Step and Save: A Wearable Technology Based Incentive Mechanism for Health Insurance

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Abstract. The market of wearables are growing explosively for the past few years. The majority of the devices are related to health care and fitness. It is embarrassing that users easily lose interest in these devices, and thus fail to improve health condition. Recently, the "be healthy and be rewarded" programs are gaining popularity in health insurance market. The insurance companies give financial rewards to its policyholders who take the initiative to keep healthy. It provides the policyholders with incentives to lead a healthier lifestyle and the insurer can also benefit from less medical claims. Unfortunately, there are hardly any studies discussing how to design the incentive mechanism in this new emerging health promotion program. Improper design would not change policyholders' unhealthy behavior and the insurer cannot benefit from it. In this paper, we propose a mechanism for this health promotion program. We model it as a monopoly market using contract theory, in which there is one insurer and many policyholders. We theoretically analyze how all parties would behave in this program. We propose a design that can guarantee that policyholders would faithfully participate in the program and the insurer can maximize its profit. Simulation results show that the insurer can improve its profit by 40% using the optimal contract.

Keywords: Wearable technology · healthcare · incentive mechanism

1 Introduction

The market of wearable devices are booming across the global. IDTechEx analysts estimate that the market will be worth \$40 billion in 2018, then accelerates to \$100 billion by 2023, and finally reaches \$150 billion by 2026 [16]. Various types of wearable devices are penetrating into our daily life, revolutionizing our clothes, watches, shoes, etc. The main functions of these smart devices are related

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C. Li et al. (Eds.): APPT 2023, LNCS 14103, pp. 326–347, 2024.
https://doi.org/10.1007/978-981-99-7872-4_19

Fig. 1. Step and save program. The insurer would reward its policyholders who take the initiative to exercise. The intention of the insurer is to let policyholders keep fit and reduce medical expenditure.

to health condition monitoring, *e.g.*, sleep quality monitoring [24], physical activity tracking [2], and smoking detection [19]. Improving the population health is an important target of wearable technology.

According to the data published by the World Bank, health expenditure accounts for 17.1% of United States GDP [3] and the number is still increasing. The rapid growth of health expenditure casts shadow on the global economics, which has caused great concern to both households and governments. A substantial portion of the diseases and deaths are caused by unhealthy dietary habits, sedentary lifestyle, tobacco and alcohol use [10]. Wearable devices can be used to monitor users' behavior and promote a healthy lifestyle. However, a survey [18] shows that more than half of consumers no longer wear their activity trackers and a third of them stop wearing the device within six months of receiving it. Consumers lack the incentives to use the device and fail to establish healthy habits.

Recently, the "be healthy and be rewarded" programs are gaining popularity in health insurance market. With the help of wearable devices, the insurer can reward its policyholders (PHs) for their healthy behaviors. The intention of the insurer is to use financial rewards to stimulate PHs to get rid of unhealthy habits and pursue a healthy lifestyle. Thus, the insurer can reimburse less amounts of medical claims and make profits. Realizing the great potentials, many insurance companies have launched similar programs. For example, AIA Vitality members can get a $7.50 Boost Juice voucher each week for engaging in physical activity. They can also get a gift card when earning enough AIA Vitality points.

Unfortunately, there are few previous works discussing how to design the incentive mechanism in these health promotion programs. Improper goals or rewards will not stimulate PHs and the insurer may not gain profit from this program. On one hand, setting a high goal or a small reward would discourage

PHs' participation; on the other hand, giving the PHs large rewards may overran the insurer's budget. For PHs, they should have enough incentive to participate in the program, and for the insurer, it wants to maximize its profits.

In this paper, we consider the scenario of "Step and Save", as shown in Fig. 1. The insurer sets step goals for the PHs, and the PHs can get discount off their insurance premiums if their average daily step counts reach the target. To maximize its profit, the insurer needs to address two challenges. First, the insurer do not have complete information about the PHs. PHs have different personal conditions (*e.g.*, workload, economic situation), termed as *type*, which is PHs' personal information and would affect their willingness to participate in physical activity. In addition to that, PHs' original daily step counts can also affect their willingness to achieve the target. However, both PHs' types and original daily step counts are not revealed to the insurer. Second, the insurer needs to guarantee that PHs would faithfully participate in the program. Only with the reasonable expectation of PHs' behavior, the insurer can maximize its profit accordingly.

To jointly tackle these two challenges, we use contract theory [5], which is effective for mechanism design under incomplete information. The intuition is to offer each PH a proper contract item, thus it will faithfully reveal its private information. The insurer would provide several contract items (*i.e.*, step goals and corresponding discounts) for the PHs to choose. The PH would select the item that maximizes its utility. After knowing PHs' behavior, the insurer can maximize its profit accordingly.

The main contributions of this paper are as follows:

- We, for the first time, theoretically analyze the insurer and PHs' behaviors in the new emerging programs in health insurance market.
- We design the optimal feasible contract that jointly considers PHs' types and original daily step counts. It can guarantee that the PHs would truthfully participate in the program and maximize the profit of the insurer.
- We conduct extensive simulations to study the performance under various scenarios. Simulation results show that the insurer can improve its profit by 40% using the optimal contract.

2 System Model

In this section, we first present our system model and define the notations that would be used in the following sections. Then we review some concepts in contract theory.

2.1 Step and Save Program

An insurance company is promoting a health program, which encourages its PHs to exercise more by giving discounts on their premiums. It provides several options (*e.g.*, $\boldsymbol{\pi} = \{[d_1, t_1], [d_2, t_2], \cdots, [d_m, t_m]\}$) for the PHs to choose. For the

PHs who choose the j-th contract item, they will get $d_j\%$ off their premiums I if their average daily steps reach the target t_j; otherwise, there is no discount for them. As there are mature techniques to authenticate/identify users [7,15,20], we assume that all the steps are taken by the legitimate PHs.

For the PHs, their daily step counts consist of two parts: S^c, the steps taken for performing daily activities (*e.g.*, get out of bed, go to dinning places) and S^e, the steps taken when participating in exercise. As pointed out in [22], S^c, the steps for daily activity (without exercise), are similar among populations. To reach the target t, they have to exercise (walk or jog) for at least $(t - S^c)$ steps.

