



# Dynamic Transmission Expansion Planning Using Adaptive Robust Optimization Under Uncertainties

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**Abstract.** The transmission expansion planning (TEP) problem is one of the perilous issues, which allows electricity transmission planners to design a cost-effective and reliable strategic model for the implementation of optimal transmission reinforcements in existing power grid networks. In this paper, a novel TEP model is proposed considering long- and short-term uncertain factors. The three-stage adaptive robust optimization (ARO) method deals with long-term uncertainties while prudently representing short-term uncertain parameters via scenarios. The formulated strategic scheme is elucidated through a modified decomposition algorithm that applies primal cutting planes and focuses on the subproblem feasible solution. The efficacy of the presented model is demonstrated through realistic case studies based on a 6-bus test system.

**Keywords:** Transmission expansion planning · Robust optimization · Renewable energy · Uncertainty · Decomposition algorithm

## 1 Introduction

The modernization and electrification of traditional power grid networks have encouraged power system researchers and engineers to predominantly concentrate on proficient, cost-effective, and reliable electricity supply from power generation units to gigantic distribution stations and then to end-users [1]. With adverse climate variations, one of the tenacious challenges is to limit global greenhouse gas (GHG) emissions [2]. In recent years, renewable energy integration has occupied a prominent position in critical agendas of many industrial countries, aiming to reduce carbon dioxide (CO<sub>2</sub>) emissions and meet the requirement of intensifying energy demand [3, 4]. The large-scale deployment of renewable energy sources (RESs) and decommission of orthodox energy resources have sparked a flurry of discussion on well-planned power transmission networks under uncertainties for dexterous power system operations. Transmission expansion planning (TEP) provides optimal and strategic decisions for the expansion and/or construction of transmission lines while reducing investment and operational costs of the power system by

observing administrative, environmental, and technical requirements [5]. Furthermore, optimization problems related to transmission network expansion compute minimum net investment cost-based expansion plans that can adequately satisfy the forecasted electric load over a pre-defined planning time horizon. Herein, the forecasted load demand and stochastic power recourses have been deliberated as major sources of uncertainty that must be taken into attention in the planning problem. Therefore, the TEP under uncertainty has engrossed ample interest in the past couple of years from the power industry for the formulation of economic robust designs that can significantly tackle all foreseeable values of net injections [6].

In TEP problems, the expansion decisions need to be calculated under the impact of uncertain factors including both long- and short- term uncertain parameters. The long-term uncertainty is usually associated with year-to-year deviations such as variations in future load demand growth and power generation capacity, while the short-term uncertainty belongs to day-to-day variability such as stochastic power production from RESs, electrical demand fluctuations, and equipment failure [7]. In the literature, there are numerous TEP models that are carried out without considering uncertainties [8–10]. For instance, the TEP approach adopted in [8] is based on the DC networks, formulated through mixed-integer non-linear programming (MINLP). In the same way, the authors formulated TEP as a mixed-integer linear programming (MILP) model and solved it via branch-and-cut approaches [9, 10]. As these expansion planning models did not consider uncertain factors, therefore, the accuracy is near to low. Thus, the TEP investment decision-making model must be articulated within an uncertain environment in order to tackle inherited risk factors.

According to recent studies, it is observed that there are various planning formulation schemes that have been attempting to model uncertainties during the analysis of TEP optimization problems [11, 12]. Over the past few years, stochastic programming (SP) and robust optimization (RO) have been vastly adopted for the characterization of uncertainties [13]. The SP approach entails exact probabilistic information and highly depends on computationally complex uncertainty discretization through scenarios. In [14], an SP-based framework is presented to deal with generation and transmission expansion planning problems under the influence of load demand variations. A MILP formulation is proposed to tackle the TEP optimization problem in a pool-based competitive electricity market, whereas, SP is employed for the realization of long-term futuristic load demand fluctuations [15]. The authors in [16] solved MINLP based TEP problem by taking into account future load demand uncertainty via a large number of scenarios by using the SP approach to reduce net investment cost and enhance reliability. In general, the SP is reliant on massive scenarios generation that may lead to computational intractability for multi-dimensional complex real-time optimization problems.

