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On the Stability of Periodic Motions of an Unbalanced Rigid Rotor on Lubricated Journal Bearings

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Abstract. A theoretical investigation is carried out on the orbital motions of a symmetrical, unbalanced, rigid rotor subjected to a constant vertical load and supported on two lubricated journal bearings. In order to determine the fluid film forces, the short bearing theory is adopted.

A method is illustrated that makes it possible to determine the analytical equation of the orbit as an approximated solution of the system of non-linear differential equations of motion of the journal axis. A procedure is also described for evaluating the stability of the solution found. Diagrams of the curves delimiting, in the working plane of the rotor σ–$m\sigma$, the areas of stability of the various periodic solutions determined are provided.

Finally, the results obtained are compared and combined with those provided by a direct integration of the motion equation made using the Runge–Kutta method.

Key words: Journal bearings, unbalanced rotor, sub-synchronous orbits.

Nomenclature

C	= radial clearance
$D = 2R$	= bearing diameter
E	= mass unbalance eccentricity
Fx, Fy	= fluid film force components
$f_i = F_i/\sigma W$	= dimensionless fluid film force components
L	= bearing length
M	= one half rotor mass
$m = MC\omega^2/\sigma W$	= dimensionless one half rotor mass
R	= bearing radius
$T = 2\pi$	= synchronous orbit period
t	= time
W	= load per bearing
X, Y, Z	= coordinates
$x = X/C$; $y = y/C$; $z = Z/L$	= dimensionless coordinates
μ	= oil dynamic viscosity
$\rho = E/C$	= dimensionless mass unbalance eccentricity
$\sigma = (\mu\omega RL/W)/(R/C)^2(L/D)^2$	= modified Sommerfeld number
$\tau = \omega t$	= dimensionless time
$\bar{\nu}$	= periodic orbit frequency
$\upsilon = \bar{\nu}2\pi/\omega$	= frequency ratio
ω	= journal angular velocity
$(\cdot)$	= dimensionless time derivative

1. Introduction

It is not uncommon in technical practice for a rotor to operate with an unbalance that is greater than the residual unbalance. This may occur for a variety of accidental reasons. In these conditions, the non-linear nature of the fluid dynamic forces exerted by the oil film makes the journal axis of the unbalanced rotor move inside the bearing with a time motion that may or may not be periodic according to the values assumed by the three parameters: mass, Sommerfeld number, and unbalance (m, σ, ρ) characterizing the system.

In the ambit of the periodic motions it is possible to identify values for the above parameters that give motions which are synchronous with the unbalancing force, or motions whose period is an integer multiple of the period of the above force.

In the ambit of the non-periodic motions these rotors typically exhibit the "oil whirl motion", which is characterized by a frequency component close but not equal to 1/2 the forcing frequency, i.e. the rotation frequency [1, 2, 8].

Finally, it has recently been shown that chaotic evolutions of the motion of the journal axis can occur [9, 11].

Although from an analytical point of view all the periodic motions that can be observed are represented by stable solutions of the differential equation of the rotor axis motion, in technical practice the behaviour of the above mentioned rotor is said to be stable only if its axis describes a fairly small orbit that is synchronous with the rotor's rotation velocity.

On the other hand, the behaviour of the rotor is said to be unstable if its axis describes any of the other motions laid out above. This terminology arises from the fact that as the angular velocity increases, the rotor of the machine first describes synchronous and small orbits due to the unbalance, followed by orbits with an integer multiple period (sub-synchronous whirl) or which are aperiodic, and in any case of a size such as to compromise the correct operation of the whole system.

Clearly, therefore, it is very useful for the system designer to know the limit stability curves of the synchronous orbit.

In the present paper a method for searching for the analytical expression of periodic orbits is illustrated. This is a general-purpose method in that it makes it possible to identify a periodic solution, if one exists, simply by varying one parameter (N) characterizing its periodicity. It thus also lends itself for a straightforward numerical implementation.

Knowing the orbits equation it is also possible to evaluate their stability and thus identify, in the operating plane σ–$m\sigma$, not only the limit stability curve of the synchronous solution but also different areas in each of which the system displays a particular behaviour.

In order to evaluate the goodness of the results obtained, the motion equations were numerically integrated using a Runge–Kutta method that, after an initial transient, provides the diagram of the orbit described by the journal. The same method has been used to obtain the curves delimiting the areas of aperiodic behaviour that cannot be predicted by the proposed analysis.

