Chapter 15
A Study of Japanese and Australian Students' Mathematical Values in Problem-Solving

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15.1 Introduction

Mathematics was invented by people, and their values have influenced its development (e.g., Kline, 1953). To do mathematics creatively, it is important to recognise people's values regarding mathematics. In mathematics education, students need to share these values to engage in proactive and creative mathematical activities. Because values are conative in nature (Seah, 2019), they help students concentrate on learning mathematics. Their connection to mathematics development helps students develop their mathematical knowledge.

Bishop (1988) argued that mathematics is not a value-free subject and that values are taught together in mathematics teaching, even if teachers are unaware of it. Bishop (1988) proposed three pairs of mathematical values that emerge from mathematics: rationalism-objectism, control-progress, and openness-mystery. These values are related to the social developments by which mathematics has developed and are fundamental aspects of Western mathematics. The particularly relevant pair of values that are driving forces for creative mathematical activity are control and progress. Control is a static desire to use mathematics to explain the natural and human-made environments. Progress is a dynamic feeling that the unknown can become known in mathematics. These values are related to feelings and attitudes that the mathematical culture has driven and reinforced.

For students to do mathematics creatively, teachers need to teach the mathematical values of control and progress in a balanced manner. Moreover, students need to appreciate these values in a balanced way (Bishop, 1988). Bishop (1988) proposed a curriculum based on projects which are time-consuming personal research based on historical situations and exemplify the values of control and progress. This curriculum is intended to focus on these values by dealing with projects related to society in the

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Y. Dede et al. (eds.), *Values and Valuing in Mathematics Education*,
https://doi.org/10.1007/978-981-99-9454-0_15

past, at present and in the future. In other words, control and progress values are conveyed through studying how mathematics is used in society. However, conveying the control and progress values in a balanced manner can come from projects and mathematics content because mathematical content is associated with several mathematical values. Dede et al. (2021) found that several mathematical values could be expressed by focusing on mathematical modelling tasks, and it may be possible to balance control and progress values through a single mathematical content. However, there have been no proposals in mathematics education on how to convey control and progress values in a balanced manner through a single mathematics content.

In Japan, mathematics is viewed as a creative activity. There is an emphasis on the values that people should be aware of when doing mathematics creatively, and the mathematical values of simplification, clarification, and unification have been proposed (Nakajima, 1971). Simplification makes concrete events easier to express, work with, and think about by mathematizing them. Clarification shows correct thought and correct relationships by logical thinking and expression. Unification captures different things in a unified way so that a single form is logically valid. In varied expressions, these values have been written into the goals of mathematics curricula in the past and recent years in Japan about the type of students they wish to nurture through mathematics education. Nakajima (1971) assumed that mathematical creation could be achieved by recognising these values and feeling that it is unpleasant not to do so. Even seemingly complex events in a problem-solving process can be simplified by expressing them using mathematical equations. It is possible to clarify an event's causes by considering it logically. Then, it is possible to develop further thinking by unifying the situation in which the idea has been applied to make it applicable to other situations.

Thus, it is necessary to appreciate the three mathematical values for the problem-solving process to become a creative activity. Japanese researches have revealed that these mathematical values appear in problem-solving contexts (e.g., Tsubomatsu, 2011) and that students consider these values in comparing various solutions for problem-solving (e.g., Hagiwara, 2007). But no research has considered problem-solving to be a product of mathematical culture. Problem-solving is an activity in mathematical culture, and developing problem-solving in one situation to apply to other situations involves learning that engages the sense of the control values and, subsequently, the progress values. In other words, problem-solving can be positioned as mathematical content that may convey control-progress values in a balanced manner. Therefore, it may be possible to link control-progress values and the mathematical values of simplification, clarification, and unification to capture the problem-solving process, but no study has been done.

This study aims to clarify what is needed for practice to convey both control and progress values in problem-solving in a balanced manner, which does not end with control values but can be felt even in progress values. To solve this problem, the following two issues are considered. First, the mathematical values proposed by Bishop (1988) and Nakajima (1971) are derived by focusing on the properties of mathematics, which are ideological concepts, and it is unclear what values students hold concerning the properties of mathematics. Therefore, students' values concerning the properties of mathematics in problem-solving need to be clarified. A survey is conducted to capture students' values in problem-solving situations. Second, the mathematical values proposed by Bishop (1988) and Nakajima (1971) are derived from the properties of Western mathematics and are inherent in mathematics content, such as problem-solving methods. This raises the question of whether mathematical values can be conveyed to students simply by studying mathematics content or whether they are influenced by the culture in which students' study and live. Therefore, suggestions are obtained on what is needed to implement practices that allow students to be creative in problem-solving and to feel control-progress values, considering the values identified in the first task. A cross-cultural comparison of students' values was conducted, focusing on students who study the same curriculum but live in different cultures. Specifically, students in Japan and Australia studying the Japanese mathematics curriculum are surveyed. The students living in Australia in this study were studying the Japanese curriculum in addition to the Australian curriculum.

The study shows how using problem-solving can draw students into values related to creativity, control, and progress in mathematics. The study identifies students' values related to the properties of mathematics and offers suggestions for how students can be creative in problem-solving. The novelty of this study is that it compares values by focusing on students from two cultures learning mathematics using the same materials. Australia is an ethnically diverse country with diverse values (e.g., Han and Seah, 2019) whereas Japan is not. While similar values may be fostered by learning mathematics using the same materials, students' cultures may influence mathematics learning. In this sense, both factors are considered in this study.

