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Interaction of shear waves with two coplanar Griffith cracks situated in an infinitely long elastic strip

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Abstract

In an earlier paper [6] we have studied the case of interaction of shear waves with a crack centrally situated in an infinite elastic strip; we, in this paper, extend the study to the case of two coplanar Griffith cracks. An integral transform method is used to find the solution of the equation of motion from the linear theory for a homogeneous, isotropic – elastic material. This method resolves the problem into an integral equation. It has been observed that shear waves with frequencies less than a parameter depending on the width of the wave guide can only propagate. The integral equation is solved numerically for a range of values of wave frequency, width of strip and the inter-crack distance. These solutions are used to calculate the dynamic stress intensity factor. The results are shown graphically.

1. Introduction

Recently great interest is being shown in studying the problems of interaction of elastic waves with singularities in the form of cracks or inclusions located in two or three dimensional configurations. We have, in a series of papers [1–5], studied the interaction of elastic waves by Griffith penny-shaped cracks located at the interface of two bonded dissimilar elastic half-spaces. All these attempts have been based on the assumption that the crack is sufficiently far from the neighbouring boundaries and hence the distribution of stresses in the solids is attributed to the crack geometry or to the wave frequency of the elastic waves. Mathematically speaking the boundaries of the solids are assumed to be at infinitely large distance from the crack. The boundary value problems of interaction of elastic waves with cracks near the free boundary are difficult to solve since they involve additional geometric parameters describing the dimensions of the solids. Such a problem in which a Griffith crack interacted by a shear wave and located in an infinitely long homogeneous, isotropic elastic strip with free lateral surface is considered in [6]. In continuation to the above papers we have considered here a problem in which a pair of coplanar Griffith cracks is situated in an infinitely long homogeneous, isotropic elastic strip, perpendicular to the lateral surfaces, and interacted by a shear wave incident normal on it.

2. Formulation of the problem

Consider an infinitely long, homogeneous, isotropic, thin elastic strip of width $2h_1$, containing two parallel Griffith cracks of infinite length and finite width. Consider a rectangular cartesian co-ordinate system (x_1, x_2, x_3) such that these cracks occupy the region $-a \leqslant x_1 \leqslant -b$, $b \leqslant x_1 \leqslant a$, $-\infty < x_2 < \infty$, $x_3 = 0$. The cracks are assumed to be excited by a normally incident antiplane shear wave originating at $x_3 = -\infty$. The displacement vector corresponding to this wave is parallel to x_2-axis. It is convenient to

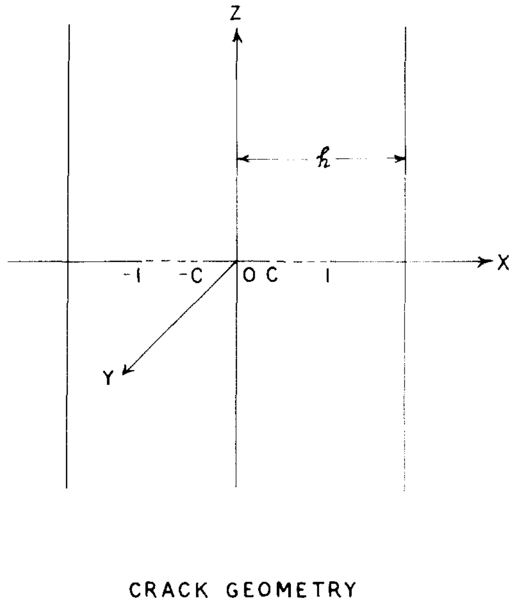


Figure 1.

normalize all lengths with respect to a which is half of the distance between the outer tips of the two cracks. Writing $x_1/a = x$, $x_2/a = y$, $x_3/a = z$, $b/a = c$ and $h_1/a = h$ the cracks are defined by $-1 \leqslant x \leqslant -c$, $c \leqslant x \leqslant 1$, $-\infty < y < \infty$, $z = 0$, as shown in Fig. 1.

Let ω be the circular frequency of the incident wave. In what follows the time dependence of all the field quantities assumed to be of the form $e^{-i\omega t}$ will be suppressed but understood. We further suppose that the two faces of the cracks do not come in contact during vibrations.

As discussed in [6] the boundary condition on the crack faces at $z = 0$ is

$$\sigma_{yz} = -q_0, \quad c \leqslant |x| \leqslant 1 \tag{2.1}$$

and we have

$$U_y = 0, \quad 0 \leqslant |x| \leqslant c, \quad 1 \leqslant |x| \leqslant h \tag{2.2}$$

where q_0 is a known constant.

The two edges of the strip are free from traction. This implies that on edges $x = \pm h$ we have the following boundary conditions

$$\sigma_{yz}(h, z) = \sigma_{yz}(-h, z) = 0. \tag{2.3}$$

The problem of determining the stress distribution reduces to that of obtaining the solution of the displacement equation

$$\frac{\partial^2 U_y}{\partial x^2} + \frac{\partial^2 U_y}{\partial z^2} + k_2^2 U_y = 0 \tag{2.4}$$

where $k_2^2 = \rho\omega^2/\mu$, μ being Lame's constant, ρ the density and ω is the circular frequency of the incident wave.

