International Journal of Fracture **64**: R53–R58, 1993.

A SIMPLE CALCULATION OF DA/DN - ΔK DATA IN THE NEAR THRESHOLD REGIME AND ABOVE

Richard W. Hertzberg
Department of Materials Science and Engineering, Lehigh University
Bethlehem, Pennsylvania 18015, USA
tel: (215) 758-4226; FAX: (215) 758-4222

Numerous studies have shown that the fatigue crack propagation (FCP) rate in metallic and non-metallic materials decreases with decreasing stress intensity factor range values until the growth rate becomes vanishingly small in what has been referred to as the threshold regime [1]. Typically, decreasing ΔK experiments have involved both decreasing maximum and minimum stress intensity factor levels (K_{max} and K_{min}, respectively), associated with a constant load ratio (typically $R^c = 0.1$). More recently, Doker and Marci et al. [2,3], and Herman et al. [4-6] demonstrated an alternative method of generating FCP data wherein tests are conducted under K^c_{max} conditions. In this instance, K_{min} levels increase with the associated R ratio increasing to levels of 0.9 or higher during the course of the test; in this manner, crack closure is eliminated and the applied stress intensity factor range is equal to the effective stress intensity factor range (ΔK_{eff}) [7]. Doker and Marci et al. characterized such closure-free data as being representative of the material's "intrinsic fatigue crack propagation response". Such data have been used to estimate the crack propagation response of physically short cracks, and long cracks that experience tensile residual stresses [4-6].

Regardless of the test method used (i.e., R^c or K^c_{max}), ASTM Standard 647-93 defines an "operative definition" of the ΔK threshold value (ΔK_{th}) at a crack growth rate of 10^{-10} m/cycle [8]. Based on the slip characteristics of a crystalline solid, it may be reasonable to define a closely related ΔK value at that driving force corresponding to a growth rate of a single Burgers vector. For purposes of identification, we may define this as ΔK_b. (For example, ΔK_b in steel and aluminum alloys would be defined at fatigue crack growth rates of 2.48 and 2.86 x E-10 m, respectively.) One may then define ΔK_b as the limit of continuous damage accumulation with crack growth increments, n**b** ($n \geq 1$) occurring when $\Delta K \geq \Delta K_b$. Any growth increment less than the mimimum unit of deformation (i.e., **b**) would correspond to discontinuous crack extension.

If one examines the closure-free "intrinsic fatigue crack propagation response" for aluminum and steel alloys, ΔK_{th} values fall roughly in the range of 1-3 MPa/m, respectively [4-6, 9-11]. It is remarkable to note that $(\Delta K_{th}/E)^2$ values (units of m) for numerous alloys correspond to the Burgers vector for each material (see Table 1). As such, it is suggested that a major portion of the da/dN-ΔK curve for a given alloy can be estimated simply by connecting a few experimental data points at a high growth rate with a single computed data point, ΔK_b, corresponding to a crack growth rate equal to the cyclic advance increment of the material's Burgers vector. Based on the data given in Table 1,

$$\Delta K_b = E\sqrt{\boldsymbol{b}} \tag{1}$$

For example, calculated da/dN - ΔK values for numerous alloys are shown in Fig. 1. The excellent agreement between measured and computed datum in the threshold regime (point A) is most encouraging.

A comparison of numerous K^c_{max}-generated da/dN - ΔK plots, some shown in Fig. 1, reveal that FCP rates under closure-free conditions tend to vary approximately with ΔK^3 (see Table 2). Accordingly, the writer suggests that it is possible to characterize the da/dN - ΔK curve at FCP rates of **b**/cyc and above in a straightforward manner wherein

$$da/dN = \boldsymbol{b}(\Delta K/\Delta K_b)^3 \tag{2}$$

where **b** = Burgers vector

ΔK = closure-free stress intensity factor range
ΔK_b = closure-free ΔK level associated with da/dN = **b**/cyc