15.2 Theoretical Framework

15.2.1 Mathematical Values

There are three types of values related to mathematics education: mathematical values, mathematics educational values, and general educational values (Bishop, 2001, 2008). Mathematical values are related to and have developed as knowledge of Western mathematics. Mathematics educational values are related to the norms in classrooms in which mathematics education is conducted. General educational values are not limited to mathematics education but arise from the demands of socialisation by society. All values are influenced by society, and various properties and objects are valued in different countries and cultures (e.g., Clarkson et al., 2019). Among these values, the mathematical values are those that students aim to appreciate through mathematics education to understand mathematics more deeply. Therefore, this study focuses on mathematical values to determine what students consider worthwhile in mathematics. The mathematical values proposed by Bishop

(1988) for Western mathematics are well known. They represent valuable properties of Western mathematics and are implicitly or explicitly believed in and sustained by people in a society. These values are inherent in mathematics and should be conveyed to students, primarily through mathematics education.

In Japan, where Western mathematics is used in mathematics education, values that should be consciously pursued have been proposed from a different perspective to that of Bishop (1988). Nakajima (1971) proposed three value types that should be pursued for students to engage in creative activities: clarification, simplification, and unification. From the standpoint that creative activities should be the aim of mathematics education, students need to think mathematically, engage in tasks, and produce new things with the feeling that they are compelled to pursue these three values. The word 'creative' used here refers to creation in the sense that students feel as if they have invented a new idea on their own and, as a result, acquire new content based on their need to work on the task. These values represent the important properties of mathematics as perceived by those who do mathematics creatively. These values, also inherent in mathematics, correspond to mathematical values. Students are expected to appreciate these values through mathematics education.

15.2.2 Mathematical Values and Problem-Solving in the Japanese Curriculum

Problem-solving can have a single answer or solution or be open-ended. In Japan, it is important to consider and compare various solutions because there are many ways to arrive at a solution, even for a single answer (e.g., Hino, 2015). To compare and summarise children's ideas, it is important to reflect on problem-solving ideas from various perspectives, not only for a deeper understanding of ideas but also to increase children's motivation to pursue ideas (Koto et al., 1992). Thus, learning mathematics to compare ideas and consider their values is important. Mathematical values function as a point of view for organising ideas in problem-solving. Depending on the perspective from which they are organised, there will be differences in how they are characterised and, consequently, in the new content created.

The fact that a problem can be solved and new content created by developing the ideas used in the solution represents the mathematical values of control and progress. Nakajima's (1971) mathematical values of clarification, simplification, and unification drive creative activities. Therefore, simplification, clarification, and unification values may be associated with the values of control and progress. Solving a problem is related to control, made possible by simplification. By clarifying and organizing the viewpoints of various solution methods and unifying them, a previously unknown problem situation will become known.

However, besides the abovementioned mathematical values, it is natural for students to become aware of other mathematical values. Creative activities can be conducted through the complex involvement of various mathematical values. In this study, practices that can convey a balance between control and progress values in problem-solving are the focus. Thus, the study is limited to the mathematical values of clarification, simplification, and unification, as well as ideas related to these values.

15.2.3 Learners' Mathematical Values in Problem-Solving

Understanding students' values aids good mathematics education, as values are a conation component and influence powerful motivating forces (Seah, 2019). Positive conative states foster students' well-being and, thus, performance in mathematics learning experiences. Focusing on students' values can improve their well-being in mathematics learning (Clarkson et al., 2010). Recent research supports teaching mathematics to align with students' values (Hill et al., 2022; Kalogeropoulos and Clarkson, 2019) to discuss students' diverse values in the classroom and increase positive feelings and engagement with mathematics.

In Japan, student values are important factors in learning. One of the goals of mathematics education in Japan today includes the development of "human nature and the ability to pursue learning" (Ministry of Education, Culture, Sports, Science and Technology, 2017, p. 18), which aims for students to learn mathematics with values, which influence mathematics learning, including cognition and thinking, and motivate students to learn. Students should appreciate the intrinsic value of mathematics because it enriches the human spirit. For example, a student who enjoys the mathematical values of unification will not learn anything by seeing different things as different but will be able to see other things as having the same structure and organize them.

Even though the goal is for students to appreciate the valuable properties of mathematics, they develop their own values not only in mathematics class but also in their culture and at home. Therefore, it cannot be said that students' values regarding the properties of mathematics are exactly the same as their conceptual values; it is natural for differences to exist among students. Therefore, learners' mathematical values are used in this study to emphasise that they are students' values related to mathematics' properties. Since mathematics education can be regarded as an enculturation process (Bishop, 1988) and students are recreators of the values of mathematical culture, their mathematical values are like, yet somewhat different, the mathematical values embedded in mathematics. Therefore, mathematical values and learners' mathematical values may differ, yet are connected. This allows us to observe the pair of mathematical values—control and progress—and their details, including subtle differences among learners.

15.2.4 Choice and Learner Mathematical Values

Students' values affect their conscious choices on how to act and can be determined by their choices. Krathwohl et al. (1973) proposed a taxonomy of internalisation in the affect domain, describing valuing at its third level. At this level, valuing through internalising specific values is consistent and stable. Learners' mathematical values are students' internalisation of the valuable properties of mathematics, appearing as consistent and stable behaviour by students.

When consciously 'selecting things of value', the criterion of choice is what the student perceives as valuable. Then, items that satisfy the criteria are selected. At this point, the criterion, the thing perceived as valuable, is what satisfies the student's desire through experience prior to the conscious act of selection. In the conscious act of selecting something of value, the students perceive both the criterion and the selected item as valuable. These can be regarded as representations of students' mathematical values. Therefore, learners' mathematical values can be captured using the criterion of choice and what is chosen in the conscious act of choosing 'what is important'.