The solution of Eqn. (2.4) for an elastic half space can be obtained with the help of Fourier cosine transform, such that the displacement at $z = \infty$ vanishes and is given by

$$U_y(x, z) = 2\int_0^\infty B(\xi)\, e^{-\beta z} \cos \xi x \, d\xi. \tag{2.5}$$

Again the solution of (2.4) for an elastic strip which is symmetrical about the z-axis can be obtained with the help of Fourier sine transforms and is given by

$$U_y(x, z) = \int_0^\infty C(\zeta) \cosh(\beta_1 x) \sin(\zeta z) \, d\zeta. \tag{2.6}$$

Thus the complete solution of (2.4) suitable for the problem stated above is obtained by superposing the two solutions (2.5) and (2.6), and is given by

$$U_y(x, z) = 2\int_0^\infty B(\xi) \, e^{-\beta z} \cos \xi x \, d\xi + \int_0^\infty C(\zeta) \cosh(\beta_1 x) \sin \zeta z \, d\zeta \tag{2.7}$$

where

$$\beta^2 = \xi^2 - k_2^2 \quad \text{and} \quad \beta_1^2 = \zeta^2 - k_2^2. \tag{2.8}$$

$B(\zeta)$ and $C(\zeta)$, unknown functions, are to be determined with the help of boundary conditions.

By using the stress and strain relations, the expression for the component of stress tensor comes out to be:

$$\sigma_{yz}(x, z) = -2\mu \int_0^\infty \beta B(\xi) e^{-\beta z} \cos \xi x \cdot d\xi + \mu \int_0^\infty \zeta C(\zeta) \cosh(\beta_1 x) \cdot \cos \zeta z \cdot d\zeta. \tag{2.9}$$

3. Derivation of the integral solution

The boundary conditions (2.1) and (2.2) lead to the following integral equations:

$$2\int_0^\infty \beta B(\xi) \cos \xi x \cdot d\xi - \int_0^\infty \zeta C(\zeta) \cosh(\beta_1 x) \, d\zeta = \frac{q_0}{\mu}, \qquad c \leqslant |x| \leqslant 1 \tag{3.1a}$$

$$\int_0^\infty B(\xi) \cos\xi x \, d\xi = 0, \quad 0 \leqslant |x| \leqslant c, \quad 1 \leqslant |x| \leqslant h. \tag{3.1b}$$

Applying the condition (2.3) on the boundary of the strip, the following relation between the unknown functions $B(\xi)$ and $C(\zeta)$ is obtained:

$$\int_0^\infty \zeta C(\zeta) \cosh(\beta_1 h) \cos \zeta z \;\; d\zeta = 2\int_0^\infty \beta B(\xi) e^{-\beta z} \cos(\xi h) d\xi$$

which, on using the inversion theorem for the Fourier cosine transform and using simple integration, yields

$$\zeta C(\zeta) \cosh(\beta_1 h) = \frac{4}{\pi} \int_0^\infty \frac{\beta^2 B(\xi) \cos \xi h}{\beta^2 + \zeta^2} \cdot d\xi. \tag{3.2}$$

Substituting the value of $G(\zeta)$ from (3.2) in (3.1a) and rearranging the terms we get the following triple integral equation for the determination of the unknown function $B(\xi)$:

$$\int_0^\infty \xi [1 + H(\xi)] B(\xi) \cos \xi x \, d\xi = P(x), \quad c \leqslant |x| \leqslant 1$$

$$\int_0^\infty B(\xi) \cos \xi x, \, d\xi = 0, \quad 0 \leqslant |x| \leqslant c, \quad 1 \leqslant |x| \leqslant h \tag{3.3b}$$

where

$$P(x) = \frac{q_0}{2\mu} + \frac{2}{\pi} \int_0^\infty \frac{\cosh(\beta_1 x)}{\cosh(\beta_1 h)} \int_0^\infty \frac{\beta^2 B(\xi) \cos \xi h}{\beta^2 + \zeta^2} \cdot d\xi, \, d\zeta \tag{3.4}$$

$$H(\xi) = \xi^{-1} \left[\left(\xi^2 - k_2^2 \right)^{1/2} - \xi \right]. \tag{3.5}$$

It can be easily seen that $H(\xi) \to 0$ as $\xi \to \infty$. In order to solve the triple integral equation (3.3) we set

$$B(\xi) = \frac{q_0}{\xi} \int_c^1 h(t^2) \sin \xi t \cdot \mathrm{d}t \tag{3.6}$$

where the function $h(t^2)$ shall soon be determined. For the interval $1 \leqslant |x| \leqslant h$, (3.3b) is automatically satisfied in view of the formula

$$\int_0^\infty \xi^{-1} \sin \xi t \cos \xi x \, \mathrm{d}\xi = \begin{cases} \pi/2, & |x| < t \\ 0, & |x| > t \end{cases} \tag{3.7}$$

and for the interval $0 \leqslant |x| < c$, (3.3b) is satisfied provided

$$\int_c^1 h(t^2) \, \mathrm{d}t = 0. \tag{3.8}$$

It is correct to write (3.3a) as

$$\frac{\mathrm{d}}{\mathrm{d}x} \int_0^\infty B(\xi) \sin \xi x \cdot \mathrm{d}\xi = P(x) - \int_0^\infty \xi B(\xi) H(\xi) \cos \xi x \cdot \mathrm{d}\xi, \quad c \leqslant |x| \leqslant 1. \tag{3.9}$$

