Journal of Elasticity, Vol. 12, No. 1, January 1982

The Hague
Printed in the Netherlands

Interaction of elastic waves with a penny-shaped crack in an infinitely long cylinder

K. N. SRIVASTAVA, R. M. PALAIYA and O. P. GUPTA

M.A. College of Technology, Bhopal-462007, India

(Received May 29, 1978; revised December 5, 1979)

ABSTRACT

This paper contains an analysis of the interaction of longitudinal waves with a penny-shaped crack located in an infinitely long elastic cyclinder. The problem is reduced to a Fredholm integral equation of the second kind which is solved numerically for a range of values of the frequency of the incident waves and the radius of the cylinder. Numerical values of the dynamic stress intensity factor at the rim of the crack have been calculated.

1. Introduction

Recently, great interest has been shown in the problems of the interaction of elastic waves with cracks situated in elastic solids. The study of these problems is motivated by their applications to seismology and exploration geophysics. Elastic waves propagating in a solid are modified by the presence of cracks, and a study of wave diffraction can yield information regarding the nature of flow in the interior of the solids.

The stress and strain fields produced in solids containing cracks due to a given static loading of various types have been discussed in great detail in the literature (see, e.g. [1]). For oscillatory loading, however, the problem becomes more complex due to the presence of additional wave length parameters, namely the wave lengths of the propagating longitudinal and shear waves. In recent years a number of such problems have been studied. Robertson [2], Mal [3, 4, 5, 6, 7], Sih and Leober [8] have studied the diffraction of longitudinal and torsional waves which are normally incident on the crack surface. All these attempts have been based on the assumption that the crack is sufficiently far away from the neighbouring boundaries, and hence the distribution of stresses in the solid is attributed to crack geometry or to the wave frequency of the elastic waves. Mathematically speaking, the boundaries of the solid are assumed to be infinitely distant from the crack. Like static problems, the boundary value problems of the interaction of elastic waves with cracks near the free boundaries are difficult to solve since they involve additional geometric parameters describing the dimensions of the solids.

In this paper we have considered one such problem in which a penny-shaped crack is situated in an infinitely long elastic cylinder and interacted by an elastic wave. As is well known, elastic waves can be decomposed into longitudinal and torsional waves. The displacement fields corresponding to the longitudinal and torsional waves are respectively parallel and perpendicular to the direction of propagation. In both cases, the amplitude of the displacement will, in general, be a function of position. In this paper our object is to consider only the longitudinal waves. The problem concerning the torsional waves shall be reported somewhere else. Using Fourier sine and cosine transforms as well as Hankel transforms, the solution of the displacement equations appropriate to our problem have been obtained. The problem is then reduced to the solution of a Fredholm integral equation of the second kind. Mal [6] has solved the problem of interaction of longitudinal waves with a penny-shaped crack situated in an infinite elastic solid. He has taken the amplitude of the longitudinal waves equal to be a constant. Sneddon and Welch [9] have studied the distribution of stress in an elastic cylinder containing a penny-shaped crack and opened by a constant pressure applied to the surface of the crack. In order to compare our results with those of Mal [6] and Sneddon and Welch [9], we perform numerical calculations by taking the amplitude of the displacement field equal to a constant. Numerical values of the dynamic stress intensity factor at the rim of the crack have been obtained for different wave frequencies and radii of the cylinder. The results are tabulated and illustrated graphically. A comparison of these results is given in the last section of this paper.

2. Formulation of the problem

Consider an infinitely long, isotropic, homogeneous elastic cylinder of radius 'a' containing a circular crack perpendicular to its axis. Consider a cylindrical polar coordinate system (r, θ, z) at the center of the crack and normalize all lengths with respect to the radius of the crack so that it is located at $z=0$, $0 \le r \le 1$. The crack is assumed to be excited by normally incident elastic waves moving in the positive direction of z-axis. Let ω be the circular frequency of the incident waves. The problem of determining the stresses in the vicinity of the crack is equivalent to that of finding the distribution of stresses in a semi-infinite cylinder $z \ge 0$, $0 \le r \le a$, when the plane boundary $z=0$ is subjected to the boundary conditions

$$\tau_{zz}(r, 0) = -p_s - p(r)\exp(-i\omega t), \qquad 0 \le r \le 1, \tag{2.1}$$

