

# Chapter 3

## Magnetic Forces



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### 3.1 Introduction

In the study of electromagnetic phenomena, this chapter delves into the domain of magnetic forces, shedding light on the intricate relationship between electric charges, currents, and fields. Central to this exploration are Maxwell's Equations [1, 2], which stand as a cornerstone of modern physics, elegantly describing the behavior of electric and magnetic fields. These equations provide a framework for comprehending how charges and currents give rise to the complex interplay of fields that encompass space.

As the exploration progresses, attention shifts to magnetic materials [3–6], each revealing distinct aspects of magnetism. Ferromagnetic materials display strong magnetic properties when subjected to an external magnetic field. Paramagnetic materials yield more subtly to the magnetic allure, while diamagnetic materials, such as water and copper, gently repel magnetic fields, exposing the fundamental nature of their properties.

In the realm of magnetic interactions, ferromagnetic materials exhibit strong magnetic properties. Under the influence of an external magnetic field, these materials demonstrate powerful magnetism. The expectation of equilibrium is disrupted within a magnetic field, as magnetic forces exert torque and translational forces on the materials. Earnshaw's theorem highlights the complexity of achieving stable equilibrium in static magnetic fields [7, 8]. However, innovation transcends these limitations by harnessing the interplay of gravity and magnetic forces to achieve stable levitation, thereby paving the way for magnetic separation and particle manipulation.

Overall, Chapter 3 provides a comprehensive examination of magnetic forces, revealing the intricate relationship between electric and magnetic fields as elucidated by Maxwell's Equations. This realm encompasses the magnetization of magnetic

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materials and the complex interplay of forces and torques that govern the movement of particles. Furthermore, this understanding of magnetism not only advances our knowledge of electromagnetism but also holds immense potential for transformative technological applications.

## 3.2 Maxwell's Equations

Maxwell's equations are a set of four equations that describe the behavior of electric and magnetic fields in space. They were developed by James Clerk Maxwell in the 1860s and played a crucial role in the development of modern physics. The equations relate electric and magnetic fields to their sources, which are electric charges and currents. The first of Maxwell's equations, Gauss's law for electric fields, is a fundamental law of electromagnetism. It states that the electric flux through any closed surface is proportional to the electric charge enclosed by the surface. Mathematically, this can be written as:

$$\nabla \cdot E = \rho/\epsilon_0 \quad (3.1)$$

where  $\nabla$  is the divergence operator. This equation relates the electric field  $E$  to the charge density  $\rho$ . The left-hand side of the equation describes the divergence of the electric field, which represents the flow of electric field lines into or out of a point in space. The right-hand side of the equation represents the charge density, which is the amount of electric charge per unit volume. The constant  $\epsilon_0$  is the permittivity of free space. This equation delineates the relationship between electric charges and the resulting electric fields that propagate throughout space. It also illustrates that the electric field is a vector field, possessing both magnitude and direction. Additionally, it follows the principle of superposition, whereby the total electric field at a given point is the sum of the individual fields produced by nearby charges. It serves as the foundation for comprehending the other three Maxwell's equations.

The second of Maxwell's equations, Gauss's law for magnetic fields, is another fundamental law of electromagnetism. It states that the magnetic flux through any closed surface is always zero. Mathematically, this can be written as:

$$\nabla \cdot B = 0 \quad (3.2)$$

where  $B$  is the magnetic field. This means that magnetic field lines never start or end, but always form closed loops. This equation shows that magnetic monopoles, which would be the magnetic analogs of electric charges, do not exist in nature.

The third of Maxwell's equations, Faraday's law of induction, is a key principle behind the operation of many electrical devices, such as generators and transformers. It states that a changing magnetic field generates an electric field. Mathematically, this can be written as:

$$\nabla \times E = -\partial B/\partial t \quad (3.3)$$

The equation presented describes the relationship between the electric field  $E$ , and the changing magnetic field  $B$ . On the left-hand side of the equation, it depicts the curl of the electric field, which signifies the tendency of electric field lines to form closed loops around a particular point in space. On the right-hand side, it represents the rate of change of the magnetic field with respect to time. It is a fundamental principle in electromagnetism and is used to explain various phenomena, such as the generation of electricity in generators and the operation of transformers.