On substituting the values of $P(x)$ and $B(\xi)$ from (3.4) and (3.6) respectively in (3.9) and on making use of the following formulae

$$\int_0^\infty x^{-1} \sin xy \cdot \sin ax \, \mathrm{d}x = \tfrac{1}{2} \log \left| \frac{y+a}{y-a} \right|$$

$$\int_0^t \frac{r J_0(\xi r) \, \mathrm{d}r}{\sqrt{t^2 - r^2}} = \frac{\sin \xi t}{\xi}$$

$$\frac{\mathrm{d}}{\mathrm{d}x} \log \left| \frac{x+t}{x-t} \right| = \frac{2t}{t^2 - x^2},$$

we get

$$\int_c^1 \frac{t h(t^2) \, \mathrm{d}t}{t^2 - x^2} = \frac{1}{2\mu} + \frac{2}{\pi} \int_c^1 h(t^2) \left[\int_0^\infty \frac{\cosh(\beta_1 x)}{\cosh(\beta_1 h)} \left\{ \int_0^t \frac{y}{\sqrt{t^2 - y^2}} \cdot \int_0^\infty \frac{\beta^2 \cos \xi h J_0(\xi y)}{\beta^2 + \zeta^2} \cdot \mathrm{d}\xi \, \mathrm{d}y \right\} \mathrm{d}\zeta \right] \mathrm{d}t - \frac{\mathrm{d}}{\mathrm{d}x} \int_c^1 h(t^2) \cdot \left[\int_0^t \frac{y}{(t^2 - y^2)^{1/2}} \left\{ \int_0^x \frac{w}{(x^2 - w^2)^{1/2}} \int_0^\infty \xi H(\xi) J_0(\xi y) J_0(\xi w) \, \mathrm{d}\xi \, \mathrm{d}w \right\} \cdot \mathrm{d}y \right] \mathrm{d}t, \quad c \leqslant |x| \leqslant 1. \tag{3.10}$$

It has been shown by Srivastava and Lowengrub [7] that the solution of the integral equation

$$\frac{2}{\pi} \int_a^b \frac{t h(t^2) \mathrm{d}t}{t^2 - y^2} = R(y), \quad a < y < b$$

is given by

$$h(t^2) = -\frac{2}{\pi} \left(\frac{t^2 - a^2}{b^2 - t^2} \right)^{1/2} \int_a^b \left(\frac{b^2 - y^2}{y^2 - a^2} \right)^{1/2} \frac{y R(y)}{y^2 - t^2} \cdot \mathrm{d}y + \frac{D}{\{(t^2 - a^2)(b^2 - t^2)\}^{1/2}}$$

with condition that R must be an even function of y so as to make the integral convergent; D is an arbitrary constant. Hence the solution of (3.10) as subjected to the condition $\int_c^1 h(t^2)\mathrm{d}t = 0$, is given by

$$h(u^2) = -\frac{2}{\pi}\left(\frac{u^2-c^2}{1-u^2}\right)^{1/2} \cdot \frac{1}{\pi\mu} \cdot \int_c^1 \left(\frac{1-x^2}{x^2-c^2}\right)^{1/2} \frac{x\mathrm{d}x}{(x^2-u^2)} + \frac{D}{\{(u^2-c^2)(1-u^2)\}^{1/2}}$$

$$-\frac{4}{\pi^2}\left(\frac{u^2-c^2}{1-u^2}\right)^{1/2} \int_c^1 h(t^2) \int_c^1 \left(\frac{1-x^2}{x^2-c^2}\right)^{1/2} \frac{x}{(x^2-u^2)} \left[\frac{2}{\pi}\int_0^\infty \frac{\cosh(\beta_1 x)}{\cosh(\beta_1 h)} \right.$$

$$\cdot \int_0^t \frac{y}{(t^2-y^2)^{1/2}} \cdot \int_0^\infty \frac{\beta^2 \cos\xi h \cdot J_0(\xi y)}{\beta^2+\zeta^2} \mathrm{d}\xi \cdot \mathrm{d}y \cdot \mathrm{d}\zeta$$

$$\left. -\frac{\mathrm{d}}{\mathrm{d}x}\int_0^t\int_0^x \left\{ \int_0^\infty \frac{\xi H(\xi) J_0(\xi y) J_0(\xi w) yw}{\{(t^2-y^2)(x^2-w^2)\}^{1/2}} \cdot \mathrm{d}\xi \right\} \mathrm{d}w \cdot \mathrm{d}y \right] \cdot \mathrm{d}x\, \mathrm{d}t. \tag{3.12}$$