15.3 Methods

15.3.1 Method for Capturing Learner Mathematical Values in Problem-Solving

This qualitative study is based on answers to a questionnaire survey in which students were asked to choose items. The learners' mathematical values are captured by analysing their responses. To explore learners' mathematical values, a questionnaire was designed in which students were asked to select the most mathematically important idea from solutions containing different ideas. In asking students to select the 'most mathematically important idea' by comparing several solution methods, they were reminded of what they perceived as valuable in mathematics, which became the criterion for selection, and the idea satisfying this criterion was selected. Thus, in describing the reasons for the choice, the value perceived by the student was mentioned, and the expression of the learner's mathematical values could be captured.

The questionnaire was developed for mathematical problem-solving. Three solution methods of varying qualities were set as choices. Learners' mathematical values were expected to emerge in the answers to 'mathematically good solution' questions in an open-ended manner. However, because this study is concerned with progress values achieved through creative activities in the problem-solving process, the aim was to capture the mathematical values of students concerning the idea of solutions of different qualities that lead to creative activities. Thus, the three solution methods differed in the quality of problem-solving and related to control and progress values, presenting them as student options. For example, if a student chose a solution method because 'it is a method that can be used in other cases', their mathematical values could be captured as 'applicability', which is less a matter of what is important in mathematics education and more what is inherent in mathematics. This expresses the value associated with covering not only the current problem but also other cases. The values are connected to Bishop's (1988) mathematical control values from the student's perspective, making it possible to closely examine them based on the characteristics of the selection criterion and selected object.

Since this is a theoretical assumption, it is possible that students would have answered without much consideration, and their mathematical values would not be expressed in the reasons for their choices. To minimise this risk, the purpose of the study was explained to the students before the survey, and they were asked to think carefully about their responses. They were asked whether they understood the idea of the choice, whether they thought the idea was good, and why. Students who made choices without due care or understanding of the solution idea and whose answers and reasons were inconsistent were excluded from the analysis. Six students selected the idea of the solution as a good idea, even though they did not understand it, and were inconsistent in their evaluation of the goodness of the solution and the statement of reasons for their choice of solution. They were excluded from the data analysis.

15.3.2 The Cultural Differences Influenced Learner Mathematical Values

This study focused on students studying the same mathematics curriculum living in different countries: Japan and Australia. Students in both locations learned Japanese mathematics content from Japanese teachers obtained from Japanese textbooks produced by the same textbook company. The Japanese government approved the textbooks. The main differences between students in the two countries were the place where they studied Japanese mathematics, the type of school in which they studied Japanese mathematics, the amount of time they spent studying Japanese mathematics (on weekdays or only on Saturdays), the qualifications of their Japanese mathematics teachers, and the culture in which they lived. The cultural differences in this study are due to Australia being an ethnically diverse country with diverse values and Japan not. Although there are Japanese schools in Australia, the students who participated in this study attended local schools on weekdays in Australia and were assumed to be more influenced by Australian culture than students who attended Japanese schools in Australia.

The students in this study who live in Japan attended a public elementary school in Tokyo located in a lush green area away from the central business district and studied Japanese mathematics textbooks almost every weekday. Their schooling was compulsory. Mathematics teachers were university or graduate school graduates with teaching licences.

The students in this study who lived in Australia were local students, including Japanese, Australians, and people of other nationalities. Some had studied mathematics in places other than Japan and Australia. They studied the content of Japanese mathematics textbooks only on Saturdays at a supplementary school in Victoria, the second-largest state in Australia, located in a lush green area. On weekdays, they studied mathematics content conforming to the Australian curriculum (Australian Curriculum Assessment and Reporting Authority, 2017) or the Victorian curriculum (Victorian Curriculum and Assessment Authority, n.d.) at their elementary schools in Victoria. Teachers of Japanese mathematics in supplementary schools were native Japanese speakers who were not required to have a teaching licence. Teachers used handouts based on Japanese mathematics textbooks to teach mathematics at the supplementary schools.

15.3.3 Research Ethics

The survey was administered with the prior consent of the teachers at the school and was communicated and explained to the students' parents as appropriate. Students participating in the survey were informed prior to the survey and told that their answers were not related to their marks at school and that they were expected to answer the questions only to the extent that they were able. Students were asked to respond to the survey if they were able to cooperate. Students who did not respond to the survey were considered to have refused to participate and were excluded from the overall data count. Student names were coded using numbers to remove personal identifiers from the data. Japanese students were prefixed with SJ and numbered consecutively using three digits, such as SJ001. Australian students were prefixed with SA and numbered sequentially, such as SA001. All data were stored electronically, and the students' responses' originals were kept locked.

15.4 Overview of the Survey

15.4.1 A Problem and Solutions in a Questionnaire

The mathematical problem-solving task was to count the number of marbles when the number of marbles increases regularly. Starting with the initial stage of the number of marbles, the first to third stages are illustrated, with the student asked to find the number of marbles in the fourth step (Fig. 15.1). This task could be answered by counting the numbers one by one, but it can also be answered simply by taking a mathematical view; there is diversity in the solution methods, and clarification can sort out the differences in solution ideas; meanwhile, unification can deepen understanding of the task phenomenon. Therefore, this task is appropriate

