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Thermally activated crack propagation – theory

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ABSTRACT
Steady state crack propagation in solids is analyzed as a thermally activated process. The fracture mechanics concept of a crack driving force is formally introduced to molecular rate theory. This representation of crack propagation appears to be, in many aspects, similar to that of the motion of a dislocation under a shear stress across thermal obstacles. The basic thermodynamic relations are derived for steady state crack propagation using assumptions similar to those well accepted in theories of deformation based on thermally activated dislocation motion.

1. Introduction

Fracture mechanics analysis predicts the conditions for which a crack propagates. It is now well established that the parameters $\mathcal{G}$ and K, respectively Irwin's crack extension force and the stress intensity factor, are the proper variables to represent the action of mechanical forces on any crack system. $\mathcal{G}$ and K are related, for plane strain fracture conditions, by

$$\mathcal{G} = \frac{K_{\mathrm{I}}^2(1-\nu)}{2\mu} + \frac{K_{\mathrm{II}}^2(1-\nu)}{2\mu} + \frac{K_{\mathrm{III}}^2}{2\mu} \tag{1}$$

where $K_{\mathrm{I,II,III}}$ are the stress intensity factors, for opening, sliding and tearing modes of fracture, μ is the shear modulus and ν is Poisson's ratio.

Consider an experiment in which a sample containing a crack is progressively loaded, then according to the Irwin [1] or the Orowan [2] theories, the crack starts to propagate when the crack extension force (or the equivalent stress intensity factor) reaches a critical value $\mathcal{G}_{cr}$ (or K_{cr})†; in plane strain, the quantities $\mathcal{G}_{cr}$ and K_{cr} are recognized as material properties independent of specimen geometry.

Although the element of time can be introduced in fracture mechanics to explain the time dependence of crack propagation at large crack velocities by taking account of a kinetic energy term, a wide range of rate-dependent fracture processes also occurs at low crack velocities, where dynamic effects are negligible. In this case, the fracture surface energy (or the critical stress intensity factor) shows a strong dependence on the temperature and the crack velocity, features which are characteristic of a thermally activated process. It is therefore not surprising that slow crack propagation and a related phenomenon, "delayed fracture", have been described in terms of thermally activated processes [3–15]. Specific models assuming detailed molecular mechanism at the crack tip as the rate controlling process have been proposed to interpret crack velocities in brittle solids. One of the most proposed reaction rate models for fracture of solid materials assumes that the crack velocity is controlled by a temperature and stress activated bond rupture process; resulting

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† In fracture mechanics, these quantities are often written $\mathcal{G}_c$ and K_c.

theories essentially differ according to the specific assumptions made on the model. A comparison of the reaction rate models for delayed fracture of Tobolsky–Eyring, Coleman, Zhurkov and Hsiao has been made by Henderson *et al.* [16] with the conclusion that bond rupture models are preferred. Knauss [17] also reviewed different expressions proposed for the kinetics of bond rupture in polymers as applied to a rate theory description of fracture. A literature survey [3–16] showed that a variety of equations describing crack velocity in the Arrhenius form of a rate activated process have been proposed and used to fit experimental data relating the crack velocity to the stress intensity factor (or to the fracture surface energy) and to the temperature. The forms of these equations are listed in Table 1. Zhurkov's rate equation for time to rupture has also been included in the table because of a probable connection between time to rupture and subcritical crack velocity.

Since it appears that crack propagation, at least stable crack propagation, behaves as a thermally activated process, it is important to consider a general model of crack propagation and to derive the formalism of a rate theory, as it applies to crack propagation. The formal rate description in the following section establishes the interrelations between the rate controlling parameters. The determination of thermodynamic parameters is discussed in the third section. An athermal crack extension component is introduced in the fourth section. In the final section, general conclusions are summarized.

TABLE 1
Expressions for the crack velocity, as found in literature*.

Reference	Crack velocity
(a) Wiederhorn *et al.* [10] and Wiederhorn and Bolz [13]	$\nu = \nu_0 \exp(-E^* + bK_1)/RT$
(b) Wiederhorn [11]	$V = \alpha \exp(-\Delta H + \lambda K_1)/RT$
(c) Evans [18]	$V = \alpha' K^n \exp(-\Delta H/RT)$
(d) Charles [6]	$v = k(\sigma_m)^n \exp(-A/RT)$
(e) Evans and Wiederhorn [12]	$V = V_0 K_1^{nT/T_0} \exp(-\Delta H/RT)$
(f) Charles and Hillig [7]†	$\nu = \nu_0 \exp(-E^* + V^*\sigma - V_M\gamma/\rho)/RT$
(g) Kies and Clark [14]	$\dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{\mathrm{d}_0^3}{h} \exp(-f^*/kT + \beta\mathscr{G}/N_0kT)\{(p+\beta)\mathscr{G} - N_0 f_s\}$
(h) Atkins *et al.* [15]	$\dot{a} = A_1 \exp(-(U - \lambda R)/kT)$
Zhurkov [4]	time for rupture: $\tau = \tau_0 \exp(U_0 - \gamma\sigma)/kT$

* See original articles for definitions of symbols.
† (For corrosion effect at tip of crack).

