Predefined-Time Event-Triggered Consensus for Nonlinear Multi-Agent Systems with Uncertain Parameter

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Abstract. In this paper, a novel predefined-time event-triggered control method is proposed, which achieved to the consistency of multi-agent systems with uncertain parameter. Firstly, a new predefined-time stability theorem is given, and the correctness and feasibility of this stability theorem are analyzed, the flexible preset time is more practical than the existed stability theorem. Compared with existing stability theorems, this theorem simplifies the conditions satisfied by Lyapunov function and is easier to implement in practical applications. Secondly, an event-triggered control strategy is designed to reduce control costs. Then, a new sufficient criterion is given to achieve the consistency of multi-agent systems with uncertain parameter based on the predefined-time stability theorem and event-triggered controller. In addition, the state consensus between nonlinear agents is completed in a predefined time, as well as the measurement error of the agent is converges to zero within the predefined time, respectively. Finally, the validity and feasibility of the given theoretical results are verified by a simulation example.

Keywords: Predefined-time Consensus · Event-triggered Control · Nonlinear Multi-agent Systems

1 Introduction

In the past decades, multi-agent systems have been widely applied in robot coordination and distributed optimization [1,2]. In the collective behavior of multi-agent systems, consensus problem is one of the basic problems in collective behavior, which has been widely studied. The speed and time of convergence are

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important subjects in the study of consensus problems. After a great deal of research, some results have been achieved [3,4].

In addition, most of the existing research focuses on the finite-time consensus [5,6] and fixed-time. For the finite-time consensus, it depends on initial conditions, which may limit practical application. In order to solve this practical problem and eliminate these limitations, the concept of fixed-time consensus was proposed [7,8]. The settling time of the fixed-time consensus is independent of the initial state. However, it relies on other system parameters, such as the eigenvalues of the Laplacian matrix, so there is a phenomenon of inflexible application. To solve these problems, the concept of predefined-time consensus [9–13] is proposed. The traditional fixed-time consensus systems can be improved by introducing predefined time parameters into the controller design process. Ref. [9] solved adaptive consensus in nonlinear multi-agent systems in switching topologies. Ref. [10] solved the predefined-time binary consistency control of output feedback for highorder nonlinear multi-agent systems. The predefined-time consensus problem of (T-S) fuzzy systems is studied in Ref. [11].

In the above research work on finite-time consensus, fixed-time consensus and predefined-time consensus, there is no way to avoid the constant communication and updating of the controller, which can lead to significant communication consumption. Event-triggered control [14–16] is a common control method, which can effectively avoid these disadvantages and has produced many important research results. In Ref. [14], dynamic event-triggered and self-triggered control of multi-agent systems were studied. A distributed dynamic event-triggered control method based on linear multi-agent system consistency of directed networks is studied in Ref. [15].

Inspired by these existing results, a predefined-time stability theorem was proposed. A controller with predefined time parameters is designed. This theorem ensuring that the settling time of the system does not depend on the initial value of the system and can be adjusted according to preset parameters. The work done in this paper has the following characteristics: Firstly, the settling time setting is more flexible, and this method is suitable for more systems and scenarios. Secondly, the design of the controller can reduce communication consumption and save resources. Finally, compared with finite-time consensus and fixed-time consensus, there is great scalability.

The rest of this article is organized as follows. The second section gives some basic graph theory, definitions and lemmas. In the third section, a new predefined-time stability theorem and a predefined-time event-triggered controller designed based on this theorem. The fourth section provides an example to demonstrate the correctness and feasibility of the above theorem and controllers. The fifth section provides conclusions and future work.

2 Preliminaries

First, graph theory is given. Then, some basic definitions and lemmas are introduced. Finally, a formulation of the problem is given.

