Stochastic Optimal Velocity Model With Two Vehicle Control Methods

Kayo Kinjo and Akiyasu Tomoeda

Abstract Automated vehicles are widely considered as an important element of future transportation systems, and their adoption is expected to reduce traffic jams. In this paper, we propose a new mathematical model, namely, the controlled-SOV (C-SOV) model, to investigate the effects of controlled vehicles on traffic flow. Our proposed model incorporates vehicle control into the well-known stochastic optimal velocity (SOV) model. We propose two control methods in this paper: the gap-based control and the flow-based control. The gap-based control method regulates the velocity of a controlled vehicle to smooth its front and rear gaps, whereas the flow-based control method regulates the velocity of a controlled vehicle to smooth its front and rear flow. The results of our simulations indicate that the gap-based control method improves traffic flow. However, depending on the strength of the control, there are density regions where the flow rate is lower than that in the case where no control is applied. On the other hand, the flow-based control method consistently improves traffic flow.

Keywords: Traffic flow, Stochastic cellular automaton model, Vehicle control method

1 Introduction

Research on traffic flow began in the first half of the 20th century, and one of its major objectives was to understand the dynamics of traffic flow and to help reduce traffic jams [1, 2, 3]. Indeed, in the past few decades, research on traffic flow has evolved through sophisticated mathematical models and field experiments. One major contribution of such research has been the elucidation of the mechanism of

Kayo Kinjo
Ochanomizu University, 2-1-1 Otsuka, Bunkyo City, Tokyo 112-8610, JAPAN,
e-mail: kayo.kinjo1@gmail.com

Akiyasu Tomoeda
Kansai University, 2-1-1 Ryozenji-cho, Takatsuki-shi, Osaka 569-1095, JAPAN,
e-mail: tomoeda@kansai-u.ac.jp

K. R. Rao et al. (eds.), *Traffic and Granular Flow '22* , Lecture Notes
in Civil Engineering 443, https://doi.org/10.1007/978-981-99-7976-9_41

spontaneous traffic jams [4], which is a famous jamming phenomenon caused by the collective movement of vehicles without any external factors such as bottlenecks, merging, and lane changing. Because there are no external factors in this spontaneous traffic jam, it is necessary to change the driving behavior of drivers to eliminate traffic jams, and driving strategies such as jam absorption driving [5] have been proposed.

Meanwhile, the traffic flow landscape has begun to change dramatically in recent years with the advent of autonomous vehicles. The widespread use of autonomous vehicles is expected to reduce traffic jams. However, it is necessary to investigate and understand the characteristics of mixed traffic between regular vehicles (manually driven vehicles) and automated vehicles because it will take some time before an environment where all vehicles are fully automated is realized, and the changes will take place in stages. Numerous studies have been conducted on mathematical models of autonomous vehicles and mixed traffic flows with regular vehicles. From the perspective of reducing traffic jams, for example, a model incorporates feedback control that relaxes the autonomous vehicle to the equilibrium traveling speed, as in the behavior of autonomous vehicles in [6]. Moreover, the model in [7] incorporates control to maintain as much distance as possible between vehicles. All of them theoretically show from mathematical models that unstable traffic flow regimes can be reduced and stabilized, suggesting the possibility of reducing traffic congestion by using autonomous vehicles.

However, these models describe the control on the basis of the speed or distance and do not explicitly focus on the flow rate. The essence of congestion mitigation lies in how to increase the number of vehicles passing through per unit of time, and maximizing the flow rate is important. Because the flow rate can be obtained by the product of speed and density, the strength of focusing on the flow rate lies in the fact that it aims to maximize the flow rate by considering the balance between speed and density. Therefore, in this paper, we propose a new model that incorporates the effects of control and clarify the impact of this control on traffic flow using the stochastic optimal velocity (SOV) model [8].

The remainder of this paper is organized as follows. In Section 2, we briefly review the SOV model. Section 3 presents our proposed controlled-SOV (C-SOV) model with two control methods. Section 4 provides the numerical results for the C-SOV model. Finally, Section 5 concludes with a discussion.