2. Analysis

The system in question is made up of a rigid, symmetrical, unbalanced rotor with a mass of $2M$ subjected to a constant radial load of $2W$ applied in its middle plane and supported on two equal lubricated journal bearings.

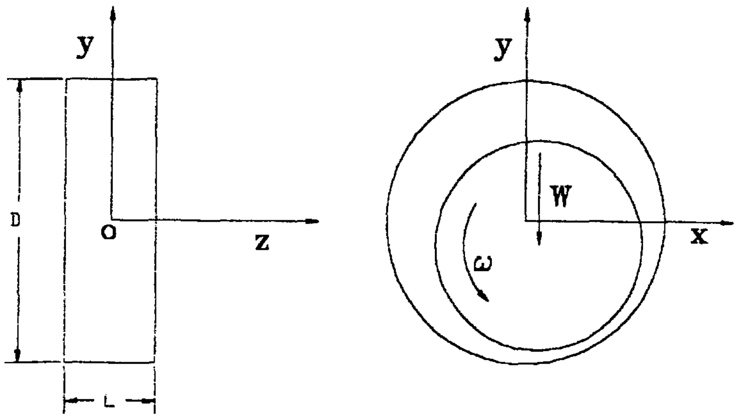

Figure 1. Notations for the journal bearing pair.

The rotor's symmetry about the middle plane makes it possible to consider only one of the parts into which the system is divided by this plane (Figure 1).

By making use of the dimensionless quantities listed in the nomenclature, the plane motion equation of the journal axis can be written in the following form:

$$m\ddot{x} - f_x(x, y, \dot{x}, \dot{y}) - m\rho \cos\tau = 0$$
$$m\ddot{y} - f_y(x, y, \dot{x}, \dot{y}) - m\rho \sin\tau + \frac{1}{\sigma} = 0. \tag{1}$$

The non-linear term that appears in (1) is made up of the fluid dynamic force components whose analytical expression, under the short bearing hypothesis, is given by [6]

$$\begin{Bmatrix} f_x \\ f_y \end{Bmatrix} = \frac{[(x-2\dot{y})^2 + (y+2\dot{x})^2]^{1/2}}{1 - x^2 - y^2} \times \begin{Bmatrix} 3xV(x,y,\alpha) - \sin(\alpha)G(x,y,\alpha) - 2\cos(\alpha)F(x,y,\alpha) \\ 3yV(x,y,\alpha) + \cos(\alpha)G(x,y,\alpha) - 2\sin(\alpha)F(x,y,\alpha) \end{Bmatrix}. \tag{2}$$

The expression of the functions G, V, F and α is given in the Appendix.

The solution $x(\tau)$, $y(\tau)$ of the system of differential equations (1) makes up the orbit described by the journal axis.

3. Determination of the Analytical Expression of the Orbit

The analytical determination of the orbit described by the journal axis is performed by searching for an approximated solution of the system of non-linear equations (1) in the form:

$$x(\tau) = x_0 + \sum_{i=1}^{k} x_{ci} \cos\left(\frac{\tau}{N}\, i\right) + x_{si} \sin\left(\frac{\tau}{N}\, i\right)$$
$$y(\tau) = y_0 + \sum_{i=1}^{k} y_{ci} \cos\left(\frac{\tau}{N}\, i\right) + y_{si} \sin\left(\frac{\tau}{N}\, i\right), \tag{3}$$

where making $N = 1$ clearly determines the synchronous solution ($T = 2\pi$), for $N \neq 1$ it determines the solution with a period NT.

Since the fluid dynamic forces must have the same periodic character along the orbit defined by (3) the Fourier series development can be adopted for them [7, 10]:

$$\begin{aligned} f_x(\tau) &= f_{x0} + \sum_{i=1}^{k} f_{xci} \cos\left(\frac{\tau}{N} i\right) + f_{xsi} \sin\left(\frac{\tau}{N} i\right) \\ f_y(\tau) &= f_{y0} + \sum_{i=1}^{k} f_{yci} \cos\left(\frac{\tau}{N} i\right) + f_{ysi} \sin\left(\frac{\tau}{N} i\right). \end{aligned} \tag{4}$$