Reaching the targets would incur cost in many ways. For example, spending more time on walking or jogging would mean less time for leisure or work [13]. It is easy to understand that only S^e would incur cost, and S^c would not. Thus, for the following discussion, we do not consider this offset value and only consider the exercising part.

The cost for each person would depend on his/her socioeconomic status, age, health situation, etc. Different PHs have different perception of how much time/comfort they sacrifice to achieve the step goal. We use θ to denote PHs' valuations over their sacrifice made for taking more steps. For PHs with large valuation, they are less willing to exercise. We assume that all PHs' valuations belong to $\boldsymbol{\theta} = \{\theta_1, \theta_2, \cdots, \theta_n\}$, where $\theta_1 > \theta_2 > \cdots > \theta_n$. According to [5], it is optimal for the insurance company to provide a contract item for each type PH. Thus, $m = n$.

Previous studies [10,23] have pointed out that physical activity can improve health status and reduce the medical expenses. We use G to denote the medical savings. For a PH who reaches t steps/day on average, the savings on its medical expense is $G(t)$. Wen *et al.* in [23] have showed that health condition is improving with increasing physical activity duration but the marginal gain is non-increasing, thus we have

$$\frac{\partial G(t)}{\partial t} > 0, \frac{\partial^2 G(t)}{\partial t^2} \leq 0. \tag{1}$$

As a common practice, only a portion of the medical expenses can be covered by the insurance. We use r to denote the reimbursement rate. We define $\bar{r} = 1 - r$.

We use C to denote the cost function. For a type-θ PH, the cost of walking t steps is $C(\theta, t)$. It is straightforward that C increases with θ and t, *i.e.*,

$$\frac{\partial C(\theta, t)}{\partial \theta} > 0, \frac{\partial C(\theta, t)}{\partial t} > 0. \tag{2}$$

It is well known that the muscle would fatigue during exercise. During walking/running periods, the speed is decreasing while the perceived difficulty is increasing. For example, the time of walking 20 thousand steps would be at least twice the time of walking 10 thousand steps and is perceived more difficult. Thus, the marginal cost is increasing with t, *i.e.*,

$$\frac{\partial^2 C(\theta, t)}{\partial t^2} > 0. \tag{3}$$

Furthermore, PHs with large valuations are more sensitive to time and comfort loss, thus we have

$$\frac{\partial^2 C(\theta,t)}{\partial\theta\partial t} \geq 0, \frac{\partial^3 C(\theta,t)}{\partial\theta\partial t^2} \geq 0 \quad (4)$$

The following paper is not limited to a concrete model of $G(t)$ or $C(\theta,t)$, but a more general discussion on the mechanism design. The insurer can substitute in the models fitting its market. For example, Duncan [10] proposed a model that G is growing linearly with physical activity engagement.

As each PH is selfish, it would choose the contract item that maximizes its utility. We use $u(\theta_i, \pi_j)$ to denote the utility of a type-θ_i PH choosing the j-th contract item.

2.2 Solution Concepts

We review the solution concepts used in this paper. The first concept is incentive compatible.

Definition 1 (Incentive Compatible). *A contract is incentive compatible if for each type-θ_j PH, it prefers to choose the contract item π_j designed for its own type,* i.e.,

$$u(\theta_j, \pi_j) \geq u(\theta_j, \pi_i), \forall i, j.$$

An accompanying concept is individual rational. We use $\pi_{Na} = [d_{Na} = 0, t_{Na} = 0]$ to denote an implicit contract, which means that a PH can choose not to participate in this program.

Definition 2 (Individual Rational). *A contract is individual rational if the utility of type-θ_j PH accepting the contract item π_j is no less than non-participating,* i.e.,

$$u(\theta_j, \pi_j) \geq u(\theta_j, \pi_{Na}).$$

The last concept is feasible contract.

Definition 3 (Feasible Contract). *A contract is feasible if it satisfies both incentive compatibility and individual rationality.*

Under feasible contract, the market is in an equilibrium. Each PH would accept the contract item designed for its type, and has no incentive to derive to another one. Similar to the PHs, the insurance company is also selfish. It will decide $\boldsymbol{\pi}$ to maximize its utility.

3 Feasible Contract Design

Before proceeding to discuss the feasibility of contract, we first study PHs' original daily step counts. Then, we give the necessary and sufficient conditions for feasible contract design, which would ensure that PHs would faithfully participate in the program.

3.1 Investigating PHs' Original Exercise Intensities

For different type PHs, they will engage in different levels of physical activity. They exercise for various reasons, *e.g.*, good body shape, high productivity in work, etc. For simplicity, we use the self-covered medical expenses to capture the self-motivation of PHs. As defined in Sect. 2.1, we use S_j^e to denote the original exercising steps for a type-θ_j PH. Its utility turns out to be

$$u(\theta_j, \pi_{Na}) = d_{Na} \cdot I + \bar{r} \cdot G(S_j^e) - C(\theta_j, S_j^e).$$

For the ease of expression, in the upcoming discussion, we will adopt the following notation:

$$u_0(\theta_j, t) = \bar{r} \cdot G(t) - C(\theta_j, t).$$

As PHs are rational, they would choose a value $t = S_j^e$ to maximize the above equation. We have the following lemma.

Lemma 1. *Before the health program, PHs with lower valuation exercise more than PHs with higher valuation,* i.e.*,*

$$S_1^e < S_2^e < \cdots < S_n^e.$$

Proof. Combing Eq. (1) and (3), we know that $\partial^2 u_0/\partial t^2 < 0$, *i.e.*, u_0 has the maximum value when $\partial u_0/\partial t = 0$. We prove by contradiction, assuming that $S_j^e > S_{j+1}^e$. We use C' and C'' to denote $\partial C/\partial t$ and $\partial^2 C/\partial t^2$ respectively.