The fourth of Maxwell's equations, Ampere's law with Maxwell's correction, relates the magnetic field to the electric current that produces it. It describes how a current flowing in a wire creates a magnetic field that circles around the wire. Mathematically, Ampere's law can be written as:

$$\nabla \times B = \mu_0 j + \mu_0 \epsilon_0 \partial E/\partial t \quad (3.4)$$

where  $\mu_0$  is the magnetic constant,  $j$  is the current density. The first term on the right-hand side of the equation represents the magnetic field produced by the current, while the second term represents the magnetic field produced by a changing electric field. This correction term was added by Maxwell to account for the fact that electric currents can also create magnetic fields, as shown by the third of Maxwell's equations. It is a fundamental equation in electromagnetism that relates the curl of the magnetic field ( $B$ ) to the current density ( $j$ ) and the rate of change of the electric field ( $E$ ) with respect to time. This equation, along with the other three Maxwell's equations, forms the foundation of classical electromagnetic theory.

One of the key contributions of Maxwell's equations to our understanding of magnetic fields is the concept of magnetic flux. Magnetic flux is a measure of the number of magnetic field lines passing through a given area. The lines form closed loops, which means that magnetic flux is conserved. This conservation law is a consequence of Gauss's law for magnetic fields, which states that the magnetic flux through any closed surface is always zero.

The concept of magnetic flux has important applications in electromagnetism, such as in the design of magnetic circuits and the calculation of magnetic forces. For example, the force exerted on a current-carrying wire in a magnetic field is given by the formula  $F = IlB \sin(\theta)$ , where  $F$  is the force,  $I$  is the current,  $l$  is the length of the wire,  $B$  is the magnetic field, and  $\theta$  is the angle between the wire and the magnetic field. Another important contribution of Maxwell's equations to our understanding of magnetic fields is the relationship between electric currents and magnetic fields. According to Ampere's law, a current flowing in a wire creates a magnetic field that circles around the wire. The strength of the magnetic field is proportional to the current and inversely proportional to the distance from the wire. In addition, the changing electric field associated with a current can also create a magnetic field, according to Faraday's law. This is the principle behind the operation of transformers, which use a changing magnetic field to induce a voltage in a secondary coil.

In summary, Maxwell’s equations provide a deep understanding of the behavior of magnetic fields and their relation to electric charges and currents. They are fundamental to the study of electromagnetism and have numerous practical applications in modern technology.

### 3.3 Magnetization and Magnetic Materials

Magnetization is the transformative process through which a material obtains magnetism in the presence of a magnetic field. This remarkable phenomenon can manifest across a vast array of materials. Magnetism, as one of the fundamental enduring attributes of matter, has captivated humanity for millennia, serving as a wellspring of exploration and advancement. Its historic influence encompasses the invention of the compass, lauded as one of the “Four Great Inventions” from ancient China, and its omnipresence in the constituent components of contemporary information and communication devices further attests to the pervasive and extensive integration of magnetism in materials.

Materials can be categorized into three simple types based on their magnetic susceptibility  $\chi$ : ferromagnetic ( $\chi \gg 1$ ), paramagnetic ( $\chi > 0$ ), and diamagnetic ( $\chi < 0$ ). The materials that are widely used in various applications are all ferromagnetic materials ( $\chi \gg 1$ ), which exhibit noticeable magnetism under external magnetic fields. However, for the vast majority of materials, their magnetism is very weak (magnetic susceptibility  $|\chi| < 10^{-5}$ ), making it difficult for them to exhibit significant magnetic response under external magnetic fields. Magnetism arises from the motion of charged particles, such as electrons, in a material. When electrons move in a certain direction, they generate a magnetic field. In a magnetized material, the magnetic moments of the electrons are aligned in the same direction, creating a net magnetic moment for the material. The strength of the magnetization is proportional to the number of magnetic moments per unit volume and their degree of alignment. The values of magnetic susceptibility for typical materials are shown in Fig. 3.1 [9]. Most matter exhibit weakly magnetic properties ( $|\chi| < 10^{-5}$ ) and are usually considered as non-magnetic materials.

