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Process region influence on energy release rate and crack tip velocity during rapid crack propagation

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Abstract. A finite element model of a plate with an edge crack is investigated. A cell model of the material, with the cell size representing some characteristic intrinsic material length, is adopted. The size of the process region depends on the number of cells that have reached a state which is unstable at load control. The results show that the growth of the process region is a main factor responsible for the lack of a unique relation between the small scale yielding energy release rate and the crack tip velocity and also for the observed constant crack velocities that are significantly below the Rayleigh wave velocity. A rapidly propagating crack appears to meet an increase of the energy flow to the crack edge per unit of time by increasing the size of the process region rather than increasing its edge velocity.

1. Introduction

Recent experimental results by, for instance, Ravi-Chandar [1], Ravi-Chandar and Knauss [2, 3, 4, 5], Kalthoff, Winkler and Beinert [6] and Kalthoff [7] have shown that dynamic crack growth at small scale of yielding could proceed under different energy release rates at the same crack velocity. These results thus contradict the previously generally accepted assumption of a small scale yielding energy release rate that is uniquely determined by the edge velocity for each material. They are particularly convincing in cases when a constant velocity prevails during a crack growth that is much longer than the length of the dissipative region.

In a previous work (Johnson [8]), a cell model (with the cell length representing some characteristic material length) of the material was introduced. At a forced increase of the cell volume the boundary loads first increased in accordance with Hooke's law, but after a certain critical volume is reached they decrease, i.e. softening takes place. During softening the cell is potentially unstable: its volume would increase indefinitely under load control. Cells that have reached such a state constitute the process region. The size of the process region was thus not predetermined. The cells were incorporated in a finite element model of a plate with an edge crack, but outside the expected process region larger elements with elastic behaviour were used. A pressure was applied at the crack faces and the subsequent crack tip history as well as the corresponding energy release rate were recorded. Simulations showed that at higher loading magnitudes a constant terminal velocity (significantly lower than the Rayleigh wave velocity) was reached while the energy release rate during the same period of time increased considerably. This is in agreement with the experimental results. The simulations further indicated that lower loading magnitudes could also result in constant velocities, but with lower magnitudes and weaker increase in energy release rate. For sufficiently low loading magnitudes, however, crack acceleration prevailed during the whole simulation time, i.e. no constant velocity was reached. In the present paper the constant velocity range is investigated in more detail. In addition (in order to find the specific influence of the expansion of the process region in the direction orthogonal to the crack propagation direction) comparisons are made with results from a model with a prescribed height.

Since only one cell row experiences softening at very low crack velocities without predetermined size of the process region, this prescribed height was chosen as one cell length.

2. Results from simulations

In the previous work (Johnson [8]), one set of simulations concerned a material characterized by Young's modulus E and Poisson's ratio 0.31 in the elastic region and by three parameters, $v_1 = 0.0020$, $v_2 = 0.048$ and $n = 1.0$ in the softening region. Parameter v_1 denotes the critical relative volume increase of a cell when softening starts while v_2 denotes the corresponding volume when softening is completed. (At this stage the cell is said to have vanished.) Softening is accomplished by multiplication of the elastic stiffness of each cell with the factor

$$\omega(v) = \frac{(v_2/v)^n - 1}{(v_2/v_1)^n - 1}, \tag{1}$$

for $v_1 \leqslant v \leqslant v_2$, where v denotes the relative volume increase of the cell. A higher magnitude of n thus corresponds to a more rapid reduction of the stiffness with increasing cell volume. A constant pressure p was applied on the crack surfaces with a short rise time, equal to 5 percent of the total