As such, no experimental FCP data are needed to describe the crack growth plot, as suggested above. Since ΔK_b = E **b**, it follows that

$$da/dN = b(\Delta K/E\sqrt{b})^3 \text{ or} (\Delta K/E)^3 (1/\sqrt{b}) \tag{3}$$

The dashed lines in Fig. 1 correspond to da/dN values computed from (3) over a range of ΔK values from ΔK_b (point A) to an arbitrarily defined level corresponding to 10 ΔK_b (point B). Again, the agreement of computed and experimental values is striking. Note the agreement with K^c_{max} (Fig. 1a-e), ΔK_{eff} (Fig. 1g), and short crack (Fig. 1f,g) results, all representative of closure-free test conditions.

There are numerous potential uses for this simple computational method for FCP data generation. These include: (1) the intrinsic closure-free FCP reponse of an untested alloy may be estimated directly, based only on knowledge of the alloy's elastic modulus and Burgers vector and the assumption that crack growth rates vary with ΔK^3; (2) the FCP response of an alloy can be computed for the case of large residual or applied tensile mean stresses such as in weldments (Fig. 1e); and (3) the short crack response of a given alloy may be estimated by simply computing the closure-free da/dN - ΔK curve, based on (3) (Fig. 1f,g).

One may, therefore, conclude that computed FCP data from (1-3) characterize the intrinsic fatigue crack propagation response of metallic alloys and are in good agreement with both closure corrected and/or K^c_{max} data. A baseline estimate of fatigue behavior for a given alloy is, therefore, established. It remains to be seen how this relation is modified to account for FCP response in the presence of crack closure.

Acknowledgements: The author appreciates the assistance of C. Ragazzo for modifying some of the figures.

REFERENCES

1. R.W. Hertzberg, *Deformation and Fracture Mechanics of Engineering Materials,* John Wiley, New York (1989).

2. H. Doker, V. Bachmann and G. Marci, in *Fatigue Thresholds,* J. Backlund, A. Blom and C.J. Beevers (eds.), EMAS (1982) 45-57.

3. G. Marci, D.E. Castro and V. Bachmann, *Journal of Testing and Evaluation* 17(1) (1989) 28.

4. W.A. Herman, R.W. Hertzberg and R. Jaccard, *Fatigue and Fracture of Engineering Materials and Structures* 11(4) (1988) 303-320.

5. W.A. Herman, R.W. Hertzberg and R. Jaccard, *Advances in Fracture Research,* 7th International Conference on Fracture (1989) 1417-1426.

6. R.W. Hertzberg, W.A. Herman, T. Clark and R. Jaccard, in *ASTM 1149,* J.M. Larsen and J.E. Allison (eds.) (1992) 197-220.