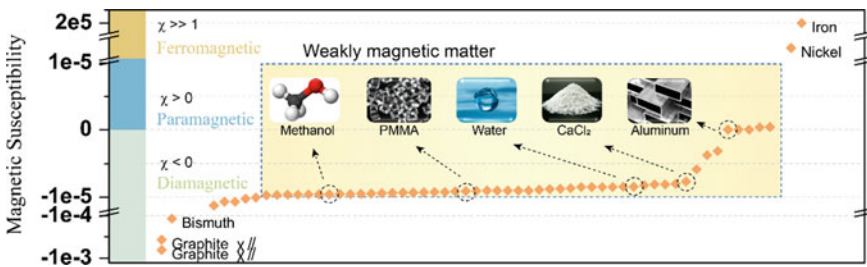


Fig. 3.1 Magnetic susceptibilities of common substances

### 3.3.1 Ferromagnetic Materials

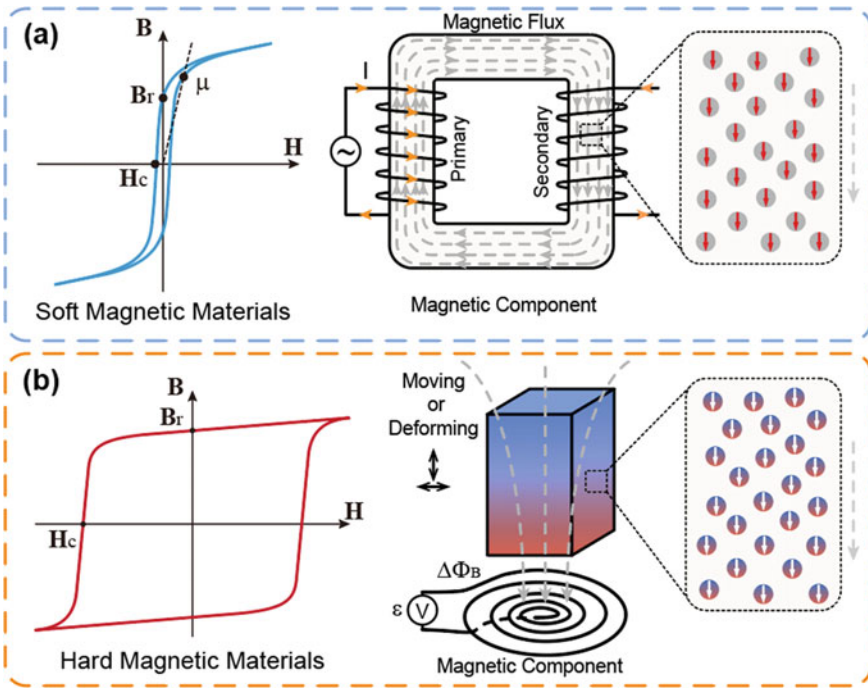
Ferromagnetic materials are materials that exhibit strong magnetization even in the absence of an external magnetic field. They have a high degree of electron spin alignment, which leads to the spontaneous alignment of magnetic moments in the same direction. The magnetic behavior of ferromagnetic materials is described by the magnetization  $M$  and the magnetic field strength  $H$ . According to the coercivity ( $H_c$ ), ferromagnetic materials can be divided into hard magnetic materials ( $H_c > 1000$  A/m) and soft magnetic materials ( $H_c \leq 1000$  A/m) [10]. Generally, once magnetized (after being exposed to a strong external magnetic field and then removed), hard magnetic materials like Alnico, SmCo alloys, and NdFeB alloys exhibit high remanence (represented by high  $B_r$ ), which allows them to retain strong magnetism. Furthermore, their high coercivity (represented by high  $H_c$ ) enables them to maintain a high residual magnetic flux density even when subjected to applied magnetic fields below their coercive field strength. In contrast, soft magnetic materials retain weak magnetism and are easily demagnetized.

