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Effective creep Poisson's ratio for damaged concrete

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Abstract. First, creep data are presented for concrete under high sustained compressive stress which is, over the long-term strength of the concrete. Creep in both axial and lateral directions is reported. Creep Poisson's ratio has remarkable change before failure, and a sharp increase of creep Poisson's ratio can be observed in the region of failure.

Secondly, a damage model is developed for the analysis of creep damage in both axial and lateral directions; effective Poisson's ratio of damaged material as a model parameter plays an important role for evaluating lateral damage, which is similar to the effective Young's modulus in evaluating axial damage.

1. Introduction

Interest in damage of concrete with directional properties has increased in recent years, largely because of the limit of the scalar variable for isotropic damage of concrete and the fact that the way cracks grow in concrete depends on the stress states; cracks have a tendency to spread in the direction normal to tension and parallel to compression. It is well known that damage in the axial direction can be evaluated with the help of the effective Young's modulus, which can be measured during the process of damage and deformation under cyclic loading [1]. However, there is still no effective way to evaluate the damage in the lateral direction particularly, for instance, when concrete is in uniaxial compression. To analyze such a problem, a knowledge of the Poisson's ratio for damaged material may be useful and necessary. It is convenient, by analogy to the Poisson's ratio for elastic strains, to relate the ratio of the corresponding creep strains in damaged concrete to an effective creep Poisson's ratio. There is no agreement on the magnitude of creep Poisson's ratio among earlier investigators [2]. Some found it remained nearly constant, others reported a tendency to increase with time. It is likely that at least a part of the discrepancy is due to differences in test conditions.

In this paper an effective creep Poisson's ratio of damaged concrete is investigated experimentally by measuring creep in both axial and lateral directions under high sustained compressive stress over the long-term strength of the concrete. The author then proposes a theoretical analysis of lateral damage for concrete under uniaxial compression based on continuum damage mechanics and phenomenological approach. A damage model is proposed in which a tensorial quantity is invoked to describe the evolution of creep damage. The effective Poisson's ratio plays an important role in this model, similar to that of the effective Young's modulus in calculating the axial damage.

2. Experiment research

2.1. Test procedure

The mix proportions of concrete tested was 1 : 2.31 : 4.44 : 0.678 for cement : fine aggregate : coarse aggregate : water. The prismatic specimens, $10 \times 10 \times 50$ cm, were sealed by wax. Storage and test temperature was $20^\circ \pm 1^\circ$C. The material constants of test specimens

Table 1. Summary of time-dependent strain data for specimens up to failure

Series	σ/f'_c	f'_c (MPa)	Axial strain (10^{-6}) Total	Axial strain Creep	Lateral strain Total	Lateral strain Creep	Time (min) at failure
YC–95	0.95	10.79	1374	1111	—	—	2.5
YC–90	0.90	11.57	1165	885	−369.3	−280.6	8.0
YC–85	0.85	11.77	1386	752	−564.1	−306.1	39.5
YC–83	0.83	12.75	2213	1017	−745.8	−342.7	78.0

included: elastic modulus $E = 25.8$ GPa, elastic Poisson's ratio $\nu = 0.14$, ultimate strength of compression at 28 days age $f'_c = 9.8 - 12.75$ MPa.

Specimens were loaded to failure in uniaxial applied compression at stress levels of 83, 85, 90 and 95 percent of ultimate. Axial and lateral creep strain were measured by electrical resistance strain gauges.

2.2. Test results and discussion

2.2.1. *Time dependent deformation up to failure*

Summary of the experimental results for creep-time failure for the specimens investigated is given in Table 1.

Figure 1 (a, b, c) shows the creep strains in axial and lateral directions versus time after loading for the specimens sustained stress at 83, 85, 90 percent of ultimate f'_c until failure.

2.2.2. *Effective Poisson's ratio*

The creep test results confirmed that creep under uniaxial compression occurs not only in the axial but also in the lateral directions. Specifically, the creep under high stress, leading to failure, is accompanied by very high lateral creep. Denote the effective Poisson's ratio ν^* by the ratio of the lateral to the axial strain on the process of creep, that is

$$\nu^* = -\frac{\varepsilon_l}{\varepsilon_a} \tag{2.1}$$

in which, ε_a is the axial strain, and ε_l the lateral strain.