Substituting (3) and (4) into the motion equation (1) gives

$$\begin{aligned} & m \sum_{i=1}^{k} \left[\left(\frac{i}{N}\right)^2 \left(x_{ci} \cos\left(\frac{\tau}{N} i\right) + x_{si} \sin\left(\frac{\tau}{N} i\right)\right)\right] \\ & \quad + f_{x0} + \sum_{i=1}^{k} \left[f_{xci} \cos\left(\frac{\tau}{N} i\right) + f_{xsi} \sin\left(\frac{\tau}{N} i\right)\right] + m\rho \cos\tau = 0 \\ & m \sum_{i=1}^{k} \left[\left(\frac{i}{N}\right)^2 \left(y_{ci} \cos\left(\frac{\tau}{N} i\right) + y_{si} \sin\left(\frac{\tau}{N} i\right)\right)\right] \\ & \quad + f_{y0} + \sum_{i=1}^{k} \left[f_{yci} \cos\left(\frac{\tau}{N} i\right) + f_{ysi} \sin\left(\frac{\tau}{N} i\right)\right] + m\rho \sin\tau - \frac{1}{\sigma} = 0. \end{aligned} \tag{5}$$

Grouping the terms in sine and cosine of the same argument, equation (5) can be put in the form:

$$\begin{aligned} & A_{x0}(\mathbf{a}) + \sum_{i=1}^{k} A_{xci}(\mathbf{a}) \cos\left(\frac{\tau}{N} i\right) + A_{xsi}(\mathbf{a}) \sin\left(\frac{\tau}{N} i\right) = 0 \\ & A_{y0}(\mathbf{a}) + \sum_{i=1}^{k} A_{yci}(\mathbf{a}) \cos\left(\frac{\tau}{N} i\right) + A_{ysi}(\mathbf{a}) \sin\left(\frac{\tau}{N} i\right) = 0, \end{aligned} \tag{6}$$

where $\mathbf{a}$ is used to indicate the vector of the $4k + 2$ unknowns:

$$\mathbf{a} = (x_0, x_{ci}, x_{si}, y_0, y_{ci}, y_{si}); \qquad i = 1 \ldots k.$$

By making the $4k + 2$ coefficients of equation (6) separately equal to zero gives the system of algebraic equations:

$$\begin{aligned} A_{x0} &= f_{x0} = \frac{1}{2N\pi} \int_0^{2N\pi} f_x \, \mathrm{d}\tau = 0 \\ A_{xci} &= f_{xci} + m \left(\frac{i}{N}\right)^2 x_{ci} + Sm\rho \\ &= \frac{1}{N\pi} \int_0^{2N\pi} f_x \cos\left(\frac{\tau}{N} i\right) \mathrm{d}\tau + m \left(\frac{i}{N}\right)^2 x_{ci} + Sm\rho = 0 \end{aligned}$$

$$A_{xsi} = f_{xsi} + m\left(\frac{i}{N}\right)^2 x_{si} = \frac{1}{N\pi}\int_0^{2N\pi} f_x \sin\left(\frac{\tau}{N}i\right)\mathrm{d}\tau + m\left(\frac{i}{N}\right)^2 x_{si} = 0$$

$$A_{y0} = f_{y0} - \frac{1}{\sigma} = \frac{1}{2N\pi}\int_0^{2N\pi} f_y\,\mathrm{d}\tau - \frac{1}{\sigma} = 0$$

$$A_{yci} = f_{yci} + m\left(\frac{i}{N}\right)^2 y_{ci} = \frac{1}{N\pi}\int_0^{2N\pi} f_y \cos\left(\frac{\tau}{N}i\right)\mathrm{d}\tau + m\left(\frac{i}{N}\right)^2 y_{ci} = 0$$

$$\begin{aligned} A_{ysi} &= f_{ysi} + m\left(\frac{i}{N}\right)^2 y_{si} + Sm\rho \\ &= \frac{1}{N\pi}\int_0^{2N\pi} f_y \sin\left(\frac{\tau}{N}i\right)\mathrm{d}\tau + m\left(\frac{i}{N}\right)^2 y_{si} + Sm\rho = 0 \end{aligned}$$

$$\text{with}\quad i = 1\ldots k \quad\text{and}\quad S = \begin{cases} 0 & \text{if } i \neq N, \\ 1 & \text{if } i = N. \end{cases} \tag{7}$$

The solution of the system of equations (7) obtained with the Newton–Raphson method provides the coefficients of the analytical expression of the approximated orbit.