2 Regular vehicles in the SOV model

To describe regular vehicles (RVs), we use the SOV model [8] because it is known to reproduce the metastable phase commonly found in practical traffic such as highway traffic and to include two solvable models: the TASEP and the ZRP. In the metastable phase, the flow would be transferred into the congested flow phase by only a subtle perturbation. Figure 1 (a) and (b) shows a fundamental diagram of the measured traffic flow on a highway and that of the SOV model, respectively. As shown in Figure 1(a), within the low-density region, the flow increases as the density increases.

However, beyond a certain density, called the critical density, the flow drops as the density continues to increase. This characteristic is also present in the SOV model, as shown in Figure 1(b).

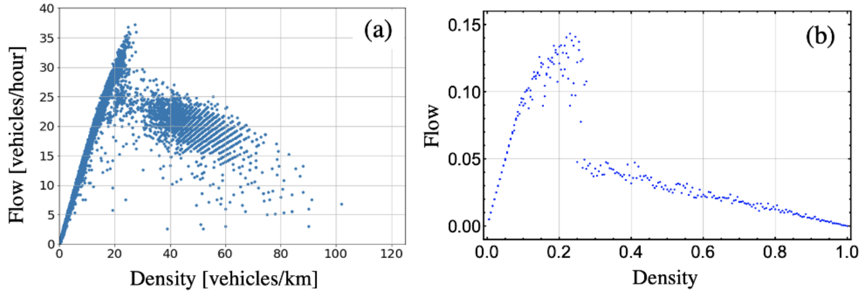

Fig. 1 (a) Measured fundamental diagram and (b) the fundamental diagram of the SOV model for a system length L of 200.

Because the SOV model is a stochastic cellular automaton model, both time and position are discretized in the model; a vehicle occupies a single cell, and its velocity is represented as the probability that it will move to the next cell, as illustrated in Figure 2(a). When the ith RV is located at position $x_i^t \in 1, 2, 3, \cdots, L$ at time step t, $v_i^{t+1} \in [0, 1]$, the velocity of the ith controlled vehicle at time step $t+1$ is given by

$$v_i^{t+1} = (1-\alpha)v_i^t + \alpha V(\Delta x_i^t). \tag{1}$$

The above expression can be regarded as the weighted average between the current velocity v_i^t and a suitable velocity for a given gap Δx_i^t, which is given by the optimal velocity function, with a proportion of $(1-\alpha) : \alpha$. Here, the optimal velocity function, as plotted in Figure 2(b), is defined as

$$V(x) := \frac{\tanh(x-c) + \tanh(c)}{1 + \tanh(c)}, \tag{2}$$

where the constant c was set to 1.0 in this paper.

3 Controlled vehicles in the C-SOV model

In this paper, we propose two vehicle control methods: the gap-based control (GC) method and the flow-based control (FC) method. Hereinafter, we refer to vehicles with the GC and the FC as GCVs and FCVs, respectively. We construct the C-SOV model such that the controls determine the velocities of the controlled vehicles as the weighted average between the velocity of the RV in Equation (1) and the target velocity specified by the control method. To ensure that velocity $v_i^{t+1} \in [0, 1]$ for the

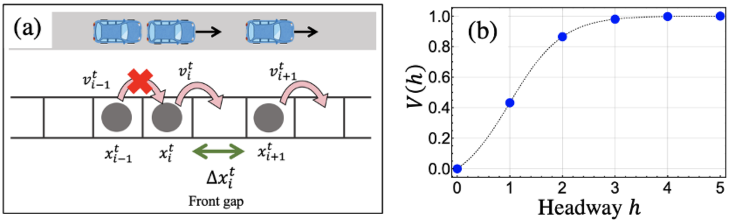

Fig. 2 (a) Schematic view used in the SOV model and (b) the optimal velocity function. Since the headway is discretized in the SOV model, the values of the optimal velocity function are represented by the blue points.

stochastic models, we introduce a parameter β to regulate the proportion of these two factors. This is expressed by

$$v_i^{t+1} = (1-\beta)\Big((1-\alpha)v_i^t + \alpha V(\Delta x_i^t)\Big) + \beta C(\cdot), \tag{3}$$

where $C(\cdot)$ represents a function that determines the target velocity through the control, which will be explained in Subsections 3.1 and 3.2. In the case when $\beta = 0$, which corresponds to the scenario where no control is applied, Equation (3) is reduced to the SOV model, as represented by Equation (1). Hereinafter, we denote the argument of the control function $C(\cdot)$ as $\Delta\tilde{x}_i^{t+1}$, which represents the gap for the controls, so that the gap is targeted at the next time step $(t+1)$.