Hard magnetic materials, also known as permanent magnets, are ferromagnetic materials with a high coercivity ( $H_c$ ) and a high remanence ( $B_r$ ). Coercivity is the measure of a material's resistance to becoming demagnetized, while remanence is the measure of the magnetic field strength that remains in a material after the external magnetic field is removed. Hard magnetic materials are used in applications where a strong and permanent magnetic field is required, such as in motors, generators, and magnetic storage devices. Examples of hard magnetic materials include alloys of rare earth metals such as neodymium-iron-boron (NdFeB) and samarium-cobalt (SmCo).

Soft magnetic materials, on the other hand, have a low coercivity and a low remanence, which means they can be easily magnetized and demagnetized. They are used in applications where a magnetic field needs to be rapidly and repeatedly switched on and off, such as in transformers, inductors, and electric motors. Examples of soft magnetic materials include iron-silicon alloys (e.g., silicon steel) and nickel-iron alloys (e.g., permalloy).

The magnetic properties of ferromagnetic materials can be described by the magnetization curve, which shows the relationship between the magnetic field strength  $H$  and the magnetization  $M$ . The magnetization curve is characterized by the saturation magnetization, which is the maximum magnetization that can be achieved in a material, and the magnetic susceptibility  $\chi$ , which is the measure of the material's ability to become magnetized in response to an external magnetic field. All these parameters can be extracted from the magnetization curve shown in Fig. 3.2.

In summary, ferromagnetic materials are widely used in various applications due to their strong magnetization. Hard magnetic materials are used in applications where a strong and permanent magnetic field is required, while soft magnetic materials are used in applications where a magnetic field needs to be rapidly and repeatedly switched on and off. The magnetic properties of ferromagnetic materials can



**Fig. 3.2** Schematic of typical magnetization and working mechanism of **a** soft and **b** hard magnetic materials and electronics components. Reproduced with permission from Ref. [11]. Copyright 2021 Wiley

be described by the magnetization curve, which is characterized by the saturation magnetization and the magnetic susceptibility.

### 3.3.2 Paramagnetic Materials

Paramagnetic materials are a class of materials that exhibit weak magnetization, meaning that they do not have any permanent magnetic moment in the absence of an external magnetic field. However, when subjected to an external magnetic field, they acquire a magnetic moment proportional to the applied field. This is due to the presence of unpaired electrons in their atomic or molecular orbitals, which have intrinsic magnetic moments that can align themselves with the external magnetic field. The degree of magnetization in paramagnetic materials is proportional to the strength of the applied magnetic field, and it disappears as soon as the external field is removed.

The magnetic susceptibility of paramagnetic materials is positive and proportional to the applied magnetic field, as described by Curie's law. According to this law, the

magnetic susceptibility is given by  $\chi = C/T$ , where  $C$  is the Curie constant, and  $T$  is the absolute temperature (K) [12]. The Curie constant is a material-dependent constant that characterizes the strength of the magnetic response to an applied field. Typically, the magnetic susceptibility of paramagnetic materials is on the order of  $10^{-6}$  to  $10^{-3}$ , which is much weaker than that of ferromagnetic materials.

One important application of paramagnetic materials is magnetic resonance imaging (MRI), which uses the interaction between the magnetic moments of protons in the human body and a strong external magnetic field to generate high-resolution images of the body's interior. Gadolinium and dysprosium are often used as contrast agents in MRI due to their strong magnetic moments and good biocompatibility. Another application is in magnetic separation, where paramagnetic particles are separated from non-magnetic particles using an external magnetic field.

In conclusion, paramagnetic materials are an important class of magnetic materials that exhibit weak magnetization that can be aligned with an external magnetic field. Their magnetic properties are characterized by the magnetic susceptibility, which is generally small and proportional to the applied magnetic field. The applications of paramagnetic materials include MRI and magnetic separation, and their typical examples include gadolinium, dysprosium, europium, and neodymium.

### 3.3.3 Diamagnetic Materials

Diamagnetic materials, also commonly treated as non-magnetic materials, have a very weak magnetization that opposes the applied magnetic field ( $\chi < 0$ ). Unlike ferromagnetic and paramagnetic materials, diamagnetic materials have completely filled electron shells, and there are no unpaired electrons to produce a magnetic moment. When subjected to a magnetic field, the electrons in the material are slightly displaced and produce a magnetic field opposite to the applied field, leading to a repulsive force between the material and the magnet.