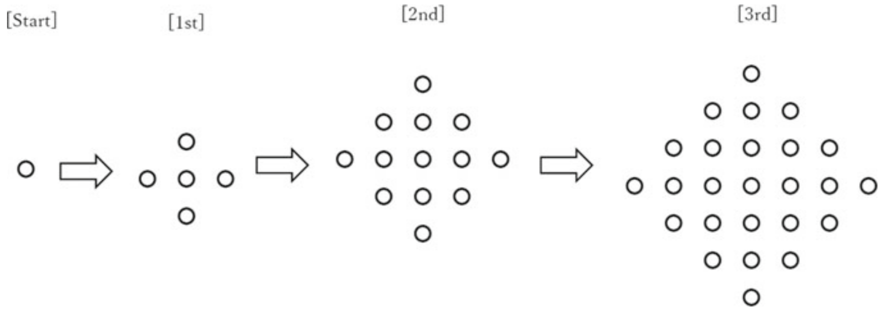

Fig. 15.1 The mathematical problem-solving task

for capturing mathematical values. The three solutions represent different ways of thinking (Fig. 15.2). Students compared the mathematical differences between these alternatives, consciously recalled what was valuable to them, and selected the 'most mathematically important' idea. The choices were presented using the word 'idea' to emphasise to the students that they should pay attention to ideas in each solution.

Idea 1 involves drawing and counting marbles. This method focuses on alignment symmetry to simplify counting the number of marbles in a circle, creating one 5×5 square and four clusters of triangles containing four marbles. Idea 2 is a way to find the pattern of how many more marbles there would be by focusing on the gradual change in the shape of the arrangement of the marbles. This method uses the number of marbles that increase in multiples of 4, that is, 4, 8, 12, and so on, and infers that the subsequent increase is 16. Idea 3 focuses on the gradual change in the shape of the marble arrangement and the mechanism for increasing the number of marbles. In this method, one marble in the initial stage is positioned at the centre, and an increasing number of marbles is seen as four triangular clusters with the same number of marbles around it. The number of marbles in each triangular cluster is 1 in the first step, $1 + 2$ in the second step, and $1 + 2 + 3$ in the third step; the next step is estimated as $1 + 2 + 3 + 4$. As the number of marbles increases as the steps progress, if the number of steps is x and the number of marbles at that step is y, the number of marbles can be captured using $y = (1 + 2 + \cdots + x) \times 4 + 1$.

Every idea has a simple and clarified explanation written with a diagram and an equation that leads to a correct answer. For example, idea 3 responds to the value of simplicity, as it aims to capture the number of marbles by linking it to the number of steps. It also responds to the value of clarification, as the number of steps is used in the formula for the number of marbles. In terms of unification, any solution idea can be applied to steps other than the fourth step in the problem or the case of different arrangements of figures. However, idea 3 explains how the number of steps is related to the number of marbles based on the four-directional symmetry of the square, and this mechanism can be used to consider the relationship between the number of steps and the number of marbles in the case of other shapes arranged in pentagons or hexagons, for example, by using the same idea (Fig. 15.3). Idea 3 is very useful for solving the problem related the relationship between the number of steps and

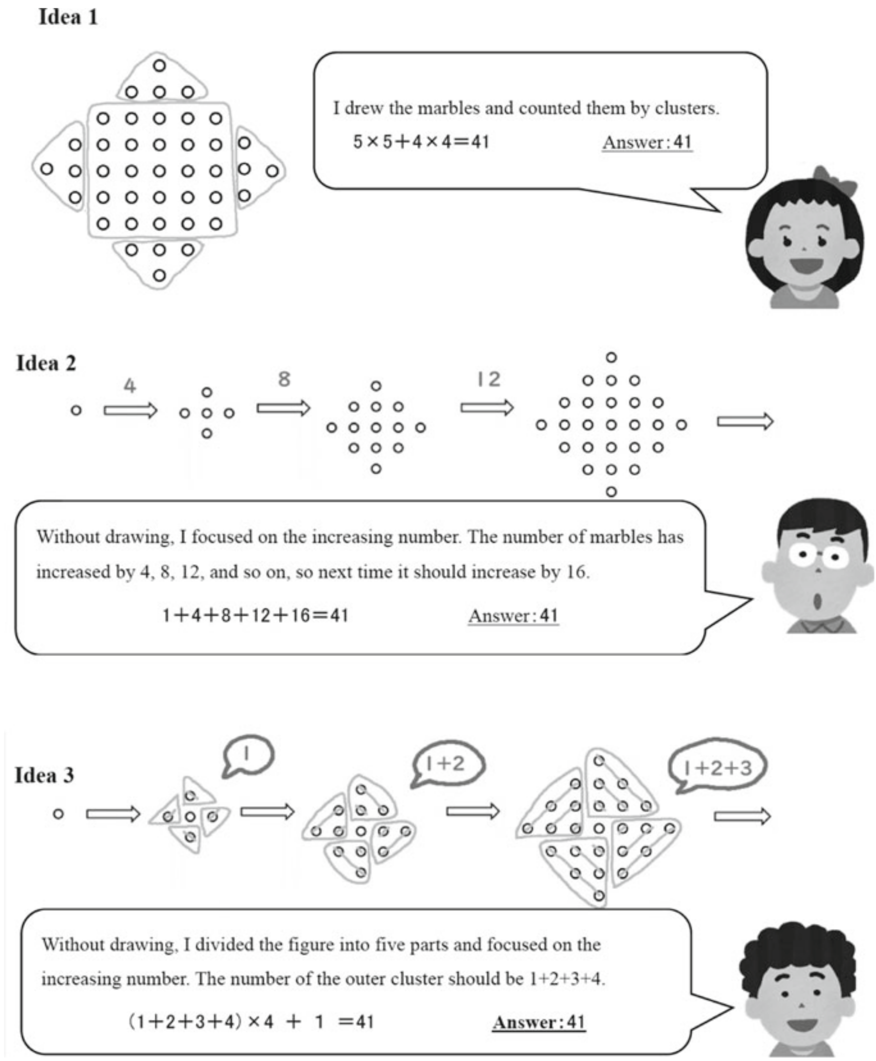

Fig. 15.2 Three different solutions

the number of marbles. Idea 3 can integrate and capture different phenomena, such as lining up in a square and lining up in other shapes, using the same mechanism, marking progress in how phenomena are perceived.