We have

$$\bar{r} \cdot G'(S_j^e) = C'(\theta_j, S_j^e),$$
$$\bar{r} \cdot G'(S_{j+1}^e) = C'(\theta_{j+1}, S_{j+1}^e).$$

For $G^{''} \leq 0$ and $C^{''} > 0$, $G^{'}$ is non-increasing and $C^{'}$ is increasing. Given $S_j^e > S_{j+1}^e$ and $\theta_j > \theta_{j+1}$, we have $G'(S_j^e) \leq G'(S_{j+1}^e)$ and $C'(\theta_j, S_j^e) > C'(\theta_{j+1}, S_{j+1}^e)$. They contradict with the above two equations.

Thus, $S_j^e < S_{j+1}^e$.

Lemma 1 explains that PHs with low valuations are more likely to engage in physical activity. This is easy to understand, as PHs with large θ have high valuations of their time or comfort that they sacrifice to participate in exercise. Thus, PHs with high valuations are more reluctant to exercise.

In addition to Lemma 1, we have the following lemma.

Lemma 2. *For a type-θ_i PH accepting a contract item $\pi' = [d', t']$, its daily step count would* s.t.

$$S_i' = \begin{cases} S_i^e & \text{if } t' < S_i^e, \\ t' & \text{otherwise.} \end{cases}$$

Proof. Its utility is

$$u(\theta_i, \pi') = d' \cdot I + \bar{r} \cdot G(S_i') - C(\theta_i, S_i') = d' \cdot I + u_0(\theta_i, S_i').$$

The PH would choose an appropriate S_i' that maximizes its utility.

As $\frac{\partial u_0(\theta_i,t)}{\partial t} = 0$ when $t = S_i^e$ and $\frac{\partial^2 u_0(\theta_i,t)}{\partial t^2} < 0$, $u_0(\theta_i, t)$ is increasing when $t < S_i^e$ and decreasing when $t > S_i^e$.

For a type-θ_i PH, if the step goal set by the insurer is less than its original exercise intensity, *i.e.*, $t' < S_i^e$, it will not decrease its exercise intensity, for $u_0(\theta_i, S_i^e) > u_0(\theta_i, t')$.

If the goal is larger than its original exercise intensity, *i.e.*, $t' > S_i^e$, it will set $S_i' = t'$ and not exceed t'. This because for $\forall t > t'$, $u_0(\theta_i, t') > u_0(\theta_i, t)$.

Lemma 2 indicates that when the goals are lower than their original daily step counts, they would maintain their exercise intensity; when the goals are higher, they would increase exercise intensity to reach the goal, but not exceed it. Thus, for a type-θ_j PH, his/her utility for choosing the j-th contract is

$$u(\theta_j, \pi_j) = d_j \cdot I + u_0 \left[\theta_j, \max(t_j, S_j^e)\right].$$

The objective of the insurer is to encourage the PHs to exercise more and be more healthy. If the insurer set the goals below PHs' original daily step counts, the monetary reward will not increase their exercise intensities. Thus, the insurer will set

$$t_j \geq S_j^e.$$

3.2 Conditions for Feasible Contract

We first introduce the following lemma, which could assist in discussing the necessary and sufficient conditions for feasible contract.

Lemma 3. *For $\theta' \geq \theta$ and $t' \geq t$, we have*

$$u_0(\theta, t') - u_0(\theta, t) \geq u_0(\theta', t') - u_0(\theta', t)$$

Proof.

$$\begin{aligned}
& u_0(\theta, t') - u_0(\theta, t) - u_0(\theta', t') + u_0(\theta', t) \\
&= C(\theta', t') - C(\theta', t) - C(\theta, t') + C(\theta, t) \\
&= \int_t^{t'} \int_\theta^{\theta'} \frac{\partial^2 C(\theta, t)}{\partial\theta\partial t}\, \mathrm{d}\theta\, \mathrm{d}t \geq 0.
\end{aligned}$$

The last line follows because the integrand is non-negative (from Eq. (4)), and $\theta' \geq \theta, t' \geq t$.

Then, we give the necessary conditions for feasible contract.

Lemma 4. *If $\boldsymbol{\pi}$ is a feasible contract, we have*

$$t_1 \leq t_2 \leq \cdots \leq t_n,$$
$$d_1 \leq d_2 \leq \cdots \leq d_n.$$

Proof. For the following discussion, without loss of generality, we assume that $i < j$.

Before proceeding to the proof, we first figure out the order between t_j and S_i^e, t_i and S_j^e. For $t_j \geq S_j^e$ and $S_j^e > S_i^e$, we have $t_j > S_i^e$. We distinguish cases when $t_i > S_j^e$ and $t_i \leq S_j^e$.

Case 1: $t_i > S_j^e$, *i.e.*, $\pi(\theta_j, \pi_i) = d_i \cdot I + u_0(\theta_j, t_i)$.

For $\boldsymbol{\pi}$ to be feasible, $\forall i, j$, it *s.t.* that

$$d_i \cdot I + u_0(\theta_i, t_i) \geq d_j \cdot I + u_0(\theta_i, t_j), \tag{5}$$
$$d_j \cdot I + u_0(\theta_j, t_j) \geq d_i \cdot I + u_0(\theta_j, t_i). \tag{6}$$

We prove by contradiction, assuming that $t_i > t_j$.

Summing up these two inequalities, we have

$$u_0(\theta_i, t_i) - u_0(\theta_i, t_j) \geq u_0(\theta_j, t_i) - u_0(\theta_j, t_j).$$

It contradicts with Lemma 3. Thus, for $i < j$, $t_i \leq t_j$.

From Inequality (6) above, we have

$$(d_j - d_i) \cdot I \geq u_0(\theta_j, t_i) - u_0(\theta_j, t_j) \geq 0.$$

The last inequality follows because $u_0(\theta_j, t)$ is decreasing when $t > S_j^e$. Thus, $d_j \geq d_i$.

Case 2: $t_i \leq S_j^e$, *i.e.*, $\pi(\theta_j, \pi_i) = d_i \cdot I + u_0(\theta_j, S_j^e)$.

It is straightforward that $t_j \geq S_j^e \geq t_i$.

Similar to (6), we have

$$d_j \cdot I + u_0(\theta_j, t_j) \geq d_i \cdot I + u_0(\theta_j, S_j^e).$$

Then we have

$$(d_j - d_i) \cdot I \geq u_0(\theta_j, S_j^e) - u_0(\theta_j, t_j) \geq 0.$$

Thus, $d_j \geq d_i$.