7. W. Elber, *ASTM STP 486* (1971) 230-242.

8. AstM Specification E647-93, *Annual Book of ASTM Standards* (1993) 679-706.

9. T. Clark, PhD dissertation, Lehigh University (1992).

10. W.A. Herman, PhD dissertation, Lehigh University (1989).

11. A. Ohta, N. Suzuki and T. Mawari, *International Journal of Fatigue* 14(4) (1992) 224-226.

12. W.N. Sharpe, Jr., J.R. Jira and J.M. Larsen, in *ASTM 1149,* J. Larsen and J.E. Allison (eds.), ASTM (1992) 92-115.

13. J.C. Newman, Ibid, 6-33.

19 November 1993

TABLE 1 THRESHOLD DATA FOR VARIOUS ALLOYS

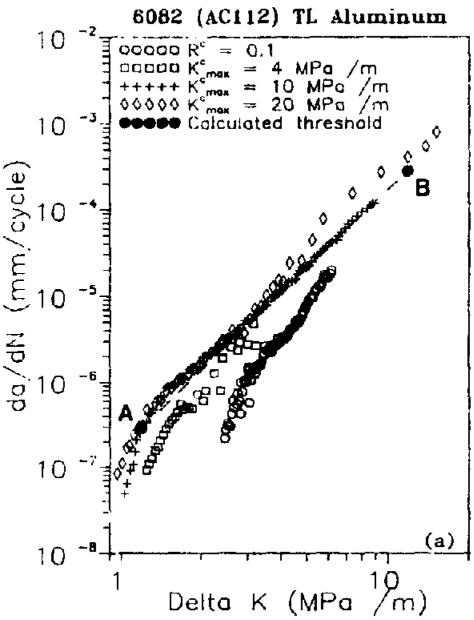
Ref.	Material	ΔK_{th} (MPa√m)	E (GPa)	$(\Delta K_{th}/E)^2$ x10^{-10}(m)	b x10^{-10}(m)	$\Delta K_{(th}/E)^2/b$
10	7075-T6	1.3	70	3.45	2.86	1.21
"	2024-T3	1.4	72.4	3.74	2.86	1.31
"	HT60	~2.5	205	1.49	2.48	0.60
"	1020	~3.2	205	2.44	2.48	0.98
"	4130	~3.5	200	3.06	2.48	1.23
"	S10C(FG)	~3.5	205	2.91	2.48	1.17
"	6005(FG)	~1.1	70	2.47	2.86	0.86
"	6005(CG)	~1.3	70	3.45	2.86	1.21
9	Astroloy	~3.2	200	2.56	2.52	1.02
"	AF42	~1.3	70	3.45	2.86	1.21
11	5083	~1.4	73.3	3.65	~2.86	1.28
"	304SS	~3.4	189	3.24	~2.54	1.28
"	SB42	~3.6	210	2.94	~2.48	1.19

