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# Fracture of brittle particles in a ductile matrix

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**Abstract**

Prerequisites for precipitate cracking in a yielding ductile matrix have been examined. A statistical model based on the fibre loading model combined with weakest link fracture theory is presented. With the model it is possible to estimate the effect of different variables on the particle fracture probability quantitatively. The predictions made are in excellent agreement with experimental results for a variety of different precipitate types. The result can be applied to calculate the probability of cleavage fracture for steels.

## 1. Introduction

Cleavage fracture is the most common catastrophic failure type at cryogenic temperatures. Because of a rising interest in arctic constructions, also the importance of understanding cleavage fracture is rising. During the past years it has been widely recognized that brittle precipitates are the main cause of cleavage fracture in metals [1,2]. The precipitates are not dangerous as such, but they become dangerous when they fracture forming microcracks. Therefore it is of great importance to be able to describe the fracture of brittle precipitates quantitatively.

Due to the importance of precipitate cracking, much work has been done to reveal the characteristics of it. The following findings concerning brittle precipitate cracking have been determined experimentally.

1. The fraction of broken particles increases with increasing matrix strain i.e. increasing flow stress [3–5].
2. The fraction of broken particles increases with decreasing temperature at constant matrix strain i.e. increasing flow stress [4,5].
3. Cracking occurs preferentially in particles oriented with their longest axis parallel to the direction of the tensile stress [3,4].
4. The orientation of cracks initiated in particles tends to be perpendicular to the direction of maximum tensile stress [3].
5. Rod shaped and platelike particles fracture more easily at their centre [4].
6. The average size of the fractured particles is larger than the average size of all particles [3].
7. Pre-yield twinning can give rise to precipitate cracking. In this case there is no clear correlation between the orientation of the cracked particles with respect to the tensile axis [4].

There have been several attempts to describe the mechanisms of brittle precipitate cracking [3–7]. Mainly two different mechanisms have been proposed to control precipitate cracking, the dislocation pile-up and the fibre loading mechanisms [4,6]. Both the suggested mechanisms are capable of describing some of the experimental findings. The dislocation pile-up model is able to describe experimental findings 1, 2 and 7 [6]. The fibre loading model is able to describe experimental findings 1, 2, 3, 4 and 5 [4]. Of the two fracture models the fibre loading model seems to be superior to the dislocation pile-up model in the post-yield region. The fibre loading model is however still unable to explain all the experimental findings. Namely, it does not explain why large particles fracture more easily than small particles.

The main problem with the existing models is that they are deterministic in nature. Stress controlled brittle fracture is always of a statistical nature and therefore the model describing brittle precipitate cracking should also be based on a statistical treatment.

In this work a statistical model describing the cracking of brittle particles in a ductile matrix is presented. The model is tested on several different materials and precipitate types and the results are shown to be quite good. The proposed model is capable of describing the experimental findings quantitatively and is also in agreement with brittle fracture theory.

## 2. The model

Brittle precipitates are basically ceramics. The fracture of brittle ceramics is rather well known and it has been found that a quantitative description of their fracture is possible by using weakest link theory based mathematics, such as the Weibull fracture probability distribution [8]. Since the precipitates are basically ceramics, their fracture probability should also be correctly described with the Weibull distribution.

The model presented here is basically a combination of the fibre loading model and the weakest link model. It is assumed that the particle stress is mainly described through the fibre loading model. Furthermore it is assumed that the particles contain randomly distributed flaws, weak spots and orientations.

According to the fibre loading model [9]

$$\sigma_{\max}^{\text{particle}} \sim \frac{l}{d} \cdot \sigma_{\text{flow}}^{\text{matrix}} \tag{1}$$

where $l$ and $d$ are particle length and diameter in the direction of $\sigma_{\text{flow}}^{\text{matrix}}$.

Relation (1) is not quite correct. The tensile stress acting on a particle is actually related to two times the matrix flow stress in shear. However, since the flow shear stress is seldom known the tensile flow stress is here used as an approximation. As relation (1) is really only used as a proportionality relation the error thus made is not very large.

