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Code-type formulation of fracture mechanics concepts for concrete

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Abstract. The new model code for the design of concrete structures of the Comite Euro-International du Béton (CEB) includes extensive information on constitutive relations for concrete and reinforcing steel. In this model code relations are also proposed to predict fracture properties of concrete on the basis of fracture mechanics concepts. In particular fracture energy G_F is given as a function of concrete grade, maximum aggregate size and temperature. In addition, bilinear stress-strain and crack opening relations are presented. In this paper these relations are verified on the basis of theoretical considerations and available experimental data.

1. Introduction

In 1978 the Comité Euro-International du Béton published the CEB–FIP Model Code 1978 for the design of reinforced concrete structures [1]. This model code served as a basis for various national codes and in particular for the Eurocode EC2 'Design of Concrete Structures' [2]. The CEB–FIP Model Code 1978 gave only limited information on material properties and on constitutive relations for concrete with the exception of creep and shrinkage which has been dealt with in an appendix.

In the process of revising the CEB–FIP Model Code it became apparent that constitutive relations both for concrete and for reinforcing steel should be an integral part of a modern code for reinforced and prestressed concrete structures. Such relations are urgently needed in particular for non-linear analyses and finite element calculations. Therefore, in the predraft of the CEB–FIP Model Code 1990 (MC 90) [3] a Section 2.1 'Concrete – Classification and Constitutive Relations' has been included which gives information on the following concrete properties:

— compressive strength, tensile strength and fracture energy;
— strength under multiaxial states of stress;
— stress-strain relations and stress-crack opening relations;
— effects of stress and strain rate on strength and deformation properties;
— effects of time on strength and deformation properties;
— effects of temperature on strength and deformation properties;
— transport of liquids and gases in hardened concrete.

The information given on fracture properties is based on fracture mechanics concepts, making use of the tremendous progress in this field during the past decade throughout the world and in various national organizations such as RILEM and ACI. Chapter 2.1 of the Predraft to MC 90 has been prepared primarily by the authors of this paper and by Dr. H.S. Müller, Bundesanstalt für Materialforschung und -prüfung, Berlin, under the auspices of CEB-Commission VIII 'Concrete Technology'. In addition, extended use has been made of the work of other groups within CEB.

In the following, relations for an estimate of fracture parameters as given in [3] as well as their justification are summarized. Since the publication of the predraft of MC 90, some minor changes in the constitutive relations have been made. They are taken into account in this paper.

2. Relations for fracture mechanics parameters given in MC 90

2.1. Parameters and input data

From the work of several investigators it follows that the behavior of concrete and reinforced concrete elements subjected to tensile stresses can be analyzed in a realistic way on the basis of the following characteristics of concrete [4], [5], [6]:

- the axial tensile strength f_{ct};
- the fracture energy G_F, defined as the energy required to propagate a tensile crack of unit area;
- stress-strain relations for increasing stresses up to the level of tensile strength and a limiting tensile strain;
- stress-crack opening relations.

Since the CEB-Model Codes are directed towards the designer, all constitutive relations have been formulated such that only parameters are used which are generally known to the designer at the stage of design. Therefore, for the prediction of fracture properties the following parameters have been taken into account:

- strength grade of the concrete expressed by its characteristic compressive strength f_{ck} [MPa];
- the maximum size of aggregates, d [mm];
- temperature of the ambient air in the range of $0°C < T < 80°C$.

The experimental data available were not sufficient to also take into account strain or stress rate effects, concrete age or the influence of sustained loads.

2.2. Relations for fracture energy

In the absence of experimental data for a particular concrete G_F may be estimated from

$$G_F = a_d \cdot f_{cm}^{0,7}, \tag{1}$$

where G_F = fracture energy [Nm/m^2]; $f_{cm} = f_{ck} + 8$ = mean compressive strength of concrete [MPa]; f_{ck} = characteristic compressive strength of concrete [MPa] defined as the 5 percent defective; a_d = coefficient to be taken from Table 1. It depends on the maximum aggregate size, d.