$$\tau_{rz}(r, 0) = 0, \qquad 0 \le r \le a, \tag{2.2}$$

$$u_z(r, 0) = 0, \qquad 1 \le r \le a \tag{2.3}$$

The curved surface of the cylinder is supposed to be free from traction. This implies that on the surface $r = a$ we have

$$\tau_{rr}(a, z) = \tau_{rz}(a, z) = 0, \qquad 0 \leq z < \infty. \tag{2.4}$$

In condition (2.1) the static pressure p_s is assumed to be sufficiently large to ensure that the two faces of the crack do not come in contact during vibrations. Since the solution of the static problem may be superimposed on the dynamical problem, condition (2.1) can be written as

$$\tau_{zz}(r, 0) = -p(r)\exp(-i\omega t), \qquad 0 \leq r \leq 1. \tag{2.1}$$

In what follows the time dependence of all the quantities assumed to be of the form $\exp(-i\omega t)$ will be suppressed.

3. The equations of equilibrium and their solutions

It is well known that the time harmonic equation for the displacement vector $\boldsymbol{u}$, in the absence of body forces, satisfies the equation

$$k_1^{-2}\nabla(\nabla \cdot \boldsymbol{u}) - k_2^{-2}\nabla \times (\nabla \times \boldsymbol{u}) + \boldsymbol{u} = \boldsymbol{0}, \tag{3.1}$$

where $k_1^2 = \rho\omega^2/\lambda + 2\mu$ and $k_2^2 = \rho\omega^2/\mu$, λ and μ are Lame's elastic constants, ρ is the density, and ω is the circular frequency of the incident waves. It may be observed that the dimensions of k_1 and k_2 are $(\text{length})^{-1}$. By taking the crack radius as our unit of measurement of length, we have made k_1 and k_2 dimensionless. In the case of symmetry with respect to the z-axis, the above equations reduce to the following two equations involving the radial and vertical components (u_r, u_z) of the displacement vector:

$$\left.\begin{aligned} &k_1^{-2}(u_{r,rr} + r^{-1}u_{r,r} - r^{-2}u_r) + k_2^{-2}u_{r,zz} + (k_1^{-2} - k_2^{-2})u_{z,rz} + u_r = 0; \\ &k_1^{-2}u_{z,rr} + k_2^{-2}(u_{z,rr} + r^{-1}u_{z,r}) + (k_1^{-2} - k_2^{-2})u_{z,rz} + u_z = 0. \end{aligned}\right\} \tag{3.2}$$

Here a comma is used to denote partial derivatives. Expressions for the components of displacement vectors with the property that the shearing stress vanishes on the $z = 0$ plane are

$$u_r(r, z) = \mathscr{H}_1[\{\exp(-\alpha z) - 2\alpha\beta\exp(-\beta z)/(2y^2 - k_2^2)\}A(y);\ y \to r], \tag{3.3}$$

$$u_z(r, z) = \mathscr{H}_0[\{y^{-1}\alpha\exp(-\alpha z) - 2\alpha y\exp(-\beta z)/(2y^2 - k_2^2)\}A(y);\ y \to r], \tag{3.4}$$

where $\alpha^2 = y^2 - k_1^2$, $\beta^2 = y^2 - k_2^2$, and $\mathscr{H}_\nu$ denotes the Hankel transform of order ν. The solution of the system of equations (3.2) for an elastic cylinder can be obtained with the help of Fourier sine and cosine transforms. The corresponding expressions for the components of the displacement vector are

$$u_r(r, z) = \mathscr{F}_c[\{B(x)I_1(\alpha_1 r) + x\beta_1 C(x)I_1(\beta_1 r)\};\ x \to z], \tag{3.5}$$

$$u_z(r, z) = -\mathscr{F}_s[\{xB(x)I_0(\alpha_1 r) + \beta_1^2 C(x)I_0(\beta_1 r)\};\ x \to z], \tag{3.6}$$

where $\alpha_1^2 = x^2 - k_1^2$, $\beta_1^2 = x^2 - k_2^2$, and $\mathscr{F}_s$ and $\mathscr{F}_c$ denote Fourier sine cosine transforms respectively.

