Weighted and Pure Dollar-Cost Averaging Strategies in Various Asset Classes

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Abstract. Dollar-cost averaging (DCA) is a popular investment strategy for retail investors, but its performance is generally inferior to lump-sum investing. In this study, we test modifications to improve the performance of DCA while keeping its simplicity, based on the Relative Strength Index (RSI) and CBOE Volatility Index (VIX). We define varying upper and lower bounds for RSI and VIX values, increasing investment to varying degrees in the asset when below the lower bound and vice versa when above the upper bound. We tested these strategies on 5 assets: Bitcoin, the S&P500, Gold, Crude oil, and the NASDAQ. We tested 300 variations of DCA strategies, based on price data from 2 January 2017 to 27 December 2021. To evaluate performance, we calculated 5-year Sharpe ratios for each tested strategy. The best performing RSI-weighted strategies outperformed pure DCA by statistically significant margins ($p < 0.05$) for 3 assets: Bitcoin (Sharpe = 1.984 vs 1.424, p = 0.0330), Crude oil (Sharpe = 1.570 vs 0.4174, p = 0.0223) and the NASDAQ (Sharpe = 1.923 vs 0.0491, p = 0.0034). This suggests that RSI-weighted strategies are viable regardless of asset class. Although all VIX-weighed strategies underpeformed pure DCA, an opposite version of our tested strategy could produce profits. Our results show that RSI-weighted DCA is a viable way to improve the profitability of DCA while keeping it simple enough for retail investors to execute.

Keywords: Dollar cost averaging · Bitcoin · S&P 500 · NASDAQ · Crude Oil · Gold

1 Introduction

Dollar-cost averaging (DCA) is a widely adopted investment strategy among retail investors. In DCA, an investor purchases an asset at regular time intervals in an effort to reduce the impact of volatility on the overall purchase [1–3]. The popularity of DCA can partially be attributed to its preventing retail investors

from attempting to time the market, and its decreased apparent risk compared to lump-sum (LS) investing [4]. The strict rule-based framework may also be beneficial to risk-averse retail investors who succumb easily to panic trading [5]. This is highly appealing as even professionally managed mutual funds rarely succeed in timing the market [6]. Another apparent advantage of DCA is that the returns obtained are less volatile compared to LS [7]. DCA may also reduce shortfall risk [8,9] and outperform LS during bear markets [10] and in assets with negatively autocorrelated returns [11]. However, traditional DCA has been theoretically and empirically shown to give lower returns than LS in most situations [12–14]. Thus, studies have tested several modifications to the traditional DCA strategy. Examples include only utilising DCA for a portfolio of assets with high dividend yield and gross profits to assets ratio [3], investing a larger amount over a shorter timespan in a strong economy and vice versa in a weak one [15], and investing more after a down month and less after an up month [16]. In this study, we propose and investigate the effectiveness of modified DCA strategies based on market momentum and volatility, using the Relative Strength Index (RSI) [17] to quantify market momentum and the CBOE Volatility Index (VIX) [18] to quantify market volatility.

1.1   Overview of Technical Terms

Relative Strength Index (RSI) [17]: A momentum indicator that measures the magnitude of recent price changes to evaluate if an asset is oversold or overbought. The RSI is displayed as an oscillator and can have a reading from 0 to 100. By convention, an RSI of 30 represents oversold conditions and a reading of 70 represents overbought conditions.

RSI calculation includes 3 components: RS, Average Gain and Average Loss. This RSI calculation is based on 14 time periods.

$$AG_1 = \frac{\sum_{n=1}^{14} G_n}{14} \tag{1}$$

$$AL_1 = \frac{\sum_{n=1}^{14} L_n}{14} \tag{2}$$

where AG_1 represents first average gain, AL_1 represents first average loss, L_n represents loss at time n and G_n represents gain at time n.