4. Analysis of the Stability of the Periodic Solutions

In order to investigate its stability the periodic solution $(x^*(\tau), y^*(\tau))$ is disturbed:

$$\begin{aligned} \tilde{x}(\tau) &= x^*(\tau) + \delta x(\tau) \\ \tilde{y}(\tau) &= y^*(\tau) + \delta y(\tau). \end{aligned} \tag{8}$$

Substituting (8) into the motion equation (1), gives

$$\begin{aligned} &m(\ddot{x}^* + \delta\ddot{x}) - f_x(x^* + \delta x, y^* + \delta y, \dot{x}^* + \delta\dot{x}, \dot{y}^* + \delta\dot{y}) - m\rho\cos(\tau) = 0 \\ &m(\ddot{y}^* + \delta\ddot{y}) - f_y(x^* + \delta x, y^* + \delta y, \dot{x}^* + \delta\dot{x}, \dot{y}^* + \delta\dot{y}) - m\rho\sin(\tau) + \frac{1}{\sigma} = 0. \end{aligned} \tag{9}$$

Expanding the fluid dynamic forces in (9) into Taylor series around x^*, y^* gives the variation equation:

$$\begin{aligned} m\delta\ddot{x} - \left(\frac{\partial f_x}{\partial x}\right)_* \delta x - \left(\frac{\partial f_x}{\partial y}\right)_* \delta y - \left(\frac{\partial f_x}{\partial \dot{x}}\right)_* \delta\dot{x} - \left(\frac{\partial f_x}{\partial \dot{y}}\right)_* \delta\dot{y} &= 0 \\ m\delta\ddot{y} - \left(\frac{\partial f_y}{\partial x}\right)_* \delta x - \left(\frac{\partial f_y}{\partial y}\right)_* \delta y - \left(\frac{\partial f_y}{\partial \dot{x}}\right)_* \delta\dot{x} - \left(\frac{\partial f_y}{\partial \dot{y}}\right)_* \delta\dot{y} &= 0. \end{aligned} \tag{10}$$

The partial derivatives appearing in equation (10) are periodic quantities with a period NT equal to that of the solution x^*, y^* and can thus be developed in a Fourier series.

$$\left(\frac{\partial f_p}{\partial q}\right)_* = K_{pq_0} + \sum_{i=1}^{k} K_{pq_{ci}} \cos\left(\frac{\tau}{N}i\right) + K_{pq_{si}} \sin\left(\frac{\tau}{N}i\right) = -\bar{K}_{pq}$$

$$\left(\frac{\partial f_p}{\partial \dot{q}}\right)_* = B_{pq0} + \sum_{i=1}^{k} B_{pq_{ci}} \cos\left(\frac{\tau}{N}\, i\right) + B_{pq_{si}} \sin\left(\frac{\tau}{N}\, i\right) = -\bar{B}_{pq}$$

$$\text{with} \quad p = x, y \quad \text{and} \quad q = x, y. \tag{11}$$

Equations (10), after the substitution (11), are Hill's equations. The instability threshold of the solution x^*, y^* with a period NT is searched for as the instability threshold of the trivial solution of the variational equation. At the threshold the above equation admit a periodic solution with a period $2NT$ [4] which has an approximated expression of the type:

$$\begin{aligned}
\delta x(\tau) &= \delta x_0 + \sum_{i=1}^{L} \delta x_{ci} \cos\left(\frac{\tau}{2N}\, i\right) + \delta x_{si} \sin\left(\frac{\tau}{2N}\, i\right) = \overline{\delta x} \\
\delta y(\tau) &= \delta y_0 + \sum_{i=1}^{L} \delta y_{ci} \cos\left(\frac{\tau}{2N}\, i\right) + \delta y_{si} \sin\left(\frac{\tau}{2N}\, i\right) = \overline{\delta y}.
\end{aligned} \tag{12}$$

Substituting the approximated solution (12) into the variational equation (10), after inserting (11), gives the residual expression:

$$\begin{aligned}
\delta\varepsilon_x &= m\ddot{\overline{\delta x}} + \bar{K}_{xx}\overline{\delta x} + \bar{K}_{yy}\overline{\delta y} + \bar{B}_{xx}\dot{\overline{\delta x}} + \bar{B}_{xy}\dot{\overline{\delta y}} \\
\delta\varepsilon_y &= m\ddot{\overline{\delta y}} + \bar{K}_{yx}\overline{\delta x} + \bar{K}_{yy}\overline{\delta y} + \bar{B}_{yx}\dot{\overline{\delta x}} + \bar{B}_{yy}\dot{\overline{\delta y}}.
\end{aligned} \tag{13}$$

