of Mortality. If medical and other costs of mortality are fixed at D, then incorporating them into the model (assuming full insurance) involves setting up a sinking fund and charging interest rD against consumer surplus in (31). Approximately 10% of all medical expenses are accounted for by people in their last year of life, or \$22,000 in 1986 dollars. Adding a generous allowance for other expenses increases the estimate of ε in table 1 by 3% and decreases the estimate of Z by 5% at $r = .10$. The adjustments are less at lower interest rates.

iii) Taxes. The survey data underlying T–R's study probably refer to before-tax earnings, so $V(a)$ should be multiplied by $(1 - \gamma)$ where γ is the marginal tax rate. For this population γ lies within [.15,.20], (see Steuerle and Hartzmark, 1981) and the values of Z and $V(\alpha)$ in tables 1 and 2 should be multiplied by .80 or .85. The estimate of ε is hardly affected by marginal taxes of this size.

iv) Cross-Section Life Table. Though the T–R data are from 1968, the 1979–1981 life table is used for $A(42,r)$ in table 1. Falling mortality rates causes cohort bias in cross-section life tables. Substantial increases in longevity during the 1970s made the 1979–1981 life table more accurate for this population. However, using the 1969–1971 life table gives estimates of ε that are only 3–6% smaller than those reported.

v) Retirement. Since earnings fall to zero during retirement and consumption changes as well, average Z in (31) is itself a weighted average of pre- and postretirement years. However, the weight on retirement years is smaller than the weight on working years for 42-year-olds, and it decreases with the rate of interest. Assuming retirement at age 65, 29% of A falls in the retirement years when $r = .01$, but only 9% of it does when $r = .08$. If $u(c,l)$ is strongly separable and $\rho = r$, ε in table 1 is underestimated by 7% at $r = .01$, by 3.5% at $r = .04$ and by 1% at $r = .08$. If $u(c,l)$ is not separable, things are more complicated. However, at respectable interest rates, the calculation above suggests small bias from this source unless ε falls dramatically during retirement.

Taken in total, all of these refinements would affect the estimates of Z and ε in table 1 by relatively little compared to their sensitivity to r. There is little professional consensus on the appropriate size of r. It ranges from the 0–1% long-run average after-tax rate of return found on individual portfolio items relevant to annuities to the 15% gross rate of return on capital implicit in aggregate production studies. A value of 8% is chosen here as middle ground, since the applicable rate for individuals should be as large as the after-tax rate on corporate capital on risk-premium considerations alone. Futhermore, Moore and Viscusi (1988) estimate only a slightly larger value in their recent study of the nonlinear interactions between wage–risk premiums and age.

Extrapolations of table 1 to $v(\alpha,a)$ in (33) must be confined to persons near the same age and income levels, since ε and Z may change markedly outside that neighborhood. Maintaining $c \approx w(1 - l)$, equation (31) implies $Z \approx w(1 - l)/\varepsilon = 2.8w(1 - l)$ for $r = .08$ and (30) implies $V(42) \approx Z \cdot A(42,.08) = 30w(1 - l)$. For each dollar increase in earnings near \$20,800, consumer surplus rises by \$2.8 and the current value of life rises by \$30. These are lower bounds because, on selection grounds, individuals with larger-than-average values of ε would find risky jobs more attractive, and also because the T–R estimates are smaller than other estimates.

Equation (33) is extrapolated in an age range six years on either side of age 42 in table 2, assuming constant income in this range (a good approximation after netting out time-series growth effects from r), and Z is held fixed at \$58,330 as estimated for $r = .08$ in table 1.[9] Column 1 shows that the value of a current risk declines with age, following the pattern of $A(a,.08)$, because Z is assumed constant in this age range. Other columns evaluate willingness to pay now by a person a years old to eliminate a prospective risk that occurs x years from now, for $x = 1, 2, 3$, and 4. These numbers decline markedly with x. A risk that is only four years in the future has \$190,000 less value (about 70%) than an immediate risk. A risk that is ten years away would have less than half the value of an immediate risk. However, estimates of ε and Z at other incomes and ages are necessary to evaluate risks for younger and older persons.[10] Again, the numbers in table 2 are on the conservative side for the same reason as mentioned above.

