International Journal of Fracture 70: R9–R14, 1995.

EVALUATION OF THE ELASTIC T-STRESS USING A HYBRID FINITE ELEMENT APPROACH

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Classical fracture mechanics is based on the premise that a single parameter such as the stress intensity factor K or Rice's path independent integral J uniquely determines the conditions at the crack tip. It is supposed that such fracture events as the initiation of crack growth and the onset of unstable crack propagation can be unambiguously characterized by a crack tip singularity which is dominant over microstructurally significant size scales. On the other hand, evidence is accumulating in the literature (e.g. [1-3]) that with K (in linear elastic fracture mechanics) or J (in elastic plastic fracture mechanics) held constant, the crack behaviour may vary with the degree of local in-plane biaxiality.

It has been suggested that a dimensionless parameter B, the stress biaxiality ratio, may be a necessary adjunct to the one-parameter description of the stress and strain field at the crack tip. The biaxiality parameter B, introduced in [3], is given by

$$B = \frac{T\sqrt{\pi a}}{K_I}, \tag{1}$$

where K_I is the stress intensity factor corresponding to the crack length, a. The term T denotes a constant-stress parallel to the crack flanks and can be related to the second term in the Williams stress function $\phi(r,\varphi)$[5],

$$\Phi(r,\varphi) = \sum_n \{(-1)^{n-1} d_{2n-1} r^{n+\frac{1}{2}} \left[-\cos\left(n-\frac{3}{2}\right)\varphi + \frac{2n-3}{2n+1}\cos\left(n+\frac{1}{2}\right)\varphi\right]$$

$$+(-1)^n d_{2n} r^{n+1}[-\cos(n-1)\varphi + \cos(n+1)\varphi] - \} \tag{2}$$

or, equivalently, to the first nonsingular term in the corresponding σ_{ij} stress distribution expansion:

$$\sigma_{ij} = \frac{K_I}{\sqrt{\pi r}} f_{ij}(\varphi) + T\delta_{ij} + \ldots \tag{3}$$

where δ_{ij} is the Kronecker delta, (r, φ) are the polar coordinates with the origin at the crack tip, and $f_{ij}(\varphi)$ represents function that follows from (2). Only the symmetric case is considered and the crack surfaces are traction free.

A two parameter fracture mechanics methodology, as suggested, e.g. in [1], rests upon the basic assumption that the structure and the test specimen encompass the same range of B values and that if two different specimens have the same T stress they will fail at the same value of applied K (or J). A knowledge of the elastic biaxiality parameter B is thus of paramount importance in order to perform structural analysis by the two-parameter approach. While the stress intensity factor K has been tabulated for a wide range of geometries and procedures for its estimation are well documented, the biaxiality parameter B is only available for a more limited number of cases, e.g. the B values for single-edged notch (SEN), double-edged notch (DEN), centre-cracked-plate (CCP) geometries have been given in [2-4].

This note introduces an effective method of calculating B, using a finite element hybrid technique for two-dimensional crack analysis. As an example, the B values are given for a C shaped tension (CST) specimen.

The method used here to calculate B utilizes the crack hybrid elements formulation developed and used by Pian [6] and others. The method uses the Hellinger-Reissner principle as a functional to formulate special crack-tip elements. A group of special hybrid elements based on theoretical principles given in [6] has been elaborated, e.g. by Kuna et al. in [7]. For the displacements and stresses in the interior of the hybrid elements, the first N terms of (2) are assumed. In the Fortran implementation CRACK2D [7], the number of terms in the Williams series to be calculated is up to 28 and the corresponding coefficients dj can be obtained directly.

The first term is then directly related to the stress intensity factor. The second one determines the T stress, see (1-3).

$$T = 4d_2 \tag{4}$$

and

$$B = \frac{4d_2\sqrt{\pi a}}{K_I} \tag{5}$$

The accuracy of the B (or T) values obtained by the hybrid element approach as implemented using CRACK2D can be compared with the results for SEN and DEN geometries given in Table 1. The first 17 terms in the series shown in (2) were used for the calculation.

Note that the present results weree obtained for a finite element mesh consisting of 38 isoparametric quadrilateral elements, 127 nodes and 1 hybrid-crack-tip element with the dimension a/5. The test calculations of hybrid elements given for other specimen geometries confirm their high accuracy and efficiency for the estimation of B.