On the contrary, the RO uncertainty modeling technique has a protuberant advantage over the SP approach; all the uncertain factors are represented in the form of robust sets instead of scenarios, therefore the size of the problem does not increase with the number of scenarios [13]. In modern research on TEP optimization problems under uncertainty, the RO approach has gained a lot of interest. Various researchers and engineers have been working on RO-based TEP methodologies. Jabr in [17] presented a RO-based TEP

approach, a polynomial uncertainty set is used to deal with load and renewable generation uncertainties. The formulated model is elucidated by a Benders decomposition (BD) algorithm for the minimization of the expansion planning cost of the power transmission systems. Similarly, a three-level robust framework is proposed in [18] by applying a polyhedral uncertainty set to regulate the fluctuation range of uncertain parameters linked with different regions of the power system, while a primal BD algorithm is used to solve the model. Mínguez and García-Bertrand [19] improved computation performance of the TEP problem via a robust polynomial uncertainty set by differentiating between long- and short- term uncertainties. This work formulates a three-level adaptive robust optimization (ARO) based approach to solve MILP problems, while a modified BD algorithm is employed to evaluate the designed approach. In another article [20], the TEP problem is addressed under long- and short- term uncertainty, whereas, the expansion problem is formulated via ARO and solved through the primal BD algorithm to enhance computational efficiency. The authors in [21] suggested ARO constituted the TEP approach by taking into account the impacts of load demand and generation capacity uncertainties. Liang et al. addressed issues of uncertainty set size and uncertainty budget amount by using a novel ARO method while providing protection against the risk associated with wind generation. Furthermore, the RO-based TEP approach is recommended in [22] to determine the optimized uncertainty budget by minimizing the uncertainty set size pertained to the risk of fluctuation in wind power generation.

After a comparative study, it is observed that references [17, 18] focused on the realization of uncertain factors by using complex polyhedral uncertainty set without considering uncertain parameter correlations. On the other hand, article [19] demonstrated that the presented methodology is computationally effective but they omitted the integrality budget uncertainty constraint. Moreover, [20] provides a dynamic robust TEP model with a limitation of the tradeoff between complexity and accuracy. The article [21] proposed an effective static robust model to find out TEP, which may lead to compromise on the accuracy of the expansion plan for the long-term planning horizon, while the work in [22] faced a tradeoff between investment cost and robustness. By keeping in view aforesaid limitations in the existing TEP, this paper introduces a novel data-driven dynamic approach to find out the optimal expansion plan for the transmission network. Our main contributions are stated below:

- (1) To design a three-staged dynamic TEP model based on ARO.
- (2) Explicitly characterize long-term uncertain factors that commonly involve multi-year financial commitments and short-term variations that include hourly operating decisions to maximize the expected profit.
- (3) Modify the decomposition algorithm based on the constraint-and-column generation method [19] to solve the presented multi-level optimization problem.

The rest of this work is structured as follows. In Sect. 2, the mathematical formulation for TEP is provided, whereas Sect. 3 describes the proposed solution algorithm. The results are discussed in Sect. 4 by analyzing a real-time case study on multiple test benches. In the end, the concluding remarks and future research direction are stated in Sect. 5.

## 2 TEP Problem Formulation

In this section, a detailed mathematical formulation for TEP optimization problem under uncertainty is discussed.

### 2.1 Framework of ARO-TEP

In the three-layered ARO-based TEP framework, the first stage minimizes the investment cost (decision variables associated with the expansion of an existing line and/or construction of a new one), the second layer considers all the realization of uncertain factors within the ambiguity set (decision variables linked with uncertain parameters), while system operators select decision variables in the third layer for the reduction in operational cost by taking into account outputs from first and second stages. Our focal goal is to attain a cost-effective expansion scheme while satisfying the worst-case uncertainties identified from the uncertainty set. The systematic approach for the recommended ARO-based TEP is illustrated in Fig. 1.

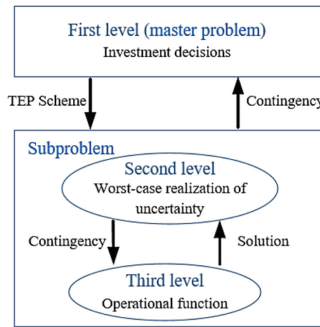


Fig. 1 Framework of ARO-based TEP

### 2.2 Problem Formulation

The ARO technique represents uncertainties in the form of uncertainty sets, computationally less complex than the SP models [13]. In this work, load demand and power generation resources have been considered as a source of uncertainty in the subjected TEP problem. Thus, the hierarchical structure of the three-layered ARO-based TEP optimization problem is provided below.

$$\min_{\Theta_1} \sum_{l \in \Omega^{L+}} I_l x_l + \max_{\Theta_2} \sum_{s \in \Omega^S} \rho^S [\mathfrak{R}(\Theta_1, \tilde{\xi})] \tag{1}$$