From the measured strain in the axial and lateral directions, effective Poisson's ratio given in Fig. 2 (a, b, c) can be obtained. It is shown that creep Poisson's ratio has a remarkable change before the creep failure occurs. Compared to the elastic Poisson's ratio, creep Poisson's ratio before failure, increases 46.7 to 112.4 percent. It suggests that, with increasing creep, internal microcracks in concrete propagate along the direction perpendicular to the direction of the compressive load, which is the main reason why creep failure occurs. So, we consider that the change in creep Poisson's ratio is one of the significant properties in assessing damage in concrete.

It should be noted that creep Poisson's ratio increases sharply only in the range of failure (see Fig. 2 b, c, specimen 1), otherwise there is no remarkable change (specimen 2). So, to measure the effective creep Poisson's ratio of damaged concrete, one should locate the gauges

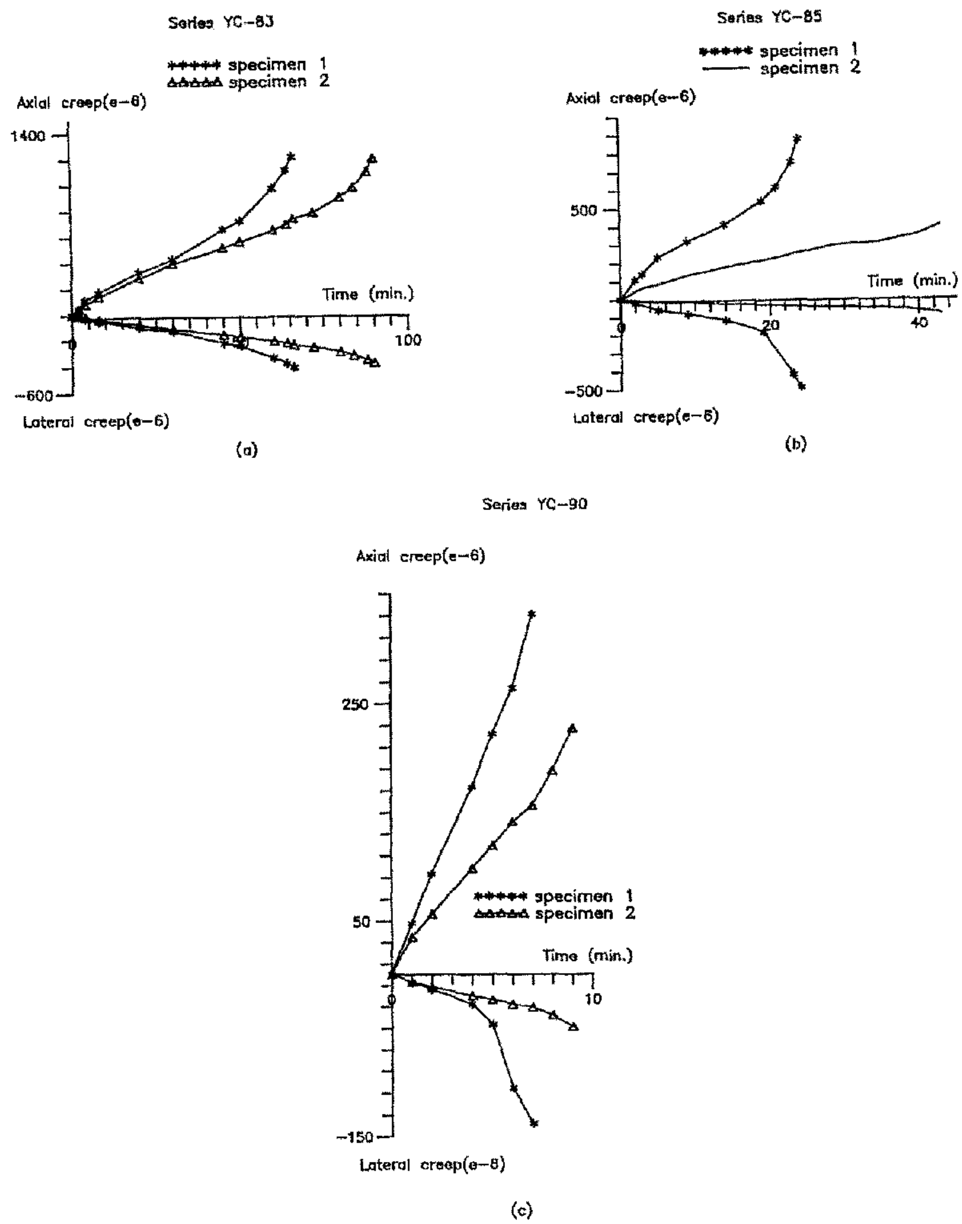

Fig. 1. Creep strain in the axial and lateral directions versus time under sustained stress at (a) 83 percent; (b) 85 percent; (c) 90 percent of ultimate f_c' until failure.

for lateral strain near the localized zone where failure occurs. The problem in doing so, of course, is that before loading and damage, the spread of the localized failure region is not known in advance.