Table 1. Coefficient, a_d, to take into account the effect of maximum aggregate size, d, on fracture energy G_F

d [mm]	a_d
8	4
16	6
32	10

2.3. Stress-strain and stress-crack opening relations for uniaxial tension

The following relations are given for the modulus of elasticity of concrete, E_c and for the mean tensile strength f_{ctm}:

$$E_c = 10^4 \cdot f_{cm}^{1/3} \tag{2}$$

and

$$f_{ctm} = 0.30 \cdot f_{ck}^{2/3}. \tag{3}$$

For uncracked concrete a bilinear stress-strain relation as expressed by (4) and (5) and shown in Fig. 1 may be used:

$$\sigma_{ct} = E_c \cdot \varepsilon_{ct} \quad \text{for } \sigma < 0.9 f_{ctm}, \tag{4}$$

$$\sigma_{ct} = f_{ctm} - \frac{0.1 f_{ctm}}{0.00015 - \dfrac{0.9\, f_{ctm}}{E_c}} \cdot (0.00015 - \varepsilon_{ct}) \tag{5}$$

where σ_{ct} = tensile stress [N/mm^2]; ε_{ct} = tensile strain; E_c = tangent modulus of elasticity of concrete [N/mm^2] to be estimated from (4); f_{ctm} = mean axial tensile strength of concrete [MPa] to be estimated from (5).

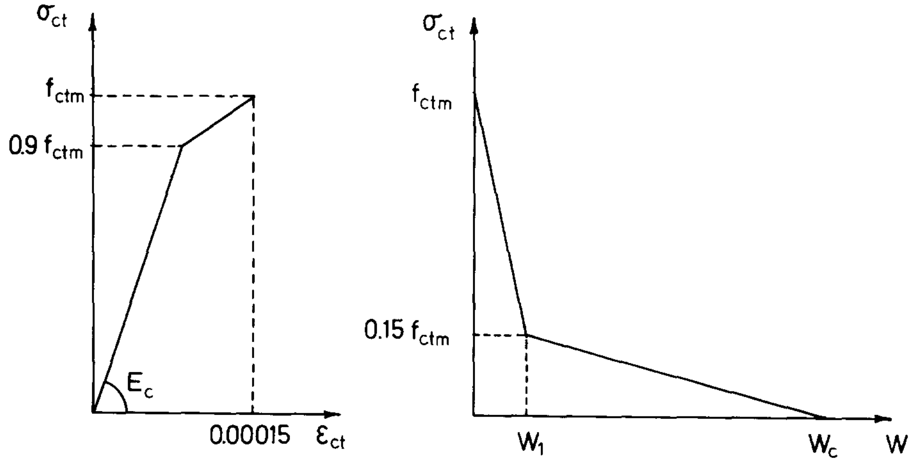

Fig. 1. Stress-strain and stress-crack opening diagram for uniaxial tension.

For cracked sections a bilinear stress-crack opening relation as described by (6), (7) and (8) and shown in Fig. 1 is proposed:

$$\sigma_{ct} = f_{ctm}\left(1 - 0.85\frac{w}{w_1}\right) \quad \text{for } 0.15 f_{ctm} < \sigma_{ct} < f_{ctm}, \tag{6}$$

$$\sigma_{ct} = \frac{0.15 f_{ctm}}{w_c - w_1}(w_c - w) \quad \text{for } 0 < \sigma_{ct} < 0.15 f_{ctm} \tag{7}$$

and

$$w_1 = \frac{G_F - 22 w_c (G_F/a_d)^{0.95}}{150 (G_F/a_d)^{0.95}}, \tag{8}$$

where w_1 = crack opening at the nick [mm] as defined in Fig. 1; w_c = crack opening [mm] for $\sigma_{ct} = 0$; G_F = fracture energy acc. to (1); a_d = coefficient to be taken from Table 1.

The crack opening w_c at $\sigma_{ct} = 0$ depends on the maximum aggregate size and may be taken from Table 2.