Subject to:

$$\sum_{l \in \Omega^{L+}} I_l x_l \leq \Pi^{L, \max} \tag{1a}$$

$$x_l \in \{0, 1\}; x_l \in \Omega^{L+} \quad (1b)$$

$$x_l = 1; x_l \in \frac{\Omega^L}{\Omega^{L+}} \quad (1c)$$

Subject to:

$$\bar{P}_{g_i}^T \in [\bar{P}_{g_i}^{T,\min}, \bar{P}_{g_i}^{T,\exp}]; \forall g_i \in \Omega^T \quad (1d)$$

$$\frac{\sum_{g_i \in \Omega^Z} (\bar{P}_{g_i}^{T,\exp} - \bar{P}_{g_i}^T)}{\sum_{g_i \in \Omega^Z} (\bar{P}_{g_i}^{T,\exp} - \bar{P}_{g_i}^{T,\min})} \leq \Gamma_z^T; \forall z \in \Omega^Z \quad (1e)$$

$$\bar{P}_{g_j}^R \in [\bar{P}_{g_j}^{R,\min}, \bar{P}_{g_j}^{R,\exp}]; \forall g_j \in \Omega^R \quad (1f)$$

$$\frac{\sum_{g_j \in \Omega^Z} (\bar{P}_{g_j}^{R,\exp} - \bar{P}_{g_j}^R)}{\sum_{g_j \in \Omega^Z} (\bar{P}_{g_j}^{R,\exp} - \bar{P}_{g_j}^{R,\min})} \leq \Gamma_z^R; \forall z \in \Omega^Z \quad (1g)$$

$$\bar{P}_d \in [\bar{P}_d^{\exp}, \bar{P}_d^{\max}]; \forall d \in \Omega^D \quad (1h)$$

$$\frac{\sum_{d \in \Omega^Z} (\bar{P}_d - \bar{P}_d^{\exp})}{\sum_{d \in \Omega^Z} (\bar{P}_d^{\max} - \bar{P}_d^{\exp})} \leq \Gamma_z^D; \forall z \in \Omega^Z \quad (1i)$$

where,  $l$  is the index of transmission lines in the set of candidate lines  $\Omega^{L+}$ ,  $I_l$  is the annualized investment cost of the transmission line and  $x_l$  is the binary variable (i.e. if  $x_l = 1$ , then line  $l$  is built). The first term of Eq. 1 is the annualized investment cost of building new transmission lines over investment decision variable set ( $\Theta_1 = x_l \in \Omega^{L+}$ ). Furthermore,  $s$  is the index of operating scenarios in the set  $\Omega^S$  and  $\rho^s$  is the weight of scenario  $s$ . The uncertainty set is denoted by  $\Theta_2$  includes uncertain parameters (i.e.  $\bar{P}_{g_i}^T; \forall g_i \in \Omega^T, \bar{P}_{g_j}^R; \forall g_j \in \Omega^R, \bar{P}_d; \forall d \in \Omega^D, \dots$ ) within ambiguity set  $\tilde{\xi}$ . In this work, the load demand and installed power generating capacity have been considered as long-term uncertainties that are represented by the polyhedral robust sets as in [20]. Constraint (1d-1e) characterizes the uncertainty set of conventional power generation capacity, Constraint (1f-1g) depicts the stochastic power production units, while Constraint (1h-1i) defines the future peak load demand. The long-term uncertain factors for generation capacities  $\bar{P}_{g_i}^T$  and  $\bar{P}_{g_j}^R$  that are belonged to the range from minimum level ( $\bar{P}_{g_i}^{T,\min}, \bar{P}_{g_j}^{R,\min}$ ) to the expected level ( $\bar{P}_{g_i}^{R,\exp}, \bar{P}_{g_j}^{R,\exp}$ ), whereas, the peak load demand  $\bar{P}_d$  at each load node fluctuates between the expected level ( $\bar{P}_d^{\exp}$ ) to its maximum level ( $\bar{P}_d^{\max}$ ). If there is no case for long-term uncertainty, then no robust protection (i.e. uncertainty budget is 0) is required. With the growth in the uncertainty budget, the uncertainty level increases and higher robust protection is needed. Moreover, the ambiguity set links to predefined geographical zones with index  $z$  in  $\Omega^Z$ . Herein, the value for uncertainty budgets  $\Gamma_z^T, \Gamma_z^R,$

and  $\Gamma_z^D$  must does not exceed the quantity of generating units and load demand within  $z$  zone. Furthermore,  $\mathfrak{H}(\Theta_1, \tilde{\xi})$  is the recourse function that anticipates operator reactions by ensuring the operating decisions feasibility over the set  $\Theta_1$  and all the realizations of uncertainties in  $\tilde{\xi}$ .