Graph Theory. The communication topology between multiple agents is usually expressed in $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, node set $\mathcal{V} = (1, \cdots, N)$, edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, edge (j, i) to indicate the presence of information flow from node j to node i, and the neighbor set of node i is denoted as $N_i = (j \in \mathcal{V} | (j, i) \in \mathcal{E}, j = i)$. Where $\mathcal{A} = [a_{ij}]$ in $R^{n \times n}$ is the adjacency matrix of $\mathcal{G}$ with the element representing the edge weight, where

$$a_{ij} = \begin{cases} 1, & if(i,j) \in \mathcal{E}, \\ 0, & otherwise. \end{cases}$$

The degree matrix is $\mathcal{D} = diag[d_1, d_2, \cdots, d_N], d_i = \sum_{j=1}^{N} a_{ij}$. The Laplacian matrix $\mathcal{L} = [l_{ij}]$ in $R^{N \times N}$ of $\mathcal{G}$ is defined as $\mathcal{L} = \mathcal{D} - \mathcal{A}$.

Suppose the origin is the equilibrium point of the following system:

$$\begin{cases} \dot{x}(t) = f(x(t), t), \\ x(0) = x_0, \end{cases} \tag{1}$$

where $f(x(t), t) : R^+ \times R^M \rightarrow R^M$ is an unknown nonlinear function, and if $f(x(t), t)$ is not continuous, the solution of the above equation system (1) can be understood in the sense of Filippov.

Definition 1. [17]. If $x(t) = 0$ is asymptotically stable and $x(t)$ can reach 0 for a finite time for any $x(0) \in R^n$, then $x(t) = 0$ is finite-time stable. For any $x(0) \in R^n$, the settling time function is $T(x(0)) = inf \left\{ T* : x(T) = 0, \forall T > T* \right\}$.

Definition 2. [18]. If the origin of the above system is asymptotically stable and there is a settling time $T(x_0) > 0$. If $\exists T_{max} > 0$ and the settling time $T(x_0) \leq T_{max}$ under any initial conditions, it is fixed-time stable.

Definition 3. [19]. $x(t) = 0$ is predefined-time (PDT) stable if two conditions are met:

(i) $x(t) = 0$ is finite-time stable;
(ii) For any constant $T_p > 0$, $sup_{x(0) \in R^n} T(x(0)) \leq T_p$. In this case, T_p is PDT.

Lemma 1. *[20]. For the strong connectivity graph $\mathcal{G}$, we have the following properties:* $x^T L x = \dfrac{1}{2} \sum_{i=1}^{M} \sum_{j=1}^{M} a_{ij} (x_i - x_j)^2$, *where* $x = [x_1, x_2, \cdots, x_M]^T$. *$\mathcal{L}$ is semipositive, assuming that the eigenvalue of $\mathcal{L}$ is labeled $0, \lambda_2, \cdots, \lambda_M$, and the second small eigenvalue $\lambda_2 > 0$. Also, if $1_n^T x = 0$, then $x^T L x \geq \lambda_2 x^T x$.*

Lemma 2. *[21]. $\Gamma(x) = \int_0^{+\infty} e^{-t} t^{x-1} dt$ represents the Gamma function. Suppose there exists a continuous function $V(\cdot) : R^n \rightarrow R_+ \cup \{0\}$, and three conditions are satisfied:*

(i) $V(x(t)) = 0 \Leftrightarrow x(t) = 0$;
(ii) $\|x(t)\| \rightarrow +\infty \Rightarrow V(x(t)) \rightarrow +\infty$;

(iii) For any non-zero $x(t) \in R^n$ and any constant $T_p > 0$,

$$\dot{V}(x(t)) \leq -\frac{\omega}{T_p}(\alpha V^q(x(t) + \beta V^r(x(t)))), \tag{2}$$

where $\alpha, \beta > 0, 0 < q < 1, r > 1$, and

$$\omega = \frac{\Gamma(\frac{1-q}{r-q})\Gamma(\frac{r-1}{r-q})}{\alpha(r-q)}(\frac{\alpha}{\beta})^{\frac{1-q}{r-q}}. \tag{3}$$

So $x(t) = 0$ is predefined-time stable, and T_p is predefined time.