The complete solution of equations (3.2) for the problems stated above is given by the equations

$$u_r(r, z) = \mathscr{F}_c[\{B(x)I_1(\alpha_1 r) + x\beta_1 C(x)I_1(\beta_1 r)\}: x \to z] \\ + \mathscr{H}_1[\{\exp(-\alpha z) - 2\alpha\beta \exp(-\beta z)/(2y^2 - k_2^2)\}A(y);\ y \to r], \tag{3.7}$$

$$u_z(r, z) = -\mathscr{F}_s[\{xB(x)I_0(\alpha_1 r) + \beta_1^2 C(x)I_0(\beta r)\};\ x \to z] \\ -\mathscr{H}_0[\{y^{-1}\exp(-\alpha z) - 2\alpha y \exp(-\beta z)/(2y^2 - k_2^2)\}A(y);\ y \to r]. \tag{3.8}$$

By using the stress-strain relations, one can calculate the components of the stress tensor. The resulting expressions are

$$\mu^{-1}\tau_{zz}(r, z) = -\mathscr{F}_c[\{(2\alpha_1^2 + k_2^2)B(x)I_0(\alpha_1 r) + 2x\beta_1^2 I_0(\beta_1 r)C(x)\};\ x \to z] \\ -\mathscr{H}_0[\{(2y^2 - k_2^2)\exp(-\alpha z) - 4\alpha\beta\exp(-\beta z)/(2y^2 - k_2^2)\}y^{-1}A(y); \\ y \to r], \tag{3.9}$$

$$\mu^{-1}\tau_{rz}(r, z) = -\mathscr{F}_s[\{B(x)2x\alpha_1 I_1(\alpha_1 r) + (2x^2 - k_2^2)\beta_1 I_1(\beta_1 r)C(x),\ x \to z] \\ -\mathscr{H}_1[2\alpha\{\exp(-\alpha z) - \exp(-\beta z)\}A(y);\ y \to r], \tag{3.10}$$

$$\mu^{-1}\tau_{rr}(r, z) = \mathscr{F}_c[(\{(2x^2 - k_2^2)I_0(\alpha_1 r) - 2\alpha_1 r^{-1}I_0(\alpha_1 r)\}B(x) \\ + 2x\beta_1\{\beta_1 I_0(\beta_1 r) - r^{-1}I_1(\beta_1 r)\}C(x));\ x \to z] \\ + \mathscr{H}_0[\{(2\alpha^2 + k_2^2)\exp(-\alpha z) - 4\alpha\beta y^2\exp(-\beta z)/(2y^2 - k_2^2)\}y^{-1}A(y); \\ y \to r] \\ -\mathscr{H}_1[2\{\exp(-\alpha z) - 2\alpha\beta\exp(-\beta z)/(2y^2 - k_2^2)\}y^{-1}A(y);\ y \to r]. \tag{3.11}$$

4. Reduction of the problem to a Fredholm integral equation

We divide the solution into two parts:

(a) *Conditions on the crack face*

We see that condition (2.2) is satisfied, while conditions (2.1′) and (2.3) lead to the dual integral equations

$$\int_0^\infty [\{(2y^2 - k_2^2)^2 - 4\alpha\beta y^2\}/(2y^2 - k_2^2)]A(y)J_0(yr)\,dy = P(r), \qquad 0 \leq r \leq 1,$$

$$\int_0^\infty (2y^2 - k_2^2)^{-1}A(y)J_0(yr)\,dy = 0, \qquad 1 \leq r \leq a,$$

where $P(r) = \mu^{-1}p(r) - (2/\pi)^{1/2}\int_0^\infty\{(2\alpha_1^2 + k_2^2)I_0(\alpha_1 r)B(x) + 2x\beta_1 I_0(\alpha_1 x)\}C(x)\,dx$. Let $2\alpha(2y^2 - k_2^2)^{-1}(k_1^2 - k_2^2)A(y) = \phi(y)$. With this assumption the above equations can be written as

$$\int_0^\infty y[1+H(y)]\varphi(y)J_0(ry)\,dy = P(r), \qquad 0 \leq r \leq 1, \tag{4.1}$$

$$\int_0^\infty \varphi(y)J_0(ry)\,dy = 0, \qquad 1 \leq r \leq a, \tag{4.2}$$

where $H(y) = \{(2y^2 - k_2^2)^2 - 4\alpha\beta y^2\}/2y\alpha(k_1^2 - k_2^2) - 1$. $H(y) \to 0$ as $y \to \infty$, so the condition for obtaining the solution of the above equations is satisfied. The second of these equations is satisfied if $\varphi(y)$ is written in terms of an unknown function $g(t)$ through the equation

$$\varphi(y) = \int_0^1 g(t)\sin(ty)\,dt = -g(1)\cos y/y + y^{-1}\int_0^1 g'(t)\cos(ty)\,dt, \qquad g(0) = 0. \tag{4.3}$$