Table 2. Crack opening w_c for $\sigma_{ct} = 0$

d_{max} [mm]	w_c [mm]
8	0.12
16	0.15
32	0.25

2.4. *Effect of temperature*

For the temperature range 0°C < T < 80°C the effect of temperature on fracture energy G_F may be estimated from (9) and (10)

$$\text{for dry concrete: } G_F(T) = G_F(1.07 - 0.0030\text{T}), \tag{9}$$

$$\text{for mass concrete: } G_F(T) = G_F(1.14 - 0.006\text{T}), \tag{10}$$

where $G_F(T)$ = fracture energy at temperature T; G_F = fracture energy at T = 20°C from (1); T = temperature in [°C].

For the temperature range considered the temperature dependence of the uniaxial tensile strength f_{ctm} does not have to be taken into account. The influence of temperature on the modulus of elasticity may be estimated from (11)

$$E_c(T) = E_c(1.06 - 0.003T), \tag{11}$$

where $E_c(T)$ = modulus of elasticity at temperature T; E_c = modulus of elasticity at T = 20°C from (4).

3. Verification of relations

3.1. Fracture energy – technological parameters

In order to evaluate the major parameters influencing fracture energy G_F the experimental data reported in [7], [8], [9], [10] have been studied carefully. Particular attention has been given to the results of round robin tests which were reported in [9]. Since the data given in [9] were partially incomplete, additional information has been obtained by direct contacts with the various investigators. This additional information as well as all other data needed for the evaluation are given in detail in [11]. From these experimental data the following technological parameters were found to be of particular significance for the fracture energy:

— compressive strength and water/cement ratio of the concrete;
— maximum aggregate size;
— concrete age.

In addition, geometrical parameters, in particular the depth of the ligament above a crack or notch are of significance. These effects will be dealt with in Section 3.2. The parameters given above also have been verified in [8] and [10] as being the most significant in influencing fracture energy.

Since water-cement ratio, concrete age and compressive strength of the concrete are interrelated and since the available data base was not sufficient to clearly distinguish between the effects of these parameters, only compressive strength and maximum aggregate size have been chosen as parameters for the Code prediction of fracture energy. For the evaluation the results of 36 experiments described in detail in [11] have been used. Figure 2 gives the relation between the mean concrete compressive strength f_{cm} and fracture energy G_F on a double logarithmic scale for maximum aggregate sizes, d, of

$$1 < d < 8\,\text{mm},$$
$$12 < d < 20\,\text{mm},$$
$$d = 32\,\text{mm}.$$

The relation between G_F and f_{cm} is best documented for $12 < d < 20\,\text{mm}$, $d_{av} = 16\,\text{mm}$. For this aggregate size (1) with $a_d = 6.0$ for $d = 16\,\text{mm}$ results in a correlation coefficient $k = 0.83$.

Figure 2 also shows the well known tendency that fracture energy increases with increasing maximum aggregate size. The same trend is true for other fracture characteristics such as K_{Ic}. However, the data base is too small to establish safe predictions for a maximum aggregate size $d < 12\,\text{mm}$ and $d > 20\,\text{mm}$. Therefore, the values of a_d for $d = 4\,\text{mm}$ and $d = 32\,\text{mm}$ given in Table 1 should be taken with caution. This is particularly true for $d = 32\,\text{mm}$ where only the result of one series of experiments is available. Therefore, additional experiments are required to ensure the relations between G_F and f_{cm} for max. aggregate sizes other than $12 < d < 20\,\text{mm}$, and in particular for $d = 32\,\text{mm}$. Where more accurate predictions are required, G_F should be determined experimentally, e.g. [12].