$$R = \min_{\Theta_3} \beta \left( \sum_{g_i \in \Omega^T} \Lambda_{g_i}^T P_{g_i}^{T,s} + \sum_{g_j \in \Omega^R} \Lambda_{g_j}^R P_{g_j}^{R,s} + \sum_{d \in \Omega^D} \Lambda_d^{\mathcal{L}} P_d^{\mathcal{L},s} \right) \quad (2)$$

Subject to:

$$\begin{aligned} & \sum_{g_i \in \Omega_n^T} P_{g_i}^{T,s} + \sum_{g_j \in \Omega_n^R} P_{g_j}^{R,s} - \sum_{l|s(l) \in \Omega_n} f_l^s + \sum_{l|r(l) \in \Omega_n} f_l^s \\ & = \sum_{d \in \Omega_n^D} \omega_d^{D,s} \bar{D}_d - \sum_{d \in \Omega_n^D} P_d^{\mathcal{L},s} : \alpha_n^s, \forall n \in \Omega^N, \forall s \in \Omega^S \end{aligned} \quad (2a)$$

$$f_l^s - x_l b_l \left( \theta_{s(l)}^s - \theta_{r(l)}^s \right) = 0 : \mu_l^{L,s}, \forall l \in \Omega^L, \forall s \in \Omega^S \quad (2b)$$

$$-f_l^{\max} \leq f_l^s \leq f_l^{\max} : \gamma_l^{L,\min,s}, \gamma_l^{L,\max,s}, \forall l \in \Omega^L, \forall s \in \Omega^S \quad (2c)$$

$$0 \leq P_{g_i}^{T,s} \leq e_{g_i}^{T,s} \bar{P}_{g_i}^T : \rho_{g_i}^{T,\min,s}, \rho_{g_i}^{T,\max,s}, \forall g_i \in \Omega^{G^T}, \forall s \in \Omega^S \quad (2d)$$

$$0 \leq P_{g_j}^{R,s} \leq e_{g_j}^{R,s} \bar{P}_{g_j}^R : \rho_{g_j}^{R,\min,s}, \rho_{g_j}^{R,\max,s}, \forall g_j \in \Omega^{G^R}, \forall s \in \Omega^S \quad (2e)$$

$$0 \leq P_d^{\mathcal{L},s} \leq e_d^{\mathcal{L},s} \bar{P}_d^D : \psi_d^{D,\min,s}, \psi_d^{D,\max,s}, \forall d \in \Omega^D, \forall s \in \Omega^S \quad (2f)$$

$$-\pi \leq \theta_n^s \leq \pi : \delta_n^{\min,s}, \delta_n^{\max,s}, \forall n \in \Omega^N, \forall s \in \Omega^S \quad (2g)$$

$$\theta_n^s = 0 : \varphi_n^s; n : ref, \forall s \in \Omega^S \quad (2h)$$

Here,  $\mathfrak{H}$  minimizes the operation cost comprises of power generating cost and load shedding cost, computed over all the short-term scenarios. The set of operating decision variables is  $\Theta_3$ , defined as  $\Theta_3 = P_{g_i}^{T,s}, \forall g_i \in \Omega^{G^T}, \forall s \in \Omega^S; P_{g_j}^{R,s}, \forall g_j \in \Omega^{G^R}, \forall s \in \Omega^S; P_d^{\mathcal{L},s}, \forall d \in \Omega^D, \forall s \in \Omega^S; f_l^s, \forall l \in \Omega^L, \forall s \in \Omega^S; \theta_n^s, \forall n \in \Omega^N, \forall s \in \Omega^S$ . In contrast to the long-term uncertainty throughout the targeted year, the short-term uncertainties pertain to the circadian demand and production variations in every node during a target planning year, represented via scenarios (such as  $s \in \Omega^S$ ). The dual variables linked with each constraint are displayed by a colon. The parameter  $\beta$  denotes the number of functional hours. Moreover, constraint (2a) imposes the power balancing equation at each node  $n$ , constraint (2b) calculates the power flow, and constraint (2c) forces the capacity limit of transmission line  $l$ . The minimum and maximum limits on demand and production capacities under diverse operational scenarios are denoted by constraints (2d, 2e, and 2f) via factors  $e_{g_i}^{T,s}$ ,  $e_{g_j}^{R,s}$ , and  $e_d^{\mathcal{L},s}$  respectively. Constraint (2g) bounds the voltage angle  $\theta_n^s$  at node  $n$  in scenario  $s$  and constraint (2h) states the reference node by fixing it at zero.