The above equation can be written as

$$h(u^2) + \int_c^1 h(t^2)\left[K_1(u^2,t^2) + K_2(u^2,t^2)\right] \mathrm{d}t = F(u^2) \tag{3.13}$$

where

$$K_1(u^2,t^2) = -\frac{4}{\pi^2}\left(\frac{u^2-c^2}{1-u^2}\right)^{1/2} \int_c^1 \left(\frac{1-x^2}{x^2-c^2}\right)^{1/2} \frac{xL_1(x,t)}{(x^2-u^2)} \cdot \mathrm{d}x \tag{3.14}$$

$$L_1(x,t) = \frac{\mathrm{d}}{\mathrm{d}x}\int_0^t\int_0^x \frac{ywL(y,w)\,\mathrm{d}w\,\mathrm{d}y}{[(x^2-w^2)(t^2-y^2)]^{1/2}} \tag{3.15}$$

$$L(y,w) = \int_0^\infty \xi H(\xi) J_0(\xi y) J_0(\xi w)\,\mathrm{d}\xi \tag{3.16}$$

$$K_2(u^2,t^2) = \frac{8}{\pi^3}\left(\frac{u^2-c^2}{1-u^2}\right)^{1/2} \int_c^1 \left(\frac{1-x^2}{x^2-c^2}\right)^{1/2} \frac{x}{(x^2-u^2)} \int_0^\infty \frac{\cosh(\beta_1 x)}{\cosh(\beta_1 h)}$$

$$\cdot \int_0^t \frac{y}{(t^2-y^2)^{1/2}} \cdot M(y,\zeta)\mathrm{d}y \cdot \mathrm{d}\zeta \cdot \mathrm{d}x \tag{3.17}$$

$$M(y,\zeta) = \int_0^\infty \frac{\beta^2 \cos\xi h \cdot J_0\ (\xi y\ )\,\mathrm{d}\xi}{\beta^2+\zeta^2}$$

$$= \frac{\pi}{2}\mathrm{e}^{-\beta_1 h} I_0(y\beta_1)\zeta^2\beta_1^{-1}, \qquad \zeta > k_2 \tag{3.18a}$$

$$= \frac{\pi}{2}\sin(\beta_1' h) \cdot J_0(y\beta_1')\frac{\zeta^2}{\beta_1'}, \qquad \zeta < k_2 \tag{3.18b}$$

$$F(u^2) = -\frac{2}{\pi}\left(\frac{u^2-c^2}{1-u^2}\right)^{1/2} \frac{q_0}{\pi\mu}\int_c^1 \left(\frac{1-x^2}{x^2-c^2}\right)^{1/2} \frac{x\,\mathrm{d}x}{(x^2-u^2)} + \frac{D}{[(u^2-c^2)(1-u^2)]^{1/2}} \tag{3.19a}$$

$$= \frac{1}{\pi\mu} \cdot \left(\frac{u^2-c^2}{1-u^2}\right)^{1/2} + \frac{D}{[(u^2-c^2)(1-u^2)]^{1/2}} \tag{3.19b}$$

and $(\beta_1')^2 = k_2^2 - \xi^2$.

The integrand in (3.16) has no poles, it has only branch point at the point $\xi = k_2$. Following the procedure in [8] the infinite integral in (3.16) can be converted into an integral with finite limits and is given by

$$L(y, w) = -ik_2^2 \int_0^1 (1-\xi^2)^{1/2} J_0(k_2\xi w) H_0^{(1)}(k_2\xi y)\, d\xi, \qquad y > w. \tag{3.20}$$

From (3.20) putting the value of $L(y, w)$ in (3.15), we get

$$L_1(x, t) = -ik_2^2 \int_0^1 \sqrt{1-\xi^2} \cos(k_2\xi x) \left\{ \int_0^t \frac{yH_0^{(1)}(k_2\xi y)}{\sqrt{t^2-y^2}} \cdot dy \right\} d\xi. \tag{3.21}$$

Further using the relation (3.18) in (3.17) and simplifying it we get

$$K_2(u^2, t^2) = \frac{4}{\pi^2} k_2^2 \left(\frac{u^2-c^2}{1-u^2} \right)^{1/2} \int_0^t \frac{y}{\sqrt{t^2-y^2}} \left[\int_0^1 \sqrt{1-s^2} \tan(k_2 \cdot sh) \right.$$
$$\cdot J_0(yk_2 s) \left\{ \int_c^1 \left(\frac{1-x^2}{x^2-c^2} \right)^{1/2} \frac{\cos(k_2 sx) \cdot x}{(x^2-u^2)} dx \right\} ds$$
$$- \int_0^\infty \frac{\sqrt{1+r^2}\, I_0(k_2 yr)}{\cosh(k_2 rh)} e^{-k_2 rh}$$
$$\left. \cdot \left\{ \int_c^1 \left(\frac{1-x^2}{x^2-c^2} \right)^{1/2} \frac{x \cosh(k_2 rx)}{(x^2-u^2)} dx \right\} \cdot dr \right] dy. \tag{3.22}$$

It may be noted that the first integral is convergent only when the dimensionless frequency k_2 is such that $k_2 < \pi/2h$. Hence it is concluded that the only shear waves with $k_2 < \pi/2h$ can propagate in an elastic strip of width $2h$. This fact is in agreement with the well known result that in a strip, only guided waves of frequencies less than a parameter depending on width of the strip can propagate.

Furthermore in order to evaluate D, the unknown constant of (3.19), we integrate (3.13) with respect to u between the limits c to 1, and using the condition (3.8) we find that

$$D = \frac{2}{\pi F} \int_c^1 g(u^2)\, du + \frac{1}{F} \int_c^1 \left[\int_c^1 h(t^2) \{ K_1(u^2, t^2) + K_2(u^2, t^2) \}\, dt \right] \cdot du \tag{3.23}$$

where

$$g(u^2) = -\frac{1}{2} \left(\frac{u^2-c^2}{1-u^2} \right)^{1/2}.$$

$F = F(\pi/2, \sqrt{1-c^2})$ is the elliptic integral of first kind and $K_1(u^2, t^2)$, $K_2(u^2, t^2)$ are given by (3.14) and (3.22) respectively.