2. Rate theory applied to steady state crack propagation

The problem to be considered is a crack propagating in a solid at a macroscopic velocity v, under the action of an external mechanical force. It will be assumed that thermally activated rate processes near the crack tip determine the crack velocity. For clarity, only fracture for mode I opening will be considered, although any combination of the three opening modes could be considered as well.

Now, let's think of a specific, thin section of the crack of width ΔX defined by the contour Σ as shown in Fig. 1; Σ is chosen large enough around the crack tip so that, at any time, it covers all regions of material undergoing transition from the

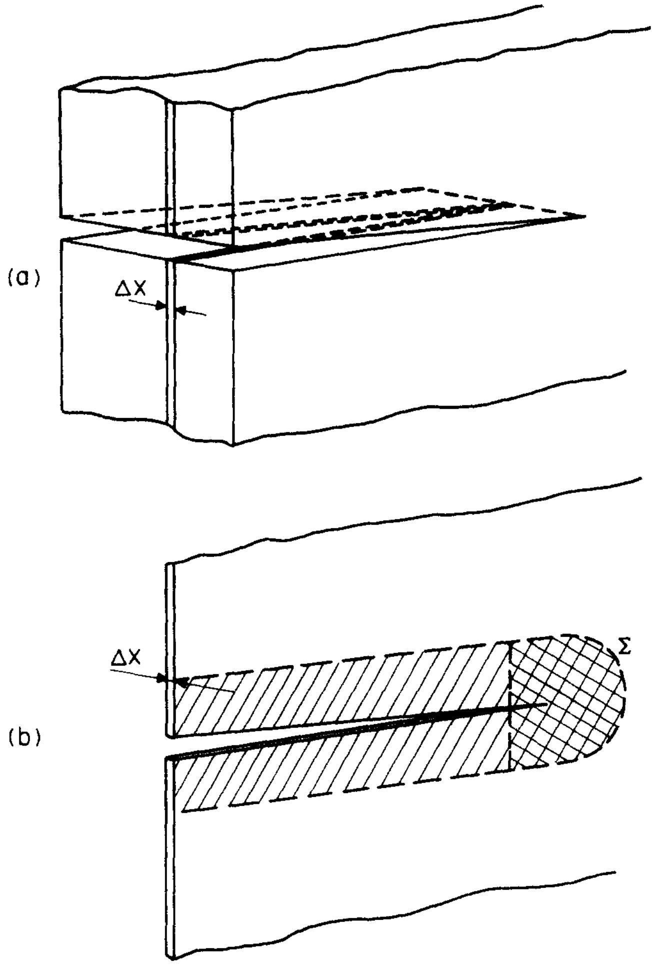


Figure 1. (a) Macroscopic crack and crack element of width ΔX; (b) crack system and crack tip region Σ of crack element.

original material to the fracture-modified material. ΔX is chosen small enough so that the crack element considered has to overcome one energy barrier at a time. It is assumed that as the crack advances, the crack element has to overcome short range energy barriers. Thus, the energy barriers are the rate controlling mechanisms and the crack motion is thermally activated. Examples of such barriers (or obstacles) might be rupture of main chain bonds in polymers, diffusion of molecular segments, diffusion of chemical components to the crack tip region, nucleation of voids, obstacles to plastic flow, lattice trapping [8], and so forth.

Following the usual procedure employed in the study of rate processes, we first consider the case of a single type of energy barrier, i.e., a single process.

Since the crack alone will be considered as the system, it is necessary to use a parameter which describes the mechanical action imposed on the crack as a whole system and does not depend on the sample geometry. Fracture mechanics analysis shows that two variables could describe this "force" on the crack: 1) the stress intensity factor K or 2) the crack extension force $\mathscr{G}$. The selection of one of these parameters is based on the following considerations:

In thermodynamics, the energy of a system is expressed as the sum of energy terms, each equal to the product of a measurable intensive variable by a measurable extensive variable. When the crack is considered as the system, the mechanical energy term could be written as the product of the stress intensity factor K (which we can admit to be an intensive variable for a crack, and can be measured), by a variable having the dimension of a $(\text{length})^{5/2}$, that is, the dimension of a surface times the square root of a length or a volume over the square root of a length; this variable is therefore difficult to picture as an extensive variable. The alternative solution of writing a term like $K/\sqrt{L} \times V$ where L is a length and V a volume, has the advantage that $K/\sqrt{L}$ can represent a stress, an intensive variable, and V has a physical meaning of an extensive variable. However, this is not a satisfactory approach because $K/\sqrt{L}$ depends on the undefined quantity L and thus does not have a unique value for a given crack, loaded with specified conditions.

