Mass evacuation planning based on mean field games theory

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Abstract This study proposes a dynamic framework for a mass evacuation in a large-scale urban area. The evacuation plan includes three main decisions: shelter (destination), departure time, and route. The first decision can be formulated as a shelter allocation problem, while the two latter decisions can be addressed by traffic assignment models. Unlike previously proposed dynamic models that solve shelter allocation and traffic assignment problems separately at the microscopic level, the proposed model addresses the population evacuation at the network level for a large but finite population of agents. We formulate the model based on an extension of infinite population game theory called Mean Field Games. The proposed dynamical system has two sets of equations: conservation equations for the traffic flow dynamics and Hamilton–Jacobi–Bellman (HJB) equation for the trajectory of the particles. The solution of the model is derived by solving the corresponding HJB with respect to the conservation equations, which is equivalent to solving a fixed-point problem. A heuristic algorithm is proposed to find the fixed point.

Keywords: Dynamic evacuation problem, Mean field games, bidimensional traffic models, Hamilton–Jacobi–Bellman PDE system, Traffic congestion

1 Introduction

Catastrophes threaten the entire population of the devastated areas and put them in high-risk situations. The best way to avoid life losses caused by these disasters is to evacuate people from areas considered as risky zones to safe areas. Order and guidance are crucial to effectively and safely managing the evacuation process. There are two main information pieces that each evacuee should have during the evacuation process: (i) the destination (shelter) and (ii) the route toward that shelter. The goal of evacuation planning problems is to optimize these two decisions. Both decisions

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K. R. Rao et al. (eds.), *Traffic and Granular Flow '22* , Lecture Notes in Civil Engineering 443, https://doi.org/10.1007/978-981-99-7976-9_9

can be made by formulating and solving two well-known mathematical problems, shelter allocation problem (SAP) and traffic assignment [8]. Each problem can follow different principles. We can categorize the models based on their principles into two groups: the first group addresses the issue of the evacuation process considering the user optimum or, in the game theory context, user equilibrium (UE). The second group formulates the evacuation as a system optimal (SO) problem. They differ mainly in their objective function. In the UE model, each traveler aims to minimize his benefit by minimizing his own cost. However, the SO principle aims to optimize the total benefit of all evacuees. The SO principle can be difficult to get people to accept, especially in evacuation situations. In the shelter allocation problem, it is also important how we formulate the shelter capacity. It can be infinite capacities like super sink or limited capacity, which can be static or dynamically changing over time. Besides, for the traffic assignment, the time-dependency is the most important character. If our formulation is time-independent, we are referring to static traffic assignment models, and otherwise, we are in the dynamic traffic assignment (DTA) context.

In this study, we consider the system optimum for shelter allocation. We assume that the information about the shelter's location and capacity is not accessible to the evacuees. Thus the authority or the system assigns the evacuees to the shelters considering the limited capacity of the shelters that change dynamically. On the other hand, we assume that the evacuees just need the shelter location, and they can choose their path, and they are not going to follow the system for the route choice. It means they want to reach their shelter as soon as possible, which is equivalent to the user equilibrium problem based on the Wardrop first principle [22]. We consider time-dependent traffic models in order to address Dynamic Population Evacuation (DPE) problems.

The proposed model consists of two main components: (i) the congestion model and (ii) the equilibrium model for evacuation. The first component corresponds to dynamic traffic models, and we use a bidimensional traffic flow model. The bidimensional model is based on a double approximation: the physical network is approximated as a continuous medium, discretized into cells, and the traffic flow is approximated as a bi-dimensional fluid, which is discretized as movements in the cells via a particle discretization. This model represents an urban traffic network as a 2D differential movement area, which enables us to not only capture the movement of the evacuees but also address the propagation of hazards in the network. The 2D model is well-adapted to large dense networks where traffic information is not available everywhere and thus allows us to model the evacuation movement from risky zones toward the shelter zones in such dense networks. The second component captures the sequence of evacuees' departures and their travel routes toward a shelter zone. Regarding the departure time choice, we extend our previous work [1] on the calculation of network equilibrium and departure via Mean Field Games (MFGs) to the 2D traffic model. For the route choice, we formulate the problem based on recent works that have been done for pedestrian evacuation [21] and [2]. The bidimensional approximation of the network in [10] permits us to build a model similar to pedestrian evacuation for traffic network evacuation.