Finally, on putting the value of D from (3.23) in (3.13) and on further simplifying it, we get the following integral equation

$$\left[(u^2-c^2)(1-u^2) \right]^{1/2} h(u^2) + \int_c^1 h(t^2) \left(\frac{4}{\pi^2} k_2^2 \right) \cdot \left[(u^2-c^2) \right.$$
$$\cdot \{ K_1^1(u^2, t^2) + K_2^1(u^2, t^2) \} - \frac{1}{F} \left\{ \int_c^1 \left(\frac{s^2-c^2}{1-s^2} \right)^{1/2} \cdot \right.$$
$$\left. \left. \cdot (K_1^1(s^2, t^2) + K_2^1(s^2, t^2)) ds \right\} \right] dt$$

$$= u^2 - c^2 + \frac{1}{F}\int_c^1 \left(\frac{s^2 - c^2}{1 - s^2}\right)^{1/2} \mathrm{d}s \tag{3.24}$$

where

$$K_1^1(u^2, t^2) = i\int_0^1 \sqrt{1-\zeta^2}\left\{\int_0^t \frac{yH_0^{(1)}(k_2\xi y)\mathrm{d}y}{\sqrt{t^2 - y^2}}\right\}$$

$$\cdot \int_c^1 \left(\frac{1 - x^2}{x^2 - c^2}\right)^{1/2} \frac{x\cos(k_2\xi x)}{(x^2 - u^2)}\mathrm{d}x \cdot \mathrm{d}\xi \tag{3.25}$$

and

$$K_2^1(u^2, t^2) = \int_0^t \frac{y}{\sqrt{t^2 - y^2}}\left[\int_0^1 \sqrt{1 - s^2}\tan(k_2 sh)J_0(yk_2 s)\right.$$

$$\cdot \left\{\int_c^1 \left(\frac{1-x^2}{x^2-c^2}\right)^{1/2} \frac{x\cos(k_2 sx)}{(x^2 - u^2)}\mathrm{d}x\right\}\mathrm{d}s - \int_0^\infty \frac{\sqrt{1+r^2}\, I_0(k_2 yr)}{\cosh(k_2 rh)}$$

$$\left.\cdot\, \mathrm{e}^{-k_2 rh}\left\{\int_c^1 \left(\frac{1-x^2}{x^2-c^2}\right)^{1/2} \frac{x\cosh(k_2 rx)}{(x^2-u^2)} \cdot \mathrm{d}x\right\}\mathrm{d}r\right]\mathrm{d}y. \tag{3.26}$$

Further let

$$h(u^2)\cdot\left[(u^2 - c^2)(1 - u^2)\right]^{1/2} = H(u^2)$$

and on making the substitutions

$$u^2 = \sin^2\phi + c^2\cos^2\phi$$

$$t^2 = \sin^2\theta + c^2\cos^2\theta$$

in (3.24), we get

$$G(\phi) + \int_0^{\pi/2} \frac{G(\theta)}{\sqrt{\sin^2\theta + c^2\cos^2\theta}} \cdot \left(\frac{4}{\pi^2}\cdot k_2^2\right)\Big[(1 - c^2)\cdot\sin^2\phi$$

$$\cdot\{K_a(\phi,\theta) + K_b(\phi,\theta)\} - \frac{1}{F}\left\{\int_c^1 \left(\frac{s^2-c^2}{1-s^2}\right)^{1/2}\right.$$

$$\left.\cdot\big(K_a(s^2,\theta) + K_b(s^2,\theta)\big)\,\mathrm{d}s\right\}\Big]\,\mathrm{d}\theta$$

$$= (1 - c^2)\cdot\sin^2\phi + \frac{1}{F}\int_c^1 \left(\frac{s^2-c^2}{1-s^2}\right)^{1/2}\cdot\mathrm{d}s \tag{3.27}$$

where

$$G(\phi) = H(\sin^2\phi + c^2\cos^2\phi)$$

$$G(\theta) = H(\sin^2\theta + c^2\cos^2\theta)$$

$$K_a(\phi,\theta) = K_1^1(\sin^2\phi + c^2\cos^2\phi,\ \sin^2\theta + c^2\cos^2\theta)$$

$$K_b(\phi,\theta) = K_2^1(\sin^2\phi + c^2\cos^2\phi,\ \sin^2\theta + c^2\cos^2\theta)$$

$$K_a(s^2,\theta) = K_1^1(s^2,\ \sin^2\theta + c^2\cos^2\theta)$$

$$K_b(s^2,\theta) = K_2^1(s^2,\ \sin\theta + c^2\cos^2\theta).$$

4. Stress–intensity factors

On putting the value of $B(\xi)$ from (3.6) in (2.9) and further solving it for $z = 0$, we get

$$\sigma_{yz}(x, 0) = -2q_0\mu\int_c^1 \frac{th(t^2)\mathrm{d}t}{(t^2 - x^2)} + X_I, \tag{4.1}$$

where

$$X_I = \mu\int_0^\infty \zeta C(\zeta)\cosh(\beta_1 x)\mathrm{d}\zeta. \tag{4.2}$$