Table 2. Value of Current and Future Risks, by Age (in 1986 Dollars)

a	$v(a,a)$	$v(a+1,a)$	$v(a+2,a)$	$v(a+3,a)$	$v(a+4,a)$
36	658,000	604,100	554,300	508,300	465,800
38	649,200	595,300	545,600	499,600	458,400
40	639,300	585,400	537,200	489,800	447,400
42	630,000	574,400	524,600	478,900	436,600
44	616,100	562,300	512,600	467,000	424,900
46	602,800	548,900	499,400		
48	588,400				

Notes: $v(a,a) = V(a) = Z \cdot A(a,.08)$, for $Z = 58,330$ from Table 1. $v(a+x,a) = (S(a+x))/(S(a)) \cdot v(a+x,a+x)/(1+r)^x$ for $r = .08$, from equation (33).

The benefit side of a project affecting age-specific mortality always respects the current values of the population. For the case at hand, total benefits are the sum $\int v(\alpha,a)\phi(a)da$, where $\phi(a)$ is the current age distribution function. Project evaluation can change as the age distribution changes. A project affecting mortality of the elderly receives a low score in a population that has relatively large numbers of young people, but the score may increase over time as the large cohort ages and the risks become more immediate.

5. Conclusions

This paper has spelled out the close connection between cross-section estimates of willingness to pay to reduce mortality risk and the valuation of changes in intertemporal survivor functions obtained from expected utility theory. It goes without saying that the most urgent needs in this area are better empirical estimates of the valuation of risks, and reconciliation of the differences in existing estimates based on different data sources.

Much work remains to be done on refining the conceptual apparatus as well. First, most models generally recognize only the two states of life and death, and do not consider illness states. This might be repaired by using a semi-Markov transition process among states, with death being the absorbing state. Since long-term disability and other serious health problems are correlated with mortality rates, risk estimates are some amalgam of the two. A more complete theory would show how to evaluate the incidence of morbidity risks. Another problem in extrapolating from table 1 to the longer-term risks in table 2 is that the manner of death may be different between immediate hazards and long-term hazards associated with lengthy periods of illness and suffering prior to death. The numbers in table 2 may be biased downward for this reason.

Second, bequests have been ignored in this study because of a belief that existing treatments are flawed. The prevailing method introduces a bequest function $B(\bar{c})$ in place of M, where $\bar{c}$ is descendants' consumption. Such a specification necessarily reduces the value of risks to life, because by leaving a large bequest, the decendent lives on through descendants. A complete analysis must incorporate the utility the person receives from dependents when alive. Surely family members are worse off when the head dies in the prime of life. A more refined treatment of altruism and preference dependencies is necessary to do justice to these issues. Since resource transfers among family members are important ways in which people cope with imperfect insurance and loan markets, such an analysis would go a long way in treating capital market imperfections as well as bequests.

Finally, the valuation formulas in (30) and (33) deal with known risks. Statistics on instantaneous exposure and risk are quite accurate in many cases, but there is much more uncertainty about the connections between exposure and changes in subsequent age-specific death rates when exposure has delayed and cumulative effects. In only a few cases, such as cigarette smoking and asbestos exposure, are the data extensive enough to determine these effects with precision. The relation between exposure and timing cannot be quantified even for such potent carcinogens as aflatoxin and vinyl chloride. Expected utility theory replaces the *pdf* $f(t)$ in (19) with a subjective *pdf* in these cases. However, the constraint (21) is affected in a different way. Uncertainty about hazards has the effect of introducing nontrivial load factors on consumption–annuity premiums because such risks are not diversifiable. The insurance company runs large risks of ruin and must charge large loads to create the necessary contingency reserves. Insurance is incomplete and may break down altogether (as it has in the case of asbestos exposure). Personal exposure to uncertain risks therefore must involve a significant degree of self-insurance, and this leads to larger valuations compared to known risks. Whether they are large enough on these grounds alone to account for the extreme social caution and prohibitions often observed among some risks remains to be shown, notwithstanding Peltzman's (1973) emphasis on the optimal production of risk information and the balancing of type I and type II errors.