If we substitute this value of $\varphi(y)$ into the first equation, we get an Abel integral equation which on inversion becomes

$$g(t) + \int_0^1 g(u)L_1(u, t)\,du = R(t), \tag{4.4}$$

where

$$L_1(u, t) = \frac{2}{\pi}\int_0^\infty H(y)\sin(uy)\sin(ty)\,dy \tag{4.5}$$

and

$$\begin{aligned} R(t) &= \frac{2}{\pi}\int_0^t r(t^2 - r^2)^{-1/2}P(r)\,dr \\ &= \frac{2}{\mu\pi}\int_0^t (t^2 - r^2)^{-1/2} rP(r)\,dr \\ &\quad - \left(\frac{2}{\pi}\right)^{3/2}\int_0^\infty \{(2\alpha_1^2 + k_2^2)\alpha_1^{-1}B(x)\sinh(\alpha_1 t) + 2x\beta_1 C(x)\sinh(\beta_1 t)\}\,dx. \end{aligned} \tag{4.6}$$

Here we have used the result [10]

$$\int_0^t r(t^2 - r^2)^{-1/2} I_0(\alpha_1 r)\,dr = \alpha_1^{-1}\sinh(\alpha_1 t).$$

(b) *Conditions on the curved surface of the cylinder*

We now complete the solution of the problem by satisfying the boundary conditions on the curved surface of the cylinder. Conditions (2.4) lead to the following relations:

$$2x\alpha_1 I_1(\alpha_1 a)B(x)+\beta_1(2x^2-k_2^2)I_1(\beta_1 a)C(x)$$
$$=-\left(\frac{8}{\pi}\right)^{1/2}\int_0^\infty \{(\alpha^2+x^2)^{-1}-(\beta^2+x^2)^{-1}\}\alpha xyA(y)J_1(ay)\,dy=i_1(x);$$

$$[(2x^2-k_2^2)I_0(\alpha_1 a)-2\alpha_1 a^{-1}I_1(\alpha_1 a)]B(x)+2x\beta_1[\beta_1 I_0(\beta_1 a)-a^{-1}I_1(\beta_1 a)]C(x)$$
$$=-\left(\frac{2}{\pi}\right)^{1/2}\int_0^\infty [(2\alpha^2+k_2^2)(\alpha^2+x^2)^{-1}-4\alpha\beta^2y^2(2y^2-k_2^2)^{-1}(\beta^2+x^2)^{-1}]A(y)J_0(ay)\,dy$$
$$+\left(\frac{8}{\pi}\right)^{1/2}\int_0^\infty [(\alpha^2+x^2)^{-1}-2\beta^2(2y^2-k_2^2)(x^2+\beta^2)^{-1}]\alpha a^{-1}yA(y)J_1(ay)\,dy=i_2(x).$$

From these equations we have

$$B(x)M(x)=2x\beta_1\{\beta_1 I_0(a\beta_1)-a^{-1}I_1(a\beta_1)\}i_1(x)-(2x^2-k_2^2)I_1(a\beta_1)i_2(x), \tag{4.7}$$
$$C(x)M(x)=-\{(2x^2-k_2^2)I_0(a\alpha_1)-2\alpha_1 a^{-1}I_1(a\alpha_1)\}i_1(x)+2x\alpha_1 I_1(a\alpha_1)i_2(x), \tag{4.8}$$

where

$$M(x)=\beta_1[4x^2\alpha_1\beta_1 I_1(a\alpha_1)I_0(a\beta_1)-(2x^2-k_2^2)^2I_1(a\beta_1)I_0(a\alpha_1)$$
$$-2a^{-1}\alpha_1 k_2^2 I_1(a\alpha_1)I_1(a\beta_1)],$$

$$i_1(x)=\frac{x(2\pi)^{-1/2}}{(k_1^2-k_2^2)}\int_0^1 [(2\alpha_1^2+k_2^2)K_1(a\alpha_1)\sinh(u\alpha_1)$$
$$-(2\beta_1^2+k_2^2)\sinh(u\beta_1)K_1(a\beta_1)]g(u)\,du,$$