### 3 Proposed Solution

According to [11, 12], and [13], the ARO-TEP optimization problem under optimization can be mathematically expressed in a compact form as;

$$\min_x \left( I^T x + \max_{u \in U} \min_{y \in \Pi(x,u)} b^T y \right) \tag{3}$$

Subject to:

$$I^T x \leq \gamma \tag{3a}$$

$$x \in \{0, 1\} \tag{3b}$$

where  $x$  is the first stage vector with binary variables, shows installment states for the transmission line,  $I$  is the vector of investment cost,  $u$  is the continuous variable of second stage that defines uncertain parameters in the uncertainty set  $U$ , vector  $b$  includes operating costs, and  $y$  is the continuous vector referring to operational variables. Moreover,  $\gamma$  represents the investment transmission expansion budget and  $\Pi(x, u)$  expresses the feasibility region as a function of  $x$  and  $u$  for the operating variables  $y$ . In the following part, the proposed three-layered optimization problem is transformed into a two stage problem by merging the second and third stage problems. This can be achieved by using the dual of the third-level problem.

#### 3.1 Subproblem

After merging the second and third stage problems in Eq. (1) by employing the KKT conditions, the resultant single level maximization problem in compact form (see [20] for detail formulation) is stated below:

$$F^{dual} = \max_{u,y,\lambda,\mu,\alpha,\varphi} b^T y \tag{4}$$

Subject to:

$$Ax + By = E \tag{4a}$$

$$I_{eq}y = d \tag{4b}$$

$$0 = b + B^T \lambda - G^T \mu + I_{eq}^T \alpha + I_{ineq}^T \varphi \tag{4c}$$

$$0 \leq K - Fx - Gy \perp \mu \geq 0 \tag{4d}$$

$$0 \leq d - I_{ineq}y \perp \varphi \geq 0 \tag{4e}$$

$$u \in U \tag{4f}$$

where constraint (4c) is obtained from differentiation of the third layer problem Lagrangian with respect to its variables  $y$  and constraints (4d) and (4e) represent the complementary conditions linked with inequality constraints.

### 3.2 Master Problem

The master problem can be expressed as

$$\min_{\Theta_1} \sum_{l \in \Omega^{L+}} I_l x_l + \eta \quad (5)$$

Subject to:

$$\sum_{l \in \Omega^{L+}} I_l x_l \leq \Pi^{L, \max} \quad (5a)$$

$$x_l \in \{0, 1\}; x_l \in \Omega^{L+} \quad (5b)$$

$$x_l = 1; x_l \in \frac{\Omega^L}{\Omega^{L+}} \quad (5c)$$

$$\eta \geq \max_{\Theta_2} \beta \sum_{s \in \Omega^S} \rho^S \left( \sum_{g_i \in \Omega^T} \Lambda_{g_i}^T P_{g_i}^{T, s, (v)} + \sum_{g_j \in \Omega^R} \Lambda_{g_j}^R P_{g_j}^{R, s, (v)} + \sum_{d \in \Omega^D} \Lambda_d^{\mathcal{L}} P_d^{\mathcal{L}, s, (v)} \right); \forall s \in \Omega^S, \forall v \leq k \quad (5d)$$

$$\begin{aligned} & \sum_{g_i \in \Omega_n^T} P_{g_i}^{T, s, (v)} + \sum_{g_j \in \Omega_n^R} P_{g_j}^{R, s, (v)} - \sum_{(l|s(l) \in \Omega_n)} f_l^{s, (v)} + \sum_{(l|r(l) \in \Omega_n)} f_l^{s, (v)} \\ & = \sum_{d \in \Omega_n^D} \omega_d^{D, s} \bar{D}_d^{(v)} - \sum_{d \in \Omega_n^D} P_d^{\mathcal{L}, s, (v)}; \forall s \in \Omega^S, \forall v \leq k \end{aligned} \quad (5e)$$

$$-(1 - x_l)M \leq \frac{f_l^{s, (v)}}{b_l} - \left( \theta_{s(l)}^{s, (v)} - \theta_{r(l)}^{s, (v)} \right) \leq (1 - x_l)M; \forall l \in \Omega^L, \forall s \in \Omega^S, \forall v \leq k \quad (5f)$$

$$-x_l f_l^{\max} \leq f_l^{s, (v)} \leq x_l f_l^{\max}; \forall l \in \Omega^L, \forall v \leq k, \forall s \in \Omega^S \quad (5g)$$

$$0 \leq P_{g_i}^{T, s, (v)} \leq e_{g_i}^{T, s} \bar{P}_{g_i}^{T, (v)}; \forall g_i \in \Omega^{GT}, \forall s \in \Omega^S, \forall v \leq k \quad (5h)$$