Lemma 4 indicates that in a feasible contract, the rewards should increase monotonically with increasing step goals. Furthermore, the insurer would set higher step goals for PHs with small valuation and give them higher rewards. The underlying intuition is that given the same amount of rewards, PHs with small valuations are willing to exercise more. Thus, the insurer tends to give out more rewards for PHs with small valuation. Next, we give the sufficient conditions for a feasible contract.

Lemma 5. $\boldsymbol{\pi}$ *is a feasible contract if it* s.t.

1. *for* $\forall i$, $t_i \geq S_i^e$,
2. $t_1 \leq t_2 \leq \cdots \leq t_n$,
3. $d_1 \geq \frac{1}{I}\left[u_0(\theta_1, S_1^e) - u_0(\theta_1, t_1)\right]$,
4. *for* $j = 2, 3, \cdots, n$,

$$d_{j-1} + A \leq d_j \leq d_{j-1} + B, \tag{7}$$

where

$$A = \frac{1}{I}\left\{u_0\left[\theta_j, \max(t_{j-1}, S_j^e)\right] - u_0(\theta_j, t_j)\right\},$$

$$B = \frac{1}{I}\left[u_0(\theta_{j-1}, t_{j-1}) - u_0(\theta_{j-1}, t_j)\right].$$

Proof. We prove by induction. We use $\boldsymbol{\pi}_n$ to denote the contract involving type-$\theta_1, \theta_2, \cdots, \theta_n$ PHs.

When $n = 1$, there is only one contract item. According to the third condition, we have

$$u(\theta_1, \pi_1) = d_1 \cdot I + u_0(\theta_1, t_1) \geq u_0(\theta_1, S_1^e) = u(\theta_1, \pi_{Na}).$$

It satisfies both incentive compatibility and individual rationality. Thus, $\boldsymbol{\pi}_1$ is feasible.

We assume that $\boldsymbol{\pi}_{k-1}$ is feasible. We will show that adding a new type θ_k PH and a new contract item $\pi_k = \{d_k, t_k\}$, the contract is also feasible.

We first show that it guarantees incentive compatibility. We first consider type-θ_k PHs. From the left inequality in Eq. (7), we have

$$d_k \cdot I + u_0(\theta_k, t_k) \geq d_{k-1} \cdot I + u_0\left[\theta_k, \max(t_{k-1}, S_k^e)\right].$$

The fact that $\boldsymbol{\pi}_{k-1}$ is feasible implies that for $i \leq k-1$,

$$d_{k-1} \cdot I + u_0(\theta_{k-1}, t_{k-1}) \geq d_i \cdot I + u_0\left[\theta_{k-1}, \max\left(t_i, S_{k-1}^e\right)\right].$$

Combing these two inequalities, we get

$$\begin{aligned}
& d_k \cdot I + u_0(\theta_k, t_k) \\
\geq\, & d_i \cdot I + u_0\left[\theta_k, \max(t_{k-1}, S_k^e)\right] \\
& + u_0\left[\theta_{k-1}, \max\left(t_i, S_{k-1}^e\right)\right] - u_0(\theta_{k-1}, t_{k-1}) \\
\geq\, & d_i \cdot I + u_0\left[\theta_k, \max\left(t_i, S_k^e\right)\right]
\end{aligned}$$

The detailed proof can be found in Appendix A.1. It indicates that type-θ_k PHs always prefer π_k over π_i for $i < k$. Next, we show that type-i PHs prefer π_i over π_k. From the right inequality in Eq. (7), we have

$$d_{k-1} \cdot I + u_0(\theta_{k-1}, t_{k-1}) \geq d_k \cdot I + u_0\left(\theta_{k-1}, t_k\right).$$

The fact that $\boldsymbol{\pi}_{k-1}$ is feasible implies that for $i \leq k-1$,

$$d_i \cdot I + u_0(\theta_i, t_i) \geq d_{k-1} \cdot I + u_0\left(\theta_i, t_{k-1}\right).$$

Combing these two inequalities, we get

$$\begin{aligned} & d_i \cdot I + u_0(\theta_i, t_i) \\ \geq\; & d_k \cdot I + u_0\left(\theta_{k-1}, t_k\right) + u_0\left(\theta_i, t_{k-1}\right) - u_0(\theta_{k-1}, t_{k-1}) \\ \geq\; & d_k \cdot I + u_0(\theta_i, t_k) \end{aligned}$$

In Appendix A.1, we also show that $A \leq B$. Up to now, we prove that $\boldsymbol{\pi}_k$ guarantees incentive compatibility.

Then, we show that $\boldsymbol{\pi}_k$ guarantees individual rationality. For $\boldsymbol{\pi}_k$ guarantees incentive compatible and $\boldsymbol{\pi}_{k-1}$ is feasible, we have

$$d_{k-1} \cdot I + u_0(\theta_{k-1}, t_{k-1}) \geq u_0(\theta_{k-1}, S_{k-1}^e).$$

Then,

$$\begin{aligned} d_k \cdot I + u_0(\theta_k, t_k) &\geq d_{k-1} \cdot I + u_0\left[\theta_k, \max\left(t_{k-1}, S_k^e\right)\right] \\ &\geq u_0(\theta_{k-1}, S_{k-1}^e) - u_0(\theta_{k-1}, t_{k-1}) \\ &\quad + u_0\left[\theta_k, \max\left(t_{k-1}, S_k^e\right)\right]. \end{aligned}$$

If $t_{k-1} > S_k^e$,

$$\begin{aligned} & d_k \cdot I + u_0(\theta_k, t_k) \\ \geq\; & u_0(\theta_{k-1}, S_{k-1}^e) - u_0(\theta_{k-1}, t_{k-1}) + u_0\left(\theta_k, t_{k-1}\right) \\ \geq\; & u_0(\theta_{k-1}, S_k^e) - u_0(\theta_{k-1}, t_{k-1}) + u_0\left(\theta_k, t_{k-1}\right) \\ \geq\; & u_0(\theta_k, S_k^e). \end{aligned}$$

The second inequality holds because $t = S_{k-1}^e$ maximizes $u_0\left(\theta_{k-1}, t\right)$. The last line follows from Lemma 3.