Assumption 1. There is a positive known constant μ such that:

$$|f(x_i(t), t)| \leq \mu. \tag{4}$$

The nonlinear multi-agent system has M agents and its communication topology is connected undirected graph. The dynamics of the agent i has the following form:

$$\dot{x}_i(t) = u_i(t) + f(x_i(t), t), \tag{5}$$

where $i = 1, \cdots, M$. $x_i(t)$ represents the state of agent i, $u_i(t)$ as the control input, $f(x_i(t), t)$ is the uncertain nonlinear functions.

3 Main Results

3.1 A New Theorem for Predefined-Time Stability

Theorem 1. *For the above system (5), if there is a continuous positive definite function $V(x(t)) : R^n \to R, T_c$ is a user-defined parameter and meets the following two conditions:*

1. *$V(x(t)) = 0 \Leftrightarrow x(t) = 0$;*
2. *For any $V(x(t)) > 0$, $1 < r < 2, a, b > 0$, satisfied:*

$$\dot{V}(x(t)) \leq -\frac{G_c}{T_c}(aV(x(t)) + bV(x(t))^{r+sign(V(x(t)-1))}). \tag{6}$$

The system (5) can then achieve predefined-time stable, where:

$$G_c = \frac{1}{(2-r)a} ln\frac{a+b}{b} + \frac{1}{ra} ln\frac{a+b}{b}. \tag{7}$$

Proof. The settling time function can be expressed as:

$$\begin{aligned} T(x(0)) &= \int_0^{T(x(0))} dt, \\ &\leq \int_0^1 \frac{T_c}{Gc}\frac{dv}{aV + bV^{r-1}} + \int_1^\infty \frac{T_c}{G_c}\frac{dv}{aV + bV^{r-1}}. \end{aligned} \tag{8}$$

case 1:

Let $W = V^{2-r}, dV = \frac{V^{r-1}dW}{2-r}$,

$$\begin{aligned} \int_0^1 \frac{T_c}{G_c}\frac{dV}{aV+bV^{r-1}} &\le \int_0^1 \frac{T_c}{G_c}\frac{1}{2-r}\frac{dW}{aW+b}, \\ &= \frac{T_c}{G_c}\frac{1}{(2-r)a}ln\frac{a+b}{b}. \end{aligned} \tag{9}$$

case 2:

Let $W = V^{-r}, dV = \frac{V^{r+1}dW}{-r}$,

$$\begin{aligned} \int_1^\infty \frac{T_c}{G_c}\frac{dv}{aV+bV^{r-1}} &\le \int_0^1 \frac{T_c}{G_c}\frac{1}{r}\frac{dW}{aW+b}, \\ &= \frac{T_c}{G_c}\frac{1}{ra}ln\frac{a+b}{b}. \end{aligned} \tag{10}$$

Thus, we can get:

$$\begin{aligned} T(x_0) &\le \frac{T_c}{G_c}(\frac{1}{(2-r)a}ln\frac{a+b}{b} + \frac{1}{ra}ln\frac{a+b}{b}), \\ &\le T_c. \end{aligned} \tag{11}$$

3.2 Multi-agent Event-Triggered Consensus

Under an event-triggered strategy with continuous communication, exist a constant p, the control input of the agent i can be constructed as:

$$u_i(t) = \frac{G_c}{T_c}[-c_1 y_i(t_k^i) - c_2(y_i(t_k^i))^{2(r+sign(\sum_{i=1}^M y_i(t)^2)/p-1))-1}], \tag{12}$$

where $c_1, c_2 > 0$, t_k^i is latest triggering time for agent i, define $y_i(t)$:

$$y_i(t) = \sum_{j=1}^M a_{ij}(x_i(t) - x_j(t)), \tag{13}$$

thus,

$$\sum_{i=1}^M y_i(t)^2 = x^T(t)L^2x(t). \tag{14}$$

We can get the following result:

$$\lambda_M(L)x^T(t)Lx(t) \ge \sum_{i=1}^M y_i(t)^2 \ge \lambda_2(L)x^T(t)Lx(t), \tag{15}$$

where $\lambda_2(L)$ is the second smallest eigenvalue of matrix L, $\lambda_M(L)$ is the maximum eigenvalue of matrix L, there must be a constant $p \in [2\lambda_2(L), 2\lambda_M(L)]$ such that $\sum_{i=1}^M y_i(t)^2 = \frac{1}{2}px^T(t)Lx(t)$.