Applying the harmonic balance method [3, 5] to the system of equations (13) gives the system of $4L + 2$ algebraic equations:

$$\begin{aligned}
\delta A_{x0}(\delta\mathbf{a}) &= \int_0^{2N(2\pi)} \delta\varepsilon_x \, \mathrm{d}\tau = 0 \\
\delta A_{xci}(\delta\mathbf{a}) &= \int_0^{2N(2\pi)} \delta\varepsilon_x \cos\left(\frac{\tau}{2N}\, i\right) \mathrm{d}\tau = 0 \\
\delta A_{xsi}(\delta\mathbf{a}) &= \int_0^{2N(2\pi)} \delta\varepsilon_x \sin\left(\frac{\tau}{2N}\, i\right) \mathrm{d}\tau = 0 \\
\delta A_{y0}(\delta\mathbf{a}) &= \int_0^{2N(2\pi)} \delta\varepsilon_y \, \mathrm{d}\tau = 0 \\
\delta A_{yci}(\delta\mathbf{a}) &= \int_0^{2N(2\pi)} \delta\varepsilon_y \cos\left(\frac{\tau}{2N}\, i\right) \mathrm{d}\tau = 0 \\
\delta A_{ysi}(\delta\mathbf{a}) &= \int_0^{2N(2\pi)} \delta\varepsilon_y \sin\left(\frac{\tau}{2N}\, i\right) \mathrm{d}\tau = 0
\end{aligned} \tag{14}$$

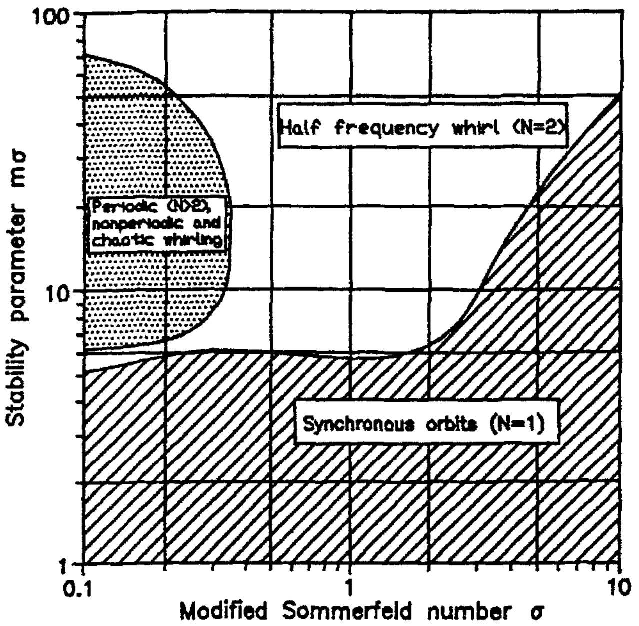

Figure 2. Stability map for $\rho = 0.2$.

with $\quad i = 1 \ldots L$

in the $4L + 2$ unknowns:

$$\delta\mathbf{a} = (\delta x_0, \delta x_{ci}, \delta x_{si}, \delta y_0, \delta y_{ci}, \delta y_{si}); \qquad i = 1 \ldots L$$

it is possible to check that the system of $4L+2$ equations (14) is uncoupled into two subsystems: one for the $2L + 2$ unknowns: δx_0, δy_0, δx_{ci}, δx_{si}, δy_{ci}, δy_{si} with i even, the other for the remaining $2L$ unknowns δx_{ci}, δx_{si}, δy_{ci}, δy_{si} with i odd. For the system equations (14) to give non-trivial solutions, one of the two determinants of the odd or even coefficients has to be equal to zero.

Since it is always the odd determinant that is nullified, the described method makes it possible to identify the curves marking the boundary between two periodic stable solutions: one with a period NT and the other with a period $2NT$.

The single terms of the determinant of the matrix $2L \times 2L$ of the coefficients of the odd subsystem of (14) have been determined by making recourse to a symbolic computing program that made it possible to perform the defined integrals and to group the coefficients of the single unknowns in a straightforward way.

For a given unbalance value the illustrated procedure makes it possible to draw limit stability curves of periodic solutions as the locus of the values σ and $m\sigma$ that nullify the determinant of the system of equations (14).

