International Journal of Fracture **80**: R37–R41, 1996.

DYNAMIC ANALYSIS OF MODE III CRACKS IN RECTANGULAR SHEETS

P.H.Wen and M.H. Aliabadi*
Wessex Institute of Technology, Southampton, UK
Tel: 44 (0)1703 293223; Fax: 44 (0)1703 292583

D.P. Rooke
Structural Materials Centre, DRA, Farnborough, Hants, UK

In this report, the fundamental solution of a mode III displacement discontinuity in a rectangular sheet for a dynamic problem is derived and stress intensity factors are calculated numerically by the use of the equivalent stress method introduced in [1].

The controlling equation for a mode III dynamic problem is

$$\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}=\frac{1}{c_2^2}\frac{\partial^2 w}{\partial t^2} \tag{1}$$

where t is the time and c_2 the velocity of elastic shear waves (c^2_2-μ/ρ, where μ is the shear modulus and ρ is the density of the material). The Laplace transform of (1) is

$$\frac{\partial^2 w^*}{\partial x^2}+\frac{\partial^2 w^*}{\partial y^2}=p^2 w^*, \tag{2}$$

where p is the Laplace transform parameter. The general solution of the above equation for the half plane $y \geq 0$ is

$$w^*=\frac{2}{\pi}\int_0^\infty A e^{-\gamma_2 y}\cos sx ds \tag{3}$$

where $\gamma_2=\sqrt{s^2+p^2/c^2_2}$ and A is an unknown which can be determined by the following conditions:

----

*On leave from the Central-South University of Technology, Changsha, People's Republic of China

$$w^*(x,0) = \begin{cases} \Delta^* & -\lambda \le x \le \lambda \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

where Δ^* is the Laplace transform of a displacement discontinuity $\Delta(t)$. The shear stresses can be written as

$$\tau_x^* = \frac{\Delta^* \mu}{\pi}\left[\frac{\partial K_0}{\partial y}\right]_{x-\lambda}^{x+\lambda} \equiv T_x^0(x,y,p)\Delta^* \tag{5}$$

and

$$\tau_y^* = -\frac{\Delta^* \mu}{\pi}\beta^2 \int_{x-\lambda}^{x+\lambda} K_0(\xi)dx - \frac{\Delta^* \mu}{\pi}\left[\frac{\partial K_0}{\partial x}\right]_{x-\lambda}^{x+\lambda} \equiv T_y^0(x,y,p)\Delta^*.$$

where $\beta = p/c_2$, $K_0(\xi)$ is the zero-order Bessel function and $\xi = \beta\sqrt{x^2+y^2}$.

If the positions of the displacement discontinuity element is (x_0, y_0) with a slant angle γ as shown in Fig. 1(a), the stress field in (x,y) coordinates is

$$\tau_y^* = [T_y^0(x',y',p)\cos\gamma + T_x^0(x',y',p)\sin\gamma]\Delta^*$$
$$\equiv F_y^*(x,x_0,y,y_0,\gamma,p)\Delta^*$$

and

$$\tau_x^* = [T_x^0(x',y',p)\cos\gamma - T_y^0(x',y',p)\sin\gamma]\Delta^* \tag{6}$$
$$\equiv F_x^*(x,x_0,y,y_0,\gamma,p)\Delta^*,$$

where x', y' are the local coordinates determined by

$$x' = (x - x_0)\cos\gamma + (y - y_0)\sin\gamma, \quad y' = (y - y_0)\cos\gamma - (x - x_0\sin\gamma.$$

The use of the superposition principle allows the solution for a mode III displacement discontinuity element in a rectangular sheet to be written in terms of a double periodic infinite set, as

$$\tau_y^* = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}[F_y^*(x, x_0 + 2mb, y, y_0 + 2nh, \gamma, p) +$$

$$F_y^*(x, -x_0 - 2mb, y, y_0 + 2nh, -\gamma, p) - F_y^*(x, -x_0 - 2mb, y, -y_0 - 2nh, \gamma, p)$$

$$-F_y^*(x, x_0 + 2mb, y, -y_0 - 2nh, -\gamma, p)]\Delta^*$$

$$\equiv f_y^*(x, x_0, y, y_0, \gamma, p)\Delta^*$$

and

$$\tau_x^* = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} [F_x^*(x, x_0 + 2mb, y, y_0 + 2nh, \gamma, p) +$$

$$F_x^*(x, -x_0 - 2mb, y, y_0 + 2nh, -\gamma, p) - F_x^*(x, -x_0 - 2mb, y, -y_0 - 2nh, \gamma, p)$$
$$-F_x^*(x, x_0 + 2mb, y, -y_0 - 2nh, -\gamma, p)]\Delta^*$$
$$\equiv f_x^*(x, x_0, y, y_0, \gamma, p)\Delta^*. \tag{7}$$