Since $h(t^2) = [(t^2 - c^2)(1 - t^2)]^{-1/2}H(t^2)$ and $t^2 = c^2\cos^2\theta + \sin^2\theta$, (4.1) can be written as:

$$\sigma_{yz}(x, 0) = -2\mu q_0\int_0^{\pi/2} \frac{H(\sin^2\theta + c^2\cos^2\theta)\mathrm{d}\theta}{(\sin^2\theta + c^2\cos^2\theta) - x^2} + X_I$$

or

$$\sigma_{yz}(x, 0) = -2\mu q_0\int_0^{\pi/2} \frac{G(\theta)\mathrm{d}\theta}{(1 - x^2)\sin^2\theta + (c^2 - x^2)\cos^2\theta} + X_I \tag{4.3}$$

where

$$G(\theta) = H(\sin^2\theta + c^2\cos^2\theta)$$

on integrating the right hand side of (4.3) by parts, we get the following:

$$\sigma_{yz}(x, 0) = \frac{-2\mu q_0}{\left[(1 - x^2)(c^2 - x^2)\right]^{1/2}} \times \left[\cot^{-1}\left(\sqrt{\frac{1 - x^2}{c^2 - x^2}} \cdot \tan\theta\right) \cdot G(\theta)\right]_0^{\pi/2} + 0(1), \quad x < c \quad \ldots (4.4\mathrm{a})$$

$$\sigma_{yz}(x, 0) = \frac{-2\mu q_0}{\left[(x^2 - c^2)(x^2 - 1)\right]^{1/2}} \times \left[\tan^{-1}\left(\sqrt{\frac{x^2 - 1}{x^2 - c^2}} \cdot \tan\theta\right) \cdot G(\theta)\right]_0^{\pi/2} + 0(1), \quad x > 1 \tag{4.4b}$$

$$\sigma_{yz}(x, 0) = \frac{\pi\mu q_0(G(0))}{\left[(1 - x^2)(c^2 - x^2)\right]^{1/2}} + 0(1); \quad x < c \tag{4.5}$$

and

$$\sigma_{yz}(x, 0) = \frac{-\pi\mu q_0 G(\pi/2)}{\left[(x^2 - c^2)(x^2 - 1)\right]^{1/2}} + 0(1); \quad x > 1. \tag{4.6}$$

The stress intensity factors N_c and N_1 at the two tips of the crack are defined by:

$$N_c = \lim_{x \to c} -(c - x)^{1/2} \cdot (-2\mu) \cdot \left[\sigma_{yz}(x, 0)\right], \quad 0 < x < c$$

$$N_1 = \lim_{x \to 1} +(x - 1)^{1/2}(-2\mu)\left[\sigma_{yz}(x, 0)\right], \quad x > 1.$$

Thus with the help of (4.5) and (4.6) we get:

$$N_c = \frac{2\pi\mu^2 q_0 G(0)}{\left[2c(1 - c^2)\right]^{1/2}} \tag{4.7}$$

and

$$N_1 = \frac{2\pi\mu^2 q_0 G(\pi/2)}{[2(1-c^2)]^{1/2}}. \tag{4.8}$$

When c tends to zero two cracks merge into one and

$$N_1 = \frac{q_0}{\sqrt{2}}[2\pi\mu^2 G(\pi/2)]$$

which is in agreement with our earlier paper [6] in which $g(1) = 2\pi\mu^2 G(\pi/2)$.

5. Numerical calculations:

The integral equation (3.27) has been solved numerically for a wide range of dimensionless frequency k_2. Using the method of Fox and Goodwin [9] (3.27) has been converted into a system of linear algebraic equations. The infinite integral involved in the kernel $K_2^1(u^2, t^2)$ has been evaluated using five point Gauss-Laguerre quadrature formula while the integrals

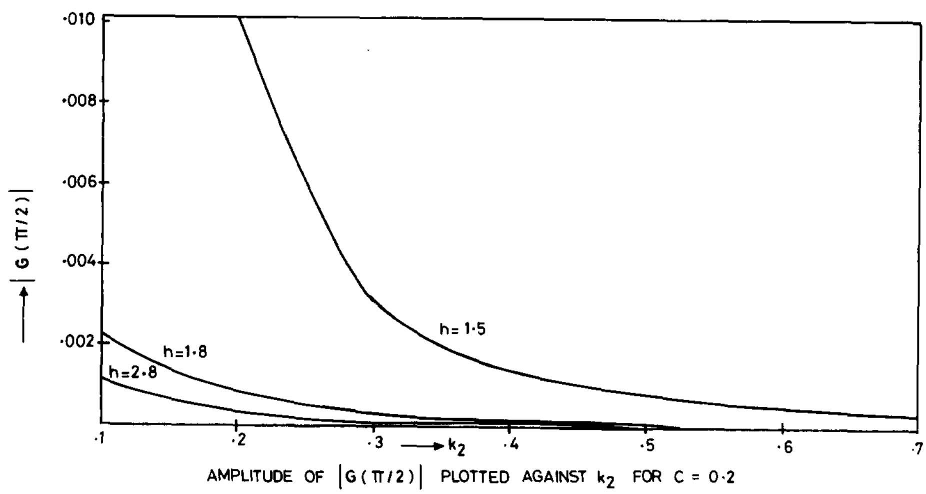


Figure 2.