The fibre loading model yields the following conclusions:

1. The particle tensile stress increases when $l/d$ increases.
2. The particle tensile stress is largest when the particle is oriented in the direction of the tensile stress.
3. The particle tensile stress is largest in the middle of the particle.
4. The particle tensile stress increases when the matrix strain i.e. flow stress increases.
5. The particle tensile stress has the same direction as the matrix tensile stress.

From the weakest link model [8] the probability of fracture $P\{fr\}$ can be written

$$P\{fr\} = 1 - \exp\left\{ -\left(\frac{d}{\bar{d}}\right)^3 \cdot \left(\frac{\bar{d}}{d_N}\right)^3 \cdot \left(\frac{\sigma - \sigma_{\min}}{\sigma_0 - \sigma_{\min}}\right)^m \right\} \tag{2}$$

where $d$ and $\bar{d}$ are particle size and mean size of the particle population respectively, $\sigma$ is the tensile stress acting on the particle, $\sigma_{\min}$ is the minimum fracture stress of the particle, $m$ is the Weibull inhomogeneity factor, and $\sigma_0$ and $d_N$ are normalizing parameters.

The weakest link model yields the following conclusions:

1. The probability of fracture increases when the particle size increases.
2. The probability of fracture increases when the stress acting on the particle increases.

Combing the fibre loading model with the weakest link model yields an explanation to all the experimental findings regarding matrix post-yield precipitate cracking.

To be able to apply (2) for a given material one has to know the particle size distribution. A relatively simple expression capable of describing the particle size distribution is [10–12]

$$P\{s\} = \frac{(\nu-2)^{\nu-1}}{\Gamma(\nu-1)} \cdot s^{-\nu} \cdot \exp\left\{-\frac{\nu-2}{s}\right\} \nu > 2 \tag{3}$$

where $s = d/\bar{d}$ and $\nu$ are a distribution dependent constant, describing the distribution inhomogeneity.

By combining (2) and (3) it is then easy to calculate the total fracture probability $P(f)$ of the particle distribution at any given stress

$$P\{f\} = \int_0^\infty P\{fr\} \cdot P\{s\} \cdot \partial s \tag{4}$$

with $s = d/\bar{d}$

Similarly it is also possible to calculate the mean size of the fractured particles versus the mean size of all particles

$$\frac{\bar{d}_f}{\bar{d}} = \frac{\int_0^\infty P\{fr\} \cdot P\{s\} \cdot s \cdot \partial s}{\int_0^\infty P\{fr\} \cdot P\{s\} \cdot \partial s} \tag{5}$$

Defining a stress parameter $f(\sigma)$

$$f(\sigma) = \left(\frac{\bar{d}}{d_N}\right)^3 \cdot \left(\frac{\sigma - \sigma_{\min}}{\sigma_0 - \sigma_{\min}}\right)^m \tag{6}$$

Equation (2) can be expressed in the form

$$P\{fr\} = 1 - \exp\left\{-\left(\frac{d}{\bar{d}}\right)^3 \cdot f(\sigma)\right\} \tag{7}$$

As long as the particle stress is assumed proportional to the flow stress of the matrix, it is possible to write, for spherical particles

$$f(\sigma) = \left(\frac{\bar{d}}{d_N}\right)^3 \cdot \frac{\left\{\sigma_{\text{flow}}^{\text{matrix}} - \sigma_{\min}^{\text{matrix}}\right\}^m}{\sigma_1^m} \tag{8}$$

where $\sigma_1$ is a constant and $\sigma_{\min}^{\text{matrix}}$ is the matrix stress corresponding to the minimum particle fracture stress. Because plastic flow in the matrix is a prerequisite for fracture, $\sigma_{\min}^{\text{matrix}}$ can be aproximated to equal the yield stress $\sigma_y$ of the matrix. The normalization constant $d_N$ can be given any realistic value, because it will be compensated through the other constant $\sigma_1$. Actually if it would be possible to extract the particles from the matrix and to test them separately, the values of $d_N$ and $\sigma_1$ could be determined experimentally. In this study the value $d_N = 1$ $\mu$m is used.

