

Chapter 1

An Overview of Values in Mathematics Education



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1.1 Introduction

To Madame M. V. Kiselyov

Moscow, October 27, 1888

You are right in demanding that an artist should take an intelligent attitude to his work, but you confuse two things: solving a problem and stating a problem correctly. It is only the second that is obligatory for the artist (Chekhov, 1888).

According to Abbott (2014), in the above prose, Chekhov's aim is to emphasise that the purpose of a creative activity of an artist is to intelligently give the 'recipient' (e.g., the reader) something to think about. On par with Chekhov, the critical and creative aim of this book is to help raise important questions about values and valuing in mathematics education, which are to help expand the possibilities of what teachers might do in their classroom practice. While this book does not provide prescriptive solutions for any particular context, specific classroom or social, cultural, ethnic setting, it articulates some of the multiple problems that we face when we take seriously the issues of values in mathematics education and their implications for practice. Sometimes, as educators, we can become rather arrogant in the way we put forward ideas on how to teach best, often assuming that our colleagues in other classrooms need to follow carefully the recipes we devised. But teaching students

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is far more complex than following a simple recipe. This is why in this book we (the editors and the authors) try to look beyond offering simple recipes for success, and instead grapple with a wider question of what should be at heart of school mathematics teaching and learning.

The field of values in mathematics education takes the view that if students were able to appreciate the relevance of mathematics and how it works, many of them would end up valuing mathematics much more and, as a consequence, would develop a feeling of empowerment through their mathematics education. That would be wonderful for all concerned (the teachers, the students, the wider society). However, as we know, many students never really develop such an understanding of and appreciation for mathematics. Could this mean that the student's own mathematical education lets them down? Although many students across the world study mathematics as a compulsory subject for many years, most leave school with the view that their mathematics adds little to their daily lives (with the exception of some practical notions such as dealing with money or attending to measurements). We contend that, if students were given an opportunity to engage with mathematics through the lens of values, they would be in a better position to grasp the interconnected ideas that lay at the centre of mathematics and its connection to real world (which we see as going beyond mere application of mathematical algorithms), and appreciate powerful ways in which mathematics can enrich people's lives in a myriad of ways.

At International Congress on Mathematical Education [ICME] 5, in 1984, in Adelaide, Australia, Ubiratan D'Ambrosio gave what has become regarded as the key plenary talk of all times. In this talk, he introduced the world-wide mathematics education community to ethnomathematics, in which he included the notion that values should become a key part of what mathematics teaching was about. Reflecting on his 1984 address, in 2004 ICME Congress, D'Ambrosio suggested further that, in part, mathematics education was "to promote citizenship, transmitting values and understanding rights and responsibilities in society" (D'Ambrosio, p. ix, 2006). On both occasions, he was emphasising the important role of societal values in the teaching and learning of mathematics. Several years later, others began to develop the implication of values further, making a distinction between societal and mathematical values, putting forward an argument that both were critical in the teaching and learning of mathematics (Bishop, 1988; Ernest, 1991).

Alan Bishop, one of the pioneers in the values field, was first to flesh out how both mathematical values and societal values play a central role in mathematics education. His attention to values was drawn during the classes run by Benjamin Bloom, during Bishop's master's study at Harvard. At that time, Bloom and colleagues, who had previously completed their book on cognition (Bloom et al., 1956), were about to publish their subsequent volume on affect, which featured values as a pivotal component (Krathwohl et al., 1964). In Bishop's earliest writing, values were seen as a core aspect of the choices mathematics teachers made at critical moments in the ongoing classroom dynamic, when they had to choose as to which teaching move to make next (Bishop & Whitfield, 1972). In the main discussions of these works, Bishop dealt with what has become known as societal values and mathematical pedagogical values. This was swiftly followed by an inclusion of values embedded

within mathematics itself (Clarkson, 2008, 2019). Of course, much has happened in research dealing with mathematics values since those early days, and research on values in mathematics education has become much richer and more profound today. Today, values are investigated by researchers worldwide in a wide range of areas, from determining the values prominent (or not) in mathematics classrooms, through to the teachers' and students' values alignment in ways of promoting mathematical wellbeing.

Values and valuing play a key role in many aspects of education, such as assessment, planning, classroom interactions (James & Pedder, 2006), choosing tasks (Levenson, 2022), and general wellbeing (Kim et al., 2020). This means that what one values and finds important in the learning and teaching of mathematics operates within the intersection of all social, cognitive, and affective aspects of school pedagogy (see Allchin, 1999; Seah, 2019a), making values a significant holistic factor in education. In the context of mathematics education, the extent to which students choose to engage (or not) with mathematics frequently depends on their inclination to embrace the curricular trends and expectations (Bishop et al., 2006). Seah (2019a) argues that such student engagement can be achieved by teachers implementing the values and valuing approach in their pedagogy; or, put differently, that an emphasis on explicitly teaching values in the mathematics classroom could help enhance students' meaningful engagement with the subject, consequently extending their learning of mathematics beyond the simple acquisition of competencies.