$$i_2(x)=-\frac{(2\pi)^{-1/2}}{(k_1^2-k_2^2)\alpha}\int_0^1 [(2\alpha_1^2+k_2^2)(2\beta_1^2+k_2^2)K_0(a\alpha_1)\sinh(u\alpha_1)$$
$$-4\alpha_1\beta_1 x^2K_0(a\beta_1)\sinh(u\beta_1)]g(u)\,du$$
$$+\frac{(2/\pi)^{1/2}}{a(k_1^2-k_2^2)}\int_0^1 [x^2K_1(a\beta_1)\sinh(u\beta_1)-(2\alpha_1+k_2^2)K_1(a\alpha_1)\sinh(u\alpha_1)]g(u)\,du.$$

The values of $i_1(x)$ and $i_2(x)$ have been evaluated with the help of (4.3) and the following integrals given in [10]:

$$\int_0^\infty y(y^2+\alpha_1^2)^{-1}\sin(uy)J_1(ay)\,dy=K_1(a\alpha_1)\sinh(u\alpha_1);$$

$$\int_0^\infty (y^2+\alpha_1^2)^{-1}\sin(uy)J_0(ay)\,dy=\alpha_1^{-1}K_0(a\alpha_1)\sinh(u\alpha_1);$$

$$\int_0^\infty \sin(ty)J_0(ay)\,dy=0, \qquad a>t.$$

If we substitute these values of $B(x)$ and $C(x)$ into (4.6), we get

$$R(t)=\frac{2}{\mu\pi}\int_0^t r(t^2-r^2)^{-1/2}p(r)\,dr-\int_0^1 [L_2(t,u)+L_3(t,u)]g(u)\,du, \tag{4.9}$$

where

$$L_2(t, u) = \frac{8}{\pi^2(k_1^2 - k_2^2)a} \int_0^\infty \frac{x}{\alpha_1 M(x)} [(2\alpha_1^2 + k_2^2)K_1(a\alpha_1)\sinh(u\alpha_1)$$
$$-(2\beta_1^2 + k_2^2)K_1(a\beta_1)\sinh(a\beta_1)]$$
$$\times[\{(2\alpha_1^2 + k_2^2)a\beta_1 I_0(a\beta_1)\sinh(t\alpha_1)$$
$$-(2\beta_1^2 + k_2^2)a\alpha_1 I_0(a\alpha_1)\sinh(t\beta_1)\}$$
$$+\{2\alpha_1^2 I_1(a\alpha_1)\sinh(t\beta_1) - (2\alpha_1^2 + k_2^2)I_1(t\beta_1)\sinh(t\alpha_1)\}]\,dx, \quad (4.10)$$

$$L_3(t, u) = \frac{2}{\pi^2(k_1^2 - k_2^2)a} \int_0^\infty [4x^2\alpha_1^2 I_1(a\alpha_1)\sinh(t\beta_1)$$
$$-(2\alpha_1^2 + k_2^2)(2\beta_1^2 + k_2^2)\cdot I_1(a\beta_1)\sinh(t\alpha_1)]$$
$$\times[2\alpha_1\{2x^2K_1(a\beta_1)\sinh(u\beta_1) - (2\alpha_1^2 + k_2^2)K_1(a\alpha_1)\sinh(u\alpha_1)\}$$
$$-a\{(2\alpha_1^2 + k_2^2)(2\beta_1^2 + k_2^2)K_0(a\alpha_1)\sinh(u\alpha_1)$$
$$-4\alpha_1\beta_1 x^2 K_0(a\beta)\sinh(u\beta)]\frac{dx}{\alpha_1^2 M(x)}. \quad (4.11)$$

By substituting this value of $R(t)$ in (4.4), we get the integral equation

$$g(t) + \int_0^1 L(t, u)g(u)\,du = \frac{2}{\mu\pi}\int_0^t (t^2 - r^2)^{-1/2} r p(r)\,dr,$$

where $L(t, u) = L_1(t, u) + L_2(t, u) + L_3(t, u)$.