Bishop's (1988) mathematical values can be conveyed through activities like those used in this study. For example, the values of rationalism can be conveyed by rationally thinking about how the number of marbles increases according to rules. In contrast, the values of objectism can be conveyed by thinking about the characteristics of how the number of marbles increases using diagrams. The values of control

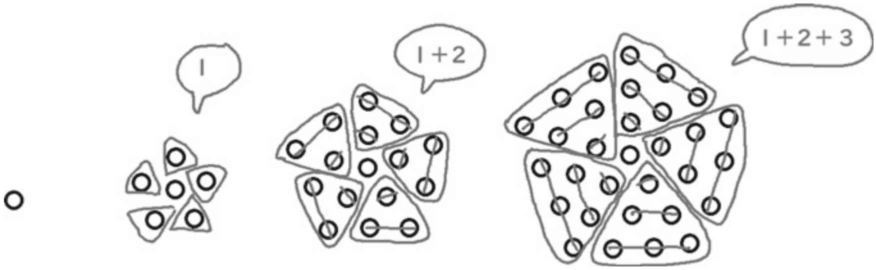

Fig. 15.3 Examples of pentagonal arrangements with the same structure

appear in explaining the number of marbles with an expression, and the values of progress appear in expanding the view adapted to the four sides of a square to the unknown problems of pentagons and hexagons. The value of openness can be expressed by explaining ideas used to solve a problem. The values of mystery can be expressed by there being many other ways of thinking about solving a problem. This study focuses on the control and progress values among mathematical values to capture how students value mathematics in problem-solving.

15.4.2 Questionnaire

After showing the solutions to the three ideas, the participants were asked to indicate (1) whether they understood idea 1; (2) whether they thought idea 1 was mathematically good, and (3) why they thought it was a mathematically good idea (or not). The same three questions (4, 5, and 6) were asked for idea 2, and the same three questions (7, 8, and 9) were asked for idea 3. For item 10, the participants were asked to choose the most mathematically important idea from ideas 1 to 3. For item 11, they were asked to describe the reasons for their choice. The Appendix x.1 shows the English version of the questionnaire.

Items 2, 5, and 8 asked whether each idea was mathematically good, maybe the same as the statement of reason in item 11, which compares the three ideas and then makes a choice, or they may be different statements. This is because it is necessary to use one common criterion to compare the three ideas. In contrast, an additional criterion may be used each time when judging whether each idea is mathematically good. The selection criteria for judging whether each idea is mathematically good may differ from those used for comparing the three ideas. Because learners' mathematical values are expressed when students consciously choose, their responses are analysed by item 11, for which students choose between the three ideas. However, as items 2, 5, and 8, which ask about each idea, and item 11 may be linked, the answers to items 2, 5, and 8 are referred to only when they are related to the content of item 11.

15.4.3 The Mathematics Curriculum Learned by Participants in Japan and Australia

The study participants were third- to sixth-grade students living in the Tokyo area of Japan and third- to ninth-grade students living in Victoria, Australia. The study participants in both countries used Japanese mathematics textbooks. In these textbooks, problem-solving is not a separate unit but is presented as a text problem in mathematics content units. In Japan, the method of dividing numbers into groups of the same number, as in idea 1, is used in the 2nd-grade multiplication unit in elementary schools (Fujii and Majima, 2020a). Students are taught to think about the meaning of the numbers in the multiplication equation by connecting them to clusters of the same number represented in the diagram. Problem-solving involving variables, such as those discussed in this study, appears in the 4th-grade textbook in a mathematics unit titled 'Investigating How Things Change', in which students consider how quantities change (Fujii and Majima, 2020b). In this unit, students examine how variable Y changes when variable X changes by organising it in a table and using a diagram. In problem situations in which a specific value of Y is required, the following methods are introduced: focusing only on how variable Y changes and finding a way to increase the value of Y (idea 2), finding an equation relating variables X and Y (idea 3), and counting by drawing a specific diagram (idea 1).

However, in Australia, there is no fixed textbook for mathematics; teachers select and use materials and printouts that conform to Australian or Victorian curricula. Therefore, referring to the Australian curriculum for mathematics, patterns, and algebra are positioned as sub-strands within the Content Strand of Numbers and Algebra. Patterns and algebra specify that writing, following, and creating number patterns should be taught in mathematics education and that students should make tables of changing events and look for patterns. For example, in a student book adapted from the Australian curriculum (Harris, 2018; Turner, 2018a, 2018b), some activities look for patterns in the sequence of numbers created by four arithmetic operations until the 4th grade. In the 5th grade, the book contains problems in finding the relationship between the number of steps and the number of blocks, and methods (e.g., idea 2) to look for patterns of increase by looking only at the number of blocks are discussed. This book also addresses the problem of determining the number of blocks required for specific steps. In the 6th grade, the search for rules regarding the number of sides in a regular polygon and the number of lines of symmetry (which may lead to idea 3) is discussed.

15.4.4 Data Collection

The survey was conducted during August–October 2022. Before the survey start, the students were informed of the purpose of the study and the survey outline and asked to fill in their honest opinions. Valid data were collected from 312 elementary school students in Japan: 71 in the 3rd grade, 62 in the 4th grade, 96 in the 5th grade, and 83 in the 6th grade. Valid data were collected from 125 students in Australia: 31 in the 3rd grade, 24 in the 4th grade, 30 in the 5th grade, 15 in the 6th grade, and 25 junior high school students in the 7th grade and above. The survey was administered to more than one grade to observe their differences and whether the content studied in each grade significantly impacted the responses.

