Density dependence of stripe formation in a cross-flow

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Abstract Qualitative observations in ecological settings along with theoretical and numerical models suggest that when two different pedestrian streams cross a shared area, stripe-like self-organised structure emerge in order to minimise collisions and facilitate the flow. Although the phenomenon has been known for relatively long time, a systematic and quantitative verification of it through controlled experiment has been performed only recently. In this work we analysed such an experiment in which the geometry was kept fixed while changing density, in order to verify if there is a minimum density for stripe formation, and more in general the dependence on density of the phenomenon. An analysis based on two different observables, namely the angle identifying the position of the first neighbour in the same flow, and an order parameter to identify the direction of the environment presenting the higher regularity (the presence of a stripe) suggests that, although the stripe formation pattern is particularly strong at intermediate densities, the tendency to walk on a diagonal stripe is present also at considerably low densities.

keywords: self-organisation. cross-flow, controlled experiment

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1 Introduction

Qualitative observations in ecological settings [1, 2], along with theoretical [3, 4, 5, 6] and numerical [7] models suggest that when two different pedestrian streams cross a shared area, stripe-like self-organised structure emerge in order to minimise collisions and facilitate the flow. Although the phenomenon has been known for relatively long time, a systematic and quantitative verification of it through controlled experiments has been missing. Nevertheless, [8] have recently verified that stripe orientation is perpendicular to the bisector of the crossing angle through a campaign of controlled experiments in which they varied the crossing geometry while keeping density fixed.

The purpose of this work is, in a sense, complementary to the one of [8], namely to understand how the cross-flow dynamics depends on density, while keeping the geometry fixed (perpendicular crossing). In our analysis we are in particular interested in verifying if there is a minimum density for stripe formation, and more in general the dependence on density of the phenomenon.

2 Experimental setting

As described in full detail in [9], we performed a controlled experiment campaign to study the dynamics of a crowd in a cross flow scenario. The campaign consisted of 38 experiment repetitions, each of them using 56 participants (male students). Nevertheless, by modifying the length of the starting area in which pedestrians lined up before entering the crossing area, it was possible to use six different (starting area) density conditions, corresponding to $\rho_I = 0.25$ (8 repetitions), $\rho_I = 0.5$, $\rho_I = 1$, $\rho_I = 1.5$, $\rho_I = 2$ and $\rho_I = 2.5$ ped/m^2 (6 repetitions each). The change in the density condition of the starting area had also obvious consequences on the pedestrian density in the crossing area (higher starting area densities corresponding to higher values of crossing area densities, refer to [9] for details). A picture frame of the experimental setting (showing highlighted stripe formations) is shown in Fig. 1 (a). Pedestrians positions were extracted from the recorded videos using the PeTrack software [10, 11]. We also obtained chest orientation data for a subset of 9 participants (by using the inbuilt gyroscopes of tablets fixed to those participants' body using a bib), but such information is not relevant to this work.

3 Relative position of first neighbours

In [9] a few observables, such as the average density in the crossing area, the velocity distribution, the relative position of pedestrians in the same and in opposite flows, and shoulder orientation, were defined to analyse how the density conditions affect the cross-flow dynamics. Between these, the observable of relevance to study the formation of the stripe pattern, is the angle between the pedestrians' walking direction (corridor axis) and the vector identifying the relative position of their first neighbour (in their same flow). As shown in Fig. 1 (b) and (c), such angle (denoted as ϕ^s) was defined with a different sign depending on which flow the pedestrian belonged to, so that positive values always identify angles in the incoming direction of the other flow. Furthermore only values of $\phi^s \in [-\pi/2, \pi/2]$ (i.e., only relative positions to pedestrians *on the front*) were considered.

The detailed definition of ϕ^s is as follows: for each pedestrian i located in the crossing area, whose position is given by vector $\mathbf{r}_i$, and whose preferred walking direction (direction to the goal) is given by unit vector $\mathbf{g}$, we consider the closest pedestrian j in the same flow and located in front of it, i.e. satisfying

$$(\mathbf{r}_j - \mathbf{r}_i, \mathbf{g}) > 0, \tag{1}$$

and define the magnitude of the angle between the relative distance $\mathbf{r}_{ij} \equiv \mathbf{r}_j - \mathbf{r}_i$ and the unit vector $\mathbf{g}$,

$$|\phi^s| = \mathrm{acos}\left[\frac{(\mathbf{r}_{ij}, \mathbf{g})}{|\mathbf{r}_{ij}|}\right]. \tag{2}$$

The sign of ϕ^s chosen in such a way to have an angle of $+\pi/2$ if pedestrians walk in a single file according to the observations reported by [1, 8] (Fig. 1 (b) and (c)).