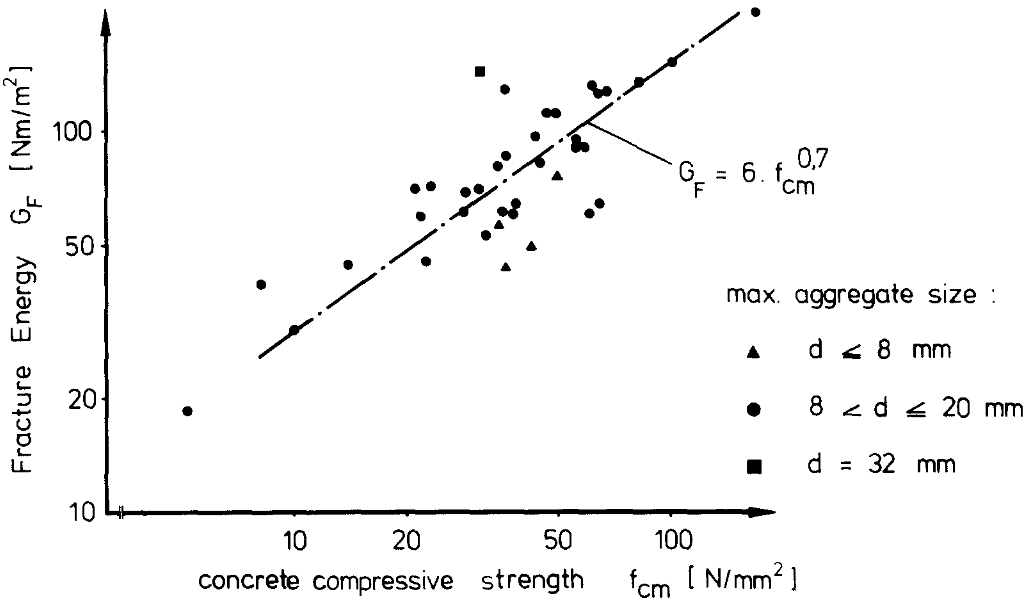

Fig. 2. Fracture energy and compressive strength of concrete.

In [7] and [15] it was shown that the characteristic length l_{ch} as defined in (12) is particularly suitable to describe the ductility and crack sensitivity of concrete:

$$l_{ch} = \frac{G_F \cdot E_{cm}}{f_{ctm}^2}, \tag{12}$$

where l_{ch} = characteristic length [mm]; E_{cm} = modulus of elasticity [N/mm²]; f_{ctm} = mean tensile strength of concrete [MPa].

From (1), (2), (3) and (12) a relation between l_{ch} and concrete compressive strength f_{cm} can be derived. It may be approximated by (13).

$$l_{ch} = 600 a_d \cdot f_{cm}^{-0.3}. \tag{13}$$

Deviating from (3) it was assumed that $f_{ctm} = 0.30 f_{cm}^{2/3}$. Figure 3 shows the experimental values of l_{ch} as a function of f_{cm} for a maximum aggregate size $12 < d_{\max} < 20$ mm. In contrast to fracture energy, the characteristic length decreases as the concrete compressive strength increases. Equation (13) describes the available experimental data reasonably well though the correlation coefficient $k = 0.72$ is lower than the corresponding value for G_F. This is not surprising since the prediction of l_{ch} from (13) also includes uncertainties in the estimate of tensile strength and modulus of elasticity.