15.4.5 Data Analysis

The collected response forms were assigned numbers so that individuals could not be identified, and the responses were organised into a list so that the contents of each number ('the most mathematically important idea' and 'reason it was the most mathematically important idea') were linked. This list was used to conduct a quantitative analysis of the data characteristics.

The number of responses for each selected idea was counted for each type. To capture differences between Japanese and Australian in selecting idea, a Chi-square test was performed as needed. An open coding procedure was used for the selection. First, keywords were extracted from the reasons for selection mentioned by all students. Next, if the keywords were similar, they were organised into labels. If several labels were similar, they were grouped. Then, one group was set as the main code, and one label as the subcode, and the reasons for the selection were classified. For example, the student in SA395 chose idea 3 by answering, 'If the question is "Answer the Xth number", I can answer it immediately and accurately'. In this case, we extracted three keywords: the Xth number, immediately, and accurately. Then, by combining the keywords with similar keywords in other students' response data, we quickly and accurately labelled each, as in the other case. These were later used as subcodes. Finally, we grouped the label unification, effect, and clarification. These three codes later became the main codes.

The validity of the keywords extracted from the student's reasons for selection is that the keywords in the reasons for selection are at least what the student considered 'mathematically important'. Because the students considered the contents of the keywords to be 'mathematically important', they selected an idea that satisfied the keywords. In other words, both the selected ideas and the keywords written in the reasons for the selection were matters that the students considered 'mathematically important'. Therefore, the learner's mathematical values were captured by extracting, classifying, and organising such keywords. Keywords described in negation form were extracted.

To ensure the reliability of the analysis, the same procedure was performed on 21 November 2022, 25 January 2023, and 27 January 2023. The order in which the students were analysed changed each time. Consequently, different parts of the analysis were examined to refine the analysis.

15.5 Findings

15.5.1 Differences Between Japan and Australia in the Type of Selected Solutions

The following is a breakdown of the types of solutions selected by the students as their most important ideas (Tables 15.1 and 15.2): The columns of each table indicate the students' grade level and the rows of the type of idea selected. For Australian students, grades 7, 8, and 9 also participated in the survey; however, because of the small total number of students, the figures are for all three grades combined. In total, 124 Australian and 307 Japanese students provided valid data.

Six Japanese students were required to settle on one solution and select two. One 6th-grade student chose ideas 1 and 2. One 4th-grade student and one 5th-grade student chose ideas 1 and 3, respectively. Three 4th-grade students chose ideas 2 and 3. One Australian 3rd-grade student who selected idea 1 evaluated it without understanding, as did one Japanese 3rd-grade student who selected idea 1, three 3rd-grade students who selected idea 3, and one Japanese 4th-grade student who selected idea 3. These students were excluded from the analysis.

The Australian students who participated in this study were likelier to choose idea 2 as their most important idea. These students accounted for approximately 69.4% of the total. These characteristics were similar throughout the school year. Among the Japanese students who participated in the survey, about 51.8% chose idea 2 as the most important idea, but ideas 1 and 3 were also present in some proportions. These characteristics were similar throughout the school year.

Table 15.1 Breakdown of the types of ideas selected by the students in Australia (n = 124)

	3rd	4th	5th	6th	7th	Total	Percentage (%)
Idea 1	5	2	4	1	3	15	12.1
Idea 2	18	20	20	13	15	86	69.4
Idea 3	5	1	5	1	6	18	14.5
n.d.	2	1	1	0	1	5	4.0
Total	30	24	30	15	25	124	100

Table 15.2 Breakdown of the types of ideas selected by the students in Japan (n = 307)

	3rd	4th	5th	6th	Total	Percentage (%)
Idea 1	17	10	35	18	80	26.0
Idea 2	32	38	44	45	159	51.8
Idea 3	8	15	17	21	61	19.9
n.d.	10	2	1	0	13	4.2
Total	67	65	97	84	313	101.9

15.5.2 Categories of Reasons for Selection

The main codes were classified into six categories by organising students' reasons for their choices: simplification, clarification, unification, technique, feeling, and effect. The simplification category is derived from an easy way of expressing, working, and thinking. The clarification category comes from the ability to explain something by attributing it to a few essential things to understand it correctly. The unification category is based on the assumptions of other problematic situations. The technique category focuses on the techniques used in each solution method, such as counting, grouping, and focusing on changes. The feeling category is an evaluation based on conformity with the students' sensory perceptions. The effect category evaluates the educational context in which abilities are gained through solutions. Sample examples of each main code are listed (Table 15.3). In every case in which ideas 1, 2, or 3 were selected, at least one response classified as simplification, clarification, or unification was included. Some students' responses were sorted into multiple codes.

The most common reasons Australian students gave for choosing idea 2 were simplification and clarification, followed by effect. Specifically, idea 2 was often chosen for such reasons as being the simplest, easy to do, or easy to understand, followed by quick, which is often chosen for the speed of the solution. The most common reason Japanese students chose idea 2 was technique, followed by simplification and effect. Specifically, for example, students often evaluated idea 2 because of its focus on the difference between variables and increasing patterns, followed by the response that it was easy to make an equation and quick.