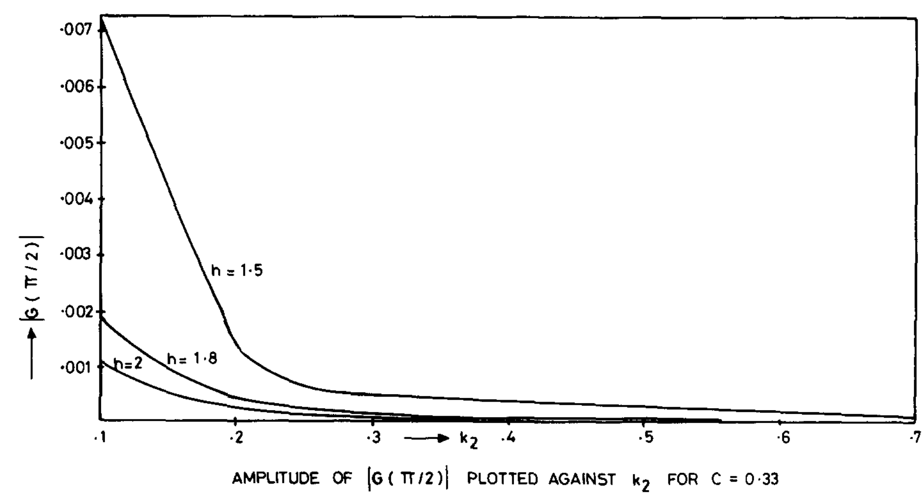
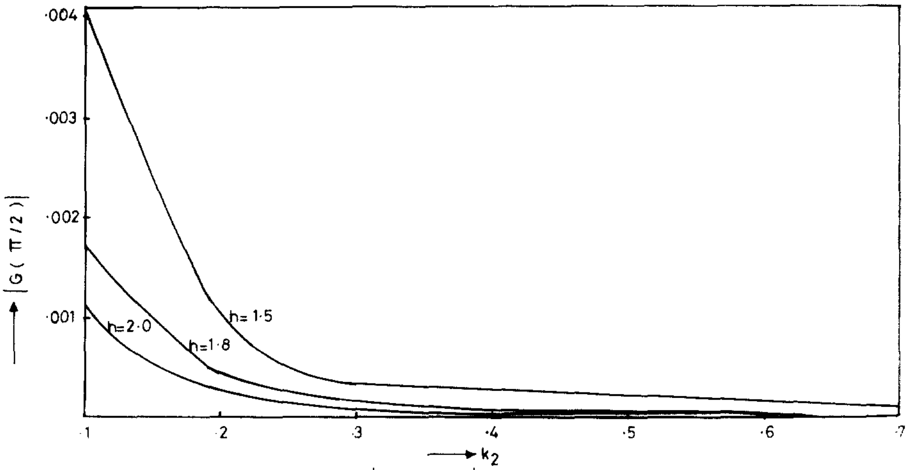


Figure 3.

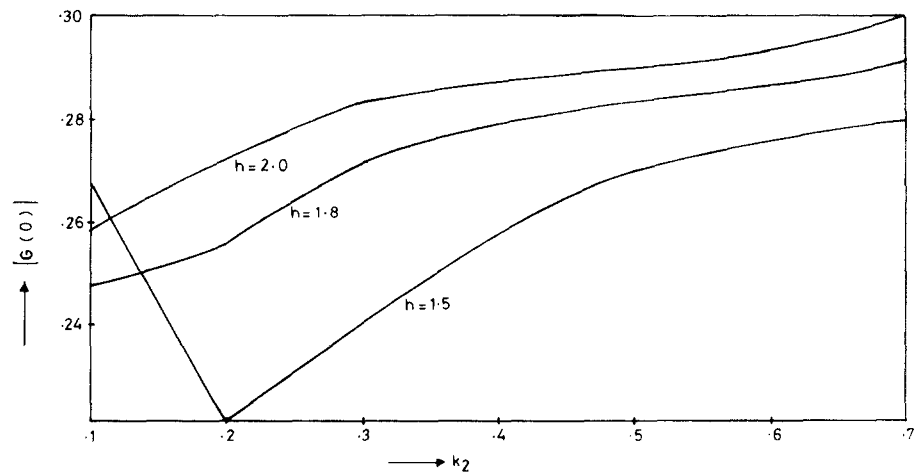


AMPLITUDE OF |G (π / 2)| PLOTTED AGAINST k_2 FOR C = 0·5

Figure 4.

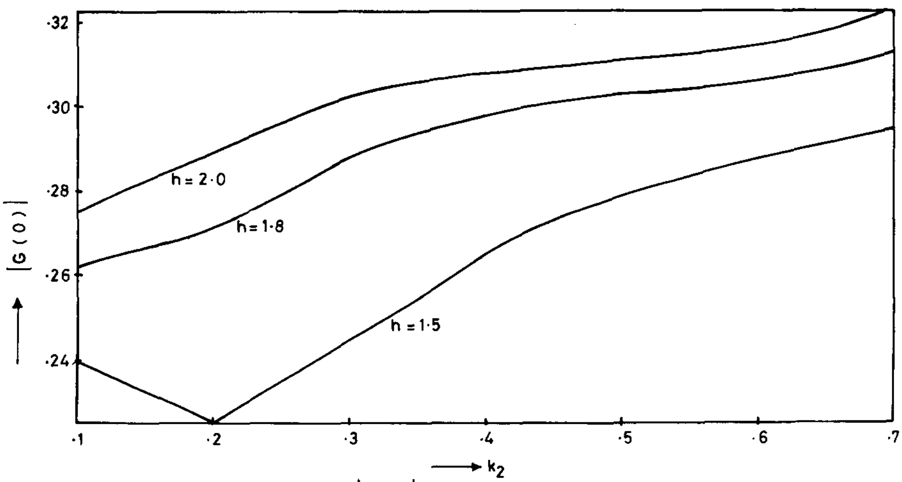
with finite limits have been evaluated using Simpson's quadrature formula. At relatively high frequencies the number of divisions in the finite interval are increased until the value of the integral reaches a stable value. A complex computer programme has been used to evaluate $vG(\phi)$. These values have been used to calculate dynamic stress intensity factors N_c and N_1 at the two tips of the crack using (4.7) and (4.8) respectively.