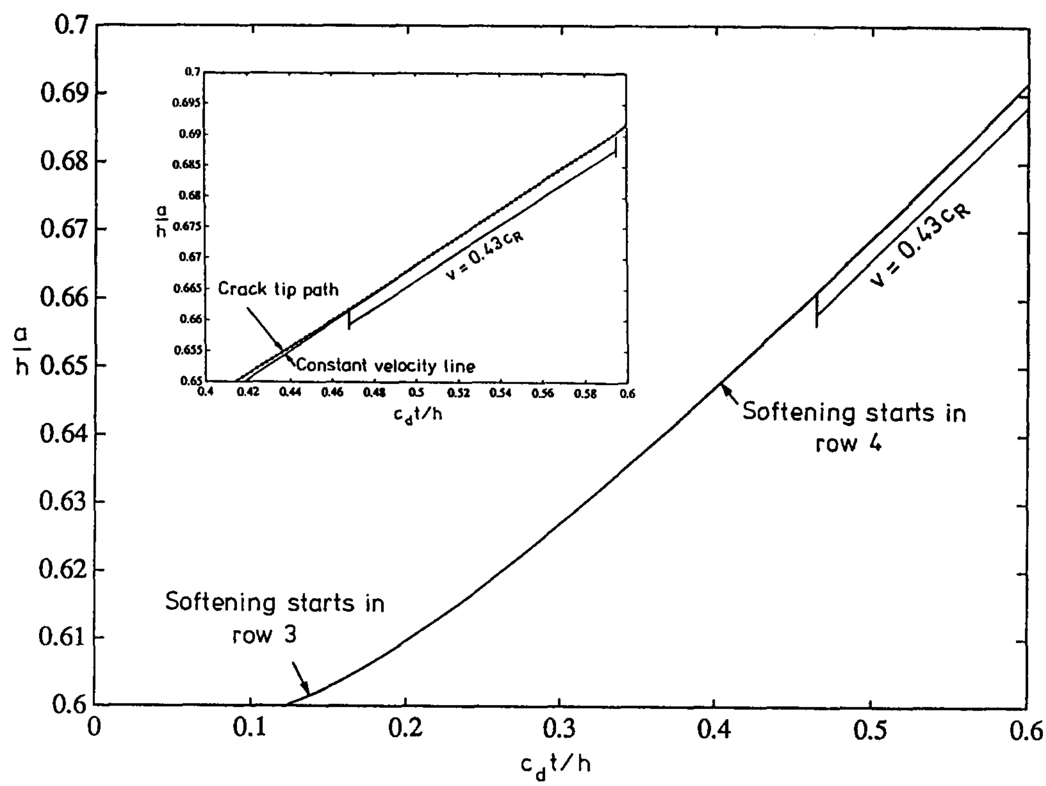
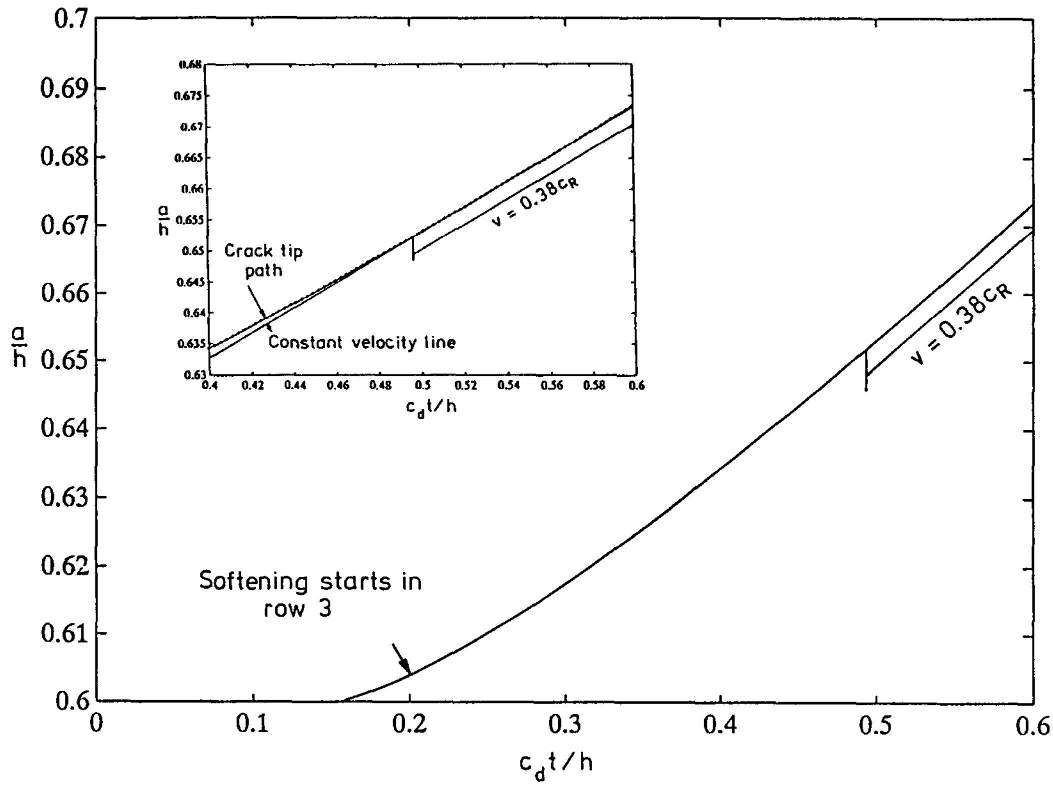
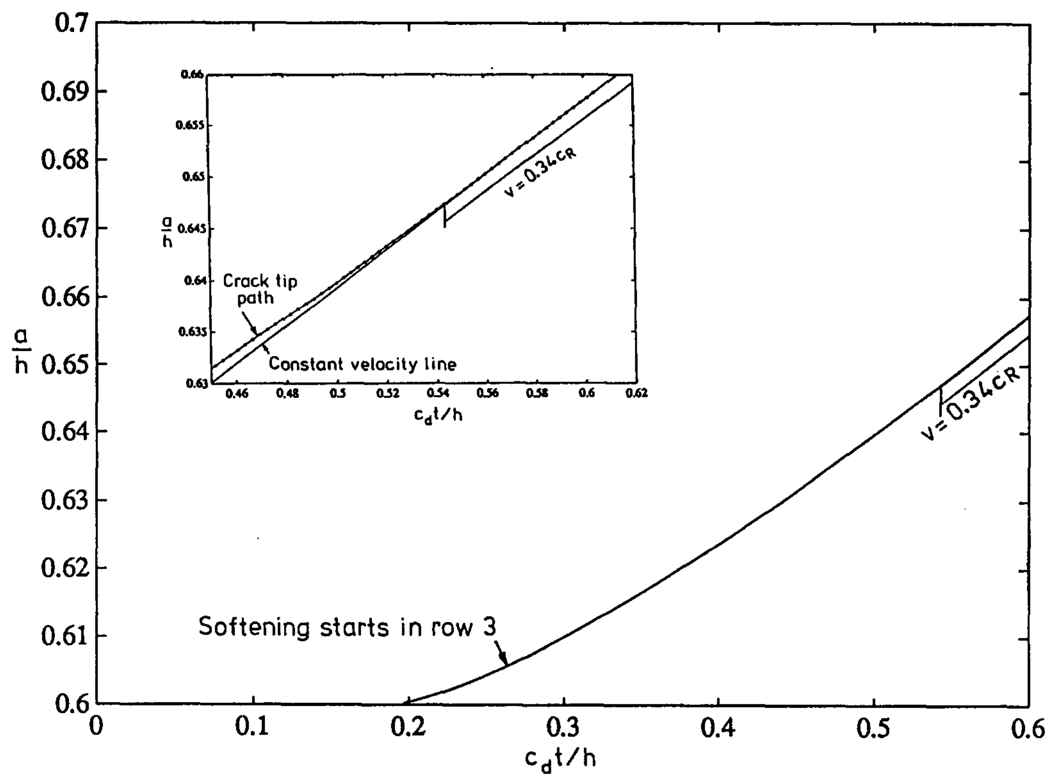