If, for rod shaped particles, the ratio $l/d$ is constant, which normally is not the case, or at least the actual distribution is known, (8) can be applied after some modifications. If, however, only the effective particle size $\bar{d}_{\text{eff}}$ is known without knowledge of the ratio $l/d$, (8) can not be applied to rod shaped particles. It was found that in such a case (8) can well be approximated as

$$f(\sigma) = \left(\frac{\bar{d}_{\text{eff}}}{d_N}\right)^{3-m} \cdot \frac{\left\{\sigma_{\text{flow}}^{\text{matrix}} - \sigma_{\min}^{\text{matrix}}\right\}^m}{\sigma_2^m} \tag{9}$$

where $\bar{d}_{eff} \approx \frac{\bar{d}+\bar{l}}{2}$ = the effective mean particle size. Equations (3)–(5) and (8) or (9) thu describe the effect of tensile stress and particle size on particle fracture probability quantita tively.

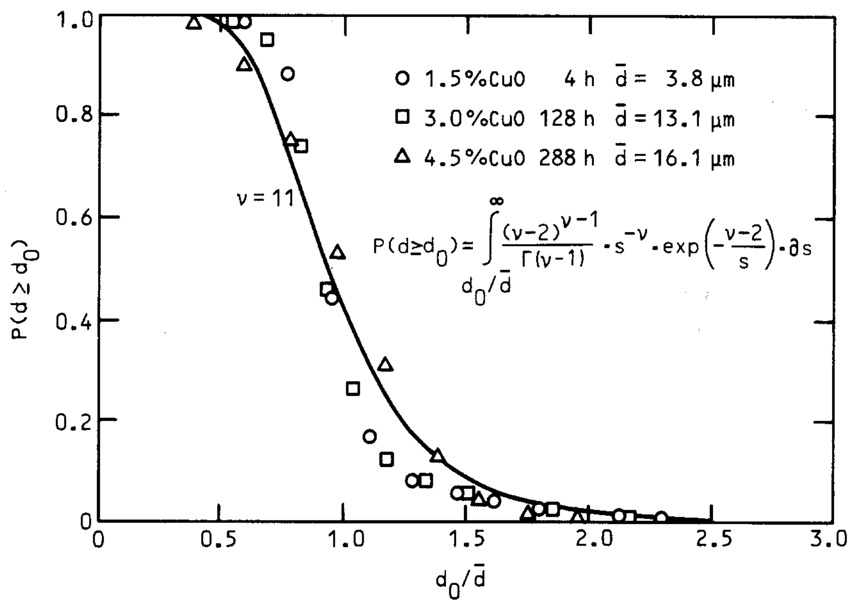


*Fig. 1.* Normalized cumulative CuO-particle size distributions [13].

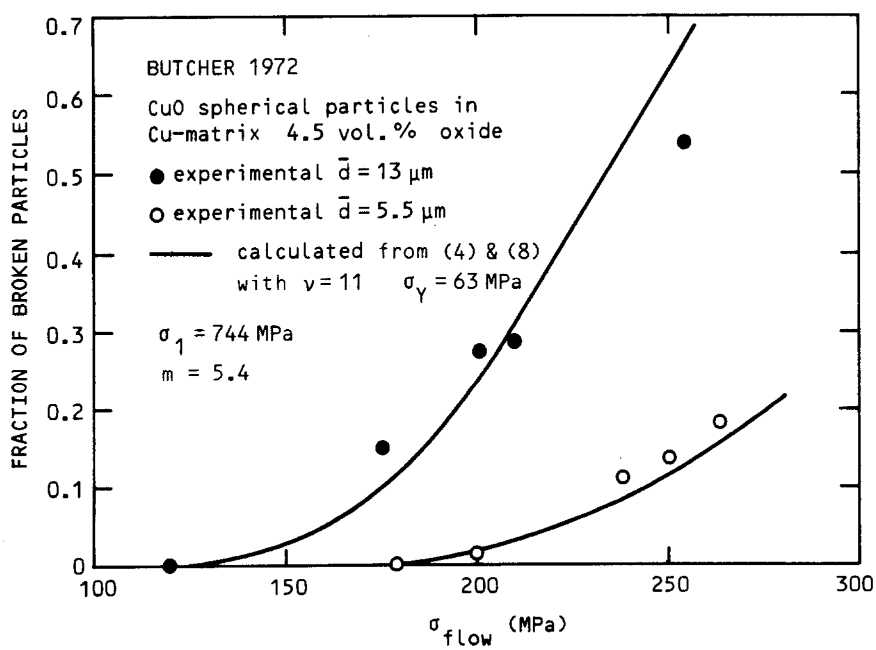


*Fig. 2.* The effect of matrix tensile flow stress on CuO-particle fracture probability [13].