If $t_{k-1} \leq S_k^e$,

$$\begin{aligned} & d_k \cdot I + u_0(\theta_k, t_k) \\ \geq\; & u_0(\theta_{k-1}, S_{k-1}^e) - u_0(\theta_{k-1}, t_{k-1}) + u_0\left(\theta_k, S_k^e\right) \\ \geq\; & u_0(\theta_k, S_k^e). \end{aligned}$$

It indicates type-θ_k PHs prefer accepting the contract over rejecting it. Thus, π_k guarantees individual rationality.

Therefore, we prove that $\boldsymbol{\pi}$ is feasible.

We learn that any contract satisfying the conditions in Lemma 5 is feasible, which indicates that PHs are willing to participate (individual rationality) and they would truthfully reveal their private information to the insurer (incentive compatibility). Given a feasible contract, the insurer can anticipate how the PHs would behave in the program and thus can maximize its profits accordingly.

4 Optimal Contract Design

The insurer is aimed at designing an optimal contract which could maximize its profit. In this section, we first give the optimal solution which could maximize the profit. However, the optimal solution may not preserve the feasibility of the contract. Then we give the optimal feasible solution which maximizes the profit and meanwhile preserves the feasibility of the contract.

4.1 Optimal Solution

The insurer's profit is given by

$$u(\boldsymbol{\pi}) = \sum_{i=1}^{n} N_i \cdot \left[r \cdot G(t_i) - r \cdot G(S_i^e) - d_i \cdot I\right]. \tag{8}$$

Thus, given any t_i, the insurer would set d_i to the lower bound in Lemma 5, *i.e.*,

$$d_1 = \frac{1}{I}\left[u_0(\theta_1, S_1^e) - u_0(\theta_1, t_1)\right],$$

$$\begin{aligned} d_i &= d_{i-1} + \frac{1}{I}\left\{u_0\left[\theta_i, \max(t_{i-1}, S_i^e)\right] - u_0(\theta_i, t_i)\right\} \\ &= \sum_{j=1}^{i} \frac{1}{I}\left\{u_0\left[\theta_j, \max(t_{j-1}, S_j^e)\right] - u_0(\theta_j, t_j)\right\} \end{aligned} \tag{9}$$

where we define $t_0 = 0$. Thus,

$$\begin{aligned} u(\boldsymbol{\pi}) = \sum_{i=1}^{n} \Bigg\{ & N_i\left[r \cdot G(t_i) - r \cdot G(S_i^e)\right] - \sum_{j=i+1}^{n} N_j \cdot \\ & u_0\left[\theta_{i+1}, \max(t_i, S_{i+1}^e)\right] + \sum_{j=i}^{n} N_j \cdot u_0(\theta_i, t_i) \Bigg\}. \end{aligned}$$

We use $f_i(t_i)$ to denote each term in the summation, as each term is only related to t_i. Thus, we can choose t_i to maximize $f_i(t_i)$ independently. We distinguish two cases, *i.e.*, $t_i < S_{i+1}^e$ and $t_i \geq S_{i+1}^e$. We use $f_i^1(t_i)$ and $f_i^2(t_i)$ to represent $f_i(t_i)$ in these two cases, respectively.

Case 1: $t_i < S_{i+1}^e$.

We have

$$f_i^{1\prime\prime}(t) = N_i \cdot r \cdot G''(t) + \sum_{j=i}^{n} N_j \cdot u_0''(\theta_i, t) < 0.$$

$f_i^1(t)$ is maximum when $t = t_i^1$ *s.t.* $f_i^{1\prime}(t_i^1) = 0$.

Furthermore, $t_i^1 \geq S_i^e$ for $f_i^{1\prime}(S_i^e) \geq 0$.

Case 2: $t_i \geq S_{i+1}^e$.

Similarly, we have

$$\begin{aligned} f_i^{2\prime\prime}(t) &= N_i \left[r \cdot G''(t) + u_0''(\theta_i, t)\right] \\ &\quad + \sum_{j=i+1}^{n} N_j \left[C''(\theta_{i+1}, t) - C''(\theta_i, t)\right] \\ &< 0. \end{aligned}$$

The last line follows because $G'' \leq 0$, $u_0'' < 0$, and $\frac{\partial^3 C(\theta,t)}{\partial\theta\partial t^2} > 0$. $f_i^2(t)$ is maximum when $t = t_i^2$ *s.t.* $f_i^{2\prime}(t_i^2) = 0$.

If $t_i^1 > S_{i+1}^e$, we know that

$$f_i^{2\prime}(t_i^1) = f_i^{1\prime}(t_i^1) - \sum_{j=i+1}^{n} N_j \cdot u_0'(\theta_{i+1}, t_i^1) > 0.$$

Then we have $t_i^2 > t_i^1 > S_{i+1}^e$. In this case,

$$f_i^1(t)_{max} = f_i^1(S_{i+1}^e) = f_i^2(S_{i+1}^e) < f_i^2(t_i^2) = f_i^2(t)_{max}.$$

The inequality follows because $f_i^2(t)$ is increasing when $t < t_i^2$.

If $t_i^1 < S_{i+1}^e$, we have that

$$f_i^{2\prime}(S_{i+1}^e) = f_i^{1\prime}(S_{i+1}^e) - \sum_{j=i+1}^{n} N_j \cdot u_0'(\theta_{i+1}, S_{i+1}^e) < 0.$$

Then we have $t_i^2 < S_{i+1}^e$. In this case,

$$f_i^2(t)_{max} = f_i^2(S_{i+1}^e) = f_i^1(S_{i+1}^e) < f_i^1(t_i^1) = f_i^1(t)_{max}.$$

The inequality follows because $f_i^1(t)$ is decreasing when $t > t_i^1$.

Thus, $f_i(t_i)$ is maximum when t_i *s.t.*

$$t_i = \begin{cases} t_i^1 \text{ if } t_i^1 < S_{i+1}^e, \\ t_i^2 \text{ otherwise.} \end{cases}$$

Thus, the optimal value of t_i are determined by θ_i, r and the PH distribution.