The subsequent calculations are based on the prior averages and the current gain and loss:

$$AG_n = \frac{(AG_{n-1} * 13) + G_n}{14} \tag{3}$$

$$AL_n = \frac{(AL_{n-1} * 13) + L_n}{14} \tag{4}$$

$$RS_n = \frac{AG_n}{AL_n} \tag{5}$$

$$RSI_n = 100 - \frac{100}{1 + RS_n} \tag{6}$$

2   Methodology

Price data from 5 assets or stock indices was used in this analysis: Bitcoin (BTC), Gold, Crude Oil, the S&P 500, and the NASDAQ. All price data was taken from 2 January 2017 to 27 December 2021.

Three DCA strategies were backtested with this data: pure DCA, RSI-weighted DCA, and sentiment-weighted DCA. Variations on weighted DCA strategies were also tested by adjusting various parameters. All DCA strategies simulated an investor buying $100 worth of the asset or index each week by default. Momentum-weighted DCA changes the amount bought based on the current RSI for the asset of interest, and sentiment-weighted DCA changes the amount bought based on the current VIX.

2.1   Pure DCA

Buying $100 USD of the asset or index every week, regardless of price, was simulated. This follows the exact definition of dollar-cost averaging.

2.2   RSI-Weighted DCA

The Relative Strength Index (RSI) was used to quantify market momentum. Lower RSI values represent negative market momentum and vice versa. We test 3 variations of a RSI-weighted strategy. We first define 3 multipliers R_{sq}, R_{sqrt} and R_{lin}, one for each strategy, to determine the amount of funds to put into the asset of interest at a particular point of time. The RSI is based on the weekly timeframe. For all calculations, RSI is rounded to the nearest integer. We define s to be the oversold RSI value and w to be the window size, the range between the oversold and overbought RSI values, and c to be the current RSI value.

$$R_{sq} = \begin{cases} 1 + \frac{(s-c)^2}{100} & c < s \\ 1 & s < c < s + w \\ 1 - \frac{(c-(s+w))^2}{100} & s + w \le c \le s + w + 10 \\ 0 & s + w + 10 < c \end{cases} \tag{7}$$

Note that the upper bound of the third condition is $s + w + 10$ to prevent R_{sq} from becoming negative.

$$R_{sqrt} = \begin{cases} 1 + \frac{\sqrt{s-c}}{100} & c < s \\ 1 & s < c < s + w \\ 1 - \frac{\sqrt{c-(s+w)}}{100} & s + w \le c \end{cases} \tag{8}$$

$$R_{lin} = \begin{cases} 1 + \frac{s-c}{100} & c < s \\ 1 & s < c < s + w \\ 1 - \frac{c-(s+w)}{100} & s + w \le c \end{cases} \tag{9}$$

We define U to be the amount of capital in USD the investor invests into the asset of interest each week.

$$U = 100 * R \tag{10}$$

We test all combinations of values s and w from the vectors $S = \{35, 40, 45, 50, 55\}$ and $W = \{20, 25, 30, 35, 40\}$.

2.3   VIX-Weighted DCA

The CBOE Volatility Index (VIX) was used to quantify market volatility. Lower VIX values represent lower volatility in the S&P500 and vice versa. Similar to Sect. 2.2, we test 3 variations of a VIX-weighted strategy. We define 3 multipliers V_{sq}, V_{sqrt} and V_{lin}, one for each strategy, to determine the amount of funds to put into the asset of interest at a particular point of time. For all calculations, the VIX value is rounded to the nearest integer. We define l to be the VIX value for low volatility and w to be the window size (the range between the values for low volatility and high volatility) and c to be the current VIX value.

$$V_{sq} = \begin{cases} 1 + \frac{(s-c)^2}{100} & c < l \\ 1 & l < c < l+w \\ 1 - \frac{(c-(l+w))^2}{100} & l+w \le c \le l+w+10 \\ 0 & c > l+w+10 \end{cases} \tag{11}$$

Note that the upper bound of the third condition is $l+w+10$ to prevent V_{sq} from becoming negative.