The growing appreciation for values in the context of education has led to an increased research interest in values and valuing in mathematics teaching and learning. This has resulted in the publishing of an extensive number of professional and research articles, including the theme of a special issue of the 'ZDM-Mathematics Education journal in 2012. These ideas have also been explored at conferences such as the setting up of Discussion Groups in the 2012 and 2016 ICME, and at the 2015 International Group for the Psychology of Mathematics Education [PME] conference. An Invited Lecture at the 2016 ICME was also dedicated to this topic. While many of the earlier contributions in values research appeared to be predominantly exploratory, recently Seah (2019b) signalled that the field has begun to move towards the intervention and application phase. Consequently, a range of ideas have been explored to date, such as fostering mathematical well-being (Clarkson et al., 2010, 2023; Hill et al., 2021) or engagement (Kalogeropoulos, 2016), teacher noticing (Aktaş et al., 2019), and the pursuit of values alignment (Kalogeropoulos & Clarkson, 2019; Kalogeropoulos et al., 2021).

This book, in many ways, can be seen as a follow up volume to an earlier edited book—*Values and Valuing in Mathematics Education: Scanning and Scoping the Territory* (Clarkson et al., 2019). While the first book brought together contributions from a range of authors that explored the territory and field of values in more general terms, this book continues that task, by shifting the emphasis onto implications for practice. This volume brings together authors' writings situated in many different countries. It draws together studies from preschool through primary and secondary schools, through to higher education level. The works presented here draw on a variety of theoretical perspectives; what they all have in common, however, is that

all of them have engaged with the idea of how, to a greater or lesser extent, their own work may influence aspects of practice. In some chapters, there is quite clear advice regarding what teachers could do in their interactions with students, with regards to mathematical values learning by their students; in other chapters, which are more concerned with students, historical developments of the area of research, or of theory, authors use a more speculative approach when discussing practice. Regardless of the structure, however, we hope that the work presented across these chapters will influence how we might interact with students, colleagues in schools and tertiary institutions, government officials who develop curricula documents, politicians, and the general public. Consequently, we hope that these values enriched discourses will lead to enabling our students to experience, understand, appreciate and use their mathematics in ways which will impact their lives and help them live with dignity in their own societies.

1.2 Formation of the Book

The aim of this book was to encourage elaborations on potential teaching strategies that could enhance students' and teachers' exploration of their values, when doing mathematics in a variety of contexts and at different educational levels. We hope that this volume may challenge our colleagues (those in schools and beyond) to think more deeply about the impact of their own mathematics values in their professional work, as well as about how articulating such values could deepen not only their own work but also their students' appreciation for these. We also hope that this volume may provide critical and creative ideas for both beginning and experienced mathematics teachers on how to engage with teaching values in their mathematics classrooms effectively. Finally, we hope that this volume may provide education stakeholders (such as researchers, curriculum developers, and policymakers) with a richer understanding of and perspective on the role of values in mathematics education, which might have an impact on richer, more meaningful and holistic ways of conceptualising classrooms and learning across different contexts.

1.3 Chapter Outlines

In our initial call for contributions for this volume we had anticipated responses from colleagues who work in a variety of different contexts across the globe, and who would be keen to engage with a wide range of issues pertaining to values and mathematics education. Among these, we also expected contributions which would deal with specific issues that had already been emphasised in literature as particularly important, such as, for example, mathematical wellbeing. What we, perhaps, did not anticipate at that stage, however, was how broad the wealth of experience and interests in the field actually was, and how challenging it would be for us to structure a coherent

classification of the contributions in the publication process of this volume. Having tackled this complex task, we have arrived at the following structure of this volume according to the following subheadings:

- Theoretical and Reflective Perspectives
- Values Alignment and Classroom Practices
- Utilising the Values Perspective in Promoting and Sustaining Student Mathematical Wellbeing
- Applying the Values Perspective to Teaching Problem Solving
- Values and Socio-cultural Contexts

Despite the breadth of the perspectives across the aforementioned sections, however, we can reassure the reader that, in keeping with the goal of shifting from theory to practice, all chapters of the book engage with such a shift, even if only in speculative terms. In what follows, incredibly grateful for the contributions from all authors, we are excited to present the reader with a rich and creative landscape of innovative research in values in mathematics education.

1.3.1 Theoretical and Reflective Perspectives

We start this volume with chapters which give a general perspective on value studies in mathematics education, from historical, philosophical and theoretical viewpoints.