On the other hand, if we use the crack extension force $\mathcal{G}$, which has the dimension of an energy/unit area, or of a force/unit length and is therefore a measurable intensive variable, to describe the mechanical driving force on the crack, the mechanical energy term corresponds to $\mathcal{G} \times A$ where A is the fracture surface area, a measurable extensive variable.

The energy term $\mathcal{G} \times A$ has the same form as the theoretical surface energy term usually used in thermodynamics or as the work performed by surface tension forces.

For the preceding reasons, $\mathcal{G}$ is the parameter that should be used to represent the mechanical intensive parameter on the crack. In the present treatment, the crack front is thus represented by a line on which a force per unit length $\mathcal{G}$ tends to move it in the forward direction; its motion is however restrained by the presence of "thermal" obstacles, i.e., obstacles, or barriers, which can be overcome by thermal activation. Whether the physical origins of those energy barriers are individual obstacles placed on a unique surface along the fracture path or whether they are some mechanisms occurring somewhere in the region Σ, does not affect the generality of the following treatment, because the energy of the whole crack element is considered. As defined above, the problem of crack motion is somewhat similar to the problem of dislocation motion over thermal barriers. It follows that the formalism of the present discussion will be inspired from that used to interpret plastic flow rate processes in terms of dislocation motion over energy barriers [19, 20].

With the assumption of thermally activated barriers, the average crack velocity can be written according to a general Arrhenius-type equation as:

$$v = v_0(\mathcal{G}, T, P, \sigma_i, s) \exp\left[-\Delta G(\mathcal{G}, T, P, \sigma_i, s)/kT\right] \tag{2}$$

where the quantity v_0 can be thought as the maximum attainable crack velocity, ΔG is the Gibbs free energy of activation, $\mathcal{G}$ is the average applied mechanical driving force per unit length of crack profile, T is the temperature, and P is the hydrostatic pressure. σ_i are external stresses not contributing to an opening mode, for example a uniform stress applied parallel to the crack front on a test sample. s are parameters describing the internal structure, for example the number of obstacles per unit area of fracture surface.

In order to simplify the problem, we will now make the important assumption, similar to that usually made in the theories of thermally activated dislocation motion [21], that the internal structure is constant. This assumption implies that in steady state, the concentration of "obstacles" referred to a unit of fracture surface area is constant. This assumption, however, allows the material in the crack tip region to be different from that outside (for example, molecular orientation in the crack tip region in a polymer), provided that this structure stays in a steady state in the range of the operating process.

From the rate theory alone, the effects of $\mathscr{G}$, T, P, σ_i and s, on v_0 cannot, in general, be found; consequently, unless it is possible from other considerations to attribute a specific dependence of one of these parameters on v_0, v_0 will be assumed to be constant for a given process. If v_0 is not constant, then v should be replaced by v/v_0 in the expressions which will be derived later.

The Gibbs free energy G of the crack element then takes the general form:

$$G = U + PV - \mathscr{G}A - \sum_i \sigma_i V_i - TS \tag{3}$$

where U is the internal energy, V is the volume, A is the fracture surface area and S is the entropy.

Now, only in order to clarify the following equations, we will consider the case where P and the σ_i's are kept constant, i.e., the only experimental variables will be $\mathscr{G}$ and T. With the previous assumptions and simplifications, the expression for the crack velocity becomes

$$v = v_0 \exp \frac{-\Delta G(\mathscr{G}, T)}{kT}. \tag{4}$$

This equation implies that, for a given material, the crack velocity depends only on $\mathscr{G}$ and T, and therefore, $\ln v$ can be written as:

$$\ln v = f(\mathscr{G}, T).$$

It follows that:

$$\left.\frac{\partial \ln v}{\partial \mathscr{G}}\right|_T \cdot \left.\frac{\partial \mathscr{G}}{\partial T}\right|_{\ln v} \cdot \left.\frac{\partial T}{\partial \ln v}\right|_{\mathscr{G}} = -1. \tag{5}$$

A useful check of the assumption that v depends only on $\mathscr{G}$ and T is to verify this last equation where each of the three partial derivatives can be measured from the experimental data.