Although this observable is aimed to identify the emergence of a "single file" stripe and may fail in describing more complex structures, we believe it to be able to describe the density regime at which stripe begin to form. Namely, if pedestrians were walking in perfect single line stripes in the direction reported by [8] it would attain exactly a value $\phi^s = +\pi/4$. Although for empirical, imperfect stripes we obviously expect a non-δ distribution, the presence of a peak at $\approx \pi/4$ in the empirical distributions of the ϕ^s observable supports the presence of a stripe pattern in the expected direction, and by studying how the distribution of ϕ^s changes with ρ_I we can understand how density affects the stripe formation process.

The empirical distributions of ϕ^s are shown, for all values of ρ_I, in Fig. 2. We may see that a peak at $\approx \pi/4$ is present for all density conditions. Nevertheless, while such peak represents the only clear maximum in the ϕ^s distributions for $\rho_I \geq 1$ ped/m^2, other clear local maxima at $\approx -3\pi/4$ and ≈ 0 are present at lower densities (such maxima could be due to the relative spatial distributions that pedestrians assume in the starting areas). Furthermore, at least at $\rho_I = 0.5$ ped/m^2, the maximum at ≈ 0 assumes a value higher than the one at $\approx \pi/4$.

Based on a comparison between in Fig. 2 (e) and (f) one could also argue that the $\pi/4$ peak is less prominent at $\rho_I = 2.5$ ped/m^2 compared to $\rho_I = 2$ ped/m^2. It is nevertheless not clear if this result is related to a weaker formation of stripes or just to the emergence of more complex patterns that may not be identified by our simple observable.

4 Order parameter: definition

In the present contribution we introduce an other observable to try to understand which direction of the environment presents the "higher order", which we may associate

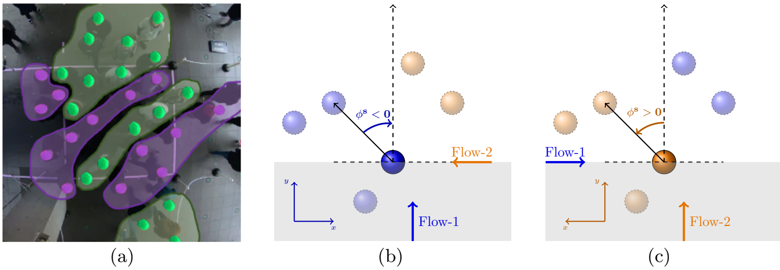


Fig. 1 (a): picture frame of the experimental setting (showing highlighted stripe formations). (b) and (c): graphical definition of ϕ^s for the different flows.

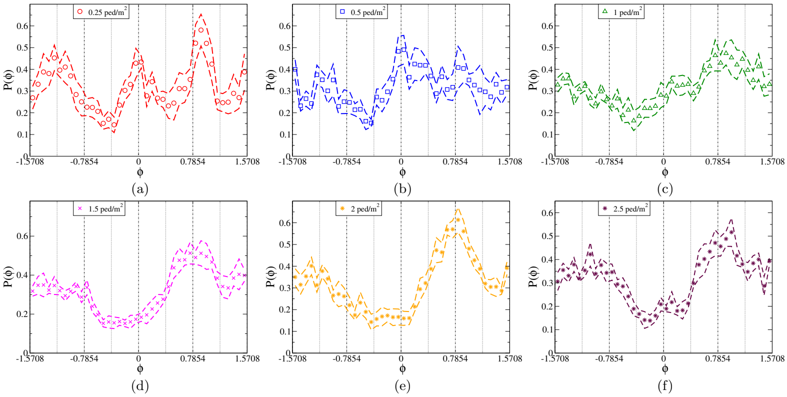


Fig. 2 Empirical distributions of ϕ^s for different values of ρ_I.

to the presence of stripes. As reported above, according to [8] for our geometrical configuration such direction should correspond to an angle of $\approx \pi/4$ with respect to the corridors' axes, but we will allow our observable to identify other possible orientations.

To be able of doing that, we rotate all the tracking data of an angle $\theta_k = k\pi/36$, $k = 1, \ldots, 36$ (e.g., multiples of 5 degrees), define a Cartesian "occupation grid" for each value of θ_k and identify which of these rotations produces a grid that is "maximally ordered" (see below for a clarification terms such as "occupation grid" and "maximally ordered").