In Chap. 2, Clarkson, sitting in his armchair and reflecting back on his own practice and on the field more generally, provides some historical accounts of the field development, while pointing to gaps and opportunities (both those that were missed and those that still arise) in the value-focused teaching and learning of mathematics. In that, he shows us that, despite rigidity and prescriptive nature of curricular designs, we can (almost) always find a wriggle room to privilege students' choosing of particular-value enactment; choosing which would consequently allow mathematical learning to flourish. In that, considering values and related pedagogies, Clarkson pays particular attention to the value of *mystery*, which he is concerned is often forgotten, or under played, in the school context of mathematics learning.

Similarly to Clarkson, who briefly alludes to an issue of conceptualisation of values in the field of mathematics education, taking a comprehensive view (in Chap. 3), Chia and Zhang draw our attention to this conceptualisation, across studies in the recent decade. Following the synthesis of 67 studies, they conclude that values definitions and conceptualisations fall broadly into cognitive, discursive and enactive perspectives—an aspect that points to the vast complexity of the concept, and one which has implications for our engagement with it in both practice and further research. In Chap. 4, Barwell and colleagues, extend the aspect of conceptualisation further, pointing towards values' orientations, which they divide into those that are individualist or collectivist. Although this conceptualisation, as they argue, might only contribute further to emphasising the complexity of the concept, it allows us to capture and think about aspects in mathematics education, which are crucial in way

of helping students understand and cope with social and ecological challenges of today's complex world. This might lead us to think about student active engagement with and meaning making about the mathematics that they learn and about world that they live in. And it is exactly the aspect of meaning making in the process of mathematics learning that Law touches upon in Chap. 5, emphasising that it is an aspect which is not only to be valued by teachers in mathematics classrooms, but the pursuit of which should necessarily be seen by teachers as their ethical responsibility towards students and their learning.

1.3.2 Values Alignment and Classroom Practices

Classroom practices that utilise approaches focusing on values alignment have been of interest since the field of values started to catch traction. The chapters in this section emphasize the importance of its exploration in five different cultural contexts: Chaps. 6 and 10 provide some results for applying values alignment in mathematics classrooms; Chaps. 7, 8, and 9 attend to an assessment of and provide suggestions for determining the current situation in values alignment in mathematics classrooms.

In this sense, Aktaş, in the Turkish context (Chap. 6), draws attention to the complex and interactive nature of inclusive education pedagogies and processes as well as general classroom practices; she proposes a practical professional development model for how mathematics teachers working in such inclusively educated mathematics classrooms can implement values alignment practices and pedagogies. In Chap. 10, Azura Abdullah examines the dominant values in Brunei mathematics Lesson Study classrooms, based on the framework proposed by Bishop (1988), emphasizes the importance of understanding these values, and reveals the possible factors underlying these values to improve mathematics education.

On the other hand, Chaps. 7, 8, and 9, all of which make use of a Values Alignment Study [VAS] questionnaire, provide valuable insights into what values alignment looks like and how it works in mathematics classrooms in different cultural contexts. These chapters reveal that students' expectations from mathematics teachers, in mathematics classrooms in different cultural contexts, are generally similar. Specifically, in Chap. 7, Pang and colleagues show that Korean students' values generally focus on problem, understanding, and review, as well as that students' values regarding learning mathematics overlap with perceived teacher values regarding teaching mathematics. Similarly, in Chap. 8, Kalogeropoulos and colleagues also reveal some similarities between student and teacher values, while at the same time concluding that Australian students expect their teachers to emphasize the process-oriented aspects of learning mathematics, as much as reasoning and collaboration. This expectation of Australian students coincides with the values of problem, understanding, and review that Korean students attach importance to when learning mathematics. In Chap. 9, Davis and colleagues compare the values of primary school students in two different cultural contexts (Australia and Ghana) and determine that students in both countries generally value mathematics positively. Another perhaps more striking emphasis in

this chapter is that, despite the stereotype that girls are less interested in mathematics than boys and do not want to pursue a job that involves mathematics, Ghanaian girls want to work in a position that requires more mathematics than Ghanaian boys.

1.3.3 Utilising the Values Perspective in Promoting and Sustaining Student Mathematical Wellbeing

The interaction between values and student wellbeing has been an issue of study for more than ten years. More specifically, one way of conceptualising student mathematical wellbeing has been through the lens of value alignment. The chapters in this section engage with these aspects, pointing to different findings in relation to student's valuing and mathematical wellbeing across three different cultural contexts. For example, in Chap. 11, Kim and colleagues illustrate Korean students' valuing of cognitive aspects of learning, which revolve around computational thinking (for younger students) and around perseverance and a growth mindset (for older students), while regarding positive emotions such as fun, excitement, interest, and happiness to be of less value. In contrast, in Chap. 13, Hill and colleagues illustrate how Australian Year 8 students value both the cognitive challenges and the positive emotions in mathematic learning, although fulfilling of these values was not achieved to the same degree (with students being least successful in fulfilling their ultimate values related to positive emotions). In addition to these, however, the Australian students in the study of Hill and colleagues, indicate high value in relationships, focusing slightly more on social interactions than the Korean students in the study of Kim and colleagues. These findings reflect both studies' concerns about the current state of affairs in students' sense of fulfilment and happiness in mathematics classrooms, recognising that the aspect of inability to fulfil specific values hinders students' mathematical wellbeing.