Brief literature review on DPE

We have done a comprehensive literature review in our previous study [9]. The results show that few studies address traffic assignment and shelter allocation problems together for DPE planning and all of them considered the static setting for traffic assignment (see e.g., [15, 14, 4, 3]). Therefore, one of the challenges is to address the complete DPE problem in the dynamic setting. In our previous study [9], we address DPE with the agent-based dynamic simulator, and we solve SAP and DTA using a two-stage optimization approach. Here, our goal is to address this problem analytically for mass population evacuation.

The main idea of our methodology is to solve SAP and DTA together, meaning that not only the DTA problem but also the SAP takes into account the congestion dynamics for the evacuation process. Therefore, the main research challenge that is already highlighted in the literature is to revise the shelter allocation solution based on the DTA calculation [20]. [9] addressed a similar problem with a simulation-based approach and optimized the shelter allocation and route choice together; however, the departure time was given for all evacuees. In this study, we optimize both departure time and route choice, and we aim to address the mass evacuation problem.

2 Methodology

We present first the SAP formulation for SO, then we describe the traffic dynamics and formulate the DPE problem based on MFGs. Table 1 presents this paper's full list of important notations.

2.1 Shelter allocation

The SAP problem is a classic problem that is well addressed in the literature, so we reformulate a well-known model by [5] based on p-median. We are minimizing the total travel time considering the dynamic capacity of shelters. The travel time of this model is updated by the DTA problem, and the output of SAP is the OD matrix that is used for the DTA calculation. The SAP model is presented in Equations (1)-(7).

In the following formulation, the α is fixed to the current time interval. Equation (1) presents the objective function to minimize the total travel time of evacuees from all origins to all chosen shelters. Constraint (2) is the conservation equation to ensure the evacuation of all the demand from origin o. Constraint (3) prevents assigning evacuees to shelters exceeding the capacity of the shelter (c_s^α), considering the used capacity at $\alpha - 1$ [17].

Table 1 Table of notations

Symbol	Definition
O	Set of origin nodes, subset of set of nodes, $O \subset N$.
S	Set of destination nodes, subset of set of nodes, $S \subset N$.
T	Set of small time intervals.
H	Total duration considered.
o	Index of origin node, $o \in O$.
s	Index of destination node, $s \in S$.
α	Time interval index.
y_s	Binary variable; it is set to 1 if shelter s is selected; 0 otherwise.
x_{os}	Number of evacuees allocated to the pair having origin o and destination s.
w_o	Amount of demand from origin o.
c_s^α	Capacity of shelter s, limit number of evacuee allocated to shelter s in time interval α.
P	Maximum number of open shelters.
x	Trip length distribution based on predefined paths
t_d	Departure time distribution
$T(t_d, x)$	Travel time distribution.
t_a	Desired arrival time distribution
$\bar{t}_a$	Actual arrival time distribution
v_t	Velocity of the system at time t.
c_t	Fraction of the total demand that traveling in the system at time t.
$z(t)$	Characteristic travel distance.
o_t	Outflow fraction of the system at time t.
$\varphi(t, \cdot)$	Probability density function of the active trips' remaining distances at t.
$\Phi(t, x)$	Fractions of active trips with trip lengths more than x at time t.
F	In-flow measure, the empirical distribution of the departures and routes.

$$\min \quad \sum_{o \in O} \sum_{s \in S} t_{os}^{\alpha} {}^* x_{os} \tag{1}$$

$$s.t. \quad \sum_{s \in S} x_{os} = w_o; \quad \forall o \in O, \tag{2}$$

$$\sum_{o \in O} x_{os} \le c_s^\alpha y_s; \quad \forall s \in S, \tag{3}$$

$$\sum_{s \in S} y_s \le P, \tag{4}$$

$$x_{os} \le w_o y_s; \quad \forall o \in O, \forall s \in S, \tag{5}$$

$$x_{os} \ge 0; \quad \forall o \in O, \forall s \in S, \tag{6}$$

$$y_s \in \{0, 1\}; \quad \forall s \in S. \tag{7}$$

Constraint (4) restricts the number of open shelters. P denotes a predetermined parameter that restricts the number of shelters that are available, inspired from [3]. Constraint (5) ensures that we do not assign evacuees to non-opened shelters. Finally, logical variable restrictions are represented in Constraints (6) and (7). For each time interval α, we are solving this linear formulation wherein the capacity is fixed and changes over time intervals. The residual shelter capacity illustrates the effect of the users' arrival at shelters. This capacity is updated $\forall s \in S$, and it is used afterward in the next time interval as follows:

$$\begin{cases} c_s^{\alpha} = c_s^{\alpha-1} - \sum\limits_{o \in O} x_{os} & \alpha \geq 1 \\ c_s^1 = c_s^0 - \sum\limits_{o \in O} x_{os} \end{cases} \tag{8}$$

where c^0 denotes the initial capacity. Note that the SAP model is an NP-hard problem [16]. Recall that the solution of the SAP is the OD matrix needed for the DTA model.

2.2 Dynamic traffic assignment

In the DTA formulation, the main goal is to capture the traffic dynamics during the evacuation. In the large-scale problem, in terms of the number of evacuees and the size of the network, macroscopic models are efficient in addressing the congestion dynamics. Here we deploy a dynamic bidimensional traffic model (known as the 2D model). The main advantage is that we can represent complex networks with heterogenous characteristics on both sides, supply and demand. The second advantage is that they can be calibrated easily based on existing data collectors. The idea behind the 2D model is to cluster the network into undifferentiated movement cells and assume that the traveler in each cell moves with the mean speed of the cell that depends on the cell characteristics and the accumulation (in other words, supply and demand) [13]. These features permit us to represent large-scale test cases as have done before in the literature for purposes other than evacuation, e.g., Paris [19, 18], San Francisco [6, 7]. Here we first present the idea behind the mathematical model that we have built. Then we formulate the MFGs-based system of equations.

The first step for the 2D model is to cluster the network into multiple cells. Here, we used MODUS zoning approach, which is basically using the information of the 4-step model to cluster the network based on discrete choice theory [11]. Note that the shape of each zone can differ from the others, and normally, we have a unique shape for each cell. But for simplicity of the model presentation, we used a pentagon to illustrate our model (Figure 1).

The cell-based model aims to hold the conservation equations for the density moving between cells. To keep track of the shelter in the calculation of the density, we disaggregate the density by shelters from the beginning and aggregate globally to check the conservation equations. But the most important part is that the lane capacity L between cells is defined. We define this parameter based on the real traffic network by accumulating the capacity of all roads at the border between two cells for both directions. For example, Figure 1 shows all the border roads that leave the green cell. To capture the dynamics of the system, we formulate the dynamics at the border of cells and inside each cell. For the flow between cells, we carried out the calculation of the speed based on the fundamental diagram of each cell, and then the interflows can be deduced as the minimum of the origin cell outflow and the inflow

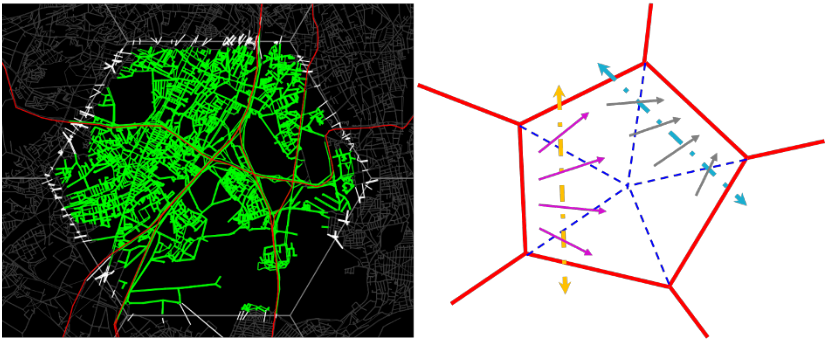

Fig. 1 Global cell dynamics: [Left] interflows, [right] intra-flows - based on [10]

of the target cell. Here we can also decompose the flow by destination to keep track of the distribution of evacuees over shelters. For the calculation of intra-flows inside each cell, we consider the heterogeneity of cell characteristics. We address isotropic and anisotropic cells. For more details, please refer to [10]. The crucial point here is where we can define our assignment variables.