## 3. Application and discussion of the model

In the following, the model is applied to spherical as well as rod shaped particles.

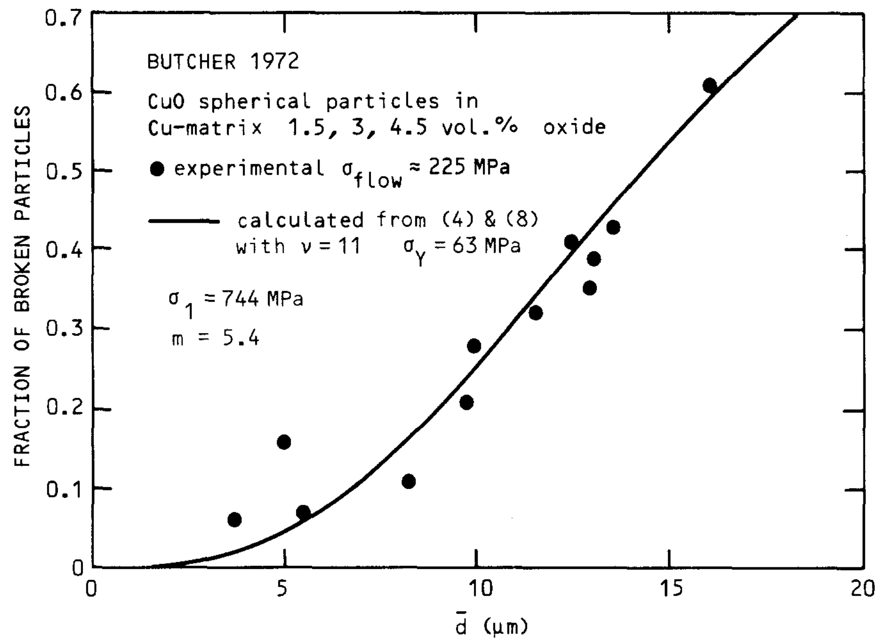


*Fig. 3.* The effect of mean particle size on CuO-particle fracture probability [13].

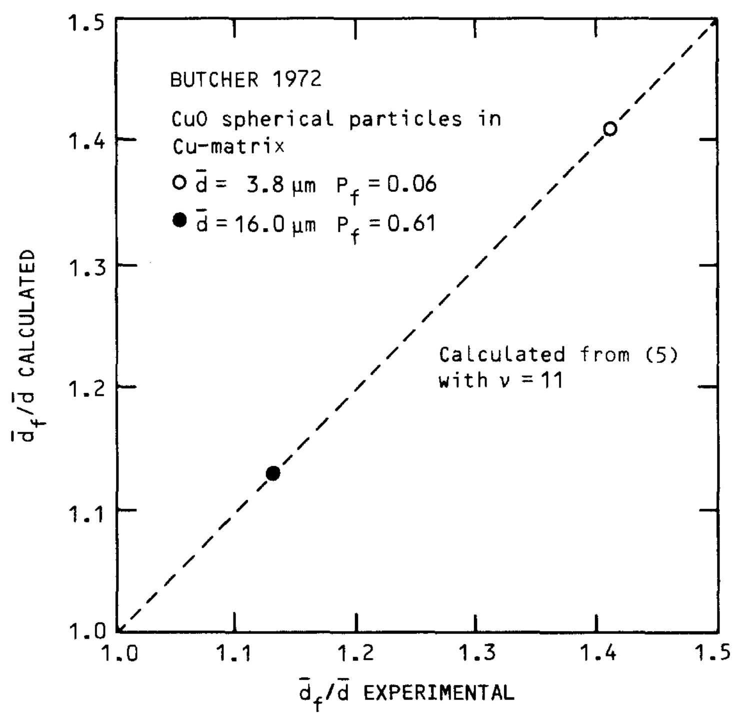


*Fig. 4.* Comparison of experimental and calculated $\bar{d}_f/\bar{d}$ ratios [13].