5. Results

By applying the method proposed over the whole normal operating range of the journal bearings, the maps shown in Figures 2 and 3, referring to dimensionless unbalance values of 0.2 and 0.3, were obtained. In the Fourier series (4), (11) and (12), k and L are equal to 4.

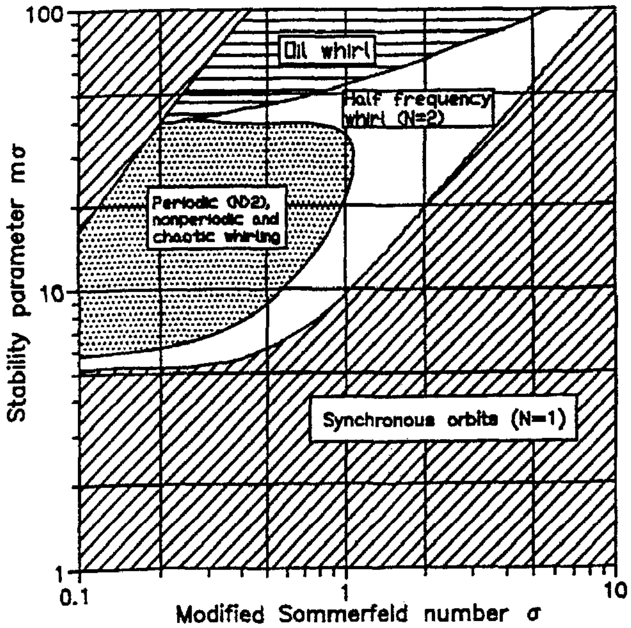

Figure 3. Stability map for $\rho = 0.3$.

In the plane σ–$m\sigma$ the areas of different dynamic behaviour of the rotor have been identified. The lower area is that of the so-called "stable operation". For values of σ and $m\sigma$ falling within this area, the unbalance makes the journal describe an orbit that is synchronous with the rotation velocity and small in size. To the top this area borders with that of the "half frequency whirl" inside which the journal motion is still a periodic motion but is characterized by a sub-synchronous component with a frequency v equal exactly to 1/2.

Inside the above area there extends another area within which the rotor displays a series of behaviours including both the periodic type with sub-synchronous fundamental frequencies in the order of 1/4, 1/8, 1/16 and so on, and the non-periodic type. In this area it is also possible to encounter chaotic behaviours. The boundary of this area has nevertheless been obtained with the proposed method as, in any case, it is the limit between two periodic solutions, one with a fundamental 1/2, the other with a fundamental 1/4.

It has been possible to check that by proceeding from the boundary into the area in question, the system reaches the non-periodic behaviour, through a number of bifurcations, and in some cases becomes chaotic.

In the case where $\rho = 0.3$, in the upper part of the map two other areas can be identified. One area inside which the journal has an almost periodic motion characterized by a large frequency component close but not equal to 0.5 which in the literature is generally indicated as "oil whirl", and another area inside which motion is once again synchronous. The journal "recovers synchronism" by describing synchronous orbits, but this time they are large in size. In the explored range, for $\rho = 0.2$, synchronism recovery and oil whirl are not observed as the area of the half frequency whirl is very large. As they represent the passage from periodic to non-periodic solutions, the curves delimiting the area of the oil whirl cannot be determined using the above illustrated method. They have been obtained point by point, by integrating the equations of motion with a fourth order Runge–Kutta method and analyzing the results.