$$V_{sqrt} = \begin{cases} 1 + \frac{\sqrt{l-c}}{100} & c < l \\ 1 & l < c < l+w \\ 1 - \frac{\sqrt{c-(l+w)}}{100} & l+w \le c \end{cases} \tag{12}$$

$$V_{lin} = \begin{cases} 1 + \frac{l-c}{100} & c < l \\ 1 & l < c < l+w \\ 1 - \frac{c-(l+w)}{100} & l+w \le c \end{cases} \tag{13}$$

Similar to Sect. 2.2, we define U to be the amount of capital in USD the investor invests into the asset of interest each week.

$$U = 100 * R \tag{14}$$

We test all combinations of values g and w from the vectors $L = \{10, 15, 20, 25, 30\}$ and $W = \{5, 10, 15, 20, 25\}$.

2.4   Evaluating Returns

We compute quarterly returns (in percentage) using the following equation:

$$R_q = \left(\frac{A * P_f}{U_S} - 1 \right) * 100\% \tag{15}$$

where A is the amount of the asset bought for that quarter, P_f is the price of the asset at the end of the quarter, and U_s is the amount in USD that the investor spent for that quarter.

To evaluate the performance of each strategy, 5-year Sharpe ratios [19] are computed using quarterly returns over yearly periods. The equation for the 5-year Sharpe ratio from quarterly returns is:

$$S = \sqrt{20} * \frac{\overline{R_b - R_a}}{\sigma_a} \tag{16}$$

where $\overline{R_b - R_a}$ is the mean of the quarterly return in excess of the quarterly risk free rate. The risk-free rate is defined as the quarterly US treasury bill rate, and σ_a is the standard deviation of excess quarterly returns for a given strategy.

We plotted 6 bubble plots for each asset, since there are 2 weighting methods (RSI and VIX) and 3 variations for each (sq, sqrt, lin). We plot the oversold value (for RSI-weighted strategies) or extreme greed value (for VIX-weighted strategies) on the x-axis, the window size on the y-axis, and the bubble size corresponds to the Sharpe ratio of the strategy.

For each asset, we determined the best-performing strategy based on 5-year Sharpe ratio. We calculated 5 annualised Sharpe ratios for the pure DCA strategy and 5 annualised Sharpe ratios for this strategy, then compared their performance using a paired 2-sample, 1-tailed t-test.

3   Results and Discussion

3.1   Bitcoin

RSI-weighted DCA For RSI-weighted strategies using the R_{sq} multiplier, strategies with higher oversold values and smaller window sizes generally have higher Sharpe ratios. The best performing strategy used oversold = 55 and window size = 20, and had a 5-year Sharpe ratio of 1.984, compared to the pure DCA Sharpe ratio of 1.424. The difference was statistically significant (p = 0.033). RSI-weighted strategies using the R_{sqrt} and R_{lin} multipliers only outperformed the pure DCA strategy slightly, if at all.