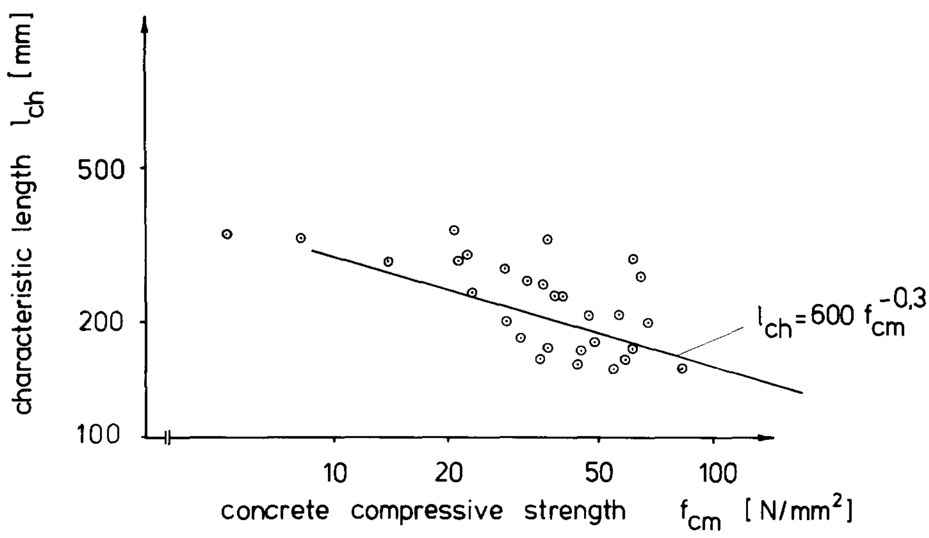

Fig. 3. Characteristic length and compressive strength of concrete.

3.2. Fracture energy – size effects

Various experiments [7–10] show that fracture energy G_F if determined experimentally according to [12] increases with increasing depth of the uncracked ligament. In [8] it was shown furthermore, that for a depth of the ligament larger than approximately 300 mm, fracture energy is little affected by a further increase of the ligament depth.

Various approaches have been proposed to take into account this size effect, in particular the size effect law developed by Bazant [e.g. 15]. Though the general validity of the size effect law is not questioned it appeared to be desirable to find a size independent approach to predict G_F for a Code type formulation irrespective of the inevitable errors which may be introduced by such a formulation. This is even more so since size effects on plain and reinforced concrete properties can be predicted even with a size independent G_F.

In [7, 16] the causes of the size dependence of G_F have been analyzed in more detail. In experiments on notched beams of different depth, however, with a constant ratio of notch depth/beam depth of 0.5 the crack propagation has been determined carefully, and the relative fracture energy G_F/G_{F0} required to propagate a crack has been determined as a function of crack length Δc. Figure 4 shows the result of this analysis. There, the relative fracture energy is given as a function of Δc for beams with uncracked ligament depths between 50 and 400 mm. Irrespective of the initial depth of the ligament the fracture energy G_F required to propagate the crack increased with increasing crack length Δc up to a crack depth of approximately 40 mm. For a further increase in crack length the fracture energy stayed constant at a level $G_F = G_{F0}$. Since the crack length at which a constant level of G_{F0} is reached is independent of the depth of the ligament the average value of G_F decreases as the depth of the ligament decreases. Table 3 summarizes the errors which occur if constant values of G_F valid for beams with a ligament depth > 800 mm are applied to beams with a smaller ligament depth. These errors are generally less than 20 percent. Thus they are small compared to other experimental errors as shown in [7, 16]. They are in the range of size dependence of the compressive strength of concrete.