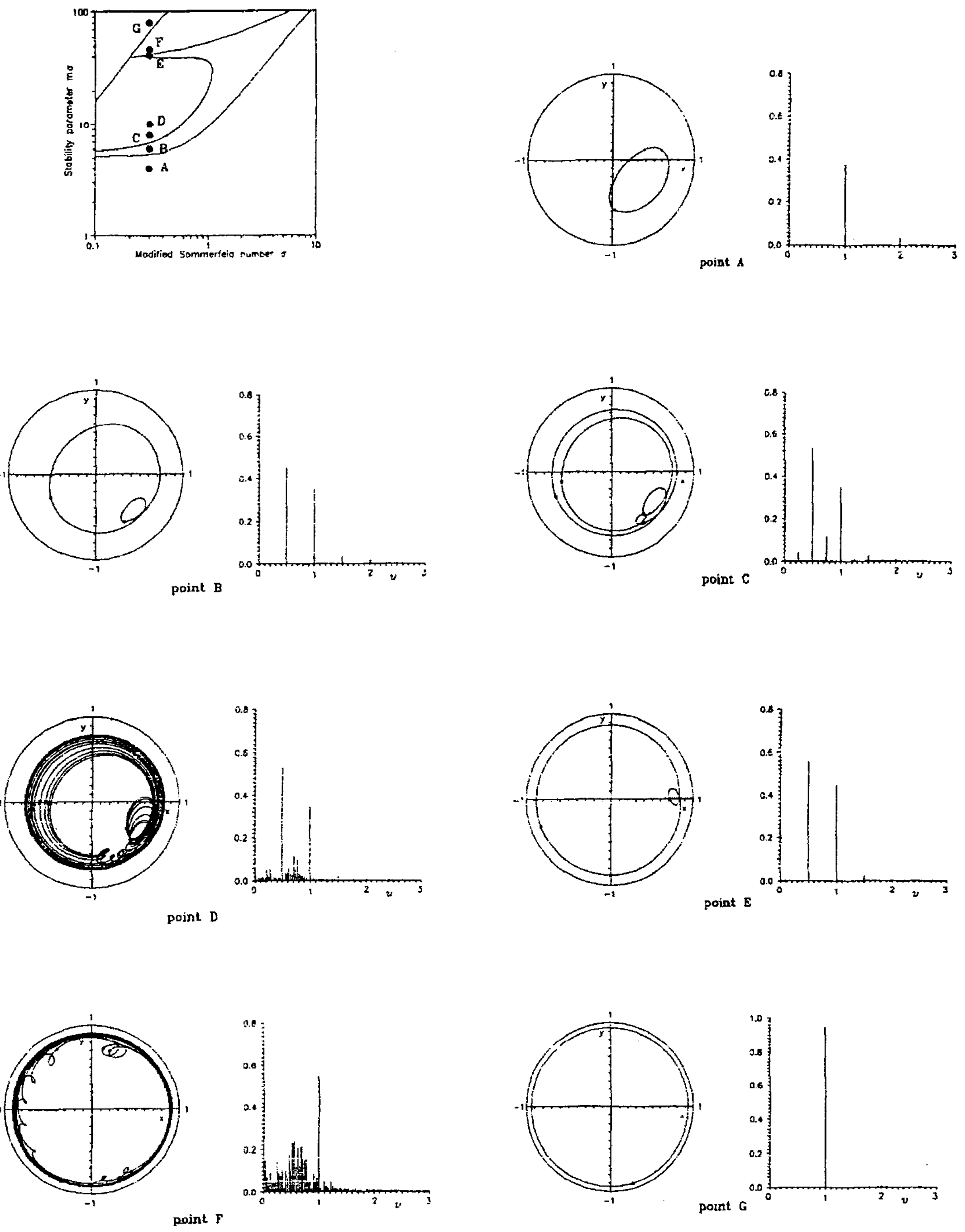

Figure 4. Journal orbits and their FFT for $\rho = 0.3$ and operating conditions corresponding to the point indicated on the map.

To give an example of the above mentioned behaviours, Figure 4 shows the trajectories described by the journal axis for operating conditions corresponding to the points indicated on the map. Alongside the trajectories in the x–y plane are shown the FFTs performed on the