We locate our assignment variables inside the cells meaning that the flow is propagating toward the next cell when it comes into the cell. To calculate the travel time inside the cell, we need to distinguish the flows based on their target cell (next cell) by the lane capacity, and for that, we have to solve an offline min-cut max-flow problem for each cell. The right pentagon in Figure 1 depicts an example of the solution for our min-cut max-flow problem. The demand and supply side can be calculated by the assignment variable that distributes the inflows. Flows yield velocities at cell boundaries and inside cells. From these velocities we deduce travel times. Based on these calculations, we can formulate and characterize our equilibrium model. Recall that the objective is to calculate the UE.

The assignment model should assign the flows to departure time intervals and the paths toward shelters. To formulate and solve the UE problem, we need to consider all evacuees' objectives together, which is equivalent to solving time-dependent dynamic programming, including the fact that the complexity of the problem exponentially increases when the number of evacuees is increasing in the system. These characteristics lead us to use Mean field games theory.

The MFG theory, introduced by [12] proposes to exploit the smoothing effect of the large numbers on each player instead of using a large number of coupled equations like classical game theory. Therefore mean-field games restate game theory as an interaction of each individual with the mass of the others. Note that the motion of the mass is the result of what each evacuee does. From the mathematical point of view, it couples two equations Hamilton–Jacobi–Bellman (HJB) equation to capture the reaction of each evacuee to the mass and the Fokker-Planck-Kolmogorov equation to consider the mass dynamics. In our case, the mass is traffic congestion, and the control is departure time and path choice of the evacuees. We define a new cost function inspired by α-β-γ scheduling preferences.

$$J(t_d; x, t_a; F) = \alpha T(t_d, x) + \beta\big(t_a - t_d - T(t_d, x)\big)_+ + \gamma\big(t_d + T(t_d, x) - t_a\big)_+. \quad (9)$$

We determine the desired arrival time of an evacuee based on its origin, and it is equal to the arrival time of a virtual evacuee who departs at the beginning of the evacuation and reaches the closest shelter (by the shortest path) based on free flow speed. It means all evacuees from the same origin have the same desired arrival time. Thus, the objective function includes the travel time and the cost of lateness for evacuee arrival.

Let us define the optimization problem for the Mean-field equilibrium, which consists of minimizing the cost for every player with respect to the mass calculated by the empirical distribution of evacuees' departure time and path choices denoted by F. We have proved the existence of the epsilon equilibrium in this setting based on [1]. As a result, the following MFG system equation is derived for continuous setting (10)-(16). The notations are described in Table 1 The equilibrium calculation process is derived based on fixed point calculation methods. To find the equilibrium, we have adapted an exact algorithm based on our previous study on the morning commute problem, wherein we proved the convergence of the algorithm [1].

$$\begin{cases} \frac{\alpha}{\alpha+\eta} + \mathbf{1}_{t_a > \bar{t}_a}\left(1 - \frac{\alpha}{\alpha+\eta}\right) \leq \frac{v_{t_d}}{v_{\bar{t}_a}} \leq 1 + \mathbf{1}_{t_a < \bar{t}_a}\left(\frac{\alpha}{\alpha+\eta} - 1\right) \text{ with solution } t_d = D(t_a, x), & (10) \\ e(D(t_a, x), x, t_a) = \frac{m(dx, dt_a)}{\partial_t D(t_a, x)}, & (11) \\ f(D(t_a, x), x) = \int_{\mathcal{T}_a} e(D(t_a, x), x, t_a)\, dt_a,\ F = f(t_d, x)\, dt_d\, dx, & (12) \\ z(t) = \int_0^t V\big(F(S_s(z))\big) ds, & (13) \\ S_t(z) = \{(\tau, \xi) \mid \tau \in [0, t] \cap \mathcal{T}_d,\ \xi \in (z(t) - z(\tau), \infty) \cap \mathcal{X}\}, & (14) \\ T(t_d, x) = z^{-1}(x + z(t_d)) - t_d, & (15) \\ F \in \mathcal{P}_{m,G}. & (16) \end{cases}$$

3 Conclusion and future work

This study proposed a mathematical framework for the DPE problem which is able to address mass evacuation in a large-scale network. We aim to evaluate the performance of our methodology on the large-scale numerical test case with evacuation KPIs such as clearance time and benchmark our model with other evacuation models. In addition, we can discretize this continuous model by particle discretization approach to address the agent-based setting.

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