If the crack face is divided into N segments and the slant angle is α, then the shear stress equations can be written as

$$\sum_{j=1}^{N} f_{kj}^* \Delta_j^* = \tau_0^{*k} \qquad k = 1,2,\ldots,N \tag{8}$$

where τ^{*k}_0 is the Laplace transform of the stress boundary value on element k and

$$f_{kj}^* = f_y^*(x_k, x_j, y_k, y_j, \alpha, p)\cos\alpha - f_x^*(x_k, x_j, y_k, y_j, \alpha, p)\sin\alpha.$$

The equivalent stresses [1] can be calculated as follows:

$$\bar{\tau}^{*k} = \frac{2\mu}{\pi} \sum_{j=1}^{N} \frac{\Delta_j^* \lambda_j}{\lambda_j^2 - 4(j-k)^2 a^2/N^2} \quad k = 1,2,\ldots,N. \tag{9}$$

The stress intensity factor is given by

$$K_{III}^*(\pm a) = \sum_{k=1}^{N} F_k^{\pm} \tau^{*k} \sqrt{\pi a}\,, \tag{10}$$

where $F_k^{\pm}$ can be found in [1]. In the Laplace transform domain a set of values of the transformation parameter p_k are chosen as $p_k = \eta + 2k\pi i/T$ (k=0,1,2,...,L) (see Durbin [2]). The parameter $\eta T = 5$ and $T/t_0 = 20$, where t_0 is unit time.

A cracked sheet is shown in Fig. 2(a), with a crack of length $2a$; the width and height of the rectangular sheet are 2W and 2H, respectively. A uniform out-of-plane shear load $\tau_0 H(t)$ acts on the crack faces. The number of elements N=40. Results for the dynamic stress intensity factor are shown in Fig. 3 for three cases, (1) $W/a = \infty$, $H/a=2$; (2)W/a-∞, $H/a=0.5$; and (3)W/a=3, $H/a=2$.

For case (1), there are more than three kinks on this curve. Each point corresponds to the time at which shear waves coming along different routes arrive at the crack tip as shown in Fig. 2(b). Route 1 is for the shear wave starting from crack tip B to travel directly to A ($\bar{t}$=2, R'_1): route 2 is for the wave starting from tip A, travelling to the strip boundary and reflecting back to tip A ($\bar{t}$=2 x 2, R'_2); route 3 is for the wave starting from tip B to travel to the strip boundary and back to tip A ($\bar{t}$ = 2 x $\sqrt{5}$, R'_3). Later kinks are due to multiple reflections.

There are more kinks for case (2) as the crack tip is closer to the free boundaries in the narrower strip. There are three main kinks according to the route the shear waves travel. The first one is at $\bar{t}$ = 2 X 0.5 (R''_1) for the wave starting from tip A to the boundary and being reflected to tip A again (route 2). Route 1 is the wave directly from tip B to A ($\bar{t}$ = 2, R''_2); route 3 from tip B to boundary and then to A ($\bar{t}$ = 2 x $\sqrt{1.25}$, R''_3).

For case (3), there is one more kink on this curve than in case (1), as there is a corner in the rectangular sheet from which the shear wave can be reflected. Route 4 is for the shear wave starting from tip A to that corner and returning to tip A ($\bar{t}$ = 2 x $\sqrt{8}$, R''_4). This extra possible reflection has only a small effect but is responsible for the subsequent divergence between the results for cases (1) and (3).

Acknowledgement: The authors acknowledge the support of the Ministry of Defence, Defence Research Agency, Farnbourough, Hants, U.K.

REFERENCES

[1] P.H. Wen, M.H. Aliabadi and D.P. Rooke, *Archive of Applied Mechanics* 66 (1996) 326-335.
[2] F. Durbin, *The Computer Journal* 17 (1974) 371-376.

21 October 1996

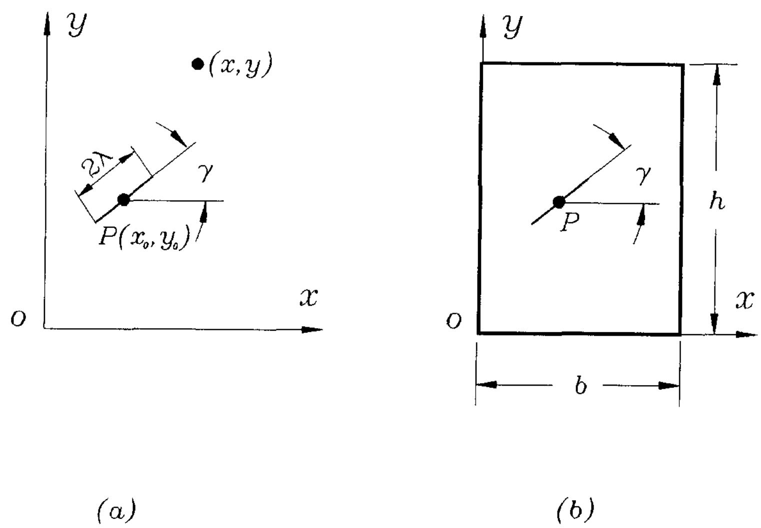

Figure 1. Displacement discontinuity element: (a) in infinite sheet; (b) in rectangular sheet.