3.1 Gap-based control (GC)

The GC is a control method for regulating the motion of a GCV by smoothing its front and rear gaps. The target velocity is defined as the value of the optimal velocity function evaluated as the average of the front and rear gaps:

$$C(\Delta x_i^t, \Delta x_{i-1}^t) = V(\Delta\tilde{x}_i^{t+1}) = V\left(\frac{\Delta x_i^t + \Delta x_{i-1}^t}{2}\right), \tag{4}$$

where Δx_i^t and Δx_{i-1}^t denote the front and rear gaps at time step t, respectively, and $\Delta\tilde{x}_i^{t+1}$ is the average of these two gaps.

3.2 Flow-based control (FC)

The FC method aims to smooth the flow of traffic by regulating the speed of an FCV. This control method is based on the assumption that an FCV will adjust its speed such that its local flow, which is defined as the product of the density and velocity, at the next time step will be equal to the average flow of the vehicles in front and behind it:

$$Q_i^{t+1} = \frac{Q_i^t + Q_{i-1}^t}{2}. \tag{5}$$

We obtain the local flow Q_i^{t+1} by its definition in terms of $\Delta\tilde{x}_i^{t+1}$, which is denoted as the gap between an FCV and the vehicle in front of it:

$$Q_i^{t+1} = \rho_i^{t+1} v_i^{t+1} = \frac{1}{\Delta\tilde{x}_i^{t+1} + 1} V(\Delta\tilde{x}_i^{t+1}). \tag{6}$$

Note that the density used in the calculation of the local flow is the reciprocal of the front gap. Combining Equations (5) and (6), we obtain the equation for $\Delta\tilde{x}_i^{t+1}$, which can be solved numerically; once the gap $\Delta\tilde{x}_i^{t+1}$ is determined, the target velocity for the FCVs is determined by evaluating the optimal velocity function:

$$C\left(\Delta\tilde{x}_i^{t+1}\right) = V\left(\Delta\tilde{x}_i^{t+1}\right). \tag{7}$$

4 Simulations

We conducted computer simulations of mixed traffic flow using the C-SOV model, which describes the behavior of controlled vehicles, and the SOV model for RVs. The initial positions of the vehicles were randomly assigned on a 200-cell circuit with periodic boundary conditions. In other words, once the number of vehicles on the circuit was determined, no vehicles were allowed to enter or exit the circuit. The simulations were terminated after 50,000 time steps.

To investigate the impact of vehicle placement on the performance of the control, we considered two different patterns of the placement of the controlled vehicles: uniform placement, in which the controlled vehicles were evenly distributed among vehicles on the circuit, and block placement, in which the controlled vehicles were grouped together as a "block." Additionally, two control methods as explained in Section 3 were implemented.

Figure 3 illustrates the fundamental diagrams for the case in which all vehicles on the circuit are controlled; the fundamental diagram of the SOV model is also shown for comparison. As shown in Figure 3(a) and (b), the flow introduced by the controls is generally higher than that of the SOV model for most density regions. However, the GC method with any parameter β does not consistently improve traffic, whereas in the density range $\rho \in [0.17, 0.25]$, the SOV model surprisingly exhibits a higher flow than that of the C-SOV model with the GC method for $\beta = 0.1$, as plotted by the green squares in Figure 3(a). This is likely due to the weak effect of the control, resulting in a few GCVs slowing down because of the vehicles behind them, even when the GCVs are able to exit a traffic jam.