$$0 \leq P_{g_j}^{R, s, (v)} \leq e_{g_j}^{R, s} \bar{P}_{g_j}^{R, (v)}; \forall g_j \in \Omega^{GR}, \forall v \leq k, \forall s \in \Omega^S \quad (5i)$$

$$0 \leq P_d^{\mathcal{L}, s, (v)} \leq e_d^{\mathcal{L}, s} \bar{P}_d^{D, (v)}; \forall d \in \Omega^D, \forall s \in \Omega^S, \forall v \leq k \quad (5j)$$

$$-\pi \leq \theta_n^{s, (v)} \leq \pi; \forall n \in \Omega^N, \forall v \leq k, \forall s \in \Omega^S \quad (5k)$$

$$\theta_n^{s, (v)} = 0; n: ref, \forall v \leq k, \forall s \in \Omega^S \quad (5l)$$

Here, M is a large enough positive constant.



### 3.3 Solution Algorithm

The proposed algorithm iteratively solved the presented three-layered ARO-TEP optimization problem. At each iteration, the master problem's optimal solution is applied to the subproblem and the optimal solution of the subproblem is used to solve master problem, continues until convergence. The steps of this iterative algorithm are stated below:

1. Set lower and upper bounds to  $Z^{lo} = -\infty$  and  $Z^{up} = +\infty$ , respectively.
2. Set the iteration counter to  $\nu = 1$ .
3. Solve the master problem (5) subject to its constraints (5a) - (5l). The resulting values of decision variables  $x^{(\nu)}$  and  $\eta^{(\nu)}$ .
4. Update the lower bound of the optimal objective function  $Z^{lo} = I^T x^{(\nu)} + \eta^{(\nu)}$ .
5. Solve the subproblem for the given value of the  $x^{(\nu)}$  obtained from (3) to get  $P_{g_i}^{T,s,(\nu)}$ ,  $P_{g_j}^{R,s,(\nu)}$  and  $P_d^{\mathcal{L},s,(\nu)}$
6. Update the upper limit by  $Z^{up} = I^T x^{(\nu)} + F^{dual,(\nu)}$ .
7. If  $Z^{up} - Z^{lo} \leq \epsilon$  then, the algorithm is terminated, else update the iteration counter and continue with step (3).

The data exchange between master and subproblem is depicted in Fig. 1. The investment decision variable  $x_i$  is transferred from master to subproblem and from subproblem to master problem, the values of  $\bar{P}_{g_i}^T; \forall g_i \in \Omega^T, \bar{P}_{g_j}^R; \forall g_j \in \Omega^R, \bar{P}_d; \forall d \in \Omega^D$  are send.

## 4 Numerical Case Studies

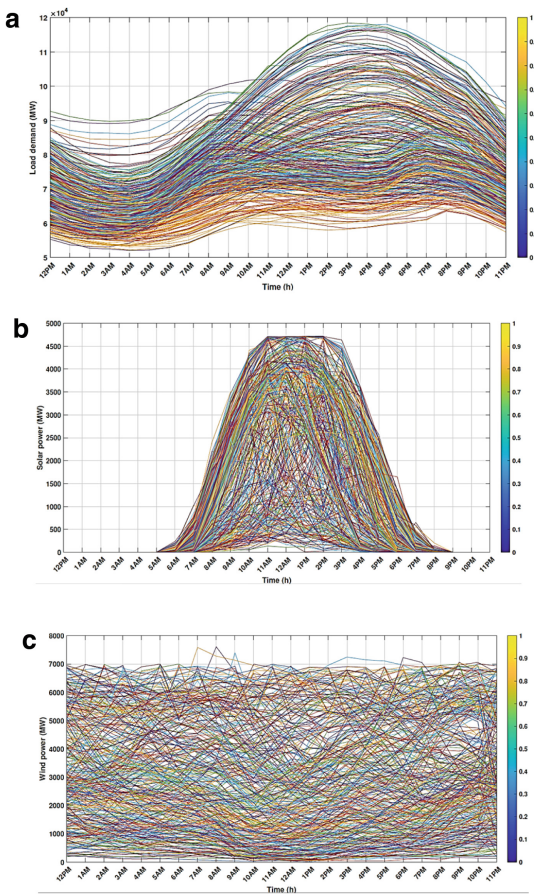
In this work, all the simulations are done in MATLAB (R2022a) and general algebraic modeling software (GAMS). The CPLEX solver is applied to elucidate the proposed ARO-TEP model on the machine with specifications: Intel (R) Core (TM) i5-CPU M 460@ 2.53 GHz 2.53 GHz with 8 GB installed RAM. Herein, the MILP problem is solved with the optimality gap of  $10^{-6}$ .