Fig. 1. The crack length against time for $v_1 = 0.0020$, $v_2 = 0.048$ and $n = 1.0$. (a) $p/E = 1.5 \cdot 10^{-3}$, (b) $p/E = 1.2 \cdot 10^{-3}$ and (c) $p/E = 1.0 \cdot 10^{-3}$. The terminal velocities and the number of cell rows that belong to the process region at different times are shown. c_d is the longitudinal wave velocity, t is time, h is specimen length and a is crack length. The original crack length is $0.6h$.



Crack tip path
Constant velocity line
v = 0.38cR
Softening starts in row 3
a/h
cdt/h



Crack tip path
Constant velocity line
v = 0.34cR
Softening starts in row 3
a/h
cdt/h

Fig. 1. Cont.

simulation time. The results showed that for $p/E = 3.0 \cdot 10^{-3}$ a constant crack velocity $V = 0.44c_R$, where c_R is the Rayleigh wave velocity, was reached rather soon after initiation. Higher loading magnitudes resulted somewhat paradoxically in a slightly lower constant velocity, apparently due to shielding effects from the rapidly expanding process region. During all these simulations, the energy release rate increased even when the crack tip velocities remained constant.

Further simulations with lower load magnitudes have now been performed. The crack length is shown against time for $p/E = 1.5 \cdot 10^{-3}$, $p/E = 1.2 \cdot 10^{-3}$ and $p/E = 1.0 \cdot 10^{-3}$ in Fig. 1. In all these cases constant terminal velocities are reached, $V = 0.43c_R$, $V = 0.38c_R$ and $V = 0.34c_R$, respectively, and as in the previous simulations the energy release rate increases even during the period of constant velocity. The approximate number of cell rows that belong to the process region at different times is indicated in the figures.

The crack path for the case with only one cell row was investigated for comparison. The results for $p/E = 3.0 \cdot 10^{-3}$ and for $p/E = 1.0 \cdot 10^{-3}$ are shown in Fig. 2. In these simulations (which are of the same length as those above) no constant velocities are reached. At the end of the simulation the velocity is $V = 0.59c_R$ in Fig. 2a and $0.36c_R$ in Fig. 2b. Still higher loading magnitudes resulted in even higher crack tip velocities. These crack velocities are significantly higher than the velocities obtained without restriction to one possible process region row, only. The energy release rate is not strictly constant, because different cell deformation patterns lead to somewhat different energy dissipation. The observed deviations from a constant energy release rate were, however, only a few percent.