VIX-weighted DCA For VIX-weighted strategies using the V_{sq} multiplier, strategies with lower low volatility values and higher window sizes generally have higher Sharpe ratios. However, all VIX-weighted strategies did not outperform the pure DCA strategy, and strategies using the V_{sqrt} and V_{lin} multipliers slightly underperformed pure DCA (Figs. 1 and 2).
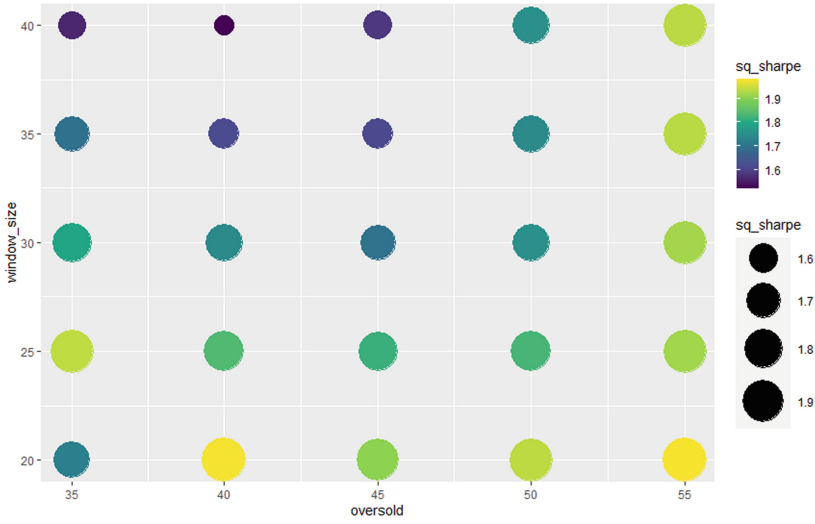
Fig. 1. Sharpe ratios for Bitcoin RSI-weighted strategies using the R_{sq} multiplier
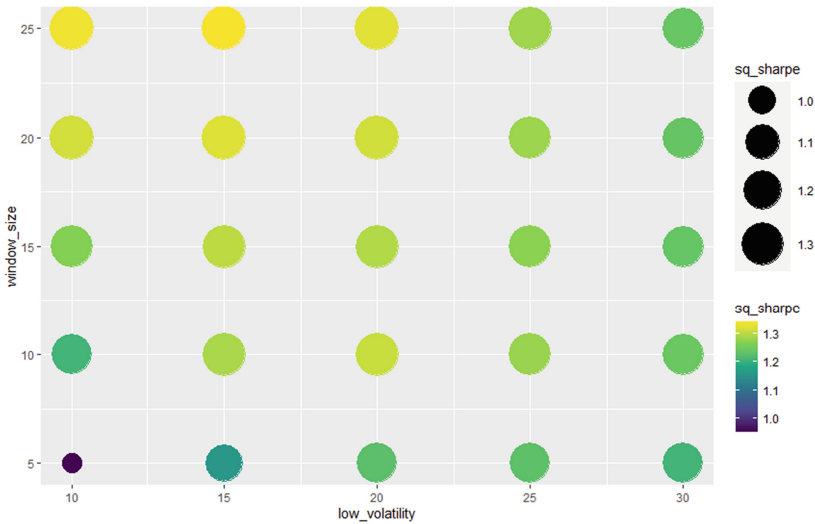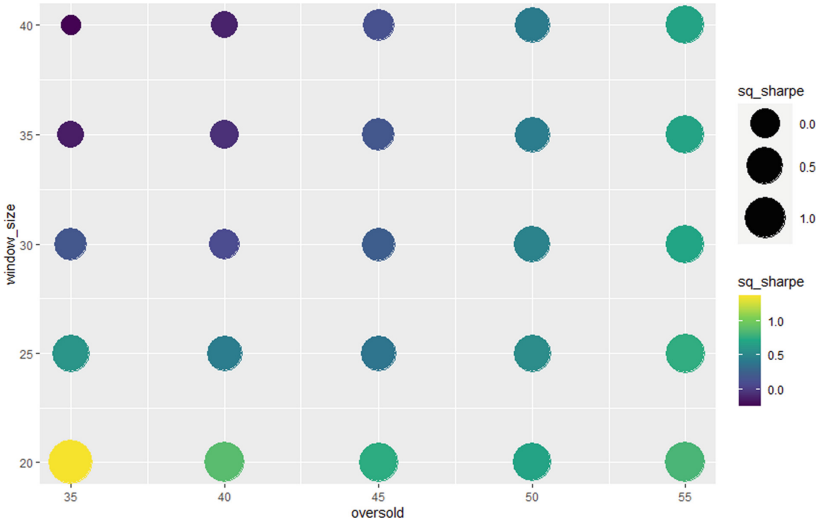

Fig. 2. Sharpe ratios for Bitcoin VIX-weighted strategies using the V_{sq} multiplier

3.2   S&P500

RSI-weighted DCA For RSI-weighted strategies using the R_{sq} multiplier, strategies with higher oversold values and smaller window sizes generally have higher Sharpe ratios. There was one outlier at oversold = 35 and window size = 20, which had the highest 5-year Sharpe ratio of 1.375, compared to the 5-year Sharpe ratio of $-$ 0.4521 for the pure DCA strategy. However, the difference is not statistically significant (p = 0.157). Strategies using the R_{sqrt} and R_{lin} multipliers outperformed pure DCA by very small margins (Figs. 3 and 4).