The most common reasons for selecting idea 3 were effect, simplification, and unification for Australian students and technique, simplification, and unification for Japanese students. Idea 3 was expressed as an equation relating the number of steps to the number of marbles. Australian students focused on the fact that the answer could be obtained by applying the number of steps and evaluating it based on its simplicity and speed. Meanwhile, idea 3 focused on the fact that the number of marbles increased in all four directions because of the symmetry of the square, and the Japanese students focused on the fact that they found a relationship between the number of steps and the

Table 15.3 List of examples for each main code

Main code	Sample
Simplification	It is the simplest and the easiest to figure out without having to do too much in my mind. (SA370)
Clarification	Because it is easy to understand $1 + 2 + 3$. (SJ13)
Unification	It is easier to think of a larger number of steps because the seventh would be $(1 + 2 + 3 + 4 + 5 + 6 + 7) \times 4 = 28 \times 4 = 112$. (SJ360)
Technique	It is a way of finding a pattern. (SJ46)
Feeling	Because I did it at the start, unlike the other ones. (SA152)
Effect	Ideas like Solution 2 would help me to be more aware of all sorts of things. (SJ185)

number of marbles by focusing on the groups and evaluated it based on its technique and simplification. In addition, unification, the value for which idea 3 was evaluated in both Japan and Australia, applies to other step-number situations. For example, the Japanese student SJ200 stated that the reason was ‘because I can quickly find the answer even if it is the 100th number’, and the Australian student SA395 stated, ‘because if I am asked to answer the Xth number, I can quickly and correctly answer the question’. Thus, other problems were assumed to be the case for other steps.

15.6 Discussion

15.6.1 A Consideration Related to Selected Idea 2

The two countries have similarities and differences regarding the types of ideas selected as most important. The most significant percentage of students in both countries chose idea 2. The difference is that an extremely large percentage of Australian students chose idea 2, whereas a certain percentage of Japanese students chose ideas 1 and 3. These results were similar throughout the school year in both Australia and Japan, suggesting that the results are not a consequence of the mathematics content of a particular grade level but instead of what students are learning in the mathematics curriculum in each country. In fact, a Chi-square test was conducted about whether students who answered item 11 selected idea 2 or not. The results revealed a significant disparity between the Australian and Japanese students ($\chi 2(1) = 11.61192548$, $p = 0.000655303$) (Table 15.4). The reason for these results may be the cultural influence of the differences in what is covered in mathematics education in each country.

Turning to the mathematics content, there are differences between Japan and Australia. In Japan, ideas 1, 2, and 3 are found in textbooks and are familiar to students. All of these ideas have been taught in math classes on weekdays and are studied carefully in class; therefore, it is assumed that they are recognised and selected as important ideas by students. Meanwhile, mathematics in the Australian curriculum includes methods that focus on the way the variable Y increases, such as idea 2 in the pattern and algebra sub-strands, and these methods were studied in weekday classes in Australia. Therefore, idea 2 is recognized as important and was selected more often. In both countries, the content of weekday mathematics classes might have influenced the types of ideas that the students selected as most important. However,

Table 15.4 Breakdown of the students who selected idea 2 or not in Australia and Japan

	Australia	Japan	Total
Selecting idea 2	86 (70.59322)	159 (174.4068)	245
Not selecting idea 2	33 (48.40678)	135 (119.5932)	168
Total	119	294	413

it is possible that something other than mathematical content influenced these results, such as teaching methods, the position of mathematics as a subject, and mathematics educational values. The relationship between the idea type and these other factors remains a topic for future research.

There were also differences between Japanese and Australian students in their reasons for choosing idea 2. The most important criterion for selecting an idea was the students' values, which functioned in the problem-solving process. Australian students' choice of idea 2 was strongly influenced by the values of simplification and clarification, while Japanese students' choices were strongly influenced by the values of technique. The reason for the differences in the values expressed as described above may be the cultural influence of differences in the content of mathematics education in each country.

In the Australian curriculum, students are expected to construct a table of values to record two possible number patterns (Australian Curriculum Assessment and Reporting Authority, 2017). In weekday classes in the Australian curriculum, Australian students focus on the variable Y and look for patterns to see what kind of changes occur, or work on tasks to find the value of Y for a specific value of X. Therefore, for example, in terms of finding a pattern, simplification is better than complexity to find a specific value of Y, and the value of clarification is considered more important. By contrast, the Japanese curriculum stipulates that expressing a figure in a formula, considering the relationship expressed in the formula, and investigating the characteristics of change or correspondence between two things should be taught in mathematics education (Ministry of Education, Culture, Sports, Science and Technology, 2017). For this reason, it is considered that Japanese students strongly value and engage in the technique of focusing on the relationship between variables X and Y and expressing the relationship in equations.

15.6.2 A Consideration Related to Selected Idea 3

While 61 students in Japan (19.9% of the students in Japan) and 18 students in Australia (14.5% of the students in Australia) chose idea 3 for various reasons, 9 students in Japan and 4 students in Australia chose idea 3 specifically because of its applicability to other situations. The reason given by these students was one characteristic: they were not only solving the fourth step but also recalled the case of a more significant number of steps and integrated them into the same situation. These students did not think that solving the 4th step alone was sufficient; instead, they recalled the case of an increased number of steps and integrated them into the same situation. In other words, these students appreciate the value of finding the value of Y in any case by considering that the same relationship can be used for various variables X. This is regarded as an expression of the mathematical value of the control.