*3.1. Spherical particles*

Butcher [13] investigated spherical CuO-particles in a Cu-matrix. He varied the CuO-content, mean particle size and tensile stress. The particle size distributions for three different cases are presented in Fig. 1. It is seen from the figure that the normalized distributions can be described quite well with (3) applying $\nu = 11$. It is also seen that $\nu$ seems to be independent of $\bar{d}$.

Using $\nu = 11$ in (3) and applying (4), (7) and (8) it is possible to describe both the effect of matrix stress (Fig. 2) as well as particle size (Fig. 3) on the particle fracture probability. Bearing in mind that the only unknown parameters having to be fitted, are the constants $\sigma_1$ and $m$ in (8), the results are very good.

Also, applying (5) to calculate the mean size of the fractured particles versus the mean size of all particles yields a perfect fit with the experimental values (Fig. 4).

Gurland [3] investigated spheroidized 1.05 percent C steel. He measured the fraction of broken particles as a function of tensile stress. Unfortunately he does not give the carbide size

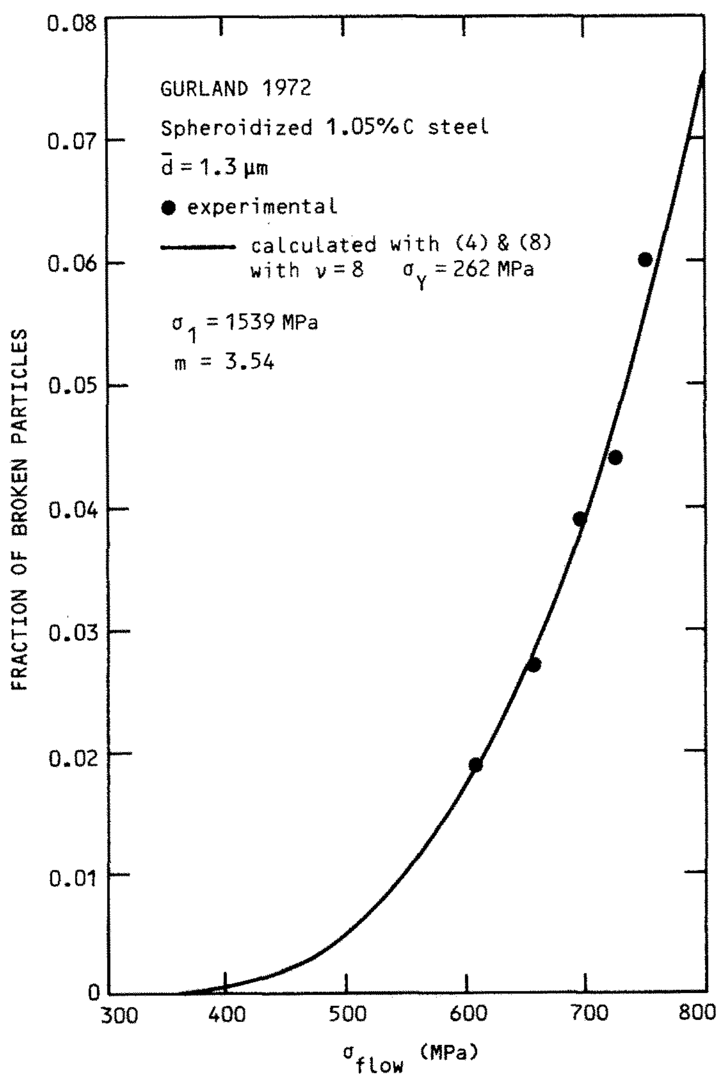


*Fig. 5.* The effect of matrix tensile flow stress on spheroidized cementite particle fracture probability [3].

distribution, but from his measured fracture size ratio $\bar{d}_f/\bar{d} = 1.69$ it can be deduced through application of (5) that $\nu = 8$.

The effect of matrix tensile stress on the cementite fracture probability is presented in Fig. 5. The theoretical result is seen to be in good agreement with the experimental findings.