The differential of the Gibbs free energy G, the Helmholtz's free energy F, the enthalpy H and the internal energy U are respectively expressed as:

$$\begin{aligned} dG &= -A\, d\mathscr{G}_i - S\, dT \\ dF &= -S\, dT + \mathscr{G}_i\, dA \\ dH &= T\, dS - A\, d\mathscr{G}_i \\ dU &= T\, dS + \mathscr{G}_i\, dA. \end{aligned} \tag{6}$$

In the above differentials, A is the area of fracture surface of the crack element, which defines its position along the fracture path. $\mathscr{G}_i$ is the local internal back force/unit crack length on the crack element and is equal to the applied force/unit crack length required to maintain the crack in equilibrium at the position A. The choice of the origin of the coordinate A is arbitrary, provided it is kept constant throughout the analysis.

From Eqn. (6), for an isothermal reversible process over a potential barrier, the derivative of the Helmholtz free energy with respect to the crack propagation area A is $\mathscr{G}_i$. The Helmholtz free energy and its derivative are schematically illustrated in Fig. 2a and 2b; also from Eqn. (6), the activation free energy is equal to:

$$\Delta G = \int_{\mathscr{G}^*}^{\mathscr{G}_c^*} A^*\, d\mathscr{G}_i \tag{7}$$

as shown in Fig. 2b. $\mathscr{G}^*$ is the crack extension force effectively used for overcoming

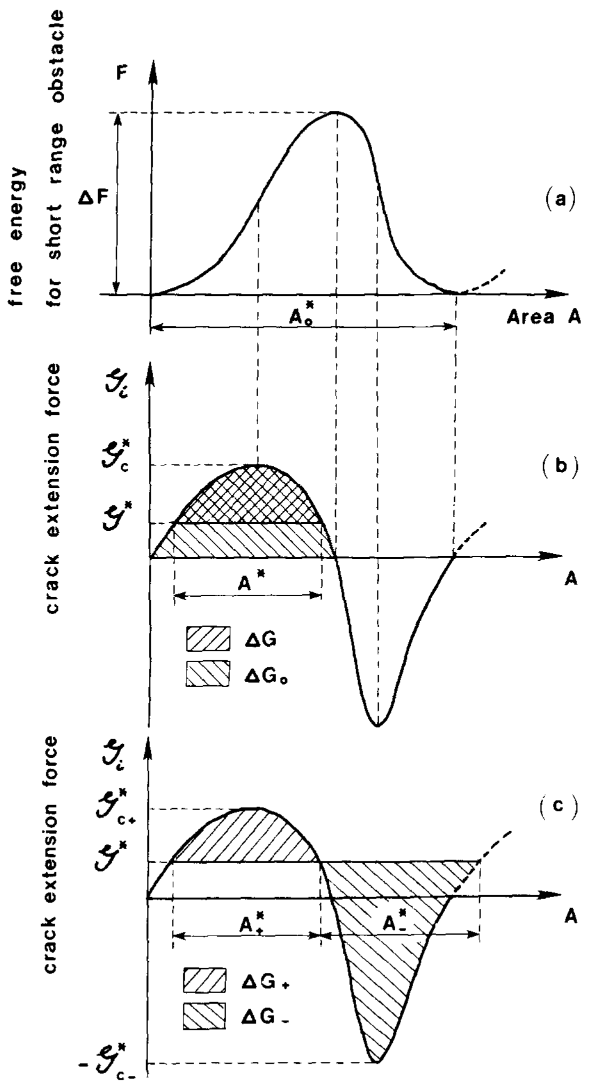


Figure 2. Energy and internal force diagram for the front of a crack element during reversible overcoming of a thermal obstacle.

the thermal obstacle and will be called "effective crack extension force"; $\mathcal{G}_c^*$ is the maximum effective crack extension force corresponding to a fully force-activated process and at which the crack velocity attains the value v_0.

Now, from Eqn. (7) and also from Fig. 2b with zero effective driving force, i.e., $\mathcal{G}^* = 0$;

$$\Delta G_0 \equiv \int_0^{\mathcal{G}_c^*} A^* \, d\mathcal{G}_i \tag{8}$$

with $\Delta G_0 = \Delta F$. Equations (7) and (8) allow ΔG to be written as:

$$\Delta G = \Delta G_0 - \int_0^{\mathcal{G}^*} A^* \, d\mathcal{G}_i. \tag{9}$$