Three different values of inter crack distance have been chosen for which c i.e. half of the inter crack distance is 0.2, 0.33 and 0.5 respectively. Further associated with each value of c there are three different values of the strip width viz. 1.5, 1.8 and 2.0. Keeping the limiting condition $k_2 < \pi/2h$ in view the values chosen for k_2 corresponding to $h = 1.5$ are from 0 to 1 in step of 0.1. Similarly in case of $h = 1.8$ and 2.0, k_2 has been taken from 0 to 0.8 and from 0 to 0.7 respectively. In Figs. 2–4 the amplitude of $|G(\pi/2)|$ has been plotted against k_2 with different values of h for $c = 0.2$, 0.33 and 0.5 respectively, while Figs. 5–7 display the graphs of $|G(0)|$ versus k_2 plotted for the same values of c and h.



AMPLITUDE OF |G (0)| PLOTTED AGAINST k_2 FOR C = 0·2

Figure 5.



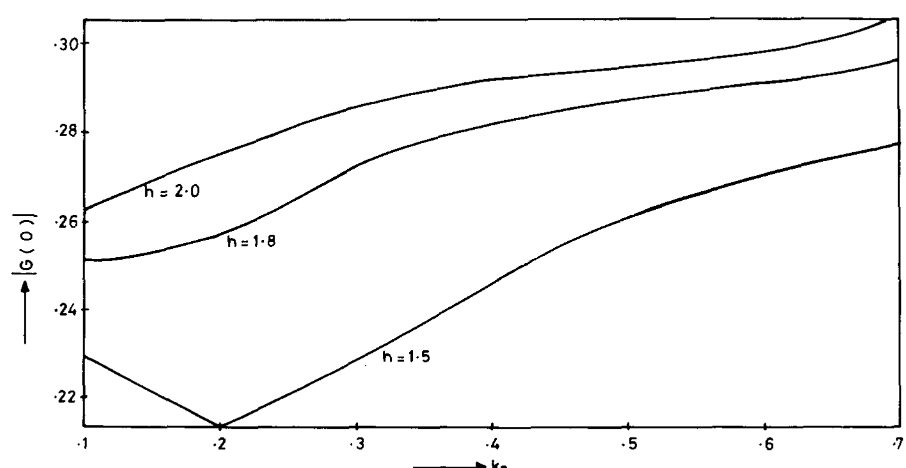
AMPLITUDE OF |G(0)| PLOTTED AGAINST k_2 FOR C = 0·33

Figure 6.

From these graphs it can be concluded that the stress intensity factor at the outer edge decreases with the increase of the frequency k_2 while at the interior edge of the cracks the stress intensity factor increases with an increase in k_2. However, the nature of the curve for $h = 1.5$ is slightly different. In this particular case the stress intensity factor decreases upto the values of $k_2 = 0.2$ then it continuously increases with the increase in the dimensionless frequency k_2.

It is interesting to compare the results with that of Jain and Kanwal [10]. The nature of the curves plotted for stress intensity factor versus the wave frequency at the outer edge of the crack is convex in nature in [10], however they are concave in our case. Furthermore at the inner edge of the crack the curves in [10] decrease with an increase in wave frequency while in our case they increase with the increasing value of wave frequency. The nature of the curve plotted for $h = 1.5$ is in accordance with the one exhibited in [10].

The difference in our curves for stress intensity factor from those of Jain and Kanwal is due to the effect of finite boundary of the strip.



AMPLITUDE OF |G(0)| PLOTTED AGAINST k_2 FOR C = 0·5

Figure 7.

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Résumé

Dans un mémoire précédent (6), on a étudié le cas de l'interaction d'ondes de cisaillement avec une fissure située au centre d'une bande infinie élastique. Dans la présente étude, on étend l'étude au cas de deux fissures coplanaires de Griffith. Une méthode de transformation intégrale est utilisée pour trouver la solution de l'équation de mouvement en partant de la théorie linéaire, pour un matériau homogène isotrope élastique. Cette méthode résoud le problème sous la forme d'une équation intégrale. On a observé que seules se propagent des ondes de cisaillement dont les fréquences sont inférieures à un paramètre dépendant de la largeur du guide d'ondes. L'équation intégrale est résolue par voie numérique pour une gamme de valeur de la fréquence d'ondes, de largeur de bande et de distance entre fissures. Les solutions sont utilisées pour calculer le facteur d'intensité de contrainte dynamique. Les résultats sont exposés par voie graphique.