The B values obtained for two types of C-shaped tension specimen (namely X/W=0.5 and X/W=0, respectively, see Fig. 1) are shown graphically in Fig. 2. Figure 2 shows that the B values are negative for CST specimens for which the ratio a/W is less than approximately 0.3. These negative B values are associated with shallow cracks and with tension-type loading, and they generally depress the maximum tensile stress at the crack tip. On the other hand, for a/W ratios greater than 0.3, the B values are positive and range up to 0.8 for a/W =0.8, see Fig. 2. This corresponds to deeply notched bending and leads to high crack tip triaxiality.

CST specimens have been used most commonly to examine the behaviour of cracks in cylinders subjected to internal pressure. Cracked cylinders are usually modelled as circular rings with the cracks emanating from the inner surfaces. Figure 3 presents the B values corresponding to various values of $a/(r_o-r_i)$ for the example of a circular ring that is under internal pressure and contains one inner radial crack. The range of B values is approximately from -0.2 to -0.5. Note that CST specimens do not meet the requirement for such negative B values. The use of C-shaped tension specimens to determine the fracture behaviour of cracked cylinders gives results that are too conservative. From the point of view of two parameter fracture mechanics, the use of single or double-edge-notch geometry is better suited to the conditions of a cracked cylinder.

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1 January 1995

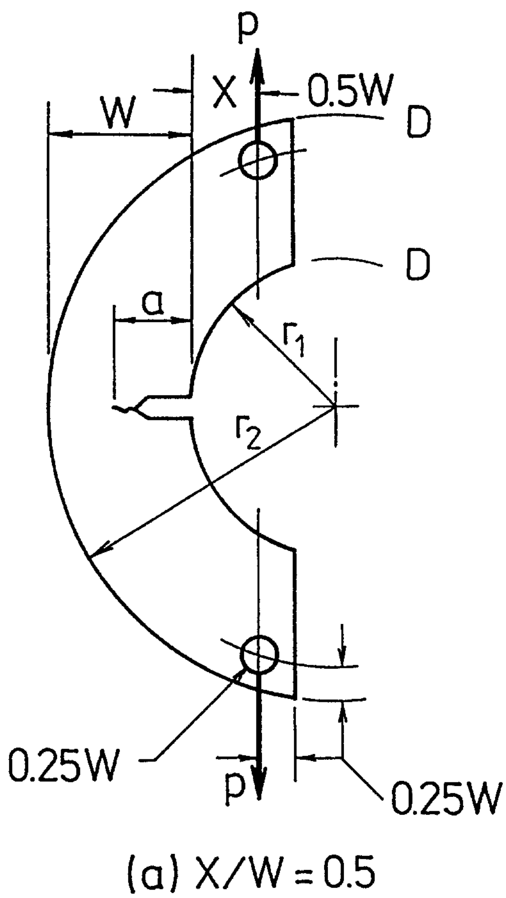

Figure 1. C-shaped tension specimen (CST).