In contrast, the C-SOV model with the FC method improved the flow in the metastable and congested flow phases, as shown in Figure 3(b), even when the effect of the control was weak, such as $\beta = 0.1$. The FC method achieved a higher flow rate than that of the GC method, as shown in the two panels of Figure 3. The shape of the fundamental diagram for the complete controlled-vehicle traffic, shown in Figure 3(b), is different from that of the measured fundamental diagram in Figure 1(b) because the FCVs are not affected by the vehicles in their rear and thus exhibit constant velocities.

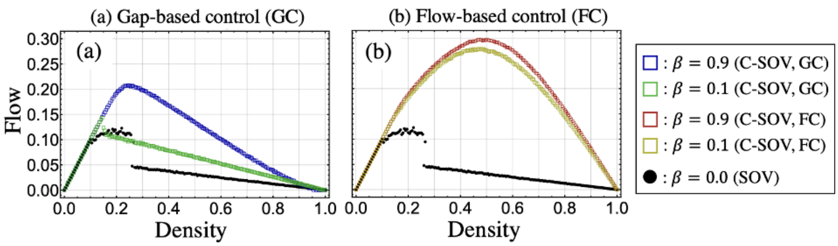

Fig. 3 Fundamental diagrams for the complete controlled-vehicle traffic. (a) Fundamental diagram when the controlled vehicles are implemented using the GC method. (b) Fundamental diagram in the case when the controlled vehicles are implemented using the FC method. The black dots represent the fundamental diagram for the SOV model, for comparison.

Figure 4 (a) and (b) shows the fundamental diagrams for scenarios in which 30% and 50% of the vehicles on the circuit are controlled vehicles, respectively. The shape of the data points indicates the placement of the controlled vehicles: the squares represent block placement and the circles represent uniform placement. For the penetration rates of the two controlled vehicles, the block placement for the GC method extends the density range of the metastable phase to higher densities, as shown in Figure 4(a) and (b). In contrast, although the uniform placement for the GC method does not extend the density range of the metastable phase, it results in a higher flow in this phase than that in scenarios where all vehicles are manually driven.

The results of our simulations revealed that the placement of the controlled vehicles has an impact on flow when using the GC method, but has none when using the FC method. To analyze these results, we plotted time-space diagrams to visualize the flow obtained from our simulations. Figures 5 and 6 present the time-space diagrams for a scenario in which 50 vehicles are on the circuit and 30% of them are controlled, which corresponds to the density of $\rho = 0.25$ in Figure 4(a). The time-space diagrams depict the trajectory of each vehicle, with the color indicating the hopping probability. Although a single large congestion is present in Figure 5(a), the scenario of block placement with the GC method in Figure 5(b) results in a higher flow than that of the case shown in Figure 5(a). This is due to the formation of smaller congestions at various locations, which prevents the formation of a single large congestion. These small congestions are caused solely by GCVs. On the other hand, in the case of the FC method, FCVs do not participate in the congestion, resulting in the absence of congestion, as seen in Figure 6.

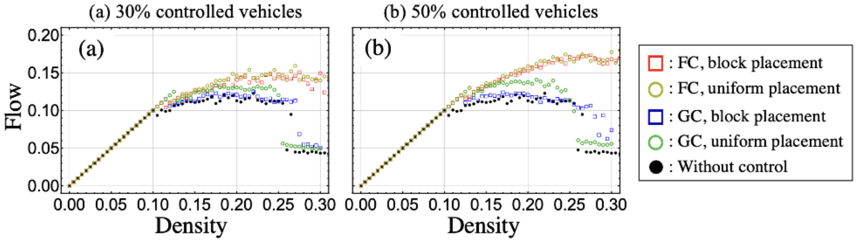

Fig. 4 Fundamental diagram of the mixed traffic flow with the controlled vehicles and RVs with different penetration rates of controlled vehicles. The black dots represent the SOV model case for comparison.