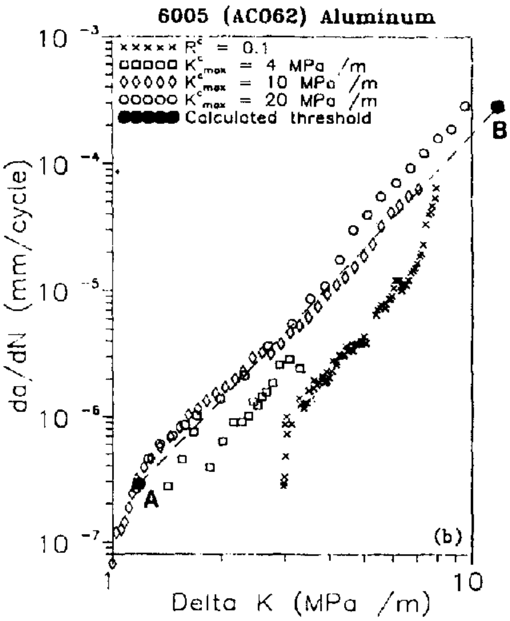
TABLE 2 FATIGUE DATA CONSTANTS

Material	Kmax level (MPa√m)	A (all data)	m (all data)	A (truncated data)*	m (truncated data)*
S10C(FG)	35	5.00E-09	3.12	5.00E-09	3.12
1020(HR)	35	5.63E-09	3.07	7.98E-09	2.93
4130(QT)	35	2.36E-08	2.59	2.36E-08	2.59
HT60	35	8.88E-08	3.14	1.33E-08	2.98
S10C(CG)	35	3.07E-09	3.24	3.07E-09	3.24
Van80	55	1.68E-08	3.00	4.45E-08	2.69
Van80	45	1.11E-08	3.22	2.21E-08	2.95
2090-T6	10	2.83E-07	1.91	1.72E-07**	2.47**
2024-T3	10	1.43E-07	2.76	1.43E-07	2.76
AC112-TL	20	1.76E-07	3.22	2.06E-07	3.11
AC112-TL	10	1.82E-07	3.04	2.47E-07	2.83
PE260	20	9.00E-08	3.44	1.01E-07	3.37
PE260	10	7.70E-08	3.73	9.98E-08	3.51
UR100-TL	20	2.41E-07	3.15	4.07E-07	2.85
UR100-TL	10	3.23E-07	2.93	3.92E-07	2.80
UR100-LT	25.4	3.35E-07	2.89	4.10E-07	2.79
UR100-LT	20	4.23E-07	2.96	5.56E-07	2.82
UR100-LT	10	2.07E-07	3.11	2.07E-07	3.11
AF42(cast)	20	1.19E-07	2.88	1.19E-07	2.88
AF42(cast)	15	2.05E-07	2.56	2.05E-07	2.56
6005	20	1.77E-07	3.24	1.77E-07	3.24
6005	10	1.60E-07	3.04	2.31E-07	2.78
7075	10	2.72E-07	3.43	2.72E-07	3.43
Astroloy	65	1.87E-09	3.31	1.87E-09	3.31
Astroloy	55	2.55E-09	2.94	2.61E-09	2.93
Astroloy	45	1.50E-08	2.34	1.50E-08	2.34

*Data excluded at $\Delta K<\Delta K_{th}$

**Data was truncated at $\Delta K>6.5$ MPa√m





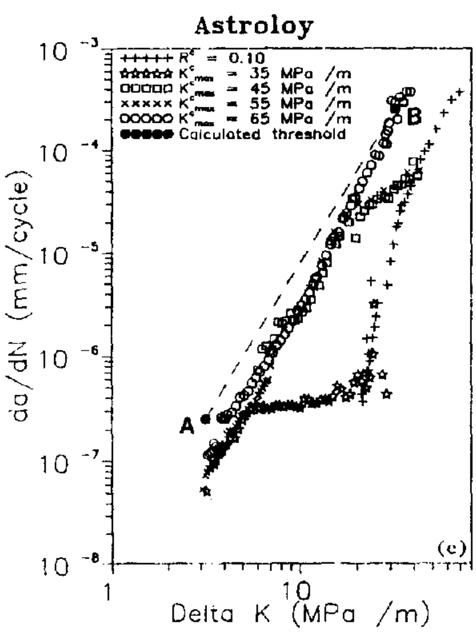
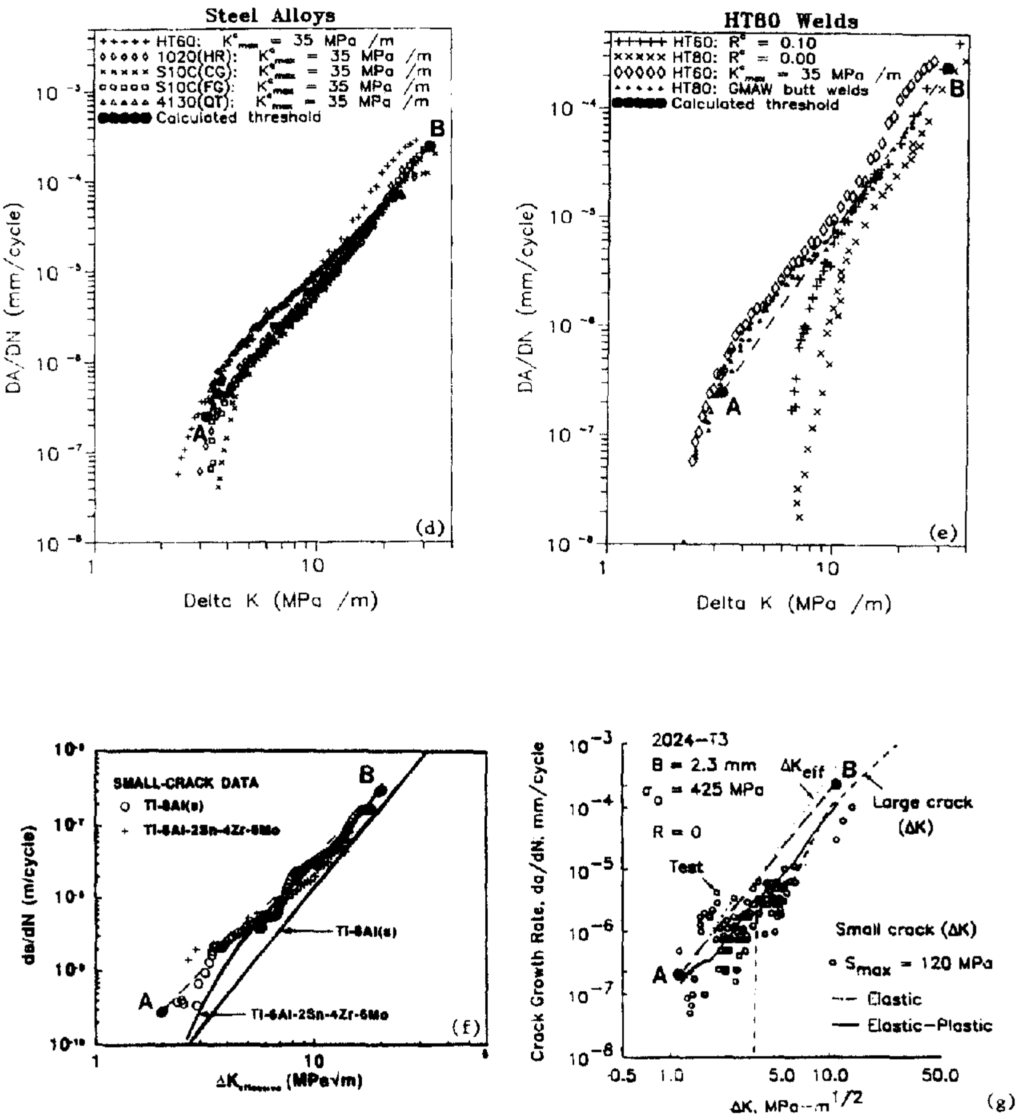