Diamagnetism is a universal property of all materials, including non-magnetic substances such as copper, silver, gold, and even water. However, diamagnetism is a very weak effect, and its magnitude is typically several orders of magnitude smaller than the other two magnetic effects. For example, the magnetic susceptibility of copper is  $-1.1 \times 10^{-5}$ , which is more than 100 times smaller than that of aluminum, a weak paramagnetic material with a susceptibility of  $2.2 \times 10^{-3}$ .

Mathematically, the magnetic susceptibility of diamagnetic materials is negative and very small, typically in the range of  $-10^{-6}$  to  $-10^{-8}$ . This value is independent of the applied magnetic field and temperature. The magnetic response of diamagnetic materials is characterized by the Larmor diamagnetic susceptibility, which is given by:

$$\chi_d = -(\mu_0 n e^2 \tau) / (6m) \quad (3.5)$$

where  $\chi_d$  is the diamagnetic susceptibility,  $\mu_0$  is the magnetic constant,  $n$  is the number density of electrons,  $e$  is the charge of an electron,  $\tau$  is the relaxation time of the electrons, and  $m$  is the mass of an electron.

Diamagnetic materials are generally weakly repelled by a magnetic field and exhibit a negative susceptibility that is independent of the magnetic field. They do not retain any magnetization when the magnetic field is removed. Some examples of diamagnetic materials include copper, silver, gold, bismuth, and graphite. Among these materials, Bismuth is one of the most diamagnetic elements, with a susceptibility of  $-1.6 \times 10^{-4}$ , and is often used as a standard reference material for measuring the magnetic susceptibility of other materials.

Superconductors, which exhibit zero resistance to the flow of electric current, are a special class of diamagnetic materials. When a superconductor is cooled below its critical temperature, it expels all magnetic flux from its interior, a phenomenon known as the Meissner effect. This behavior is due to the formation of Cooper pairs, which are pairs of electrons that are bound together and behave as a single entity. Superconductors have a wide range of applications in fields such as power generation and transmission, magnetic levitation, and medical imaging. The development of high-temperature superconductors in the 1980s has made these materials more practical and economical for various applications.

In summary, diamagnetic materials are non-magnetic materials that exhibit weak repulsion in the presence of a magnetic field. Their magnetic susceptibility is negative and independent of the magnetic field. Superconductors, which are a special class of diamagnetic materials, exhibit zero resistance to electric current and expel magnetic fields from their interior. Despite their weak magnetic properties, diamagnetic materials have a wide range of applications in various fields, including biology, materials science, and physics.

## 3.4 Force on Magnetic Materials

### 3.4.1 Magnetic Force

When a material is placed in a magnetic field, it experiences a force known as the magnetic force. The magnitude and direction of this force depend on the magnetic field strength, the magnetic moment of the material, and the angle between the field and the moment. The magnetic force can be divided into two components: (i) the torque or moment, which tends to align the magnetic moment of the material with the field, and (ii) the translation force, which tends to move the material in a direction perpendicular to the field.

The torque or moment on a magnetic dipole in a magnetic field is given by the expression:

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \boldsymbol{B} \quad (3.6)$$



where  $\boldsymbol{\tau}$  is the torque or moment,  $\boldsymbol{\mu}$  is the magnetic moment of the dipole, and  $\mathbf{B}$  is the magnetic field. The cross product symbol  $\times$  indicates that the torque is perpendicular to both the magnetic moment and the magnetic field. The torque tends to align the magnetic moment of the dipole with the field, causing the dipole to rotate until it is aligned with the field.

The translation force on a magnetic dipole in a magnetic field is given by the expression:

$$\mathbf{F} = (\nabla(\boldsymbol{\mu} \cdot \mathbf{B}))/\mu_0 \quad (3.7)$$

where  $\mathbf{F}$  is the translation force,  $\mu_0$  is the magnetic constant, and  $\nabla$  represents the gradient operator. The translation force is perpendicular to both the magnetic moment and the magnetic field, and its direction depends on the gradient of the field. In regions of high field gradient, the translation force is strong and can cause the dipole to move rapidly in a direction perpendicular to the field.