Meanwhile, because idea 3 expresses the mechanism of the relationship between the number of steps and the number of marbles, we can consider applying this idea not only to other problems involving the number of steps but also to problems involving other shapes by keeping these mechanisms in mind. In other words, students can perceive unknown problems as known problems if they have the same mechanism of relations. In this sense, this idea could be viewed as an expression of progress values in mathematics. However, there was no reference to other shapes in the reasons why the students chose idea 3. In other words, no student in this study judged idea 3 to be important from the progress viewpoint. This indicates that what is necessary for students to be able to perceive progress values is to understand that even the premise of a problem can be considered a changeable factor and that the problems can be viewed in an integrated manner. In this example, it is necessary not only to consider the number of steps in the problem as a variable but also to consider the given figure, which is the premise of the problem, as a variable and to view the problems of other figures in an integrated manner so that they could be included as part of the same set.

Students bring their own values to mathematics. If these values correspond to the progress values proposed by Bishop (1988), they can relate unknown problems to known ones and systematically construct knowledge. If the values created by the students correspond to the unification proposed by Nakajima (1971), it is possible to learn mathematics creatively based on the knowledge already built by the students. Mathematics is not mysterious knowledge discovered by great mathematicians that must be received but is a product created by people in a culture. Students are expected to appreciate the values of Bishop (1988) and Nakajima (1971) and be able to embody them in mathematics.

15.7 Suggestions for Practice

Idea 2 was selected most often by the students; idea 3 could lead to progress values. Based on the above discussion of student selection of these ideas, some suggestions for practice are offered.

Based on the discussion of students choosing idea 2, the instruction content may influence the choice of ideas they consider most important. The content taught on weekdays in mathematics education is perceived by students as worthwhile learning because it is carefully taught over time. What is taught and focused on in terms of weekday mathematics education content helps convey to students what mathematical ideas are valuable and what is valued in mathematics. Taking time to teach mathematics content significantly impacts the development of students' values. To make students recognise the importance of ideas, such as idea 3, and to make them value thinking creatively and making progress, it is effective to spend time in mathematics classes and organise lessons that focus on the idea and emphasise the value of creativity. For students, ideas mentioned over time in class are more effective in fostering a sense of value about the content than those seen or introduced only once in a book. The results of this study reaffirm the importance of teacher practice.

Based on the discussion of the students who chose idea 3, solving problems is an objective in problem-solving, but it is difficult to achieve progress through creative thinking if students only solve one required problem, which is the goal of the problem-solving process. Students are expected to understand not only phenomena in terms of variables but also the mechanism of the relationship between variables X and Y, and then to integrate phenomena with the same mechanism, thereby making progress in understanding the phenomena of the problems. In this study, both the Japanese and Australian students tended to regard as valuable the fact that they could derive answers to the questions they were asked and the ability to derive answers to questions at other steps of the process. In both Japan and Australia, students need to appreciate the value of developing an integrated view through problem-solving processes to engage in activities associated with progress values.

The following suggestions are offered on the teacher instruction required for students to obtain a sense of progress values. First, teachers should discuss various solution methods for content in weekday mathematics classes. In particular, idea 3, which focuses on the relationship between variables X and Y, and clearly explains the mechanism of these relationships, should be discussed in class. This is because the mechanism clarifies the structure of the arrangement, provides a framework for viewing the arranged marbles, and can be used for any arrangement with the same structure. Second, in problem-solving, students should not stop finding the answer to a question but rather spend time considering the solution's value after obtaining it. Teachers should discuss various ideas with students and share the values of ideas that integrate the required problem with other problems of figures that have not yet been faced and convince students that these ideas can be used for other figure arrangements, such as idea 3. These practices are expected to help students acquire an essential view of how the marbles are arranged and how the number of marbles increases, and of feeling that problems requiring the same structural arrangement are no longer unknown problems. This practice is related to Bishop's (1988) progress values.

15.8 Limitations and Further Research

This study focused on problem-solving situations and captured students' mathematical values through their responses to the survey questions. I set up a problem-solving situation that can be viewed from a variable perspective, with three options of different ideas. By doing so, I was able to capture what ideas students valued and for what reasons, and I was able to make practical suggestions. However, there are limitations to this study, and further research is needed.

First, the values expressed by the students are limited to problem-solving situations dealing with variables such as those in this study. This does not necessarily mean students express values in problem-solving situations or choices. Students' values are stable, which means they would reflect the same values if given the same problem and choices. The students' values captured in this study were relatively superficial, depending on the questions and choices. To deepen understanding of students' values, it is necessary to collect and analyse multiple datasets, for example, by giving students a variety of questions and open-ended answer choices.

Next, students' values were identified as far as the survey could capture them. This was based on the assumption that the students understood the survey's intent, thought carefully about the survey questions and options, and answered the questions by confronting themselves. Although the survey and designed questions were explained to students to help this assumption hold true, the results of this study are limited. To capture students' values more accurately and in more detail, researchers should be in the classroom, understand the classroom culture, get to know the students well, and combine other factors that may need to be investigated in the future.

Acknowledgements This study was supported by JSPS KAKENHI Grant Number JP19K14222. This work was supported by JSPS KAKENHI Grant Number JP19K14222. Any opinions expressed herein are those of the author and do not necessarily represent the views of JSPS KAKENHI.

Appendix 1 The English Version of the Questionnaire

(1) Circle your condition and let me know your view about idea1.

1. Do you understand the idea? Yes · No

2. Do you think this is a mathematically good idea? Yes · No

(Please give reasons…)

(2) Circle your condition and let me know your view about idea2.

1. Do you understand the idea? Yes · No

2. Do you think this is a mathematically good idea? Yes · No

(Please give reasons…)

(3) Circle your condition and let me know your view about idea3.

1. Do you understand the idea? Yes · No

2. Do you think this is a mathematically good idea? Yes · No

(Please give reasons…)

(4) Which idea do you think is the most mathematically important?

Idea 1 · Idea 2 · Idea 3

(Please give reasons in detail…)

Thank you for your cooperation.

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