### *3.2. Rod shaped inclusions*

Gangulee and Gurland [7] investigated elongated rod shaped silicon particles in aluminium-silicon alloys. They measured the fraction of broken silicon particles as a function of effective mean particle size and matrix flow stress. They do not give the particle size or length ratio distributions, but $\nu$ can be calculated from measured $\bar{d}_f/\bar{d}$ values combined with the total fracture probability $P\{f\}$ through application of (5). The value of $\nu$ as a function of effective mean particle size is presented in Fig. 6.

The fact that $\nu$ changes as a function of effective mean particle size is thought mainly to be due to the fact that the ratio $l/d$ changes as a function of the mean particle size. This could, however, not be confirmed since no information of the ratio $l/d$ in this case is given.

Using the appropriate values of $\nu$ and applying (4), (7) and (9) it is possible to describe the effect of stress (Fig. 7) and effective particle size (Fig. 8) on the particle fracture probability. There seems to be a slight discrepancy in the theory and experiments regarding the effect of mean particle size. This may however be due to the fact that some of the particle sizes were partly modified by 0.15 percent Na addition. Such a change in the particle chemistry is bound to change their fracture properties. Unfortunately there is no information regarding which specimens have the Na addition.

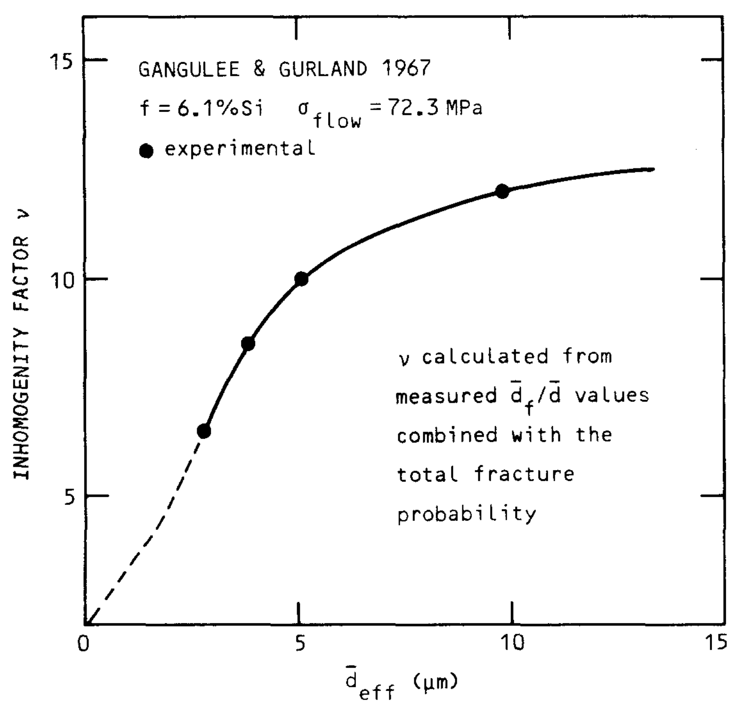


*Fig. 6.* The effect of effective mean silicate particle size on size distribution inhomogeneity factor $N$ [7].