Fig. 3. Sharpe ratios for S&P500 RSI-weighted strategies using the R_{sq} multiplier

VIX-weighted DCA For VIX-weighted strategies using the V_{sq} multiplier, Sharpe ratios increase with increasing low volatility values for smaller window sizes, but Sharpe ratios decrease with increasing low volatility values for larger window sizes. All VIX-weighted strategies underperformed the pure DCA strategy, including those using V_{sqrt} and V_{lin} multipliers.

3.3   Gold

RSI-weighted DCA For RSI-weighted strategies using the R_{sq} multiplier, strategies with higher oversold values generally have higher Sharpe ratios regardless of window size. The best performing strategy used oversold = 55 and window size = 20, and had a 5-year Sharpe ratio of $-$ 0.1983, compared to the pure DCA Sharpe ratio of $-$ 0.7464. However, the difference is not statistically significant (p = 0.0991). RSI-weighted strategies using the R_{sqrt} and R_{lin} multipliers outperformed pure DCA by very small margins (Fig. 5).
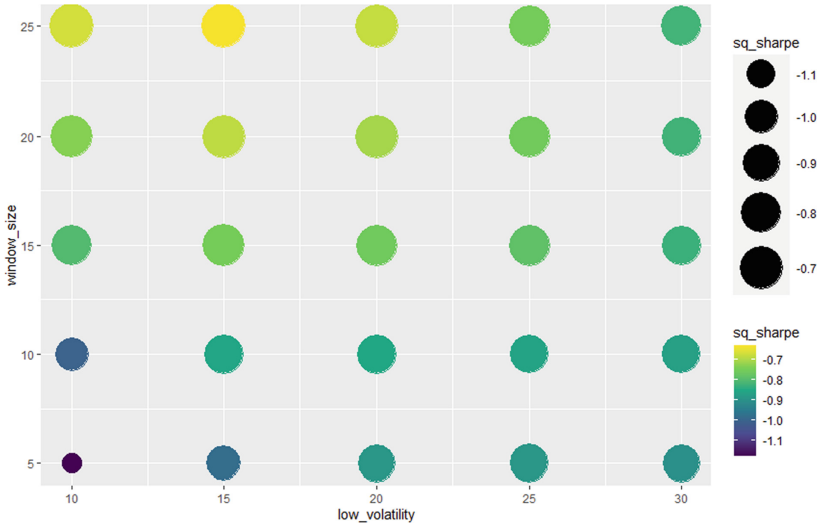
Fig. 4. Sharpe ratios for S&P500 VIX-weighted strategies using the V_{sq} multiplier
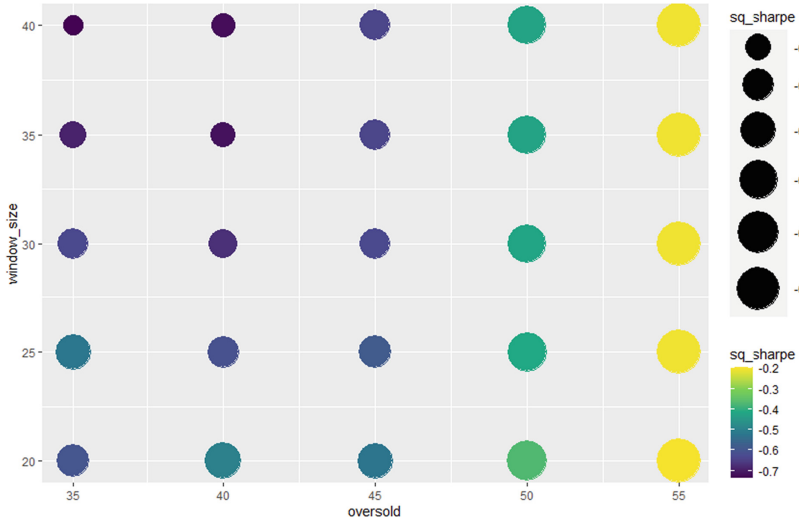