Fig. 1. FCP data based on both experimental results (R^c or K^c_{max}) and computed values (Eqtns 1-3). Elastic modulii and Burgers vectors given in Table 1 unless specified. Note excellent agreement between computed values of ΔK_b-b/cyc datum (point A) and K^c_{max} data and the strong correlation of computed data lines (A-B) with K^c_{max}, ΔK_{eff}, and/or short crack experimental data. a) 6082 aluminum alloy[9] (E = 70 GPa, **b** = 0.286nm); b) 6005 aluminum alloy[9]; c) Astroloy[9]; d) several steel alloys[10]; e) HT 80 welds[10]; f) titanium alloys[12] (E = 116 GPa, **b** = 0.295nm); g) 2024-T3 aluminum alloy[13]


Steel Alloys
HT60: K*max = 35 MPa √m
1020(HR): K*max = 35 MPa √m
S10C(CG): K*max = 35 MPa √m
S10C(FG): K*max = 35 MPa √m
4130(QT): K*max = 35 MPa √m
Calculated threshold
DA/DN (mm/cycle)
Delta K (MPa √m)
(d)
HT80 Welds
HT60: R* = 0.10
HT80: R* = 0.00
HT60: K*max = 35 MPa √m
HT80: GMAW butt welds
Calculated threshold
DA/DN (mm/cycle)
Delta K (MPa √m)
(e)
SMALL-CRACK DATA
Ti-8Al(s)
Ti-6Al-2Sn-4Zr-6Mo
da/dN (m/cycle)
(f)
2024-T3
B = 2.3 mm
σo = 425 MPa
R = 0
ΔKeff
Large crack (ΔK)
Test
Small crack (ΔK)
Smax = 120 MPa
Elastic
Elastic-Plastic
Crack Growth Rate, da/dN, mm/cycle
ΔK, MPa-m1/2
(g)