Before proceeding, we have a couple of problems to deal with:

1. Stripes are not to be expected to be stable in time, or to occur always in the same place.
2. The original geometry is not rotationally invariant (as clearly visible in Fig. 1 (a)).

The proposed solution is, schematically, the following definition of an "order parameter"[12]:

1. The order parameter is not computed based on an average velocity field, but uses an "instantaneous occupation grid".
2. To compute the order parameter we use data tracked only inside a disk or radius 1.5 m (regaining thus rotational symmetry).
3. Our definition of the order parameter takes in account the presence of empty cells (which are necessarily present in a "instantaneous occupation grid", in particular at low densities.

The detailed definition is as follows. We create a 6 by 6 cell "occupation grid" $M_{ij}(t)$. Since such a grid is computed at each tracking time t, we named it "instantaneous". Each grid cell is a 0.5 by 0.5 meters square, and the grid value $M_{ij}(t)$ corresponds to the signed difference of the pedestrians belonging to each flow found in the cell at t.

Namely, if we have $U_{ij}(t)$ pedestrians moving down/up and $R_{ij}(t)$ moving left/right, we define

$$M_{ij}(t) = U_{ij}(t) - R_{ij}(t). \tag{3}$$

The grid U and R may be computed using data rotated for each possible angle $\theta = \theta_k$, and we may refer to the grids computed using the angle θ as $U(t, \theta)$ and $R(t, \theta)$ (and correspondingly define the grids $M(t, \theta)$ through their difference).

We may now sum over rows (or columns) and define

$$M_i^x(t,\theta) = \sum_j M_{ij}(t,\theta),\ M_j^y(t,\theta) = \sum_i M_{ij}(t,\theta), \tag{4}$$

(by scanning on the x and y directions we need only to rotate by up to $\pi/2$ to get the full π information).

We also define the total number of pedestrians in a row/column as

$$T_i^x(t,\theta) = \sum_j U_{ij}(t,\theta) + R_{ij}(t,\theta),\ T_j^y(t,\theta) = \sum_i U_{ij}(t,\theta) + R_{ij}(t,\theta). \tag{5}$$

We define the row/column order parameter as follows (to simplify we provide only the row definition)

$$O_i^x(t,\theta) = \begin{cases} 0, \text{ if } T_i^x(t,\theta) \le 1 \\ 1, \text{ if } |M_i^x(t,\theta)| \ge 6 \\ \left(\frac{M_i^x(t,\theta)}{6}\right)^2, \text{ otherwise} \end{cases}. \tag{6}$$

Finally we define the grid order parameter $O^x(t,\theta)$ ($O^y(t,\theta)$) as the average over rows (columns) with non-zero $T_i^x(t,\theta)$ ($T_j^y(t,\theta)$).

For each density condition ρ_I, experiment repetition j and angle rotation θ we average all instantaneous $O(t,\theta)$ values to obtain the experiment repetition order parameter

$$O^x(\rho_I, j, \theta),\ O^y(\rho_I, j, \theta). \tag{7}$$

We notice that based on our definition we actually have

$$O^x(\rho_I, j, \theta) = O^y(\rho_I, j, \theta + \pi/2), \tag{8}$$

so that we may drop the x, y superscripts.

Time observations are not to be considered independent, but we may expect their average to be normally distributed. We thus average over repetitions and express such averages as

$$O(\rho_I, \theta) \pm \varepsilon(\rho_I, \theta), \tag{9}$$

$\varepsilon(\rho_I, \theta)$ being standard errors.

5 Order parameter: results

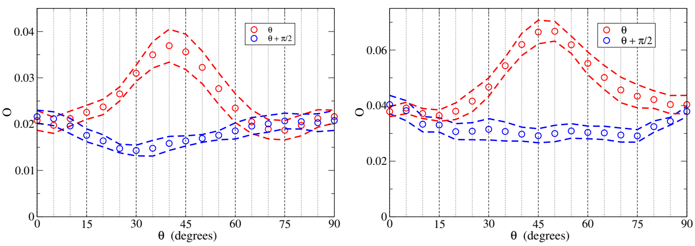
We can study which rotation angles generate the highest O by plotting them at fixed ρ_I (Figs 5- 5). The results are in agreement with the theory with peaks in the 40 – 60 degrees range.

Although the different θ values are not obviously independent, we may still use a student t test as a quantitative tool to compare the difference between the data rotated of $\theta = \pi/4$ (for which we expect a maximum) with the "un-rotated" ones at $\theta = 0$, and with the those rotated with the "opposite value" $\theta = 3\pi/4$, as for the latter we expect much lower O values. The corresponding p values are shown in Fig. 6 (a). We may observe that the order emerges for all ρ_I values.