In addition to the magnetic force on individual dipoles, there is also a force on a bulk sample of magnetic material, known as the magnetic force. The magnetic force on a material can be described mathematically using the following equation:

$$\mathbf{F} = \nabla(\mathbf{m} \cdot \mathbf{B}) \quad (3.8)$$

where  $\mathbf{m}$  is the magnetic moment of the magnetic material. The dot product ( $\cdot$ ) between  $\mathbf{m}$  and  $\mathbf{B}$  indicates the projection of the magnetic moment along the direction of the magnetic field.

In general, the magnetic force on a material can be broken down into three components: (i) the torque due to the misalignment between magnetic moment and external magnetic field; (ii) the force due to the external magnetic field gradient. The torque/force due to the magnetic field gradient is the dominant effect for materials under a static or quasi-static magnetic field. It is important to note that even non-magnetic materials experience forces or torques when exposed to a varying magnetic field, as a result of the phenomenon of eddy currents. Eddy currents are a significant phenomenon that occurs in electrically conductive materials when subjected to changing magnetic fields. These currents circulate within the material, creating their own magnetic fields, which in turn induce additional electrical currents. This interplay between the original magnetic field and the induced currents results in the generation of forces and torques, which can be utilized to manipulate non-magnetic but conductive objects [13]. An interesting observation is that non-contact dexterous manipulation can be achieved on objects that contain electrically conductive material but not necessarily a significant amount of ferromagnetic material. This is possible due to the generation of eddy currents in the conductive material when exposed to time-varying magnetic fields. The interaction between these eddy currents and the magnetic field creates forces and torques that can be utilized for manipulation. Previous studies have used this phenomenon to induce drag on objects passing through a static magnetic field or to exert force on an object in a specific direction using a dynamic field. However, the application of this principle for dexterous manipulation of conductive

objects was not explored until 2021. Researchers presented a novel application where multiple rotating magnetic dipole fields are utilized to enable six degrees of freedom manipulation of conductive objects. Through dimensional analysis, along with multi-physics numerical simulations and experimental verification, the forces and torques on a conductive sphere within a rotating magnetic dipole field are characterized [13].

For a uniform magnetic field, the magnetic force on a material is zero, since there is no magnetic field gradient. However, in a non-uniform magnetic field, the force can be significant and can be used to manipulate and control the motion of magnetic materials. One important consequence of the magnetic force on materials is Earnshaw's theorem, which states that it is impossible to achieve stable equilibrium of a collection of point magnetic dipoles in a static magnetic field, using only permanent magnets. This theorem is important in the design of magnetic levitation systems, as it implies that additional control mechanisms, such as feedback control or electromagnets, are necessary to achieve stable levitation.

It is important to note that the magnetic force on a material is dependent on the magnetic properties of the material. Diamagnetic materials, which have a negative magnetic susceptibility, experience a weak repulsive force in a magnetic field. Paramagnetic materials, which have a positive magnetic susceptibility, experience a weak attractive force in a magnetic field. Ferromagnetic materials, which have a large positive magnetic susceptibility, experience a strong magnetic force that can result in magnetization and magnetic hysteresis.

### 3.4.2 Diamagnetic Force

When a diamagnetic material is placed in an external magnetic field, the magnetic moments of its atoms will experience a torque that tends to align them antiparallel to the applied field. This alignment results in a small negative magnetization of the material that opposes the applied field. In contrast to paramagnetic and ferromagnetic materials, the magnetic susceptibility of diamagnetic materials is independent of temperature and is typically several orders of magnitude smaller in magnitude. The diamagnetic susceptibility of a material is a universal property that arises from the response of its electron cloud to the applied magnetic field.