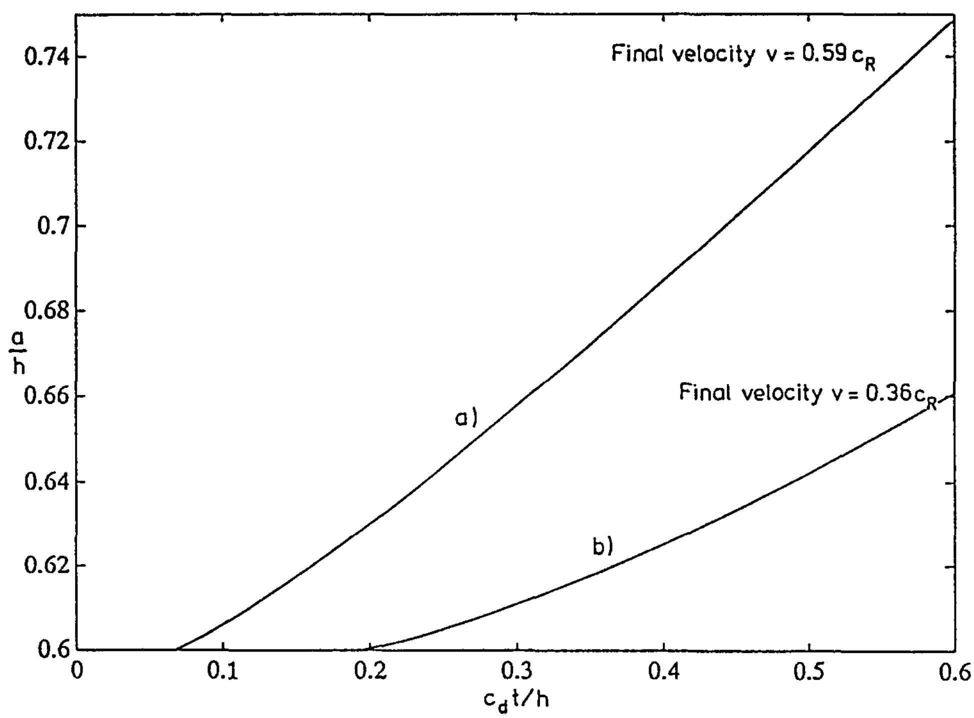


Fig. 2. Material characteristics are the same as in Fig. 1. No constant velocity region is obtained when softening is allowed in only one row. The crack length against time for $v_1 = 0.0020$, $v_2 = 0.048$ and $n = 1.0$. Softening is only allowed in the lowest cell row. (a) $p/E = 3.0 \cdot 10^{-3}$ and (b) $p/E = 1.0 \cdot 10^{-3}$.

3. Discussion and conclusion

The simulations generally show an acceleration to a constant velocity, whose magnitude is higher for higher loading magnitudes, except for the highest loads, when a maximum possible velocity seems to be reached or even a slight decrease of the constant velocity with increasing loading magnitude is observed. When the constant velocity is reached, the height of the process region is several cell lengths, i.e. its size is no longer determined by the characteristic cell length. The whole range of constant velocities is significantly lower than the Rayleigh wave velocity (in all simulations less than half of it). The energy release rate increases during both crack acceleration and during the subsequent period of constant crack tip velocity. The magnitude of the crack velocity is thus not determined by the energy release rate. Instead, it appears to depend on the whole history of the process region development. These results are different from those obtained with softening allowed in only one cell row. Then acceleration continues to higher velocities and no constant velocity phase is reached. The acceleration is larger, the larger the loading magnitude is. The energy release rate remains approximately constant (decreases slightly) with this model. This is apparently more in agreement with results from models with singular crack tips but not in agreement with experimental results.

The results seem to confirm that the process region constitutes the main factor responsible for the lack of a unique relation between the small scale yielding energy release rate and the crack tip velocity. The growth of the process region also appears to be responsible for the observed lower velocities, which are significantly below the upper theoretical limit. The lack of a unique relation between the small scale yielding energy release rate and crack edge velocity might be regarded as a necessary requirement for the appearance of a constant velocity phase under increasing energy release rate. However, it does not provide sufficient explanation for the apparent fact that the crack is meeting an increase of the energy flow to the crack edge per unit of time by increasing the size of the process region rather than by increasing its edge velocity.

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