



Bearings-Only Passive Localization in Unmanned Aerial Vehicle Formation Based on Mathematical Model

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Abstract. In recent years, with the improvement of electronic technology, the maturity of satellite positioning system and the emergence of new UAVs, UAVs have entered a stage of rapid development. In order to ensure the concealment of UAVs and avoid external interference as much as possible, passive location and tracking system has gradually become a research hotspot when UAVs are flying in formation. The method of passive location has strong anti-interference ability and good concealment, which is a bridge between positioning technology and algorithm performance analysis. In this paper, aiming at the problem of how to maintain the formation of UAV cluster in the process of formation flight, the positioning coordinates of UAV are obtained by optimizing the particle swarm optimization algorithm, and the appropriate positioning model is constructed by using the bearing-only passive positioning algorithm. The position adjustment scheme of UAV is studied and extended to other formation scenarios.

Keywords: Bearings-only passive positioning · Mathematical model · Unmanned aerial vehicle

1 Introduction

With the improvement of electronic technology and the maturity of satellite positioning system, UAV has entered a stage of rapid development (see [1–4]). The method of passive location has strong anti-interference ability and good concealment, which is a bridge between location technology and algorithm performance analysis. In order to accurately locate the UAV and adjust the UAV to the ideal position, the passive location tracking system has gradually become a research hotspot. Reference [5] utilized geometric knowledge to locate unmanned aerial vehicles, and demonstrated the impact of positioning geometric parameters on estimation bias through analysis and numerical examples. References [6, 7] proposed an optimization algorithm for the distribution of positioning stations in the adaptive matching reconnaissance area, which effectively reduced the TDOA positioning error in the reconnaissance area and obtained more accurate positioning results. A new system supporting search and rescue activities was reported, and its performance was tested in on-site experiments simulating real search scenarios in [8].References [9, 10] provided a particle swarm optimization algorithm,

which accelerated the convergence speed of the algorithm, but the flexibility of the algorithm is low. This paper intends to use the method of bearing-only passive positioning to adjust the position of the UAV, that is, several UAVs in the formation transmit signals and the other UAVs passively receive signals, from which the direction information is extracted for positioning, so as to adjust the position of the UAV. Each UAV in the formation has a fixed number, and the relative position relationship with other UAVs in the formation remains unchanged.

2 Effective Localization of Unmanned Aerial Vehicles Under Symmetric Models

In the initial case of a slight deviation in the position of the drone, it is assumed that a drone is located at the center of the circle and up to three drones are around to transmit the signal. As shown in Fig. 1, the positions of FY00 (0) and FY01 (01) are fixed, and other UAVs are adjusted for azimuth. Calculate the angle between each drone site of the regular nine-sided based on 0 and 01 relative to the 0 and 01 drones. Because it is a regular nine-sided, it can be directly adjusted by using the relationship between the angle under the regular nine-sided and the current angle. Adjust from near to far, bundle the symmetry point, that is, when the drone 0 and 01 are fixed, first determine the orientation of the drone 02 to determine whether the drone 02 has a deviation. If the other drones have no deviation, we have the signal angle $\beta_1 = \frac{9}{2\pi}$.

When the drone 02 has deviation and the FY00 drone is at the center of the circle and the 01 drone is fixed, based on the drone number information, we can obtain $\beta'_1 = (b - a) * \frac{9}{2\pi}$, the arc length without deviation $L = r * \beta_1$ and the deviated arc length $L' = r' * \beta'_1$. Let $Q = \beta'_1 - \beta_1$.

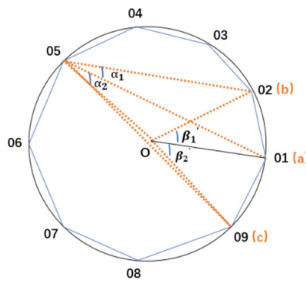


Fig. 1. Adjustment orientation of UAV

When $Q > 0(Q \neq 0)$, $D > 0$, it indicates that drone 02 has an upward angle deviation and a deviation from the center of the circle (drone FY00) radius deviation towards the outside of the track. At this time, it is necessary to send a signal to drone 02 to adjust it to the correct orientation;

When $Q > 0(Q \neq 0)$, $D < 0$., it indicates that drone 02 has an angle deviation upwards and a radius deviation towards the center of the circle (drone FY00) in the