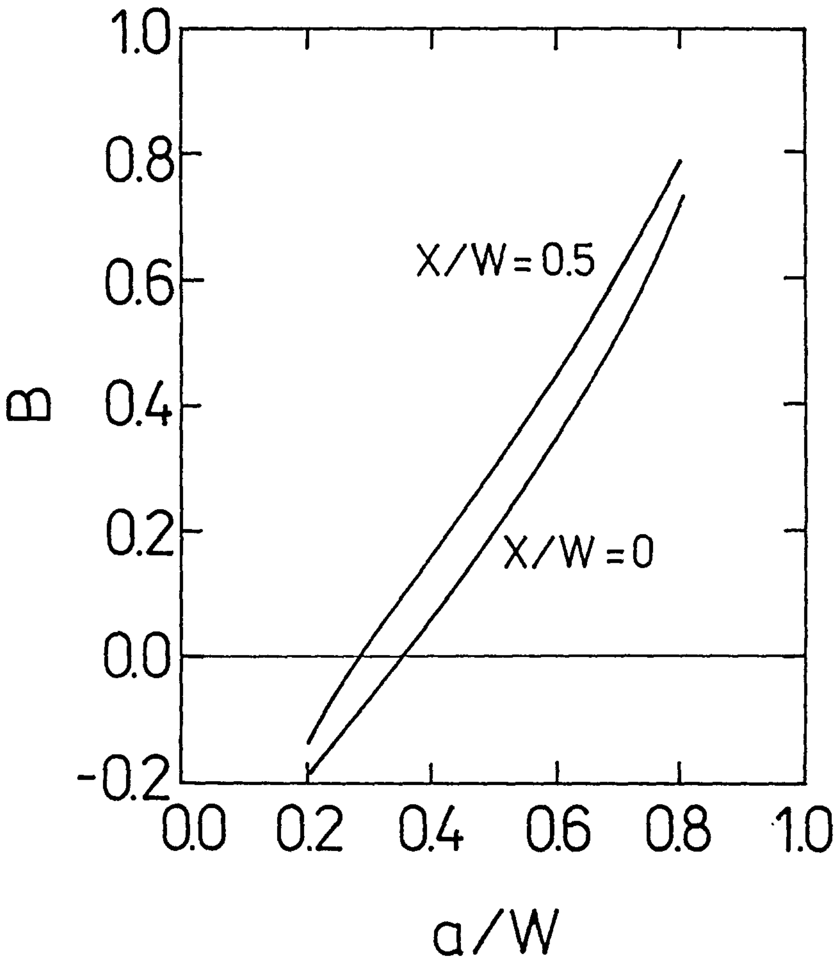

Figure 2. Values of the dimensionless biaxiality parameter B for CST specimens as a function of a/W. The results hold for the case wherein $r_2/r_1=2$.

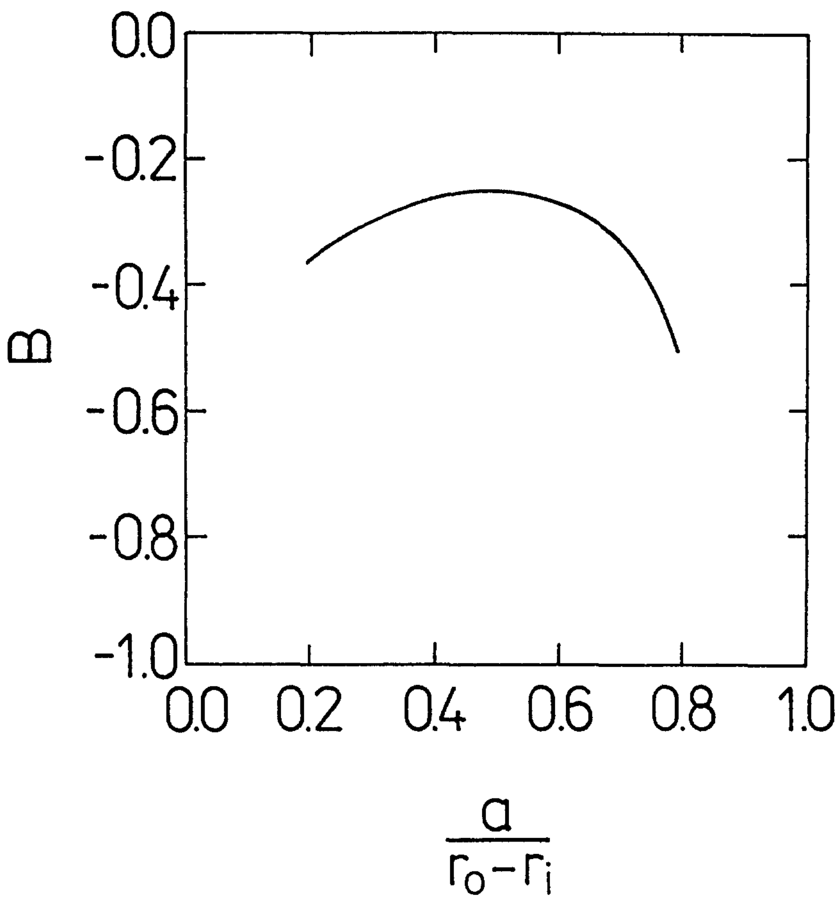

Figure 3. Values of the parameter B for a circular ring that is subjected to internal pressure and contains an inner radial crack. The symbols r_i and r_o represent the inner and outer radii (r_o/r_i=2), respectively, and a represents the crack length.