For a forward motion of the crack front element, the corresponding crack front velocity becomes

$$v_+ = v_0\, \mathrm{e}^{-\Delta G_+/kT} \tag{10}$$

and for a reversible backward process:

$$v_- = v_0\, \mathrm{e}^{-\Delta G_-/kT} \tag{11}$$

where ΔG_+ and ΔG_- are the activation Gibb's free energies respectively for the forward and for the backward motion of the crack over a thermal obstacle. Thus, for a given $\mathscr{G}^*$ (see Fig. 2c), the net crack velocity becomes:

$$v = v_+ - v_-$$

or:

$$v = v_0 \left\{ \exp - \left(\int_{\mathscr{G}^*}^{\mathscr{G}^*_{c+}} A^*_+ \,\mathrm{d}\mathscr{G}_i \right) \Big/ kT - \exp - \left(\int_{-\mathscr{G}^*_{c-}}^{\mathscr{G}^*} A^*_- \,\mathrm{d}\mathscr{G}_i \right) \Big/ kT \right\}. \tag{12}$$

Following the arguments of Li on dislocation velocity [22], we have, for a reversible crack motion under a zero effective driving force $\mathscr{G}^*$, a net zero crack velocity:

$$v = v_0 \left\{ \exp - \left(\int_{0}^{\mathscr{G}^*_{c+}} A^*_+ \,\mathrm{d}\mathscr{G}_i \right) \Big/ kT - \exp - \left(\int_{-\mathscr{G}^*_{c-}}^{0} A^*_- \,\mathrm{d}\mathscr{G}_i \right) \Big/ kT \right\} = 0 \tag{13}$$

which gives the condition

$$\Delta G_0 \equiv \int_0^{\mathscr{G}^*_{c+}} A^*_+ \,\mathrm{d}\mathscr{G}_i \equiv \int_{-\mathscr{G}^*_{c-}}^{0} A^*_- \,\mathrm{d}\mathscr{G}_i. \tag{14}$$

At very small values of $\mathscr{G}^*$ so that A^*_+ and A^*_- are approximately independent of $\mathscr{G}^*$, we have from Eqn. (11):

$$\Delta G_+ = \Delta G_0 - A^*_+ \mathscr{G}^*$$

and (15)

$$\Delta G_- = \Delta G_0 + A^*_- \mathscr{G}^*$$

with

$$A^*_+ + A^*_- = A^*_0. \tag{16}$$

Therefore, from Eqns. (10), (11) and (15),

$$v = v_0\, \mathrm{e}^{-\Delta G_0/kT} (\mathrm{e}^{\mathscr{G}^* A^*_+/kT} - \mathrm{e}^{-\mathscr{G}^* A^*_-/kT}), \tag{17}$$

and, for a symmetric barrier, where $A^*_+ = A^*_- = A^*$, one obtains an Eyring-type expression:

$$v = 2 v_0\, \mathrm{e}^{-\Delta G_0/kT} \sin h \frac{\mathscr{G}^* A^*}{kT}. \tag{18}$$

On the other hand, if ΔG_- is sufficiently larger than ΔG_+, then v_- is much smaller than v_+ and the net solution of the crack is given by

$$v = v_0\, \mathrm{e}^{-\Delta G_+/kT} = v_0 \exp - \left(\int_{\mathscr{G}^*}^{\mathscr{G}^*_c} A^*_+ \,\mathrm{d}\, \mathscr{G}_i \right) \Big/ kT \tag{19}$$

The condition of ΔG_- being large compared to ΔG_+ can be obtained at large values of $\mathscr{G}^*$. In some cases, the backward process may be very improbable and therefore be negligible because of the nature of the obstacle. For example, if in a polymer such obstacles are main chain bonds of highly strained molecules that can be ruptured, the reversible recombination of the broken bonds of the same molecule is very unlikely because the broken parts may be far apart. It should be noted that whenever the overall process is controlled by the forward process, the condition of reversibility has to apply only during the forward activation process. From Eqn. (19) and (9), we then have

$$v = v_0\,\mathrm{e}^{-\Delta G_0/kT}\exp\left(\int_0^{\mathcal{G}^*} A_+^*\,\mathrm{d}\,\mathcal{G}_i\right)\Big/kT \tag{20}$$

3. Determination of thermodynamic parameters

The derivative of Eqn. (20) with respect to $\mathcal{G}^*$, at constant T, yields

$$\left.\frac{\partial \ln v}{\partial \mathcal{G}^*}\right)_T = \frac{A_+^*}{kT}. \tag{21}$$

The activation area is therefore obtained from the dependence of the crack velocity on $\mathcal{G}^*$ at constant temperature.