In order to compare our results with those of Mal [6] and Sneddon and Welch [9], we shall take $p(r) = p_0 = \text{constant}$. By setting $g(t) = (p_0/\mu)g_1(t)$, we see that the above equation becomes

$$g_1(t) + \int_0^1 L(t, u)g_1(u)\,du = \frac{2t}{\pi}. \quad (4.12)$$

It is interesting to note that for 'a' tending to infinity the above equation reduces to the integral equation obtained by Mal [6] for an infinite solid containing a circular crack which is excited by normally incident longitudinal harmonic waves. When $\omega \ll 1$ it can be shown that

$$L_1(t, u) = O(\omega^2),$$

$$L_2(t, u) + L_3(t, u) = \frac{4}{\pi^2}\int_0^\infty [\{F(ax) - 1\}\{xu\cosh(xu)\sinh(xt)$$
$$+ xt\cosh(xt)\sinh(xu)\}$$
$$+\sinh(xt)\sinh(xu)\{2F(ax) - 3 + 2\eta - a^2x^2\}$$
$$- x^2ut\cosh(xt)\cosh(xu)]\{1 + O(\omega^2)\}\frac{dx}{G(x)},$$

where $G(x) = x^2 I_0^2(x) - (2 - 2\eta + x^2)I_1^2(x)$ and $F(x) = x^2 I_0(x)K_0(x) + (2 - 2\eta + x^2)\times I_1(x)K_1(x)$.

In the static case, when $\omega = 0$, Eq. (4.12) reduces to the integral equation obtained by Sneddon and Welch [9, page 412].

The dynamical stress intensity factor is defined by the equation

$$N = \lim_{r \to 1+} |(r-1)^{1/2} \tau_{zz}(r, 0)|_{1 \leq r \leq a} \tag{4.13}$$

From Eq. (3.9), with the help of (4.3), (4.7) and (4.8), it can be easily shown that

$$\tau_{zz}(r, 0) = (r^2 - 1)^{-1/2} p_0 g_1(1) + O(1).$$

Hence the stress intensity factor is given by the equation

$$N = p_0 |g_1(1)| / \sqrt{2}. \tag{4.14}$$

The integral Eq. (4.12) is solved numerically in the next section. The solution is used to calculate the numerical values of the stress intensity factor.

5. Numerical solutions

Numerical calculations have been done for a particular example. In this example, we have taken $\rho = 2.7$ gm/cm^3, $\eta = 0.339$ and $E = 0.75 \times 10^{11}$ dyne/cm^2, where η is the Poisson's ratio and E the Young's modulus of the elastic material. Computations have been carried out for different values of the dimensionless wave frequency k_2 varying from 0 to 8. Five different values of the radius of the cylinder have been chosen, viz., 1.5, 2.0, 5.0, 6.5 and 8. It may be noted that the integrands of (4.5), (4.10) and (4.11) defining L_1, L_2 and L_3, respectively have no poles, but only branch points. In the numerical calculation of these integrals, account of this fact has been made and a complex computer programming has been done in which α, β, α_1 and β_1 have been interpreted in the following way (compare Robertson [2]):

$$\alpha = (y^2 - k_1^2)^{1/2}, \quad y > k_1 \qquad \beta = (y^2 - k_2^2)^{1/2}, \quad y > k_2$$
$$= -i(k_1^2 - y^2)^{1/2}, \quad 0 \leq y \leq k_1; \qquad = -i(k_2^2 - y^2)^{1/2}, \quad 0 \leq y \leq k_2;$$

$$\alpha_1 = (x^2 - k_1^2)^{1/2}, \quad x > k_1 \qquad \beta_1 = (x^2 - k_2^2)^{1/2}, \quad x > k_2$$
$$= -i(k_1^2 - x^2)^{1/2}, \quad 0 \leq x \leq k_1; \qquad = -i(k_2^2 - x^2)^{1/2}, \quad 0 \leq x \leq k_2.$$

For the calculation of the integrals L_1, L_2 and L_3, the five-point Gauss–Laguerre formula was found suitable and gave stable values of the integrals and of the integral equation (4.12). For solving the integral equation (4.12), the standard method of Fox and Goodwin [11] was applied. By virtue of relation (4.14), these solutions determine the dynamic stress intensity factor. Results are given in the following table and illustrated graphically in the figure. When $k_2 = 0$, the problem reduces to the elastostatic problem considered by Sneddon and Welch [9].