The force exerted on a diamagnetic material by a magnetic field can be calculated using the magnetic energy density  $U$  of the material in the field. The magnetic energy density is defined as the energy per unit volume required to create the magnetic field. In terms of the magnetic field strength  $H$  and the magnetic induction  $B$ , the magnetic energy density is given by:

$$U = 1/2\mu_0(H \cdot B) \quad (3.9)$$

where  $\mu_0$  is the magnetic constant, which has a value of  $4\pi \times 10^{-7}$  T m/A. The force  $F$  on a small volume element  $\delta V$  of a diamagnetic material is given by the negative gradient of the magnetic energy density:

$$\mathbf{F} = -\nabla U = -\mu_0(\nabla \mathbf{H}) \cdot \mathbf{B} \quad (3.10)$$

where  $\nabla$  is the gradient operator. This equation shows that the force on a diamagnetic material is proportional to the gradient of the magnetic field, and is thus strongest in regions of high field curvature.

In 1842, Earnshaw conclusively demonstrated that a collection of point particles cannot be maintained in a stable equilibrium solely through the interaction forces governed by the classical inverse square law. This principle, commonly referred to as Earnshaw's theorem, affirms that a mechanical equilibrium structure composed exclusively of gravitational, electrostatic, and static magnetic forces is incapable of achieving stable suspension. Earnshaw's theorem states that a stable equilibrium cannot be attained by relying solely on electric and magnetic forces. Specifically, it asserts that it is impossible to hold a system of charged particles or magnets in stable equilibrium through any combination of electric and magnetic fields. This theorem also carries the implication that levitating a diamagnetic material using solely magnetic fields is unachievable due to the persistent destabilizing force acting on the material.

Nonetheless, it is feasible to levitate a diamagnetic material by harnessing a combination of magnetic and gravitational forces. In 1939, German physicist Braunbeck achieved stable levitation in a static magnetic field by utilizing diamagnetic materials possessing a negative magnetic susceptibility. This breakthrough supplemented Earnshaw's theorem and paved the way for the advancement of levitation technology using diamagnetic materials. Subsequently, researchers have been diligently working on constructing magnetic fields with more substantial gradients, ranging from 15 to 30 T, in order to achieve stable suspension for an expanded range of materials. These materials include metallic antimony, water, and even organic life forms. By meticulously arranging a system of permanent magnets and diamagnetic materials, it becomes possible to establish a stable levitation configuration wherein the diamagnetic material is suspended within the magnetic field.

### 3.5 Magnetic Buoyant Force

The potential energy of a material in a magnetic field is shown in Eq. 3.11:

$$E = -\frac{1}{\mu_0} \vec{m} \cdot \vec{B} = -\frac{\chi_s - \chi_m}{2\mu_0} B^2 V \quad (3.11)$$

where  $\chi_s$  and  $\chi_m$  are the magnetic susceptibilities of the material and the surrounding medium, respectively.  $\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$  is the vacuum permeability,  $B$  is the external magnetic field strength at the location, and  $V$  is the sample volume. From this, it can be seen that to achieve a stable suspension state, the magnetic potential energy must have a minimum value. The stable position must satisfy the condition:

$$\nabla^2 E = -\frac{\chi_s - \chi_m}{2\mu_0} \nabla^2 B^2 > 0 \quad (3.12)$$

Since  $\nabla^2 B^2 \geq 0$  has been proven, the above equation can be simplified to:

$$\chi_s - \chi_m < 0 \quad (3.13)$$

It can be seen from this that a sample with a magnetic susceptibility less than that of the surrounding medium can achieve stable suspension under the action of magnetic forces. Professor Ikezoe achieved stable levitation of water droplets using a 60-fold compressed air, which increased the ratio of the magnetic susceptibilities of water droplets and air from  $\sim 0.042$  to  $\sim 2.5$  [14]. As a result, the required magnetic field strength and gradient for stable levitation were greatly reduced to  $BdB/dz = 420 \text{ T}^2/\text{m}$ . This allows the magnetic levitation method to no longer be limited to the use of super-strong magnetic field devices or materials with high diamagnetic susceptibility, laying a solid foundation for its widespread applications.

The magnetic buoyant force is a type of magnetic force that arises when a magnetic field is applied to a fluid containing magnetic particles/ions. This force is due to the interaction between the magnetic field and the medium surrounding sample, which can cause them to move or be suspended in the medium. The magnetic buoyant force has been studied in the context of magnetic separation and particle manipulation.

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