Now consider derivatives of (20) with respect to temperature at constant $\mathcal{G}^*$:

$$\begin{aligned}\left.\frac{\partial \ln v}{\partial T}\right)_{\mathcal{G}^*} &= \frac{\Delta G_+}{kT^2} - \frac{1}{kT}\left(\frac{\partial \Delta G_+}{\partial T}\right)_{\mathcal{G}^*}\\ &= \frac{\Delta G_+}{kT^2} + \frac{\Delta S_+}{kT}\\ &= \frac{\Delta H_+}{kT^2}\end{aligned} \tag{22}$$

or

$$\left.\frac{\partial \ln v}{\partial (1/T)}\right)_{\mathcal{G}^*} = \frac{-\Delta H_+}{k}. \tag{23}$$

ΔS_+ and ΔH_+ being the entropy and enthalpy of activation. Equation (23) represents the well known procedure for obtaining the enthalpy of activation by plotting the rate of the process versus $1/T$.

In order to calculate the expressions relating different experimental and thermodynamic parameters, Jacobian algebra can be conveniently used, as suggested by Li [20]. A list of partial derivatives of the functions to be considered is presented in Table 2. From this table, any other partial derivative of any tabulated variable can readily be obtained from the following relation:

$$\left.\frac{\partial X}{\partial Y}\right)_Z = \frac{\begin{vmatrix} \left.\frac{\partial X}{\partial T}\right)_{\mathcal{G}^*} & \left.\frac{\partial X}{\partial \mathcal{G}^*}\right)_T \\ \left.\frac{\partial Z}{\partial T}\right)_{\mathcal{G}^*} & \left.\frac{\partial Z}{\partial \mathcal{G}^*}\right)_T \end{vmatrix}}{\begin{vmatrix} \left.\frac{\partial Y}{\partial T}\right)_{\mathcal{G}^*} & \left.\frac{\partial Y}{\partial \mathcal{G}^*}\right)_T \\ \left.\frac{\partial Z}{\partial T}\right)_{\mathcal{G}^*} & \left.\frac{\partial Z}{\partial \mathcal{G}^*}\right)_T \end{vmatrix}} = \frac{J(X,Z)}{J(Y,Z)}. \tag{24}$$

For example, by applying the relation (24) and using Table 2, one finds:

$$\left.\frac{\partial \mathcal{G}^*}{\partial T}\right)_{\ln v} = \frac{-\Delta H_+}{A_+^* T}. \tag{25}$$

TABLE 2

Jacobian table for activation parameters related to crack propagation.

f	$(\partial f/\partial T)_{\mathscr{G}^*}$	$(\partial f/\partial \mathscr{G}^*)_T$
T	1	0
$\mathscr{G}^*$	0	1
$\ln v_+$	$\Delta H_+/kT^2$	$A^*_\ddagger/kT$
ΔG_+	$-\Delta S_+$	$-A^*_\ddagger$
ΔH_+	C_+	$T\gamma_+ - A^*_\ddagger$
$A^*_\ddagger$	γ_+	θ_+
ΔS_+	C_+/T	γ_+
ΔF_+	$-\Delta S_+ + \mathscr{G}^*\gamma_+$	$\mathscr{G}^*\theta_+$
ΔU_+	$C_+ + \mathscr{G}^*\gamma_+$	$T\gamma_+ + \mathscr{G}^*\theta_+$

Thus, the enthalpy of activation can also be obtained from the temperature dependence of the crack extension force, at a constant crack velocity.

A few examples of possible relations between $A^*_\ddagger$ and $\mathscr{G}^*$ leading to linear relations between ln v and $\mathscr{G}^*$, or simple functions of $\mathscr{G}^*$, are summarized in Table 3; the dependence of the corresponding activation energy on $\mathscr{G}^*$ at constant temperature is also shown; these results are immediate by integrating Eqns. (21) and (9). From the experimentally observed dependence of the crack velocity on the effective crack extension force $\mathscr{G}^*$, it is therefore, in principle, possible to define a relation between the activation area $A^*_\ddagger$ and $\mathscr{G}^*$.

The types of relations as given in cases (a), (b) and (d) of Table 3 between crack velocity and crack extension force, or equivalent relations between crack velocity and stress intensity factor with the approximation that $K \propto \mathscr{G}^{1/2}$, are frequently found in literature. Examples (a), (b) and (f) in Table 1 correspond to type (d) relation in Table 3, whereas (c), (d) and (e) relations in Table 1 follow a (b) type dependence in Table 3 and (h) relation in Table 1 is associated with an (a) type behavior in Table 3.