The measurement error of the agent i can be defined as:

$$E_i(t) = c_1(y_i(t_k^i)) + c_2(y_i(t_k^i))^{2(r+sign(\sum_{i=1}^{M} y_i(t)^2/p-1))-1} - c_1 y_i(t) - c_2(y_i(t))^{2(r+sign(\sum_{i=1}^{M} y_i(t)^2/p-1))-1}. \quad (16)$$

Combining the control input (12) and the measurement error (16), the control input can be sorted out:

$$u_i(t) = \frac{G_c}{T_c}[-E_i(t) - c_1 y_i(t) - c_2(y_i(t))^{2(r+sign(\sum_{i=1}^{M} y_i(t)^2/p-1))-1}]. \quad (17)$$

The event-triggered function of the agent i is constructed as:

$$g_i(t) = |E_i(t)| - \varepsilon c_1 |y_i(t)| - \varepsilon c_2 |y_i(t)|^{2(r+sign(\sum_{i=1}^{M} y_i(t)^2/p-1))-1}, \quad (18)$$

where $\varepsilon \in (0, 1)$ is the trigger parameter and can be selected later. Therefore, for the agent i, an event is fired when $g_i(t) \geq 0$. Its controller updates at its own event time $t_0^i, t_1^i, \cdots$.

Remark 1. In previous studies, commonly used trigger conditions have been designed based on error or communication time. When the error is too large or the communication time is too long, the trigger condition is reached, and the trigger occurs. The trigger condition used in this article is designed according to the error, and when $|E_i(t)| \geq c_1|y_i(t)| + c_2|y_i(t)|^{2(r+sign(\sum_{i=1}^{M} y_i(t)^2/p-1))-1}$, the system is triggered. Different from previous research, $c_2|y_i(t)|^{2(r+sign(\sum_{i=1}^{M} y_i(t)^2/p-1))-1}$ takes into account the superposition situation under small errors, and long-term small error superposition will also lead to event triggering, so it can be said that it is compatible with the limitations of large errors and long periods of untriggered.

Theorem 2. *Suppose Assumption 1 holds, and the following condition is satisfied:*

$$\mu \leq \frac{c_1 \lambda_2(L) G_c}{2T_c}. \quad (19)$$

Thus, the multi-agent system (5) *is stable at a predefined time* T_c*.*

Proof. Construct the following Lyapunov function:

$$V(t) = \frac{1}{2}x^T(t)Lx(t), = \frac{1}{4}\sum_{i=1}^{M}\sum_{j=1}^{M} a_{ij}(x_i(t) - x_j(t))^2. \quad (20)$$

For simplicity, let's write $V(t)$ as V.

According to the above equation:

$$\sum_{i=1}^{M}\sum_{j=1}^{M}a_{ij}(x_i(t)-x_j(t))^2=4V. \tag{21}$$

From Eqs. (14), (15) and (20), we get:

$$\sum_{i=1}^{M}y_i(t)^2=pV. \tag{22}$$

Take the derivative of V:

$$\begin{aligned}
\dot{V} &= x^T(t)L\dot{x}(t),\\
&= \sum_{i=1}^{M}y_i(t)(u_i(t)+f(x_i(t),t)),\\
&= \frac{G_c}{T_c}\sum_{i=1}^{M}y_i(t)[-E_i(t)-c_1(y_i(t))-c_2(y_i(t))^{2(r+sign(\sum_{i=1}^{M}y_i(t)^2/p-1))-1}]\\
&+\sum_{i=1}^{M}y_i(t)f(x_i(t),t),\\
&\leq -\frac{G_c}{T_c}[\sum_{i=1}^{M}|y_i(t)||E_i(t)|+c_1\sum_{i=1}^{M}(y_i(t))^2+\\
&c_2\sum_{i=1}^{M}(y_i(t))^{2(r+sign(\sum_{i=1}^{M}y_i(t)^2/p-1))}]+\sum_{i=1}^{M}\sum_{j=1}^{M}a_{ij}(x_i(t)-x_j(t))f(x_i(t),t),\\
&\leq -\frac{G_c}{T_c}[\sum_{i=1}^{M}|y_i(t)||E_i(t)|+c_1\sum_{i=1}^{M}(y_i(t))^2+\\
&c_2\sum_{i=1}^{M}(y_i(t))^{2(r+sign(\sum_{i=1}^{M}y_i(t)^2/p-1))}]+\mu\sum_{i=1}^{M}\sum_{j=1}^{M}a_{ij}(x_i(t)-x_j(t))^2,\\
&\leq -\frac{G_c}{T_c}[c_1\sum_{i=1}^{M}(y_i(t))^2+c_2(pV)^{(r+sign(V-1))}]+4\mu V,\\
&\leq -\frac{G_c}{T_c}[c_1(2\lambda_2(L))V+c_2(pV)^{(r+sign(V-1))}-\frac{4\mu T_c}{G_c}V],\\
&= -\frac{G_c}{T_c}[(c_1(2\lambda_2(L)-\frac{4\mu T_c}{G_c})V+c_2(pV)^{(r+sign(V-1))}],\\
&= -\frac{G_c}{T_c}[a_1V+c_2p^{r+sign(V-1)}V^{r+sign(V-1)}],
\end{aligned} \tag{23}$$

where $a_1=2c_1\lambda_2(L)$, then, the following discussion needs to be done:

(1) when $V<1$; $p^{r+sign(V-1)}=p^{r-1}, V^{r+sign(V-1)}=V^{r-1}$,

(2) when $V > 1$; $p^{r+sign(V-1)} = p^{r+1}, V^{r+sign(V-1)} = V^{r+1}$,
(3) when $V = 1$; $p^{r+sign(V-1)} = p^{r}, V^{r+sign(V-1)} = V^{r}$,

where $b_1 = c_2 min\left\{p^{r-1}, p^{r+1}, p^{r}\right\}$, then by using Theorem 1 and Theorem 2, it gives:

$$\dot{V} \leq \begin{cases} -\frac{G_c}{T_c}(a_1 V + b_1 V^{r-1}), & V < 1, \\ -\frac{G_c}{T_c}(a_1 V + b_1 V^{r+1}), & V > 1, \\ -\frac{G_c}{T_c}(a_1 V + b_1 V^{r}), & V = 1. \end{cases} \tag{24}$$

Thus,

$$\dot{V}(x(t)) \leq -\frac{G_c}{T_c}(a_1 V(x(t)) + b_1 V(x(t))^{r+sign(V(x(t)-1))}). \tag{25}$$

So, referring to Theorem 1, when $\dot{V} \leq -\frac{G_c}{T_c}(a_1 V + b_1 V^{r+sign(V-1)})$, the settling time $T \leq T_c$.

The proof is completed.

Remark 2. Compared with the finite-time consensus and fixed-time consensus, the predefined-time consensus has prominent advantages. First of all, the predefined-time is more flexible and can be set according to specific application scenarios and adjusted according to system requirements. Finite-time and fixed-time are usually fixed or can only be adjusted within a certain range. Secondly, the predefined-time is more adaptable to the dynamic system. However, the limited time and fixed-time may not adapt to the system changes in time. Therefore, this paper proposes a predefined-time event-triggered consensus.