Finally, the definition of the order parameter O provided above as a clear *direct* dependence on density (since it depends on the number of occupied cells), and thus it is not suitable to compare different ρ_I values. We may nevertheless try to circumvent this problem by using a normalised

$$O^N(\rho_I, \theta) \equiv \frac{O(\rho_I, \theta)}{O(\rho_I, 0)}, \Rightarrow O^N(\rho_I, 0) = 1 \, \forall \rho_I. \tag{10}$$

O^N is reported in Fig. 6 (a) for all ρ_I and θ values. We may see that the effect is comparable between densities values although appears to be stronger at intermediate ones.



(a) (b)

Fig. 3 Dependence of the order parameter O on the rotation angle θ for (a): $\rho_I = 0.25$ ped/m^2; (b): $\rho_I = 0.5$ ped/m^2. The blue curves show values for $\theta + \pi/2$, while dashed curves show standard error bars.

6 Conclusions

In order to understand the effect of density on the formation of stripes in a orthogonal cross-flow, we examined data from a set of controlled experiment using two observables: the distribution of the relative angle to first neighbours ϕ^s and an order parameter O. The combined analysis strongly suggests that stripe formation is happening at the theoretically predicted angle $\pi/4$. While the study based on the relative angle suggested that the local (neighbour) effect is quite weak at low (starting area) densities ρ_I, the study using the order parameter has shown that the tendency to walk on a diagonal stripe is present already at $\rho_I = 0.25$ ped/m^2.

Based on the current analysis the effect appears to be particularly strong at the intermediate values of $\rho_I \in\approx [1, 2]$, but it may be argued that the proposed indicators are not adequate to describe multi-line stripes or other similar formations that may emerge at high density. For future studies we plan to use also a clustering-algorithm based approach.

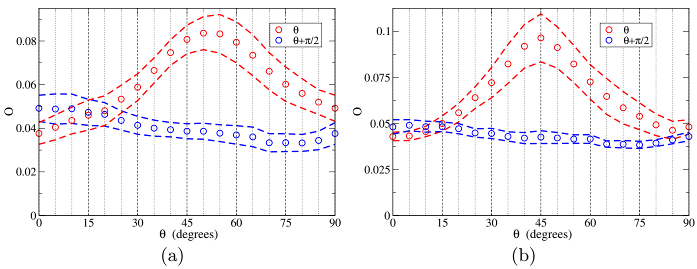


Fig. 4 Dependence of the order parameter O on the rotation angle θ for (a): $\rho_I = 1$ ped/m^2; (b): $\rho_I = 1.5$ ped/m^2. The blue curves show values for $\theta + \pi/2$, while dashed curves show standard error bars.

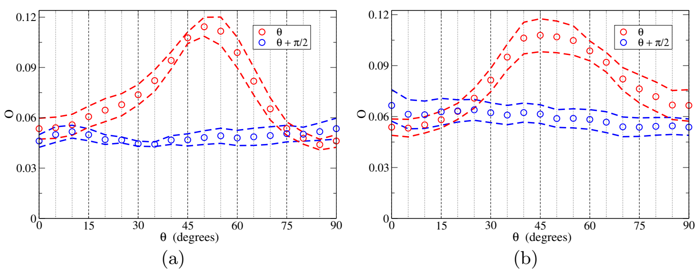


Fig. 5 Dependence of the order parameter O on the rotation angle θ for (a): $\rho_I = 2$ ped/m^2; (b): $\rho_I = 2.5$ ped/m^2. The blue curves show values for $\theta + \pi/2$, while dashed curves show standard error bars.

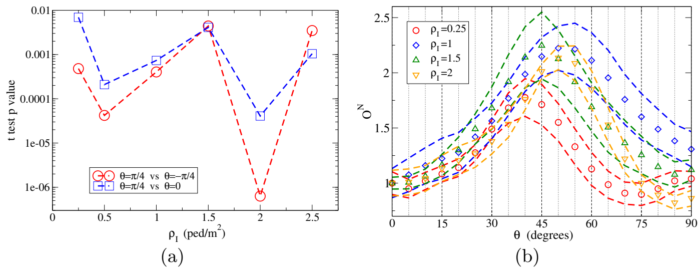


Fig. 6 (a): p values for a t test comparing $\theta = \pi/2$ to $\theta = 0$ (blue) and to $\theta = -\pi/2$ (red). (b): Comparison of the normalised order parameter O^N for different values of ρ_I.

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