### 4.1 Data

The presented strategic planning methodology is tested to demonstrate its generality on the modified version of the IEEE 6-bus test system comprises of 6 buses, 8 branches, 5 dispatchable energy units, 4 non-dispatchable units, and 5 domestic load. In our model, a set of 6 candidate transmission lines is considered. For the long-term uncertain factor bounds, we have made following assumptions.

1. The maximum futuristic peak load demand is  $\bar{D}_d^{\max} = 1.5\bar{D}_d^{\exp}$  in the targeted planning year.
2. Moreover, the minimum future generating capacity of installed conventional power is 80% to its expected bounds such as  $\bar{P}_{g_i}^{T,\min} = 0.8 \bar{P}_{g_i}^{T,\exp}$ , whereas the minimum level of future stochastic generating capacity is 50% to its expected limit i.e.  $\bar{P}_{g_j}^{R,\min} = 0.5\bar{P}_{g_j}^{R,\exp}$  ..

In short-term uncertainties, the daily load demand and power production fluctuations at each node of the considered system are taken, represented through scenarios that epitomize operating conditions in every hour of the planning year. Note that by taking into account 8760 h of the target year may lead to intractability. Therefore, this paper adopts a clustering technique, known as the K-means algorithm [23]. In order to circumvent intractability condition, the number of representative hours are selected via K-mean clustering techniques for the sake of simplicity. The historical data related to the Illinois hub for the 2022 year is considered. More specifically, the hourly datasets of the load demand are taken from the MISO [24], while the statistics related to solar and wind energy units are obtained from the NREL and PJM Interconnection [25, 26], as shown in Fig. 2



**Fig. 2** a Hourly historical load demand datasets, b Hourly historical datasets for Solar units, c Hourly historical datasets for Wind turbine units

After obtaining the required historical datasets, the traditional K-means clustering algorithm is applied to curtail the considered year (2022) into representative days. The variation in the conventional power production plants (i.e., thermal unit) is much less than the stochastic units, therefore its scenarios are represented by random numbers within bound of  $[0.85, 1]$ . Each scenario that signifies a operation condition comprises of  $e_{gi}^{T,s}$ ,  $e_{gj}^{R,s}$ , and  $e_d^{L,s}$ . The generation system is divided into east and west zones with same demand in both regions. For the analysis of uncertainty budget impacts, range of budget for future generating is  $[0, 9]$  and for the future peak demand is  $[0, 90]$ .

## 4.2 Result

After applying K-mean algorithm, 8 clusters corresponding to 8 representative days and 192 operating conditions have been selected. The acquired data of 24-h-length operational conditions for each representative day is stated in Table. 1.

Additionally, the scenarios weight set is given as  $\rho^S = \{0.106, 0.124, 0.140, 0.143, 0.110, 0.138, 0.116, 0.123\}$ . Table 2 shows different three investment budgets.

From the Table 2, it is clear that with the increase in the investment budget,  $I_l$  is increasing with new lines built in order to relieve transmission congestion and net cost keeps decreasing (also reported in [20]).

## 5 Conclusion

By considering the theoretical form of the presented ARO-TEP model and stated case study, it can be deduced that the proposed model offers optimal robust transmission plans by providing protection against long- and short- term uncertainties. Through the K-means clustering technique, short-term uncertainties can be accurately represented and formulated model remains tractable. The three-layer ARO-TEP formulation is transferred into a dual-staged problem by using KKT condition and then effectively solved via the primal BD. Results illustrate that TEP decisions highly dependent on realization of uncertain factor and the availability of investment budget. Note that same model can be easily implemented with any case studies such as 12-bus test system, 33-bus test, and 118-bus test system. In the future research directions, we will test our model for other test benches and will enhance the computational efficiency.