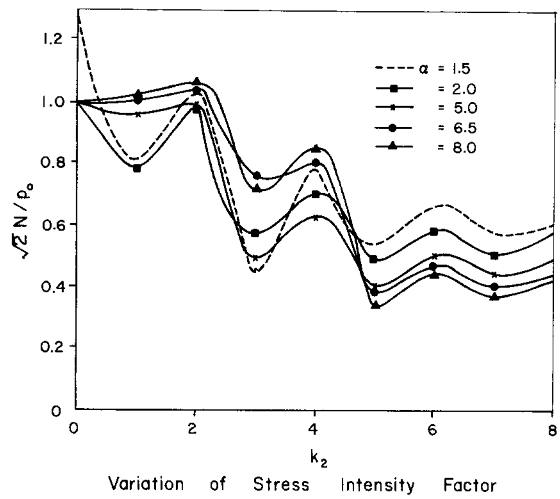

Figure 1.

k_2 / a	0	1	2	3	4	5	6	7	8
1.5	1.227	0.802	1.152	0.418	0.782	0.537	0.675	0.573	0.602
2.0	1.027	0.784	1.120	0.563	0.714	0.482	0.592	0.503	0.578
5.0	1.002	0.950	1.026	0.483	0.632	0.403	0.513	0.435	0.484
6.5	1.000	1.050	1.210	0.782	0.813	0.362	0.481	0.412	0.421
8.0	1.000	1.120	1.341	0.713	0.892	0.320	0.450	0.330	0.403

6. Discussion of the results

It is interesting to note that for $k_2 = 0$, the problem discussed here reduces to elastostatic problem studied by Sneddon and Welch [9]. The numerical values of the stress-intensity factor are in good agreement with those calculated from the results derived by Sneddon and Welch [9] for the static problem. It will be interesting to compare the results of this paper with those derived by Mal [6]. Mal has studied the interaction of elastic waves with a penny-shaped crack situated in an infinite elastic solid. A comparison of these results will yield valuable information about the effect of finite boundaries on the distribution of stresses. A look at the graph showing the variation of stress intensity factor with k_2 for the problem studied by Mal [6] shows that the value of the stress intensity factor first increases from its elastostatic value, reaches a maximum value, then it decreases and shows an oscillatory behaviour. In contrast to this in our case the value of the stress intensity factor, for values of $a < 5$, first decreases from its elastostatic value, reaches a minimum value, then increases. After reaching a maximum value at $k_2 = 2$, in our case also the curves show an oscillatory character similar to that of Mal [6]. For $a = 5$, the curve has a slightly different character. With further increase in the values of the radius of the cylinder this difference becomes more pronounced and their shape becomes somewhat similar to that given in the paper of Mal [6]. This shows that as the radius of the cylinder increases the effect of the finite boundary goes on decreasing.

REFERENCES

[1] Sneddon, I. N. and M. Lowengrub, Crack Problem in the Classical Theory of Elasticity. John Wiley, New York 1969.
[2] Robertson, I. A., Diffraction of a plane longitudinal wave by a penny-shaped crack. *Proc. camb. Phil. Soc.* 63 (1967) 229.
[3] Mal, A. K., Dynamic stress intensity factor for an axisymmetric loading of a penny-shaped crack. *Int. J. Engng. Sci.* 6 (1968) 623.
[4] Mal, A. K., Dynamic stress intensity factor for non-axisymmetric loading of the penny-shaped crack. *Int. J. Engng. Sci.* 6 (1968) 725.
[5] Mal, A. K., Interaction of elastic waves with a Griffith crack. *Int. J. Engng. Sci.* 8 (1970) 763.
[6] Mal, A. K., Interaction of elastic waves with penny-shaped crack. *Int. J. Engng. Sci.* 8 (1970) 381.
[7] Mal, A. K., Diffraction of elastic waves by a penny-shaped crack. *Quart. App. Math.* 26 (1968) 231.
[8] Sih, G. C. and Leober, J. F., A class of wave diffraction problems involving geometrically induced singularities., *J. Math. Mech.* 19 (1969/70) 327.
[9] Sneddon, I. N. and Welch, J. T., A note on the distribution of stress in a cylinder containing a penny-shaped crack. *Int. J. Engng. Sci.* 1 (1963) 411.
[10] Erdelyi, A. et al., *Tables of Integral Transforms*, Vol. 2. McGraw Hill 1954.
[11] Fox, L. and Goodwin, E. T., The numerical solution of non-singular integral equations. *Philos, Trans. Ser. A.* 24 (1953) 501.