3. Theoretical analysis

It has been confirmed by the experiment that changes in effective Poisson's ratio of damaged concrete ν^* reflect the time dependent property of damage in concrete and provide insight into

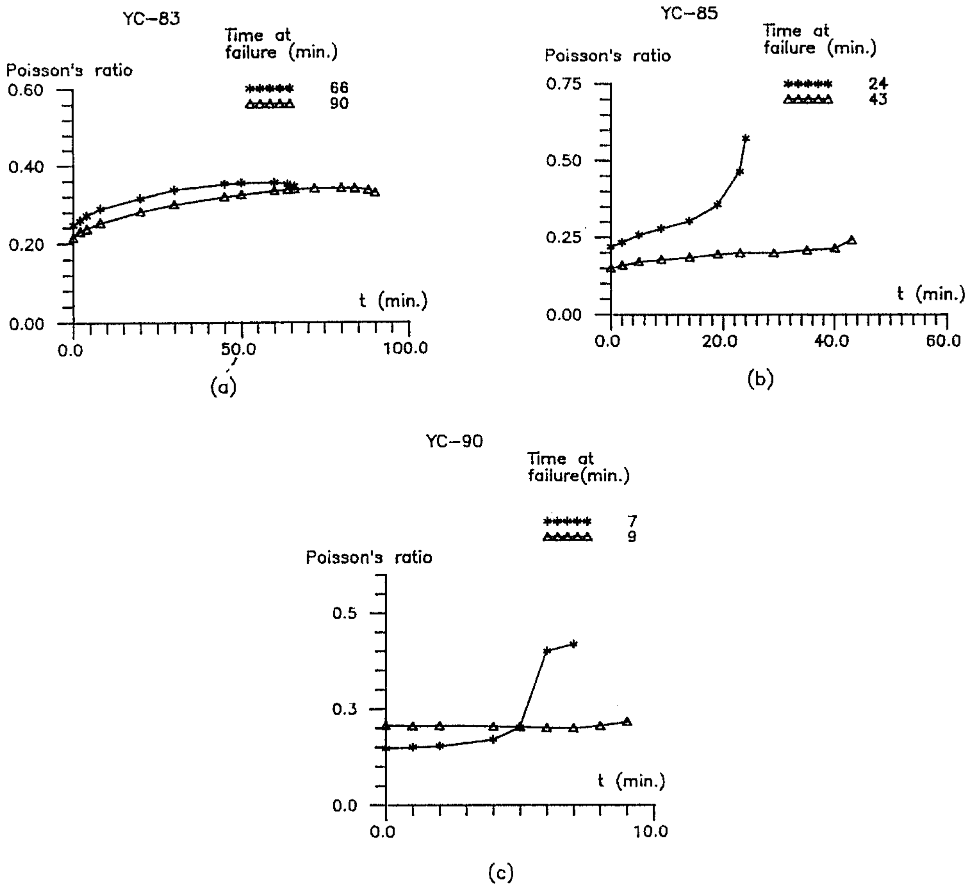

Fig. 2. Creep Poisson's ratio versus time under sustained stress at (a) 83 percent; (b) 85 percent; (c) 90 percent of ultimate f_c' until failure.

the analytical character of the damage tensor $\mathbb{D}$. Therefore, effective creep Poisson's ratio ν^* should be an important parameter in the damage model provided below.

3.1. DAMAGE MODEL

The components of the damage tensor D_{ij} are defined as

$$\mathrm{d}A_i^* = (\delta_{ij} - D_{ij})\,\mathrm{d}A_j, \tag{3.1}$$

where $\mathrm{d}A_i^*$, $\mathrm{d}A_i$ are the differential areas in the fictitious undamaged and actual damaged configuration, respectively [3]. Equation (3.1) represents a transformation of the area $\mathrm{d}A_j$ to $\mathrm{d}A_i^*$ due to damage.