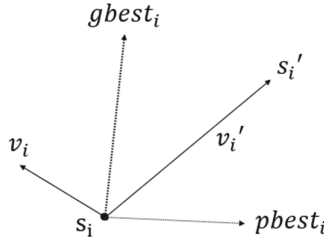


Fig. 2. The update position of particles

orbit. At this time, it is necessary to send a signal to drone 02 to adjust it to the correct orientation;

When $Q > 0 (Q \neq 0)$, $D = 0$, it indicates that drone 02 only has an upward angle deviation. At this time, it is necessary to send a signal to drone 02 to adjust it to the correct orientation;

When $Q < 0 (Q \neq 0)$, $D > 0$, it indicates that drone 02 has an angle deviation downwards and a radius deviation from the center of the circle (drone FY00) towards the outside of the track. At this time, it is necessary to send a signal to drone 02 to adjust it to the correct orientation;

When $Q < 0 (Q \neq 0)$, $D < 0$, it indicates that drone 02 has an angle deviation downwards and a radius deviation towards the center of the circle (drone FY00) in the orbit. At this time, it is necessary to send a signal to drone 02 to adjust it to the correct orientation;

When $Q < 0 (Q \neq 0)$, $D = 0$, it indicates that drone 02 only has downward angle deviation. At this time, it is necessary to send a signal to drone 02 to adjust it to the correct orientation.

At this time, the position of UAV 02 has been adjusted. Since UAV 09 should be in a symmetrical position with UAV 02, the position of UAV 09 should also be adjusted. In addition, since the positions of UAV 01 and UAV 02 have been determined and are symmetrical, the positions of UAV 0 and UAV 01 are fixed. Then, (the circumferential angle or central angle corresponding to the same arc is equal), that is, UAV 05 can also receive the signal of adjusting the position, and so on, the other UAVs with deviations will receive the signal of adjusting the position accordingly.

In summary, if the UAV position of the transmitted signal has no deviation, in addition to FY00 and FY01, at least one UAV is required to transmit the signal to achieve effective positioning of the UAV.

3 Particle Swarm Optimization Algorithm

The solution initialized by the particle swarm optimization algorithm is a random solution. The particles will update themselves according to the two values obtained by tracking to find the optimal solution. By optimizing the particle swarm optimization algorithm, the information data of each UAV is iteratively processed, that is, a massless particle is designed to simulate a UAV in the UAV cluster and extend to N-dimensional space. The UAV FY00 has only two parameters in the dimensional space: speed and

position, where the speed represents the speed of movement and the position represents the direction of movement. Let s_i and v_i ($i = 1, 2, \dots, N$) be the position system and the flight speed system, either.

The following is the updated speed and position formula:

$$v_i = v_i + k_1 * rand() \times (pbest_i - s_i) + k_2 * rand() \times (gbest_i - s_i) \quad (1)$$

$$s_i = s_i + v_i \quad (2)$$

The position of the particles is updated as shown in Fig. 2. The speed of a drone v_i is usually limited to the range of $[-v_{max}, v_{max}]$. $rand()$: Random numbers between 0 and 1. Let s_i be the current position of the particle. The learning factor (k_1, k_2) is usually 2, which can enable particles to possess self and social cognitive abilities. Since the dynamic can obtain better optimization results than the fixed value, the following formulas are obtained:

$$v_i = c * v_i + k_1 \cdot rand() \times (pbest_i - s_i) + k_2 \cdot rand() \times (gbest_i - s_i) \quad (3)$$

As an inertia factor, c takes a non negative value. When c is large, the global optimization ability is strong and the local optimization ability is weak; When c is small, the global optimization ability is weak and the local optimization ability is strong. Denote $pbest$ and $gbest$ the global and local optimal positions of the particle swarm, respectively.

When $k_1 = 0$, particles became models of only society.

$$v_i = c * v_i + k_2 \cdot rand() \times (gbest_i - s_i) \quad (4)$$

When $k_2 = 0$, particles became only a model of cognition.

$$v_i = c * v_i + k_1 \cdot rand() \times (pbest_i - s_i) \quad (5)$$

The dynamic changes linearly in the search process of the particle swarm optimization algorithm, and can be dynamically changed according to a measure function of its performance-linear decreasing weight strategy.

$$c^{(u)} = \frac{(c_{ini} - c_{end})(G_k - g)}{G_k} + c_{end} \quad (6)$$

In the Formula (6), G_k represents the maximum number of iterations, c_{ini} represents the initial inertia weight, and c_{end} represents the inertia weight when iterating to the maximum evolution algebra. Let $c_{ini} = 0.9$ and $c_{end} = 0.4$.