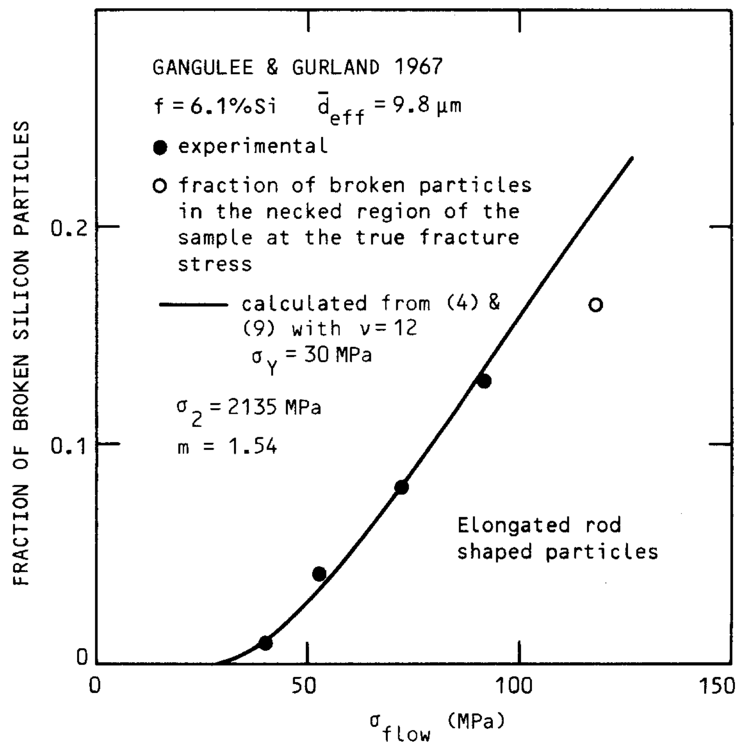


*Fig. 7.* The effect of matrix tensile flow stress on rod shaped silicon particle fracture probability [7].

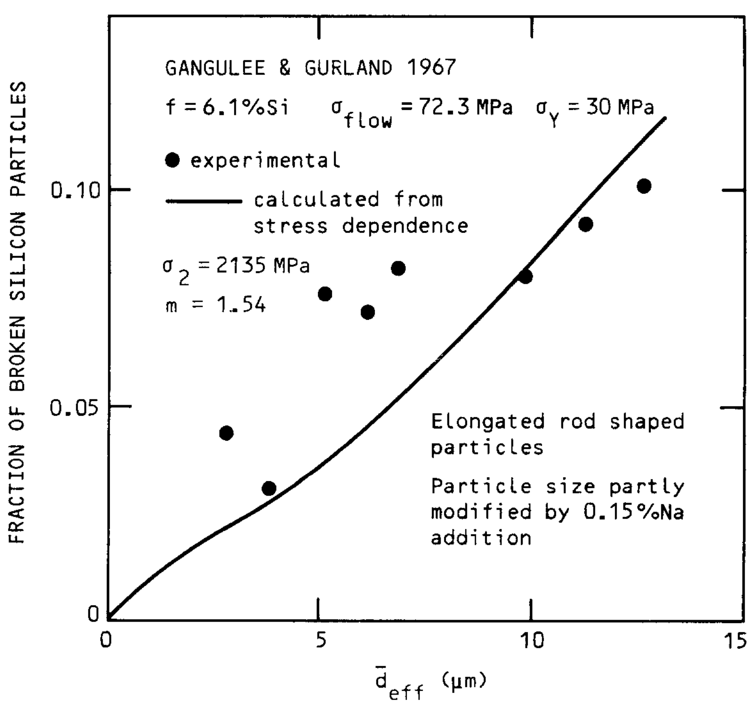


*Fig. 8.* The effect of effective mean particle size on rod shaped silicon particle fracture probability [7]. Calculated from stress dependence in Fig. 7. Inhomogeneity parameter $\nu$ as according to Fig. 6.

Bearing in mind that (1) is only an approximation the results of the model are very promising. Using the actual shear flow stress instead of the tensile flow stress in the calculations would probably yield even better results.

The model can be directly applied to make more accurate calculations to predict the probability of actual cleavage fracture of the metal matrix. This can be done simply by combining the procedure presented here with, for example, the WST-model [10].

## 4. Summary and conclusions

The fracture of brittle precipitates in a ductile yielding matrix have been explained through a statistical model. The model presented is a combination of the fibre loading model and weakest link type statistics. The model has been shown to be capable of describing the precipitate cracking correctly both qualitatively as well as quantitatively. The main conclusions based on the model are as follows.

1. The fibre loading model describes well the stress acting on a precipitate in matrix post-yield situations.
2. The size effect observed in precipitate cracking can be correctly described through the statistical weakest link model.

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## Résumé

On examine les conditions pour lesquelles un précipité fragile se rompt dans une matrice ductile en déformation plastique. On présente un modèle statistique basé sur les modèles de chargement d'une fibre et combiné avec la théorie de la rupture de la liaison la plus faible. Avec ce modèle, il est possible d'estimer quantitativement l'effet de diverses variables sur la probabilité de rupture d'une particule. Les prédictions qui sont avancées sont en excellent accord avec les résultats expérimentaux, pour une gamme de divers types de précipités. Les résultats peuvent être appliqués au calcul de la probabilité de rupture par clivage dans le cas des aciers.