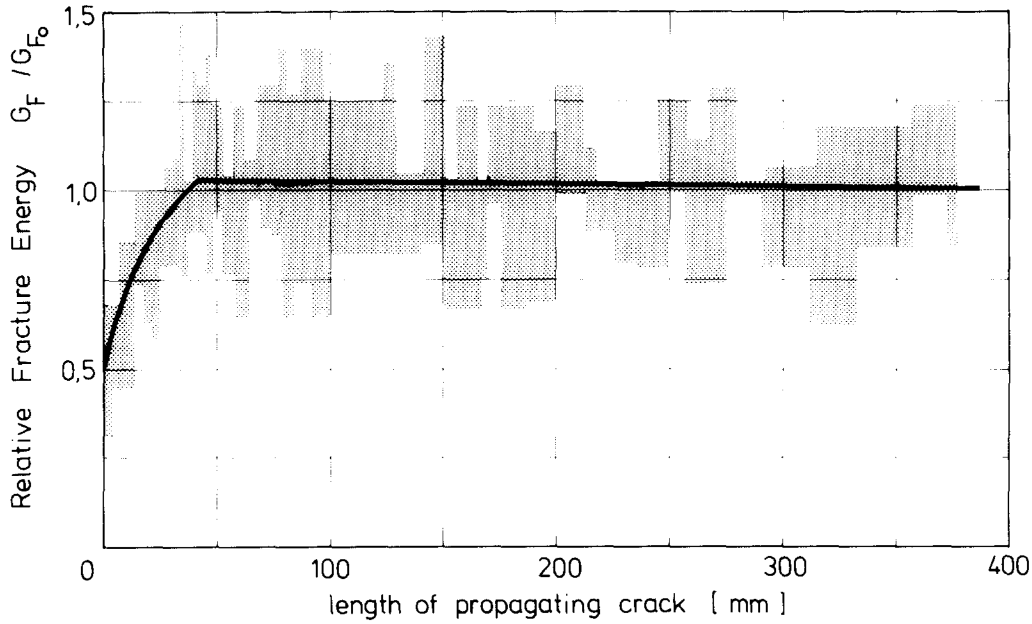

Fig. 4. Fracture energy as a function of length of propagated crack.

Table 3. Fracture energy G_F as a fraction of fracture energy of a deep beam G_{F0}; $a/d = 0.5$

	G_F/G_{F0}	
depth of beam (mm)	concrete $d_{max} = 32$ mm	mortar $d_{max} = 4$ mm
100	1.16	1.08
200	1.08	1.04
400	1.04	1.02
800	1.02	1.01
1000	1.00	1.00

3.3. Fracture energy – temperature effects

The data base to evaluate the effect of temperature in the range of 0°C < T < 80°C on fracture energy is small [7, 15]. Figure 5 summarizes the available results. There, fracture energy G_F at a given temperature is expressed as a fraction of G_F at $T = 23$°C and plotted as a function of the temperature at testing, T. From Fig. 5 it follows that fracture energy decreases linearly with increasing temperature. In addition, the moisture state of the concrete is of significance: dry concretes are less temperature sensitive than wet concretes.

Figure 5 also shows the relations given in MC 90 to describe the effect of temperature on G_F (9–10). They are in close agreement with the available test data for a temperature range 0°C < T < 80°C.

In [7, 16] the theoretical basis for a linear relationship between G_F and T in the above temperature range has been given. It is based on the relation between potential energy of bonding and temperature. As shown in [7, 16], (9) and (10) allow extrapolations up to

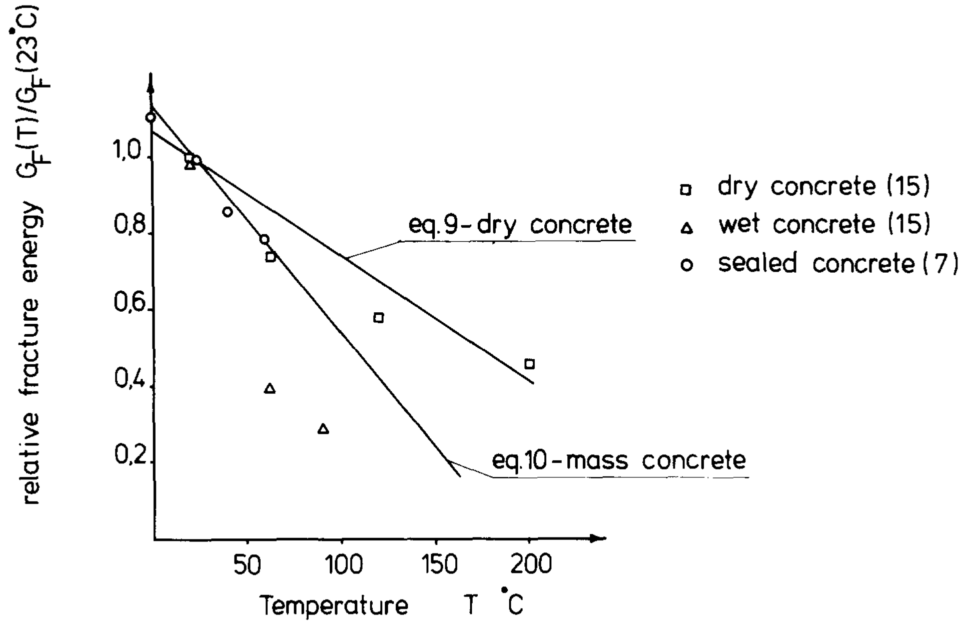

Fig. 5. Effect of temperature on fracture energy of concrete.