**Table 1:** Resultant data for 24-h-length operational condition.

Variables	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$	$t_8$	$t_9$	$t_{10}$	$t_{11}$	$t_{12}$	$t_{13}$	$t_{14}$	$t_{15}$	$t_{16}$	$t_{17}$	$t_{18}$	$t_{19}$	$t_{20}$	$t_{21}$	$t_{22}$	$t_{23}$	$t_{24}$	
Demand factor (p.u)	$d_1$	0.40	0.34	0.31	0.29	0.28	0.30	0.31	0.38	0.40	0.42	0.45	0.51	0.53	0.55	0.57	0.58	0.58	0.58	0.58	0.55	0.50	0.50	0.48	0.47
	$d_2$	0.29	0.27	0.26	0.25	0.25	0.27	0.32	0.37	0.37	0.36	0.34	0.33	0.30	0.28	0.28	0.27	0.25	0.25	0.29	0.31	0.27	0.27	0.30	0.31
	$d_3$	0.40	0.38	0.37	0.35	0.36	0.36	0.40	0.45	0.45	0.45	0.44	0.41	0.38	0.35	0.34	0.32	0.32	0.31	0.35	0.39	0.36	0.36	0.38	0.42
	$d_4$	0.27	0.27	0.24	0.23	0.24	0.29	0.37	0.43	0.40	0.39	0.39	0.35	0.32	0.31	0.29	0.27	0.26	0.27	0.31	0.33	0.30	0.27	0.29	0.30
	$d_5$	0.48	0.44	0.46	0.38	0.38	0.40	0.43	0.50	0.52	0.56	0.56	0.54	0.50	0.50	0.51	0.51	0.53	0.53	0.53	0.52	0.50	0.49	0.52	0.52
	$d_6$	0.64	0.57	0.52	0.48	0.48	0.50	0.52	0.58	0.63	0.69	0.76	0.79	0.79	0.79	0.78	0.78	0.80	0.79	0.79	0.78	0.75	0.74	0.74	0.72
	$d_7$	0.14	0.14	0.14	0.14	0.16	0.20	0.27	0.32	0.28	0.26	0.24	0.21	0.20	0.20	0.18	0.18	0.17	0.16	0.16	0.16	0.15	0.14	0.16	0.15
	$d_8$	0.15	0.15	0.15	0.14	0.15	0.21	0.29	0.33	0.31	0.31	0.29	0.26	0.24	0.23	0.22	0.20	0.19	0.18	0.19	0.20	0.20	0.19	0.19	0.17
Solar unit capacity factor (p.u)	$d_1$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$d_2$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$d_3$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$d_4$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$d_5$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$d_6$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$d_7$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$d_8$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

(continued)

**Table 1:** (continued)

Variables	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$	$t_8$	$t_9$	$t_{10}$	$t_{11}$	$t_{12}$	$t_{13}$	$t_{14}$	$t_{15}$	$t_{16}$	$t_{17}$	$t_{18}$	$t_{19}$	$t_{20}$	$t_{21}$	$t_{22}$	$t_{23}$	$t_{24}$	
Wind unit	$d_1$	0.23	0.20	0.18	0.19	0.19	0.17	0.15	0.14	0.10	0.10	0.10	0.10	0.10	0.11	0.11	0.12	0.14	0.13	0.14	0.13	0.18	0.18	0.21	0.26
capacity	$d_2$	0.85	0.84	0.83	0.83	0.82	0.82	0.75	0.76	0.78	0.83	0.84	0.84	0.86	0.87	0.84	0.81	0.80	0.78	0.80	0.80	0.83	0.80	0.80	0.80
factor (p.u)	$d_3$	0.35	0.35	0.34	0.34	0.33	0.31	0.32	0.30	0.28	0.29	0.27	0.27	0.26	0.28	0.28	0.28	0.30	0.30	0.31	0.32	0.34	0.34	0.37	0.37
	$d_4$	0.41	0.38	0.39	0.40	0.38	0.39	0.37	0.31	0.30	0.28	0.26	0.26	0.29	0.32	0.31	0.30	0.30	0.31	0.29	0.30	0.34	0.38	0.40	0.40
	$d_5$	0.52	0.49	0.48	0.53	0.57	0.56	0.55	0.50	0.48	0.50	0.47	0.49	0.51	0.52	0.55	0.60	0.64	0.58	0.59	0.65	0.64	0.64	0.64	0.66
	$d_6$	0.15	0.12	0.12	0.13	0.12	0.15	0.15	0.12	0.12	0.09	0.05	0.05	0.05	0.06	0.07	0.10	0.15	0.15	0.13	0.14	0.16	0.16	0.17	0.17
	$d_7$	0.42	0.40	0.42	0.44	0.47	0.41	0.39	0.31	0.27	0.25	0.25	0.22	0.23	0.25	0.27	0.27	0.29	0.24	0.24	0.26	0.30	0.39	0.40	0.40
	$d_8$	0.72	0.69	0.72	0.75	0.77	0.77	0.72	0.70	0.70	0.75	0.75	0.75	0.76	0.74	0.76	0.72	0.74	0.75	0.70	0.73	0.75	0.78	0.76	0.73

**Table 2:** Impact of investment budget

$\Pi^{L,\max}(M)$	1	2	3	4
Lines built	5–6	3–6	2–4, 3–6	3–4, 3–6
$I_T(M\$)$	0.7	1.6	3	3.4
Total cost (M\$)	1590	1271	1089	1019

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