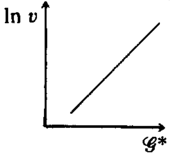
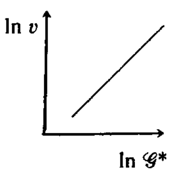
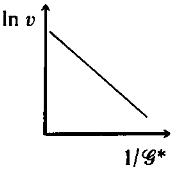
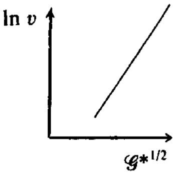
We have so far considered only a single operative thermally activated process. The combined effects of simultaneous processes on the rate and on the thermodynamic parameters have been discussed by Li [23] and can be directly applied to crack propagation described by the foregoing model. For illustration, a crack velocity versus crack extension force relation as represented in Fig. 3 can be explained in terms of three consecutive processes (one athermal process I, and two thermal processes II and III) coupled in parallel with a thermal process IV.

4. Athermal crack extension forces

The existence of an applied $\mathscr{G}$, or the equivalent applied stress intensity factors K, below which crack propagation does not occur is observed in many materials [11, 13, 24–26]. These experimental measurements suggest that there is an athermal component to $\mathscr{G}$. The relation between $\mathscr{G}^*$ and the externally applied and measured crack extension force $\mathscr{G}$ is shown in Fig. 4. The addition of the crack extension forces was established from the following considerations.

When the crack is moving, the material in the crack tip region undergoes transitions from a low energy state, i.e. original material, to a high energy state. The fracture modified material has new surfaces and contains more defects such as dislocations, broken bonds, etc. This energy transition is equivalent to a structural change in the material and results in an increase in the energy of the crack system

TABLE 3
Examples of the dependence of $\ln v$ and ΔG_+ on $\mathcal{G}^*$ according to the dependence of $A^*_\ddagger$ on $\mathcal{G}^*$, at constant temperature.

$A^*_\ddagger$ Note: c may be a function of T	$\ln v$	ΔG_+
(a) $A^*_\ddagger = c =$ constant	linear with $\mathcal{G}^*$ slope $= c/kT$ 	$\Delta G_0 - c\mathcal{G}^*$
(b) $A^*_\ddagger = c/\mathcal{G}^*$	linear with $\ln \mathcal{G}^*$ (power law) slope $= -c/kT$ 	$\Delta G_0 - c \ln \mathcal{G}^*$
(c) $A^*_\ddagger = c/\mathcal{G}^{*2}$	linear with $1/\mathcal{G}^*$ slope $= c/kT$ 	$\Delta G_0 + c/\mathcal{G}^*$
(d) $A^*_\ddagger = c/\mathcal{G}^{*1/2}$	linear with $\mathcal{G}^{*1/2}$ slope $= 2c/kT$ 	$\Delta G_0 - c\mathcal{G}^{*1/2}$

which is proportional to the crack area. The mechanical work provides this energy and is equal to it in a reversible process.

For example, in the case of reversible crack propagation in nearly perfectly brittle material such as during cleavage experiments on some crystals [27], the increase in the material energy, per unit fracture area, for one fracture surface is equal to the thermodynamic surface energy, γ. Therefore the crack is in an equilibrium state, zero velocity, when the applied crack extension force $\mathcal{G}$ is equal to 2γ. When $\mathcal{G}$ is increased, and if crack propagation is thermally activated, the crack extension force component $\mathcal{G}^*$ equal to $\mathcal{G} - 2\gamma$ is used to overcome the thermal obstacles.

More generally in a non-perfectly brittle material the energy per unit area of a crack surface of the material going through the crack tip region is increased by 2γ plus an additional amount 2Γ corresponding to changes in the state of the bulk material close to the fracture surface. Then the thermal component of the crack extension force is equal to $\mathcal{G} - 2\gamma - 2\Gamma$.

High energy obstacles that cannot be overcome by thermal activation may also be

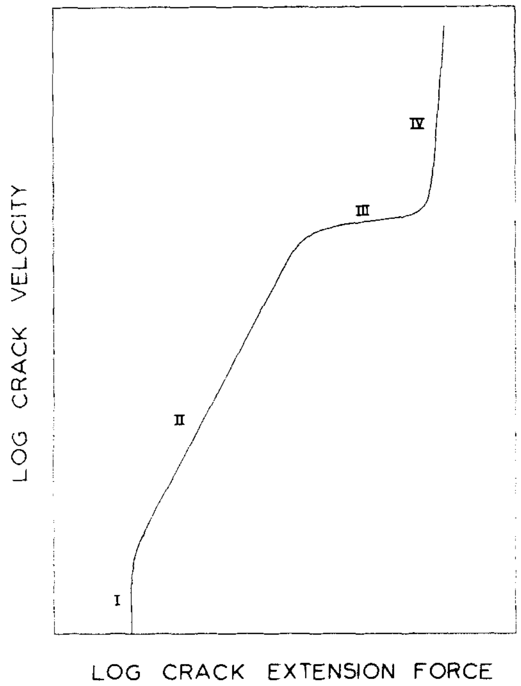


Figure 3. Consecutive and parallel processes in crack propagation.