Fig. 5. Sharpe ratios for Gold RSI-weighted strategies using the R_{sq} multiplier

VIX-weighted DCA For VIX-weighted strategies using the V_{sq} multiplier, strategies with higher low volatility values tend to have higher Sharpe ratios. The best performing strategy used low volatility = 30 and window size = 25, and had a 5-year Sharpe ratio of $-$ 0.6323, compared to the pure DCA Sharpe ratio of $-$ 0.7464. However, the difference is not statistically significant (p = 0.4053). VIX-weighted strategies using the V_{sqrt} and V_{lin} multipliers generally underperformed pure DCA by very small margins (Fig. 6).
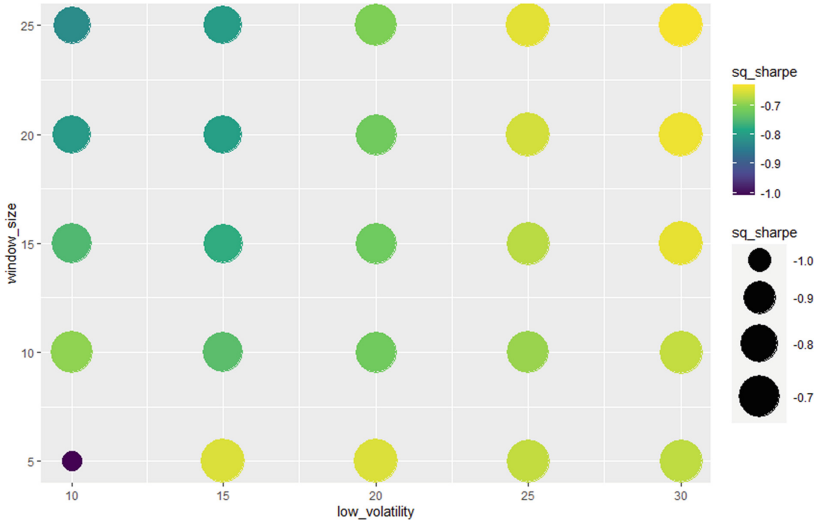

Fig. 6. Sharpe ratios for Gold VIX-weighted strategies using the V_{sq} multiplier

3.4 Crude Oil

RSI-weighted DCA For RSI-weighted strategies using the R_{sq} multiplier, strategies with higher oversold values and lower window sizes tend to have higher Sharpe ratios (Fig. 7).

There is an outlier at oversold = 35 and window size = 20, which was the best performing strategy and had a 5-year Sharpe ratio of 1.570, compared to the pure DCA Sharpe ratio of 0.4174. The difference is statistically significant (p = 0.0223). Strategies using the R_{sqrt} and R_{lin} multipliers generally outperformed pure DCA by small margins.