4 UAV Adjustment Scheme

By optimizing the particle swarm optimization algorithm, each UAV is iteratively processed, the UAV (particle) information is tracked to find, the distance between adjacent UAVs is fixed, and the design formation is adjusted. When the plane formation is known, the position of a part of the UAV is adjusted to complete the change of another formation (two-dimensional or three-dimensional).

Firstly, two vertices are determined. Let the vertices be FY00 and FY01, send signals to other UAVs, and other UAVs with slight deviation receive signals. Assuming that there are n UAVs, the distance between adjacent UAVs on the straight line is equal, and the formation diagram is shown in Fig. 3.

Using the idea of step-by-step following and greedy algorithm, the jump problem in the greedy algorithm is cited. Each element in the array is expressed as 1, and the update reaches the farthest distance to complete the coverage of all formations. Since the adjacent UAVs of the formation are equal in distance, three adjacent UAVs determine a regular triangle $\alpha_i = 60^\circ$.

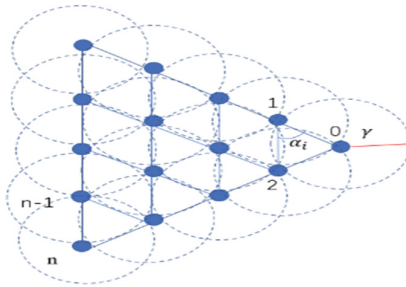


Fig. 3. UAV formation

Each UAV is positioned step by step, and the formation is completely covered when it is detected directly. There are two-dimensional formations into three-dimensional formation. Assuming that there are n UAVs, when n is odd, there is no UAV at the vertex, when n is even, there is a UAV at the vertex, and the serial number of the vertex UAV is FY00, which changes the serial number of the UAV in the counterclockwise direction.

In the case that the drone has been woven into a regular triangle (on the same plane), the interior is evenly distributed through an equilateral triangle (composed of 10 drones, which can be divided into three layers). The rising height is the first layer of the adjacent distance of the outermost UAV: composed of 6 UAVs with a spacing of 60, numbered FY0 (1–6). The second layer is composed of three UAVs with a spacing of 60, numbered FY0 (7–9). The third layer: the center position of the equilateral triangle composed of three UAVs in the second layer is numbered FY00. We can reach the shape of the pyramid by raising the same height layer by layer. The first step is to raise the drone number 7–9, taking No.1 and No.0 as the standard points, to test whether there is a deviation between 9 and the standard angle (connecting 0, 1 into a straight line, connecting 1 and 9 into a straight line, the angle between the two straight lines). The second step: Raise the drone number 0 at the same altitude as the first step. Connect drones 0 and 9, and connect drones 7 and 9. Compare whether the included angle is the standard 60° . If not, repeat the above steps to obtain an accurate position. The third step: The 10 drones will eventually form a pyramid shape.

Since the distance between adjacent UAVs is constant, the formation can form a regular triangular prism. The top view and elevation of the formation are shown in Figs. 4 and 5.

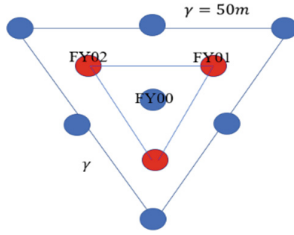


Fig.4. Top view of formation

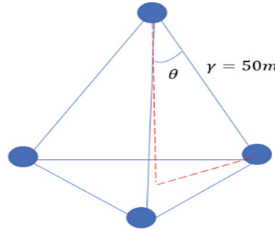


Fig. 5. Facade elevation diagram

The formation needs to determine the correct position of a surface of the UAV, by the correct position of the UAV to judge whether the next layer of aircraft adjacent layer UAV offset. Since the figure composed of the formation is a regular polygon, it is known that only the spatial angle between the position UAV and the upper layer is required.



Fig. 6. Chessboard formation

5 Example of the Chessboard Model

It is assumed that the initial formation of the UAV formation is similar to the checkerboard shape (as shown in Fig. 6 above, taking 24 UAVs as an example, it can be more). The checkerboard is composed of countless squares. Set the side length of the square to 1, and the drone located in the center of the chessboard will still be FY00. Other drones will start from a length of 1 units directly above FY00 and form in a clockwise direction, returning to their original positions according to the drone number.