An energy approach which relates the stress/strain state of the damaged configuration to a fictitious undamaged one is applied to determine the actual stresses σ_{ij} and strains ε_{ij} in the damaged state by using the effective stress σ_{ij}^* and effective strain ε_{ij}^*. Their relations can be written as [4]

$$\sigma_{ik} = \sigma_{ij}^*(\delta_{jk} - D_{jk}), \tag{3.2}$$

$$\varepsilon_{ik}^* = \varepsilon_{ij}(\delta_{jk} - D_{jk}). \tag{3.3}$$

In view of (3.2) and (3.3), the creep constitutive equations for the damaged concrete are known once ε_{ij}^* and σ_{ij}^* are related [4]

$$\varepsilon_{ij}^*(t) = C_{ijkl}(0)\sigma_{kl}^*(t) - \int_{\tau_1}^{t} \frac{\partial C_{ijkl}(t, \tau)}{\partial \tau} \sigma_{kl}^*(\tau)\, \mathrm{d}\tau, \tag{3.4}$$

in which the creep tensor $C_{ijkl}(t, \tau)$ is symmetric

$$C_{ijkl}(t, \tau) = C_{klij}(t, \tau). \tag{3.5}$$

Let the time rate of the creep damage tensor be denoted as $\dot{\mathbb{D}}$, the functional form of its components is given by

$$\dot{D}_{ij} = \dot{D}_{ij}(D_{kl}, \sigma_{kl}, \dot{K}_{kl}, t), \tag{3.6}$$

where $\dot{K}_{ij}$ represent

$$\dot{K}_{ij} = A_{ijkl}\dot{D}_{kl}, \quad (i, j \neq k, l). \tag{3.7}$$

To consider all quantities in the principal directions, say i, (3.6) becomes

$$\dot{D}_i = \dot{D}_i(D_j, \sigma_j, \dot{K}_j, t). \tag{3.8}$$

The components of the fourth order tensor A in (3.7) now reduce to

$$A_{ij} = g\left(\frac{\sigma_k}{\sigma_l}\right) a_{ij}, \tag{3.9}$$

such that A_{ij} acts as an influence coefficient representing the damage rate in the j-direction affecting that in the i-direction. The scalar function g in (3.9) is determined experimentally and a_{ij} is a coefficient denoting the influence in the j-direction, which is equivalent to the Poisson's effect. The limits on g are

$$g\left(\frac{\sigma_j}{\sigma_i}\right) = \begin{cases} 0, & \text{for } \sigma_j/\sigma_i = 0 \\ 1, & \text{for } \sigma_j/\sigma_i \to \infty \end{cases} \tag{3.10}$$

Moreover, the damage evolution equation (3.8) can be written as [5]

$$\dot{D} = -\gamma^{(i)} D + f(\sigma) + \dot{K}. \tag{3.11}$$

3.2. Effective Poisson's ratio and lateral damage

For uniaxial loading with $\sigma_{11} = \sigma$, (3.2) simplifies to

$$\sigma_{11}^* = \frac{\sigma}{1 - D_1}, \qquad \sigma_{ij}^* = 0, \quad \text{for } i \neq j \neq 1, \tag{3.12}$$

let the subscripts 1 and 2 denote quantities in the axial and lateral direction, respectively, while the subscripts e and c denote the elastic and creep response. The total axial and lateral strain thus take the forms

$$\varepsilon_1(t) = \varepsilon_1^e + \varepsilon_1^c, \qquad \varepsilon_2(t) = \varepsilon_2^e + \varepsilon_2^c. \tag{3.13}$$

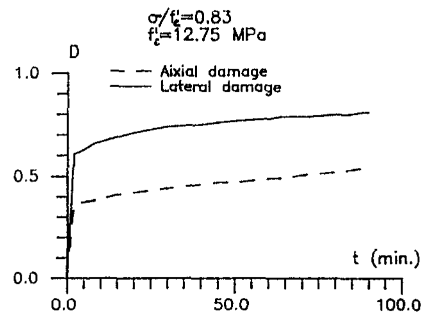

Fig. 3. Creep damage in the axial and lateral directions for concrete under uniaxial compression.