4 Simulation Result

We will use an example to illustrate the usability of the proposed consensus algorithm. The undirected strong connectivity graph of the five agents is shown in Fig. 1.

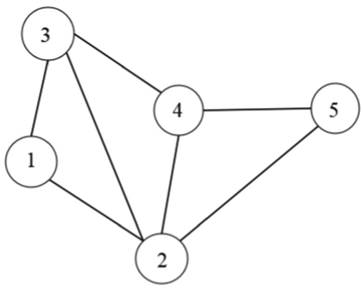


Fig. 1. An undirected graph of five agents.

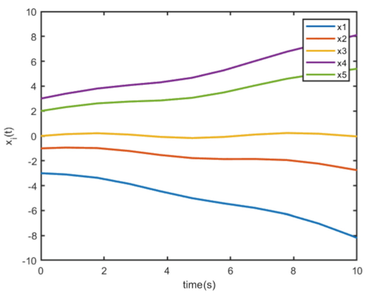


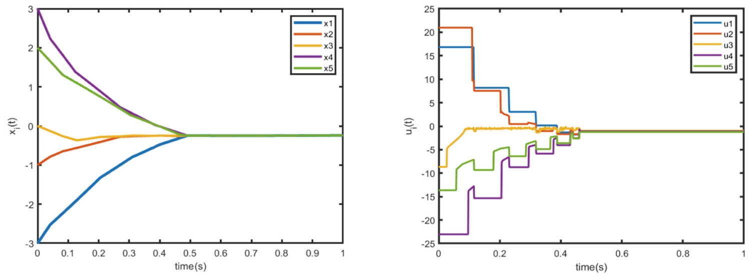
Fig. 2. The state trajectory of five agents without a controller when $T_c = 0.5$.

The dynamic model of agent i is written by:

$$\dot{x}_i(t) = u_i(t) + 0.5x_i(t) + 0.2sin(t). \tag{26}$$

Suppose the initial state of the five agents is $x(0) = [-3; -1; 0; 3; 2]^T$, and nonlinear function $f(x_i(t), t)$ satisfies Assumption 1 with $\mu = 0.2$. The state trajectories of the five agents without controllers are shown in Fig. 2. As shown in Fig. 2, when the five agents are not affected by the controller, the state cannot be consensus. It is possible that the state trajectory of an agent disappears. According to the undirected graph, L can be written as the following formula.

$$L = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 \\ -1 & 4 & -1 & -1 & -1 \\ -1 & -1 & 3 & -1 & 0 \\ 0 & -1 & -1 & 3 & -1 \\ 0 & -1 & 0 & -1 & 2 \end{bmatrix}. \tag{27}$$



(a) (b)

Fig. 3. (a) Represents the state trajectories of the five agents when $T_c = 1$, (b) represents the control input of the five agents when $T_c = 1$.

Under controller (12), select parameters $c_1 = 2; c_2 = 1; r = 1.6; \varepsilon = 0.6$. Fig. 3 and Fig. 4 respectively represent the state trajectories and control inputs of the five agents when T_c takes different values. It can be seen from Fig. 2 and Fig. 4 (a) that the system state of the agent can be consistent under the action of the controller. It can be seen from Fig. 3 (a) and Fig. 4 (a) that under the same conditions, the system state of the agent changes with different predefined-time parameter T_c, and both achieve consistency within T_c.

As can be seen from Fig. 3 (b) and Fig. 4 (b), the control inputs of the five agents are different when T_c takes different values. When the error is too large, the trigger condition (18) is reached, the trigger occurs, and the control input changes. As the controller takes effect, the error gradually decreases, and the control input will also reach stability and finally tend to 0. The figure below shows the fluctuation of the error measurement error of the five agents in $T_c = 1$, indicating that each of the five agents has its own event-triggered time.