In the case where $t_i^1 < S_{i+1}^e$, $t_i = t_i^1 < S_{i+1}^e < t_{i+1}$. In the case where $t_i^1 > S_{i+1}^e$, $t_i = t_i^2 > t_i^1 > S_{i+1}^e$. In both cases, $t_i > S_i^e$. However, in the second case, there is possibility that $t_i > t_{i+1}$, which violates the second condition for feasible contract.

4.2 Optimal Feasible Solution

In this subsection, we will show how to adjust the optimal solution in Sect. 4.1 to satisfy the sufficient conditions for feasible contract.

We first claim that $f_i(t_i)$ is concave, the proof of which can be found in Appendix A.2. Then we borrow the following proposition from [14].

Lemma 6. *Let $f_i(t)(1 \le i \le k)$ be concave functions on t and $f_i(t)$ is maximum when $t = t_i$. If $t_1 \ge t_2 \ge \cdots \ge t_k$, then $\hat{t}_1 = \hat{t}_2 = \cdots = \hat{t}_k$ where*

$$\left\{\hat{t}_1, \hat{t}_2, \cdots, \hat{t}_k\right\} = \underset{\hat{t}_1, \hat{t}_2, \cdots, \hat{t}_k}{\arg\max} \sum_{i=1}^{k} f_i(t), \text{s.t.}\, \hat{t}_1 \le \hat{t}_2 \le \cdots \le \hat{t}_k.$$

Give Lemma 6, we can adjust an infeasible sequence of $\{t_i\}_{1 \le i \le n}$ to make it satisfy the conditions in Lemma 5.

If $\{t_i\}_{1 \le i \le n}$ is infeasible, then there must be a subsequence $\{t_j, t_{j+1}, \cdots t_k\}$ that is decreasing. It can be replaced by a nondecreasing sequence $\left\{\hat{t}_j, \hat{t}_{j+1}, \cdots \hat{t}_k\right\}$ according to Lemma 6. This step can be done iteratively until there is no decreasing subsequence, which indicates that second condition for feasible contract is preserved. Then, we will show that the first condition is also preserved.

Lemma 7. *The non-decreasing sequence $\left\{\hat{t}_i\right\}_{1 \le i \le n}$* s.t. *$\hat{t}_i \ge S_i^e$.*

Proof. If $\{t_i\}_{1 \le i \le n}$ is nondecreasing, then $\hat{t}_i = t_i \ge S_e^i$. If a subsequence $\{t_j, t_{j+1}, \cdots t_k\}$ is decreasing, then

$$t_j > t_{j+1} > \cdots > t_k \ge S_k^e > \cdots > S_{j+1}^e > S_j^e.$$

We have $f_j'(S_k^e) > 0, f_{j+1}'(S_k^e) > 0, \cdots, f_k'(S_k^e) \ge 0$. We write $F(t) = \sum_{i=j}^{k} f_i(t)$. Then $F'(S_k^e) = \sum_{i=j}^{k} f_i'(S_k^e) > 0$, indicating that $F(t)$ is increasing when $t = S_k^e$.

Thus, $\hat{t}_j = \hat{t}_{j+1} = \cdots = \hat{t}_k > S_k^e > \cdots > S_{j+1}^e > S_j^e$.

Combining Lemma 5 and 6, we conclude that $\left\{\hat{t}_i\right\}_{1 \le i \le n}$ is the optimal feasible solution. Given $\left\{\hat{t}_i\right\}_{1 \le i \le n}$, the insurer can set the rewards as in Eq. (9). In this way, PHs will truthfully participate in the program and the insurer can maximize its profits.

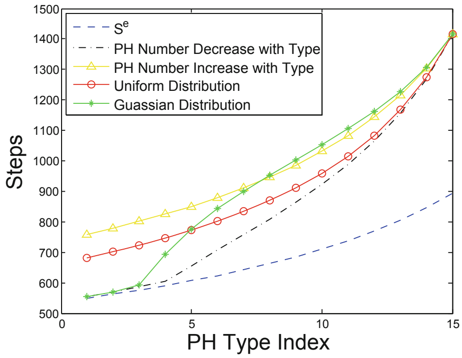
5 Numeric Results

In this section, we show the simulation results.

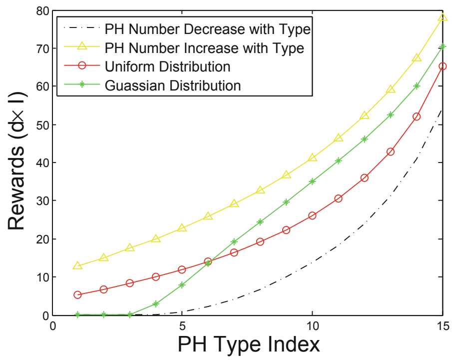
In [10], Duncan gave the model for savings estimation. With regard to physical activity, Duncan reported that each year a PH can save \$306 for per hour high intensity activities per week. We choose two models for $G(t)$: $G_1(t) = c_1 \log(t+1)$ and $G_2(t) = c_2\sqrt{t}$. They both satisfy the properties of G. We estimate that high intensity activity is equivalent to 100 steps/min. Based on this information, we can estimate c_1 and c_2.

Similarly, we choose two models for $C(\theta, t)$: $C_1 = \theta t^2$ and $C_2 = \theta t^3$. They also satisfy the properties of C. In [13], Finkelstein *et al.* reported that if paid \$9.7, participants are willing to exercise for one more hour than the control group. We set this average valuation to be the mean value of θ. The remaining θ values are evenly distributed in the range $\left[0.5\bar{\theta}, 1.5\bar{\theta}\right]$.

By default, we use G_1 and C_1 and set $r = 0.6$, $n = 15$.



(a) Step goals under different PH distribution-s.



(b) Rewards under different PH distribution.

Fig. 2. Contract under four different PH distributions.