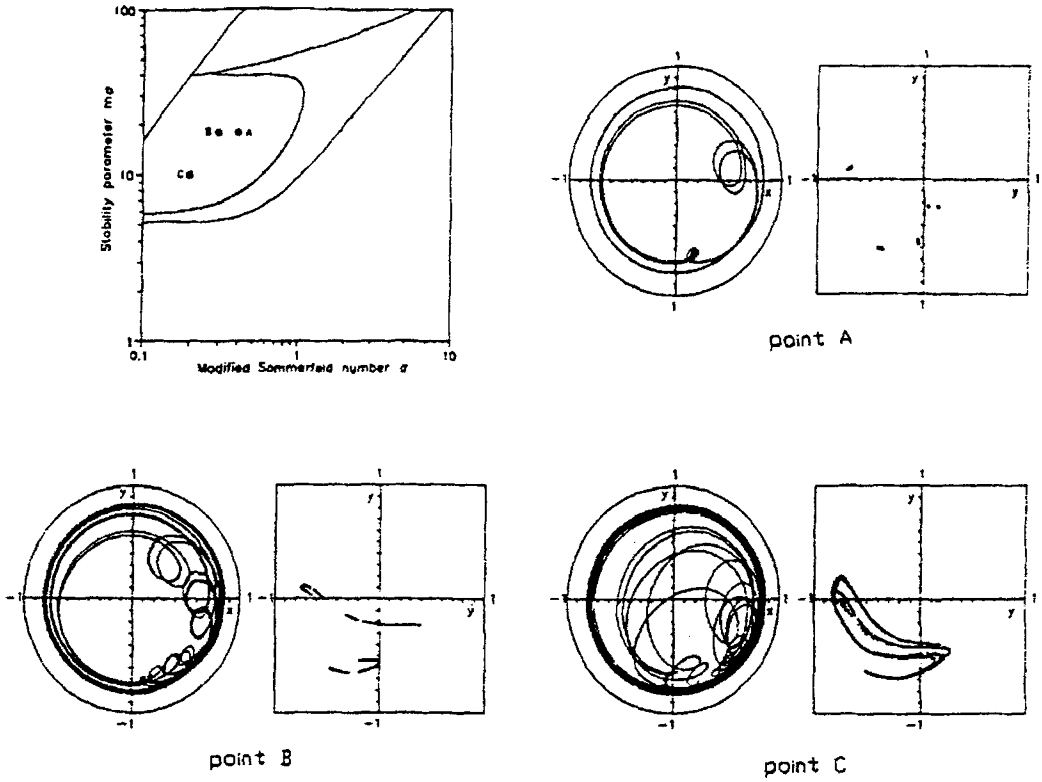

Figure 5. Journal orbits and their attractors in the phase plane (y–$\dot{y}$) for $\rho = 0.3$ and operating conditions corresponding to the points indicated on the map.

component along the y axis of the motion which make it possible to obtain an unequivocal definition of the motion.

Other examples of trajectories are provided in Figure 5. In this case, we have chosen operating conditions contained in the area of motions with a fundamental less than 1/2. Alongside the trajectories in the x–y plane are shown the attractors in the phase plane (y–$\dot{y}$). The attractors refer to 2500 journal revolutions after a sufficient number of initial revs needed for the transient to be extinguished, whereas the orbits refer to a few dozen revolutions after the initial revs. An analysis of the results shows that, in case A the motion is periodic with a fundamental frequency of 1/8, while in case B the motion can be said to be non-periodic because of the shape of the attractor; and finally, in case C the motion is chaotic, as is clearly indicated by the attractor's characteristic structure.

6. Conclusions

A method has been described for determining the operating maps of rigid, symmetrical and unbalanced rotors on lubricated journal bearings. In these maps, for each pair of modified Sommerfeld number and stability parameter values, it is possible to identify the type of orbit and the relative frequencies that the journal axis describes inside the bearing. Naturally, for the system to operate correctly, the rotor must describe a synchronous and small orbit, and so the conditions falling in that lower area of the maps must be achieved in the design phase.

The upper areas each bound different behaviours that the non-linear system displays. These behaviours must be avoided as they are characterized by large orbits that are not synchronous but, rather, have a more large harmonic content that can cause dangerous vibrations in the whole system. The curves, obtained using the proposed methods, have been tested at several points. The curve delimiting the area of synchronous orbits is very accurate whereas the one delimiting the area of the various sub-synchronous orbits is of less precision.

This inaccuracy, which does not affect the quality of the results, is due to the approximation inherent to the method and may be reduced even if this entails a greater computing overhead.

In the above conditions, the size of the orbits and their harmonic content are such as to require a greater number of terms in the series developments used if improved accuracy is to be attained. In particular, the curve for the case with an unbalance of 0.2 delimits an area that is much larger than the one indicated by the tests performed. The corresponding curve for the case with an unbalance of 0.3 is more accurate as the uncertainty is limited to the area in which it joints the oil whirl curve.

Appendix

$$G(x,y,\alpha) = \frac{2}{(1-x^2-y^2)^{1/2}}\left(\frac{\pi}{2} + \tan^{-1}\frac{y\cos(\alpha) - x\sin(\alpha)}{(1-x^2-y^2)^{1/2}}\right)$$

$$F(x,y,\alpha) = \frac{x\cos(\alpha) + y\sin(\alpha)}{1-(x\cos(\alpha)+y\sin(\alpha))^2}$$

$$V(x,y,\alpha) = \frac{2 + (y\cos(\alpha) - x\sin(\alpha))G(x,y,\alpha)}{1-x^2-y^2}$$

$$\alpha = \tan^{-1}\frac{y+2\dot{x}}{x-2\dot{y}}\,.$$

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