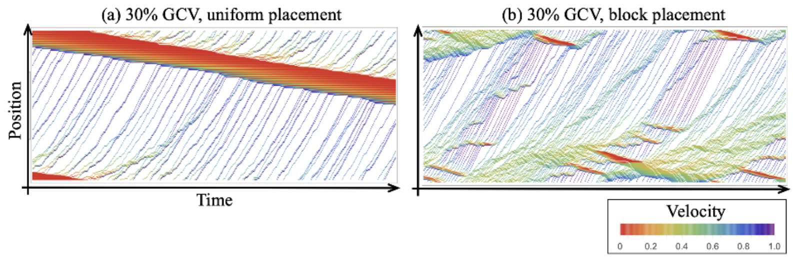

Fig. 5 Time-space diagrams for mixed traffic flow with GCVs and RVs when the GCVs are placed (a) uniformly and (b) with on a block. The number of the vehicles is 50 on the circuit, and the penetration rate of the GCVs is 30%.

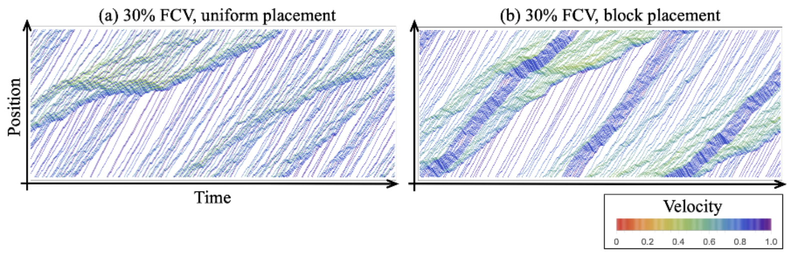

Fig. 6 Time-space diagrams for mixed traffic flow with FCVs and RVs when FCVs are placed (a) uniformly and (b) with on a block. The number of the vehicles is 50 on the circuit, and the penetration rate of the FCVs is 30%.

5 Conclusion

In this paper, we propose the C-SOV model, which incorporates vehicle control into the well-known SOV model of traffic flow in the form of a stochastic cellular automaton. The C-SOV model combines a manually driven vehicle with control $C(\cdot)$. Here, we presented two control methods: the GC method and the FC method. The GC method regulates the velocity of the controlled vehicle to smooth its front and rear gaps, whereas the FC method regulates the velocity of the CV to smooth its front and rear flows.

It was found that some GCVs slow down after exiting the congestion when the strength parameter β is small. This may be due to the CVs being affected by the vehicles in their rear during the congestion. However, because the FC method considers not only the front and rear gaps but also their velocities, FCVs do not slow down after exiting the congestion. This can be observed in the comparison of the fundamental diagrams when the two control methods are introduced for the complete CV traffic, which shows that the density that realizes the maximum flow differs between the two methods. Additionally, for mixed traffic with CVs and RVs, FC achieves a higher flow than that of GC, regardless of vehicle placement. When the vehicle control is GC with block placement, the formation of small congestions solely by CVs prevents the formation of a single large congestion in the system.

Acknowledgment

This research was partially supported by JSPS Grant-in-Aid for Scientific Research (C) JP17K05147, the Graduate Program Science and Technology for Global Leaders at Ohcanomizu University, and the Kansai University Fund for Supporting Young Scholars, 2022.

References

1. D. Chowdhury, L. Santen, and A. Schadschneider: *Phys. Rep.*, **329**, 199–329 (2000).
2. D. Helbing: *Rev. Mod. Phys.*, **73**, 1067-1141 (2001).
3. A. Schadschneider, D. Chowdhury, and K. Nishinari: StochasticTransport in Complex Systems from Molecules to Vehicles (Elsevier, Amsterdam, 2010).
4. Y. Sugiyama, et al.: *New J. Phys.*, **10**, 033001 (2008).
5. R. Nishi A. Tomoeda, K. Shimura, and K. Nishinari: *Trans. Res. B*, **50**, 116–129 (2013).
6. S. Cui, B. Seibold, R. Stern, and D. B. Work: *in Proc. IEEE Intell. Veh. Symp. (IV)*, 1336-1341, (2017).
7. W.X. Zhu and L.D. Zhang: *Physica A*, **496**, 274-285, (2018).
8. M. Kanai, K. Nishinari and T. Tokihiro: *Phys. Rev. E*, **72**, 035102-1, (2005).