temperatures as low as $-170°C$. For $T > 80°C$ the actual relation between G_F and T is non-linear so that an extrapolation of (9) and (10) is no longer permissible. In [15] a model for the relation between G_F and T is given which is based upon activation energy considerations. Both models differ little in the temperature range $0°C < T < 80°C$. However, the model given in [15] does not allow extrapolations to lower temperatures since such an extrapolation would result in considerable overestimates of G_F at lower temperatures [7, 16].

3.4. Stress-strain and stress-crack opening relations

The description of the stress-strain properties of concrete subjected to tensile stresses is based primarily upon a proposal initially made by Petersson [13]. This proposal consists of a linear stress-strain relation of uncracked concrete and a linear stress-crack opening relation for cracked concrete. In reality the stress-strain relation for uncracked concrete is not entirely linear. Therefore, a bilinear function expressed by (4) and (5) has been chosen to take into account the non-linear behavior at stresses $\sigma_{ct} > 0.9 f_{ctm}$. A constant strain at maximum stress $\varepsilon_{ctmax} = 0.00015$ has been assumed since no systematic effects on this parameter could be found.

A variety of formulations have been tested to describe the strain softening behavior of the cracked concrete. Of particular significance is the question to which extent the stress-crack opening relations found experimentally could be simplified for a code-type formulation. Therefore, various calculations have been carried out which were based on the fictitious crack model described in [13].

From the literature and in particular from the results of the aforementioned round robin tests load-deflection relations for various types of concretes and specimen geometries have been taken. Together with other concrete properties, in particular modulus of elasticity, tensile strength and fracture energy, theoretical load-deflection relationships have been calculated based on bilinear stress-crack opening relations of the shape shown in Fig. 1. The parameter w_c, i.e. the maximum crack opening at zero stress, and the crack opening at the nick w_1 of the bilinear $\sigma - w$ relation have been varied in order to obtain an acceptable agreement between theoretical and experimental load-deflection curves. The following simplifying assumptions were made as a result of these trial calculations:

1. The maximum crack opening at zero stress has little effect on the calculated load-deflection relations. It, therefore, would be an unnecessary complication to express w_c as a direct function of G_F or f_{ctm}. However, the maximum crack opening increases with increasing maximum aggregate size as expressed by Table 2.
2. The agreement between experimental and theoretical load-deflection relations is strongly influenced by the slope of the initial part of the bilinear $\sigma - w$ relation. However, variations of the nick in the $\sigma - w$ relation are less significant. Therefore, a value of $\sigma_{ct}(w_1) = 0.15 f_{ctm}$ may be employed.

From the condition that

$$G_F = \int_{w=0}^{w=w_c} \sigma_{ct}(w) \cdot \mathrm{d}w$$

the value of w_1 can be calculated:

$$w_1 = \frac{G_F - \frac{w_c}{2} \cdot \sigma_{ct}(w_1)}{0.5 f_{ctm}} \tag{15a}$$

and

$$w_1 = \frac{G_F - \frac{w_c}{2} \cdot 0.15 f_{ctm}}{0.5 f_{ctm}}. \tag{15b}$$

Expressing f_{ctm} in terms of fracture energy on the basis of (1) and the modified (3), (8) is obtained which gives the crack opening at the nick in terms of fracture energy G_F and max. aggregate size. Thus the stress-crack opening behavior of concrete can be described by (6)–(8).

These equations depict the well-known characteristics for concrete loaded in tension, in particular

— decreasing non-linearity of the stress-strain relations with increasing compressive strength;
— decreasing slopes of the stress-crack opening relations with increasing compressive strength.