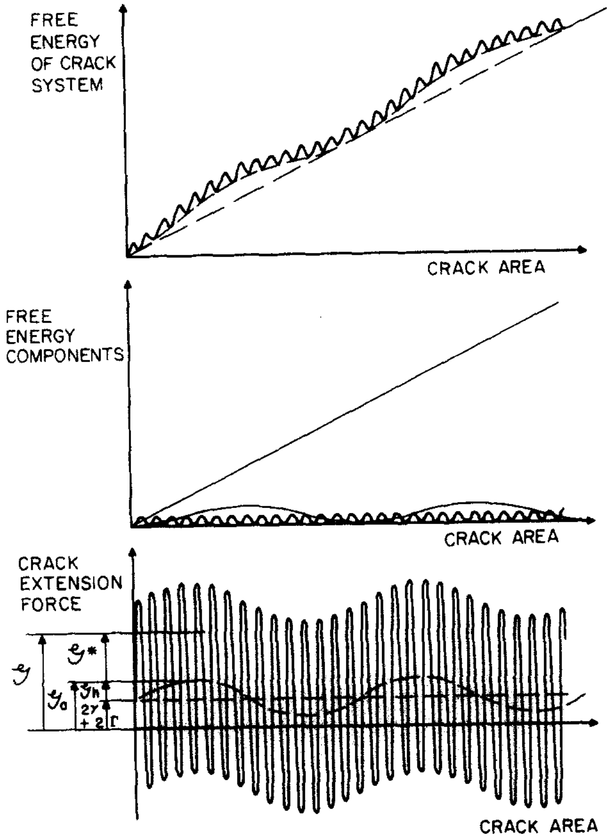


Figure 4. Components for the free energy and for the crack extension force.

encountered by the propagating crack. These obstacles will influence the crack propagation. Such obstacles may result from local variations in the material structure. Designate $\mathcal{G}_h$ the crack extension force component required to overcome these high energy obstacles. Thus, $\mathcal{G}^*$ is then equal to

$$\begin{aligned} &\mathcal{G}^* = \mathcal{G} - 2\gamma - 2\Gamma - \mathcal{G}_h \\ \text{or}\quad &\mathcal{G}^* = \mathcal{G} - \mathcal{G}_a. \end{aligned} \tag{26}$$

Equation (26) is similar, in its form and in its origin, to the relation widely used in dislocation motion theories which states that the internal stress τ on the dislocation consists of the sum of an athermal component τ_μ and a thermal component τ^* [28]. The value of $\mathcal{G}_a$ should then correspond to the value of $\mathcal{G}$ at which the crack velocity is zero.

5. Conclusion

Steady state crack propagation is described as the motion of a crack front controlled by energy barriers. Arguments are given to select the crack extension force $\mathcal{G}$ rather than the stress intensity factor K for describing the mechanical force on a crack system. The crack front progresses under the action of the crack extension force, which, if we neglect the thermodynamic surface energy of the material, is equal to the sum of an athermal component $\mathcal{G}_a$ and a thermal component $\mathcal{G}^*$. Each of these components corresponds to the force per unit length of crack front, respectively a) required to overcome long-range, high energy, obstacles that cannot be activated by thermal fluctuation and b) used to assist thermal activation of short-range barriers. Accordingly, the term $\mathcal{G}_a$ should correspond to the value of the applied crack extension force below which crack propagation does not occur. The activation area, defined as the area swept by a crack front during the thermal activation event is allowed to depend on the effective crack extension force. Interrelations between thermodynamic activation parameters are derived. Most of published crack velocity dependences on mechanical force are shown to correspond to different crack extension force dependences on the activation area.

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RÉSUMÉ

La propagation stationnaire d'une fissure dans un solide est analysée en tant qu'un processus d'activation thermique. Le concept d'une force d'extension de la fissure, utilisé en mécanique de la rupture, est formellement introduit dans la théorie cinétique moléculaire. Cette représentation de la propagation d'une fissure apparaît, en maint aspects, similaire à celle du mouvement d'une dislocation sous l'effect d'une contrainte de cisaillement à travers des obstacles thermiquement franchissables. On dérive les relations thermodynamiques fondamentales pour la propagation stationnaire d'une fissure, à partir d'hypothèses simplificatrices similaires à celles couramment acceptées dans les théories de déformation basées sur le mouvement thermiquement activé des dislocations.