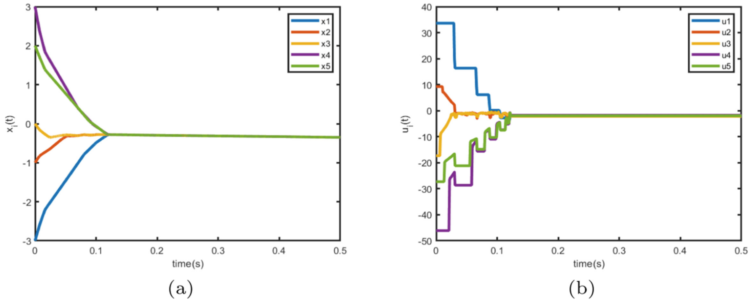


Fig. 4. (a) Represents the state trajectories of the five agents when $T_c = 0.5$, (b) represents the control input of the five agents when $T_c = 0.5$.

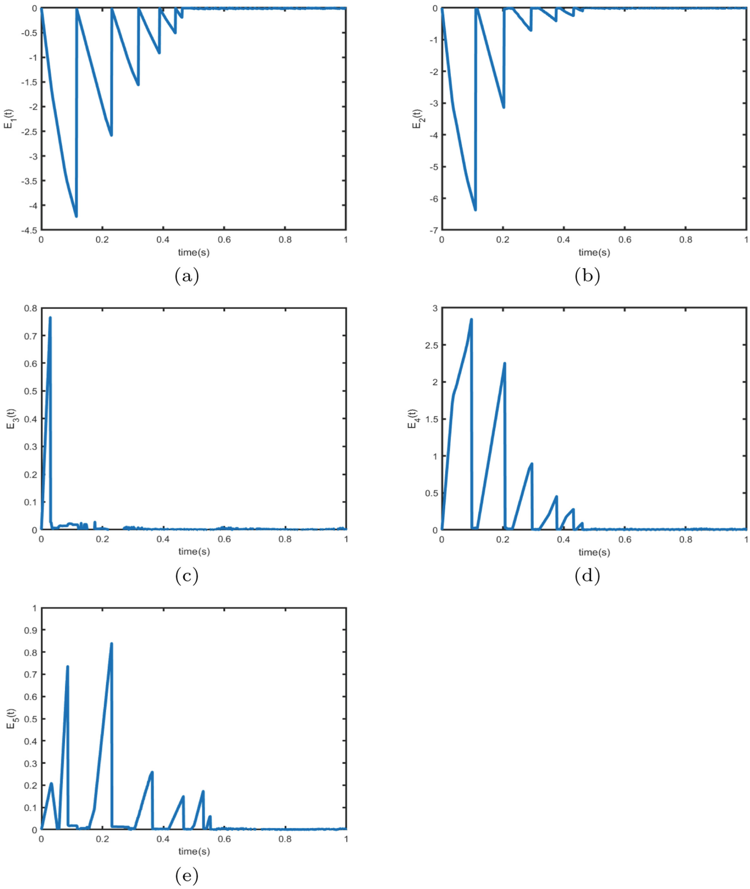


Fig. 5. The fluctuation of measurement error when the five agents communicate continuously under the controller.

As shown in the Fig. 5. First, under the action of the controller (12), the state trajectory of the multi-agent system can be consistent, and the control input of each agent is different due to the setting of trigger conditions. Second, when the predefined time parameters are different, the time to achieve consistency is also different. Finally, the measurement error of the five agents tend to 0 for a predefined time T_c.

5 Conclusion

This work investigates the predefined-time event-triggered consensus problem for nonlinear uncertain multi-agent systems. By designing the event-triggered controller, communication consumption is greatly reduced. The proposed predefined-time consensus differs from the existing finite-time consensus and fixed-time consensus in that it does not depend on the initial conditions and the parameters are adjustable. In addition, we demonstrate its feasibility for predefined-time stability conditions. In the future, we hope to investigate the consensus problem between predefined-time leaders-follower in linear multi-agent systems. First, the problem of leader-follower consistency is more practical than what this manuscript examines. Second, in the application, the leader-follower is easy to implement.

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