In Fig. 6 load deflection curves determined experimentally and reported in [7] are compared to theoretical load-deflection curves calculated on the basis of the fictitious crack model [14] and (1)–(8). Acceptable agreement has been obtained.

In addition to the bilinear functions for the stress-crack opening relations given above also continuous functions have been developed at our institute. Similar functions have been reported

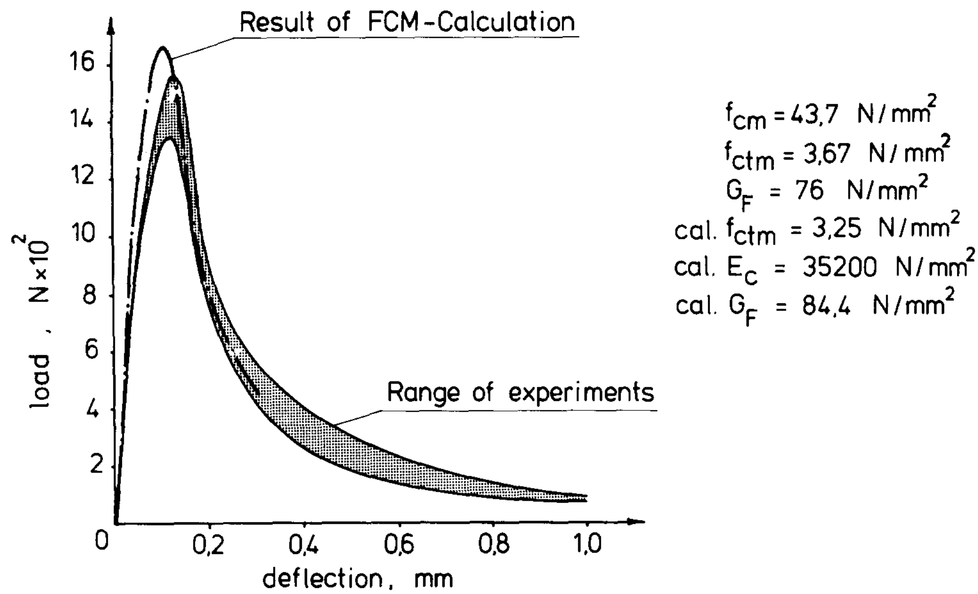

Fig. 6. Comparison of experimental and calculated load-deflection curves of plain concrete beams, 3-point-bending. $1 \times d \times b = 500 \times 100 \times 100$ mm; notch depth: 50 mm; max. aggregate size: 16 mm.

in the literature [17, 18]. It also has been proposed to express w, in terms of G_F/f_{ctm} [19]. However, there is not sufficient experimental evidence based on tests on concentrically loaded concrete specimens that such interrelations exist and are of major significance. It must be pointed out in this context that the formulations given above are primarily based on load-deflection measurements of notched beams. Though comparison with the few experimental data from concentric tension tests indicates acceptable agreement, further experimental data in this field are urgently needed.

4. Summary and conclusions

In the CEB–FIP Model Code 1990 various constitutive relations to describe the properties of concrete are given. This section includes information on the fracture properties and stress-deformation characteristics of concrete loaded in tension. Among the fracture parameters under discussion fracture energy G_F, bilinear stress-strain relationships for the uncracked concrete and bilinear stress-crack opening relations for the cracked concrete have been chosen. Despite the fact that only parameters generally known to the designer, i.e. strength grade and maximum aggregate size, have been chosen as input data, acceptable agreement between prediction and experimental results has been obtained.

It is considered a major breakthrough that fracture mechanics data are included in an international concrete code, and it is hoped that this new approach will open new avenues for more realistic ways in non-linear structural analysis. But in deriving these code-type formulations of concrete fracture properties the lack of experimental studies oriented towards practical engineering applications became evident. It is hoped that in the planning of future research this gap in our knowledge will be kept in mind.

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Author's Note Added in Proof

In the revised version of MC 90 to be published in 1991 the expressions (1) to (11) as well as Tables 1 and 2 have been slightly altered in order to make these relations dimensionally compatible.