VIX-weighted DCA For VIX-weighted strategies using the V_{sq} multiplier, Sharpe ratios of strategies with lower low volatility values tend to increase as window sizes increases, but strategies with higher low volatility values tend to have similar Sharpe ratios regardless of window size. Regardless of the type of multiplier used, none of the VIX-weighted DCA strategies outperformed pure DCA, which had a Sharpe ratio of 0.4174 (Fig. 8).
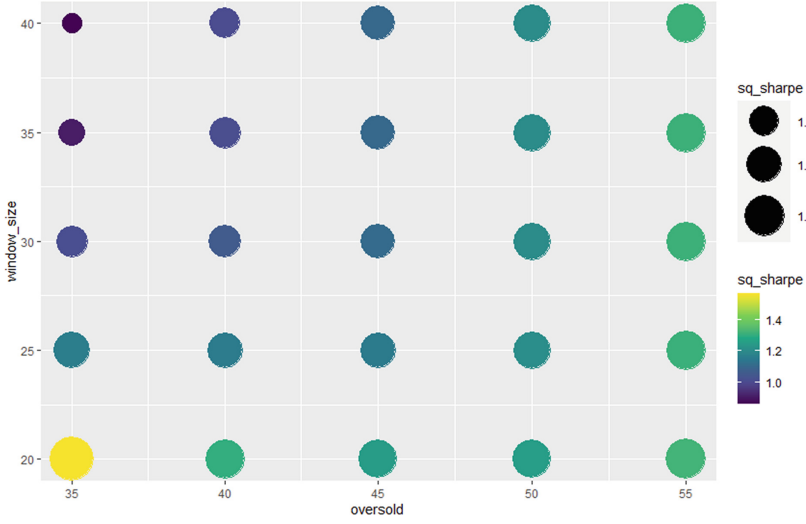
Fig. 7. Sharpe ratios for Crude Oil RSI-weighted strategies using the R_{sq} multiplier
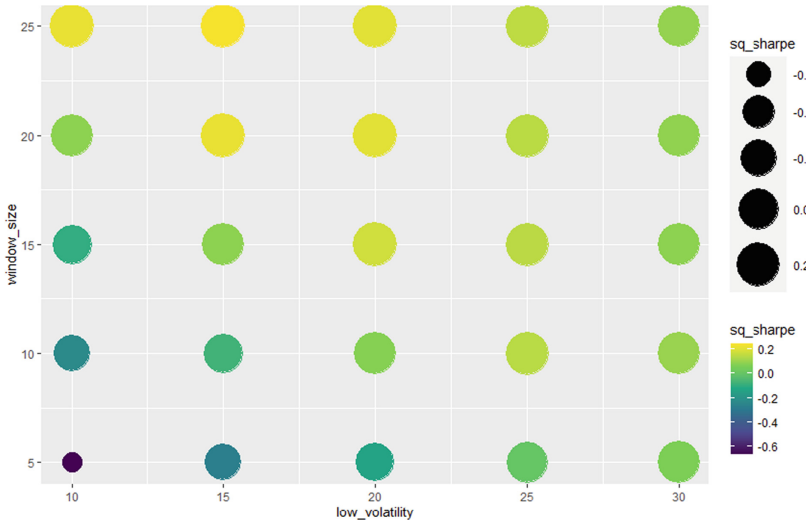

Fig. 8. Sharpe ratios for Crude Oil VIX-weighted strategies using the V_{sq} multiplier

3.5   NASDAQ

RSI-weighted DCA For RSI-weighted strategies using the R_{sq} multiplier, Sharpe ratios of strategies with lower oversold values tend to increase as window size decreases, but strategies with higher oversold values tend to have similar Sharpe ratios regardless of window size. The best performing strategy used oversold = 35 and window size = 20, and had a 5-year Sharpe ratio of 1.923, compared to the pure DCA Sharpe ratio of 0.0491. The difference was statistically significant (p = 0.0034). Strategies using the R_{sqrt} and R_{lin} multipliers outperformed pure DCA by small margins (Fig. 9).
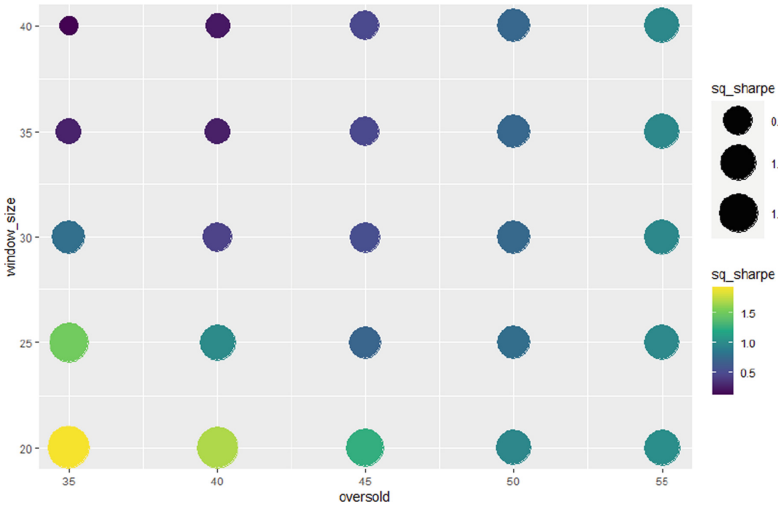