A similar aspect is emphasised by Zhong and colleagues (Chap. 12), who affirm the necessity for students to fulfil all seven values (discussed in Hill's framework) in order to achieve mathematical wellbeing. Having established that, the authors then reach beyond the exploration of value fulfilment and student mathematical wellbeing, and elaborate on practices which teachers can employ to help foster such wellbeing.

1.3.4 Applying the Values Perspective to Teaching Problem Solving

Although problem-solving has been a topic in mathematics teaching and learning for decades, it has only recently been linked explicitly to exploring values in mathematics. The chapters here offer several ideas that advance this connection. In this regard, in Chap. 14, Baba and his colleagues assess socially open-ended problems,

which they bring in from their previous studies (see Baba, 2007; Baba & Shimada, 2019), analysing normative and socio-critical mathematical modelling tasks against Bishop's (1988) six universal activities. They propose an important and useful link between the context of mathematical modelling, mathematical problem solving, and values in mathematics education. In Chap. 15, Yamazaki compares the mathematical values of students studying the same Japanese mathematics curriculum in Japan and Australia in the context of problem-solving. This comparison reveals that even though students in both countries study the same curriculum, their choices in the problem-solving process and the values underlying these choices differ depending on culture.

1.3.5 Values and Socio-cultural Contexts

Socio-cultural contexts influence education in general and mathematics teaching in particular. The chapters in this section further explore these contexts on mathematics teaching/learning through mathematics values.

The context for the Chap. 16 is pre-school children in Japan. Here, Nakawa sets out to explore social values that the children act on when engaged in mathematical activities. Interestingly, a number of these social values are precursors to mathematical values (e.g., fairness, rule-making), and pedagogical mathematical values, such as 'sympathy for others', which will hopefully help with working cooperatively in groups in school years. In Chap. 17, Novikasari and colleagues also examine social values, but this time in the context of how the different values are embedded among three ethnic groups of Indonesia. The university preservice teaching students were asked to develop mathematical modules of work that drew on their social values that were foundational to their own ethnic group.

Culturally speaking, it can be much easier for values to be shared when the teacher and students come from similar backgrounds, and when the classroom is located within their specific space. If this is not the case, educational context poses challenges to learners. This is an issue which Street touches upon in Chap. 19, where she examines the experience of two women of colour on their undergraduate mathematics studies in the United States of America, which was highly gendered and racialized. In her analysis of these experience, Street suggests five areas of values misalignment based on sociohistorical Western mathematical values (meaning of mathematics, mathematics as fast-paced, innateness of mathematical ability, mathematics is competitive, and mathematics as a gatekeeper), and concludes with discussion of possible change in practice that might lead to support more equitable mathematics environments for women of colour.

Facing a similar issue, in Chap. 18, Hunter studies a classroom located in New Zealand, but one run by a 'European New Zealand' teacher and with a student group which includes a mix of Pacific Islanders. In this study, she illustrates how the teacher succeeds in making the students feel positively in her classroom environment by using a culturally sustaining mathematics pedagogy, based on prompts and actions that

make implicit and explicit connections to mathematics educational values including family, respect, collaboration, reciprocity, inclusion, and belonging (all crucial values embedded in Pacific Island cultures).

Over the last 10 years, more than 20 countries have participated in the international *What I Find Important (in my mathematics learning) [WIFI]*¹ survey. Although several chapters in other sections of this book touch upon the WIFI study, the final two chapters in this section deal with the WIFI results explicitly, focusing particularly on ‘the magic pill’ item from Section C of the survey.

In Chap. 20, set in Turkey, Dede and colleagues examine the results from the WIFI survey of grade 9 students who reported that they ‘felt good when doing mathematics’. Here, as the authors report, the Turkish students privilege ability, effort, mathematics concepts, fun, teacher, and materials that enhance thinking. In Chap. 21, Österling and colleagues, study the idea of the ‘magic pill’ with Swedish students from Grade 5 and 9. Although these students seem to privilege similar aspects, such as ability, numbers, calculation processes and being fast, a number of students also include ingredients such as ‘unicorns’ in their magic pill. This makes the authors wonder whether there should be a change in teaching practice, one which reduces the emphasis on exclusionary practices, such as encouraging students to complete their mathematical problems ‘fast’, but includes “an ounce of joy and happiness” (in one of the participant’s words) in their teaching.

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¹ Information on this survey can be found at <https://thirdwavelab.education.unimelb.edu.au/>

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