Notes

1. The value of lotteries for dealing with indivisibilities and nonconvexities has been increasingly recognized in the past few years. Bergstrom (1974) was the first economist to notice the possible optimality of lotteries for mortality risks. Marshall (1984) emphasized preferences for life-risk lotteries in the context of bequests. Bergstrom (1986) gives a superb account of the case for a draft lottery over a voluntary army. Townsend (1986) presents the most complete general equilibrium theory yet available. Viscusi (1979) showed value for uncertainty in the present problem based on incomplete information and option value, which is a different basis for risk-preferring behavior. Notice that there is possible moral hazard in a randomization scheme. It is in the interests of a person to agree to the scheme ex ante and then to defect from it ex post, given that everyone else follows through with it. If everyone defects then it is the case that $v < 0$. As usual, some form of commitment is required to eliminate this problem.

2. It is easy to show that the indifference curves between W and p are convex so that v is decreasing in p: the greater the hazard rate, the more a person is willing to pay to reduce it. This must be qualified in case (ii), where the convexified indifference curve has a zero slope at high values of p and willingness to pay is zero there. Finally, differentiate (6) with respect to W to obtain $\partial v/\partial W = s/\varepsilon p > 0$, where $s = -u'' \cdot c/u'$ is the coefficient of relative risk aversion: safety is a normal good.

3. The statement is loose because the connection between ε and the intertemporal elasticity of substitution is lost if $|M|$ is large.

4. Shephard and Zeckhauser (1984) consider a lending but no borrowing constraint and simulate valuations that are very close to the full annuities case. Other possible ways of specifying such constraints may give much different results.

5. The marginal conditions for constraints $h^*(t) \leqslant h(t)$ where $h^*(t)$ is chosen to convexify preferences are

$$\lambda \int_t^\infty \left\{ \frac{1-\varepsilon}{\varepsilon} c(\tau) + w(\tau)(1 - l(\tau)) \right\} S(\tau) e^{-r(\tau - t)} d\tau \geqslant 0, \text{ all } t.$$

The intergral enters because a change in $h(t)$ has a permanent effect on future values of S (see below). The *discounted* expected surplus at every age must be nonnegative or else randomization is desirable. Notice that this condition allows instantaneois surplus (in curly braces) to take on some negative values, so long as the sum is positive or zero. Since this condition places a lower bound of zero on the value of life as before, it is ignored hereafter.

6. If $\rho = r$, then (22) implies that $Z'(t+a) = w'(t+a)(1 - l(t+a))$ and $Z(t)$ varies with the wage rate. If $\rho \neq r$, the expression for Z' is much more complicated and not very helpful.

7. The regression coefficient of the weekly wage on excess death risk is about $4 per .001 risk increment in 1968 dollars. The average person in the sample worked 50 weeks per year, for an increment of $200 per year per .001 unit of excess risk. Dividing by .001 and multiplying the result by 3.15 to convert to 1986 dollars yields V = $630,000.

8. Estimates of V based on industry rather than on occupation risks are as much as five times (!) larger than those in T–R. The implied estimates of Z are five times larger and the estimates of ε are one-fourth the size of those in table 1. Interested readers are invited to adjust the estimates in table 2 correspondingly.

9. The numbers in table 2 refer to the steady state. The transitional problem of recontracting annuities among existing cohorts after the risk has been eliminated is ignored here.

10. Suppose exposure changes the flow death rate by $\Delta(t)$ from age α onward. Here $\delta S_a(t) = 0$ for $t < a$ as before, but $\Delta(t)$ now has cumulative effects on subsequent survival rates from (28), so $\delta S(t) = S_a(t) \int_a^t \Delta(\tau)d\tau$ for $t \geqslant a$. Substituting into (26) gives a valuation formula for any age-risk pattern. In distinction to the stock experiment in (29), where the weight on future survival rates is reduced by discounting, approximation errors are greater for these flow changes because the cumulation of effects in $\delta S_a(t)$ for $t \geqslant a$ offsets the discount factor, and expected surplus at much older ages gets much greater weight. More precise knowledge of Z and ε are required to implement these more elaborate experiments.

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