5.1 PH Distributions

As shown in Sect. 4, the optimal solution varies with PH distribution. We show how the insurer would optimize the design under different PH distributions. Although the optimal design is generic for all kinds of PH distributions, here we discuss four cases: 1) PHs' number decreasing with type, *i.e.*, $N_1 < N_2 < \cdots < N_n$; 2) PHs' number increasing with type, *i.e.*, $N_1 > N_2 > \cdots > N_n$; 3) Uniform distribution; 4) Gaussian Distribution.

In Fig. 2, we show the optimal contract under four different type distributions. S^e does not depend on PH distribution. When PHs' number is decreasing with type (*i.e.*, $N_1 < N_2 < \cdots < N_n$), most PHs have small valuations. For the purpose of profit maximization, the insurer would set low rewards for large valuation PHs, so as to reduce small valuation PHs' interest in these contract items. Accordingly, it can set high step goals for small valuation PHs and maintain the rewards as low as possible. The situation is on the opposite when PHs' number is increasing with valuation (*i.e.*, $N_1 > N_2 > \cdots > N_n$). Compared to the previous case, it would set high step goals for large valuation PHs, so as to encourage the majority of PHs to exercise as much as possible. The case with uniform type distribution falls between the previous two cases. For Gaussian distribution, the small index part is close to the first case and the large index part is close to the second case.

5.2 Reimbursement Rate

Reimbursement rate varies in different health plans. For example, the silver plan will cover 70% of medical expenses while a gold plan will cover 80%. We show how the optimal contract would vary with the reimbursement rate. We assume that PHs' types follow Gaussian distribution.

Figure 3(a) shows that when the reimbursement rate is large, the insurer prefers PHs to exercise more than the case when the reimbursement rate is small. This is intuitive because when the insurer has to cover a large portion of medical expenses, it would like its PHs to exercise more and be healthy.

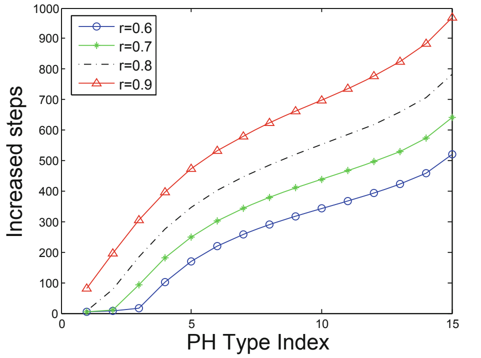
Conversely, when the reimbursement rate is large, PHs are reluctant to exercise. Thus, as shown in Fig. 3(b), the insurer has to give them higher rewards to stimulate them.

5.3 Insurer's Profits

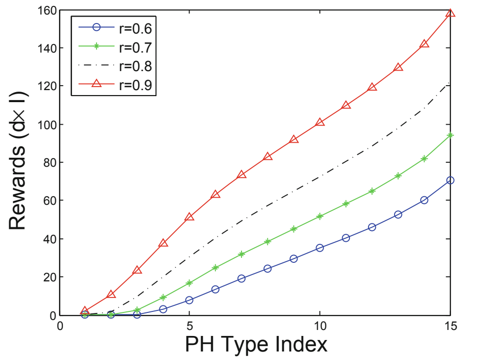
We study the insurer's profits in this subsection. We compare the optimal contract with the baseline, in which the insurer does not separate different types of PHs, neither consider their original daily step counts, and provides only one contract item $[d^*, t^*]$ for all PHs. We use exhaustive search to find the optimal d^* and t^*.

In Fig. 4, we show the insurer's profit in the baseline and the optimal contract under four combinations of $G(t)$ and $C(\theta, t)$.

1. $G_1(t) = c_1 \log(t+1), C_1(\theta, t) = \theta t^2$;
2. $G_1(t) = c_1 \log(t+1), C_2(\theta, t) = \theta t^3$;
3. $G_2(t) = c_2 \sqrt{t}, C_1(\theta, t) = \theta t^2$;
4. $G_2(t) = c_2 \sqrt{t}, C_2(\theta, t) = \theta t^3$.



(a) Increased exercise level under different reimbursement rates.



(b) Rewards for PHs under different reimbursement rates.

Fig. 3. Contract under four different reimbursement rates.

Figure 4 shows that compared with the baseline, in all four cases, the insurer can achieve higher profits using the optimal contract design. The results are averaged over the four distributions in Sect. 5.1.

According to [5], it is optimal to design a contract item for each type PHs. In this problem, paid the same amount of rewards, PHs with small valuation will exercise more than the PHs with large valuation. Thus, the insurer will prefer to allocate the financial incentives to PHs with small valuations. In the baseline mechanism, the insurer can not optimize the allocation. Furthermore, if $t^* < S_k^e$ for type-θ_k PHs, they can get the rewards without increasing their physical activity intensity. The rewards to these PHs will not change their behavior, and thus no savings for the insurer. On average, the profits in optimal contract design is 1.39 times of the profits in the baseline.

We also compare the optimal design with the baseline when n varies from 5 to 20. The results are shown in Fig. 5. On average, the insurer can improve its profit by 40%.

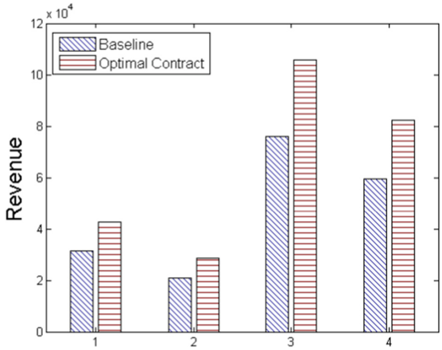


Fig. 4. The insurer's profits in the baseline and optimal contract.

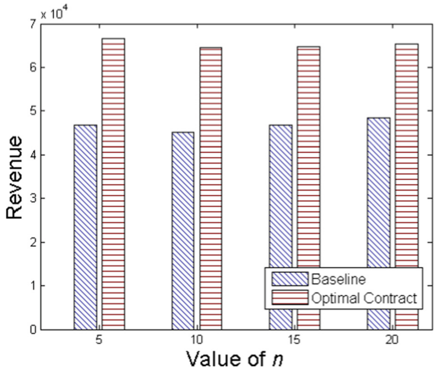


Fig. 5. The insurer's profits when n varies from 5 to 20.