Fig. 9. Sharpe ratios for NASDAQ RSI-weighted strategies using the R_{sq} multiplier

VIX-weighted DCA For VIX-weighted strategies using the V_{sq} multiplier, Sharpe ratios of strategies with lower low volatility values tend to increase as window size increases, but strategies with higher low volatility values tend to have similar Sharpe ratios regardless of window size. Regardless of the type of multiplier used, none of the VIX-weighted DCA strategies outperformed pure DCA, which had a Sharpe ratio of 0.0491 (Figs. 10 and 11).

4   Conclusion

Across all asset classes, weighted strategies using the R_{sq} and V_{sq} multipliers generally had large differences in Sharpe ratio compared to the corresponding pure DCA strategy. However, strategies using other multipliers generally had very small differences in Sharpe ratio compared to the pure DCA strategy. All VIX-weighted strategies underperformed the pure DCA strategy except for the
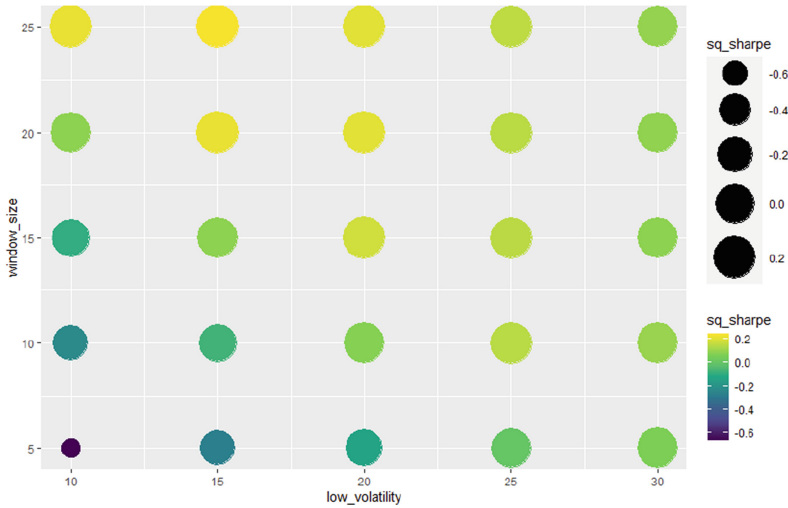
Fig. 10. Sharpe ratios for NASDAQ VIX-weighted strategies using the V_{sq} multiplier

Best R_{sq}	Bitcoin	S&P500*	Gold *	Crude Oil	NASDAQ
Pure DCA Sharpe Ratio	1.424	-0.4521	-0.7464	0.4174	0.0491
Sharpe Ratio	1.984	1.375	-0.1983	1.570	1.923

Fig. 11. Table of best performing pure DCA and RSI-weighted strategies for the 5 assets. * indicates difference is not statistically significant.

best performing VIX-weighted strategy for Gold, but the difference was not statistically significant (p = 0.4053). This suggests that increasing the amount invested when volatility is low is a subpar strategy. The VIX can be regarded as a proxy for investor sentiment regarding the S&P500, where lower volatility corresponds to higher greed in the market. Thus, our results show that investing more into an asset when sentiment is greedy performs worse than pure DCA. Investing more when sentiment is fearful is likely to outperform pure DCA. It is also worth noting that there may be some anticorrelation between gold and the S&P500, since investing more into gold when sentiment in the S&P500 is greedy outperformed the pure DCA strategy for gold (although the difference was not statistically significant).

3 assets had RSI-weighted strategies that outperformed pure DCA and had statistically significant differences: Bitcoin, Crude Oil and the NASDAQ. The other 2 assets (Gold and the S&P500) had an RSI-weighted strategy that outperformed pure DCA, but the difference in Sharpe ratio was not statistically significant. This suggests that the asset classes tested (Cryptocurrency, Stocks and Commodities) all benefit from a RSI-weighted DCA strategy.

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