The UAV formation is adjusted by changing the radius of the trajectory circle of the UAV formation. The specific method strategy is shown in Fig. 7.

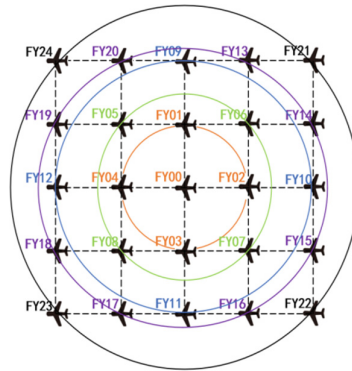


Fig. 7. UAV formation adjustment

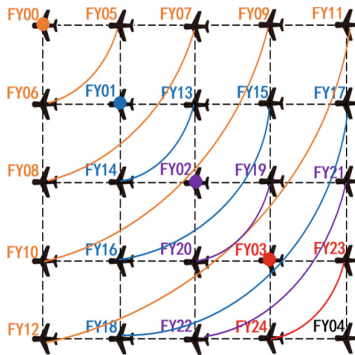


Fig. 8. Chessboard formation

Step 1: Draw a circle with l radius and the center of the circle is still the FY00 site. The intersection of the circle and the chessboard is the UAV site. Four intersections can be located to four UAVs. There is an intersection with the chessboard directly above FY00, which is recorded as FY01, and the other three intersections in the clockwise direction are recorded as FY02, FY03 and FY04;

Step 2: Set a circle with FY01 as the center and a radius of $\sqrt{2}l$. At this time, the circle has four intersection points with the chessboard. The starting position is still clockwise from the top of FY00 to determine other UAV sites. The numbers are FY05, FY06, FY07 and FY08;

Step 3: Set a circle with FY01 as the center and a radius of $2l$. At this time, the circle has four intersection points with the chessboard. The starting position is still clockwise from the top of FY00 to determine other UAV sites, numbered FY09, FY10, FY11, FY12;

Step 4: Set a circle with FY01 as the center and a radius of $\sqrt{5}l$. At this time, there are 8 intersections between the circle and the chessboard. The starting position is still clockwise from the top of FY00 to determine other UAV sites, numbered FY13, FY14, FY15, FY16, FY17, FY18, FY19, FY20;

Step 5: Then set a circle with FY01 as the center and radius of $2\sqrt{2}l$. At this time, the circle has four intersections with the chessboard. The starting position is still clockwise from the top of FY00 to determine other UAV sites, numbered FY21, FY22, FY23, FY24.

At this time, the chessboard-shaped formation mode is successfully adjusted into a layer-by-layer ring-shaped formation. Similarly, for the chessboard formation problem, if any point is selected as the location of FY00 (as shown in Fig. 8). The adjustment process is as follows:

Step 1: The chessboard is on a plane at this time, and the diagonal of the chessboard is numbered one by one from the upper left corner to the lower right corner, that is, FY00, FY01, FY02, FY03 and FY04. The FY01 is moved up a unit length in the direction of the reader, the FY02 is moved down a unit length in the opposite direction of the reader, and the FY03 is moved up a unit length in the direction of the reader;

Step 2: Firstly, a 1/4 arc is drawn for the radius with FY00 as the center of the circle. The intersection points of the arc and the chessboard are FY05 and FY06 from the top right to the bottom left. When the radius is doubled, the center of the circle is unchanged, and then the arc is drawn. At this time, the intersection points of the arc and the chessboard are FY07 and FY08; the radius is expanded again, the center of the circle is unchanged, and the arc is drawn again. At this time, the intersection points of the arc and the chessboard are FY09 and FY10; the radius is expanded again, the center of the circle is unchanged, and the arc is drawn again. At this time, the intersection points of the arc and the chessboard are FY11 and FY12;

Step 3: Taking FY01 as the center of the circle, quarter arc is drawn for the radius. The intersection points of the arc and the chessboard are FY13 and FY14 from the top right to the bottom left.

Step4: Taking FY02 as the center of the circle, quarter arc is drawn for the radius. The intersection of the arc and the chessboard is FY19 and FY20 from the upper right to the lower left.

Step 5: Finally, FY03 is used as the center of the circle, and quarter arc is drawn for the radius. The intersection of the arc and the chessboard is FY23 and FY24 from the upper right to the lower left.

Finally, the chessboard-shaped formation mode is successfully adjusted to a three-dimensional shape.

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