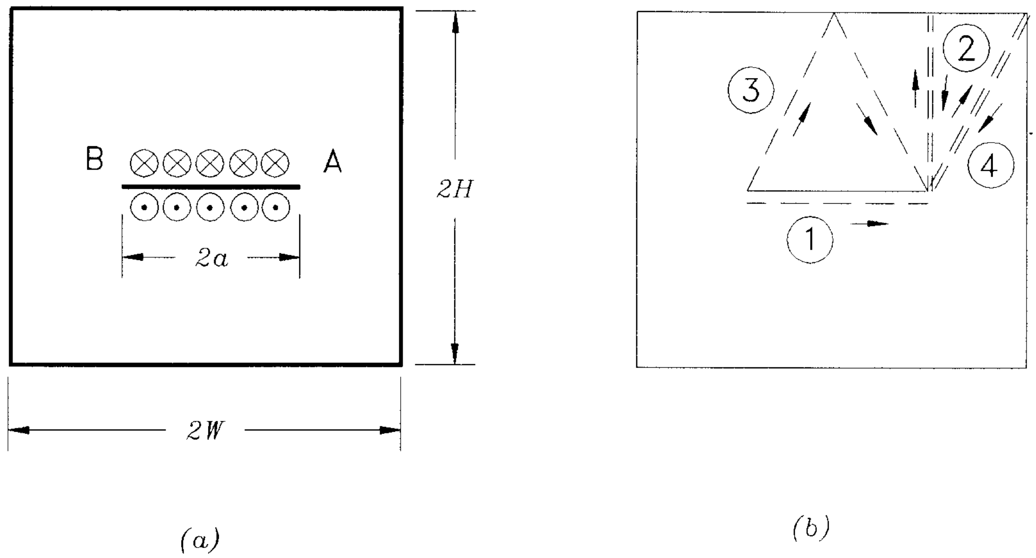

Figure 2. Rectangular sheet and shear wave travelling routes.

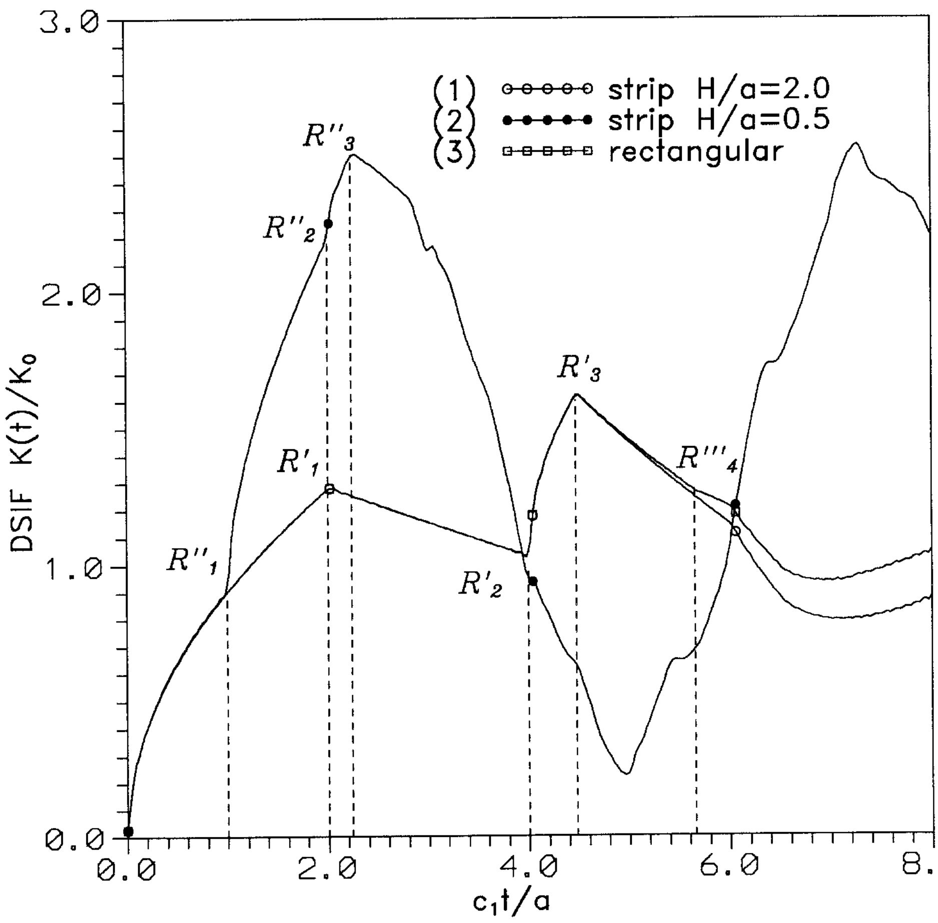

Figure 3. Dynamic stress intensity factors for a crack in rectangular sheet.