It can be identified with (3.4) that

$$\varepsilon_2^e = -\nu_e^* \varepsilon_1^e, \qquad \varepsilon_2^c = -\nu_c^*(t, \tau_1)\varepsilon_1^c, \tag{3.14}$$

in which ν_e^* and ν_c^* are, respectively, the effective elastic and creep Poisson's ratio and they are given as

$$\nu_e^*(t) = \nu_e(t)\frac{1 - D_1}{1 - D_2},$$

$$\nu_c^*(t) = \frac{1 - D_1}{1 - D_2}\int_{\tau_1}^{t} \frac{\partial}{\partial \tau}\left(\frac{\nu_e(\tau)}{E(\tau)} + \nu_c(t, \tau)c(t, \tau)\right)\frac{\mathrm{d}\tau}{1 - D_1}$$

$$\times \left[\int_{\tau_1}^{t}\left(\frac{1}{E(\tau)} + c(t, \tau)\right)\frac{\mathrm{d}\tau}{1 - D_1}\right]^{-1}. \tag{3.15}$$

For the special case, the elastic and creep Poisson's ratio are constant, that is $\nu_e = \nu_c = \nu$, so (3.15) yields

$$\nu_e^*(t) = \nu_c^*(t) = \frac{1 - D_1(t)}{1 - D_2(t)}\nu, \tag{3.16}$$

the lateral component of damage $D_2(t)$ becomes

$$D_2(t) = 1 - \frac{\nu}{\nu_c^*(t)}\{1 - D_1(t)\}. \tag{3.17}$$

For plain concrete specimens tested (series YC–83), the data for $\nu_c^*(t_i)$ was used to obtain $D_2(t_k)$, which is shown in Fig. 3. It is shown that damage in the lateral direction D_2 is greater than D_1 in the axial direction. What this means is that D_2 would first reach the failure threshold and lateral damage should be responsible for the creep failure of concrete at uniaxial compression.

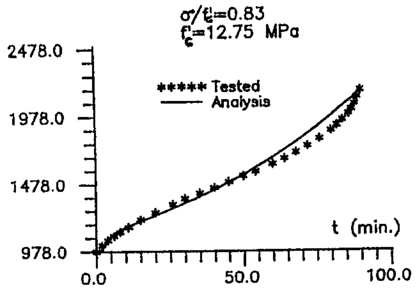

Fig. 4. Axial creep strain for specimens YC–83 and short-strength $f_c' = 12.75$ MPa.

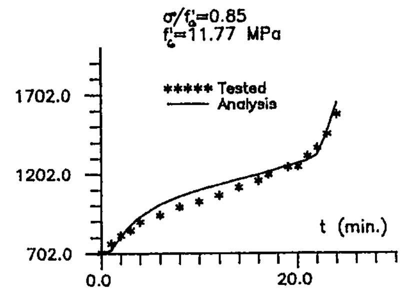

Fig. 5. Axial creep strain for specimens YC–85 and short-strength $f_c' = 11.77$ MPa.

3.3. Evaluation of creep for damaged concrete and comparison with test data

With the proposed damage evolution equation (3.11) and creep constitutive equation (3.4), the equations for uniaxial loading can be written as

$$\begin{aligned}
\dot{D}_1 &= -\gamma D_1 + f_1(\sigma), \\
\dot{D}_2 &= a\dot{D}_1, \\
\varepsilon_1(t) &= \frac{\sigma}{(1-D_1)^2 E(t)} - \frac{1}{1-D_1}\int_{\tau_1}^{t} \frac{\partial}{\partial \tau}\left(\frac{1}{E(\tau)} + c(t,\, \tau)\right)\frac{\sigma\, \mathrm{d}\tau}{1-D_1}, \\
\varepsilon_2(t) &= -\nu \varepsilon_1^*,
\end{aligned} \tag{3.18}$$

where $\nu^* = \nu_e^* = \nu_c^*$ is the effective Poisson's ratio of damaged concrete, and $c(t,\, \tau)$ is the specific creep at time t under a constant stress applied at time τ.

Using the above equations, the author calculated the creep strain for plain concrete under long term sustained strength. The calculation results and comparison with experimental data for series YC–83 and YC–85 are shown in Figs. 4 and 5; the theoretical analysis is shown to agree well with experimental data.

4. Conclusions

This work presents the effective creep Poisson's ratio of damaged concrete and its application to evaluating damage in the lateral direction by experimental observation and theoretical analysis. It appears that:

1. The creep under high stress, leading to failure, is accompanied by very high lateral creep, and the effective creep Poisson's ratio for damaged concrete has a remarkable change and sharp increase in the region of failure.
2. Effective Poisson's ratio of damaged materials as an important model parameter can be used to evaluate damage in the lateral direction for concrete and possibly other brittle materials in which material damage has directional properties.
3. For concrete under uniaxial compression, damage in the lateral direction is found to be more than in the axial direction and would presumably first reach its failure threshold laterally.

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