6 Discussion

6.1 Investment in Wearable Devices

In this paper, we assume that every PH owns an activity tracker, such as Nike+, Fitbit, etc. In practice, wearable devices are not so widespread yet. Some insurers may distribute fitness trackers to its PHs for free [4], others may cooperate with device vendors and provide discount off these devices [1].

The cost for wearable device can be regarded as a constant. The insurer's profit should be Eq. (8) minus a constant term. However, the constant term would not change the optimal solution to the maximization problem. Thus, the optimal solution is valid when the insurer takes the cost of devices into account.

6.2 Privacy Concerns of PHs

In this paper, we do not consider the privacy concerns of PHs and assume that PHs are willing to share their fitness data with the insurer. We believe that this assumption is reasonable for three reasons. First, studies show that users are willing to trade personal information for better experience and savings. A study from IBM [17] reveals that consumers are willing to share their location, mobile number and social handle with retailers for personalized shopping experience. Similarly, Cisco [6] reports that 74% of consumers would allow driving habits to be monitored to save on insurance/service maintenance. Second, unlike blood pressure or glucose levels, the daily step counts are not sensitive information. Last, the users only need to share the aggregate statistics, such as monthly or annual average, not all the details. The insurer cannot infer further private information about the PHs.

7 Related Works

Extensive works study the effectiveness of financial incentives for health behavior change, including weight loss, smoking cessation and attendance for vaccination or screening [13,21]. There is evidence showing that finical incentives can encourage healthy behavior change. However, the effectiveness depends on various factors, such as ages, socioeconomic status, etc [13]. In this study, we focus on physical exercise intensity and use *type* to separate different cost-effectiveness groups of populations.

There are also a number of research related to health-care intervention programs [10,11]. In [10], Duncan present a literature review of population health management programs, reporting that investment in health management programs can bring financial returns, including both savings for medical expenditure and improved productivity at workplace. In this paper, we discuss how to encourage PHs to participate in physical activity. From the insurer's point of view, we give the optimal contract design, maximizing the insurer's profits.

Contract theory is widely used in job market, supply chain planning and insurance market. Besides, it also has applications in spectrum trading [8,12,14] and smartphone collaborative computing [9]. In our case, the PHs' incentives comprise of two parts: the finical rewards from the insurer and the self-motivation to keep fit, which complicates the discussion. We jointly consider these two sources of incentives and present the optimal contract design.

8 Conclusion

In this paper, we study the emerging health promotion programs in insurance markets and model the market using contract theory. We theoretically analyze how the insurer and PHs would behave in this program. We give the optimal feasible contract design, which can guarantee that PHs would truthfully participate in the program and maximize the profit of the insurer. We conduct extensive simulations to study the performance under various scenarios. Simulation results show that the insurer can improve its profit by 40% using the optimal contract. It provides a promising solution to tackle the increasing health expenditure all over the world.

Acknowledgements. This research is supported in part by the Key-Area Research and Development Program of Guangdong Province (No. 2020B0101390001) and in part by the National Natural Science Foundation of China (No. 62002150).

A Appendices

A.1 Supplementary Proof for Lemma 5

Here we provide supplementary proof for Lemma 5. As $i \leq k-1$, $t_{k-1} \geq t_i$.

To prove that

$$\begin{aligned} & u_0\left[\theta_k, \max(t_{k-1}, S_k^e)\right] + u_0\left[\theta_{k-1}, \max\left(t_i, S_{k-1}^e\right)\right] \\ & -u_0(\theta_{k-1}, t_{k-1}) \\ \geq & \, u_0\left[\theta_k, \max\left(t_i, S_k^e\right)\right], \end{aligned}$$

we distinguish five cases:

Case 1: $t_i \geq S_k^e > S_{k-1}^e$.

$$u_0\left(\theta_k, t_{k-1}\right) + u_0\left(\theta_{k-1}, t_i\right) - u_0(\theta_{k-1}, t_{k-1}) \qquad \geq u_0\left(\theta_k, t_i\right).$$

The inequality follows from Lemma 3.

Case 2: $S_{k-1}^e \leq t_i < S_k^e$ **and** $t_{k-1} \geq S_k^e$.

$$\begin{aligned} & u_0\left(\theta_k, t_{k-1}\right) + u_0\left(\theta_{k-1}, t_i\right) - u_0(\theta_{k-1}, t_{k-1}) \\ \geq & \, u_0\left(\theta_k, t_{k-1}\right) + u_0\left(\theta_{k-1}, S_k^e\right) - u_0(\theta_{k-1}, t_{k-1}) \\ \geq & \, u_0\left(\theta_k, S_k^e\right). \end{aligned}$$

The first inequality holds because $u_0(\theta_{k-1}, t)$ is decreasing when $t > S_{k-1}^e$.

Case 3: $S_{k-1}^e \leq t_i < S_k^e$ **and** $t_{k-1} < S_k^e$.

$$u_0\left(\theta_k, S_k^e\right) + u_0\left(\theta_{k-1}, t_i\right) - u_0(\theta_{k-1}, t_{k-1}) \geq u_0\left(\theta_k, S_k^e\right).$$

The inequality holds because $u_0(\theta_{k-1}, t)$ is decreasing when $t > S_{k-1}^e$, and then $u_0\left(\theta_{k-1}, t_i\right) > u_0(\theta_{k-1}, t_{k-1})$.

Case 4: $t_i < S_{k-1}^e$ **and** $t_{k-1} \geq S_k^e$.

$$\begin{aligned} & u_0\left(\theta_k, t_{k-1}\right) + u_0\left(\theta_{k-1}, S_{k-1}^e\right) - u_0(\theta_{k-1}, t_{k-1}) \\ \geq & \, u_0\left(\theta_k, t_{k-1}\right) + u_0\left(\theta_{k-1}, S_k^e\right) - u_0(\theta_{k-1}, t_{k-1}) \\ \geq & \, u_0\left(\theta_k, S_k^e\right). \end{aligned}$$

The first inequality holds because $u_0(\theta_{k-1}, t)$ is decreasing when $t > S_{k-1}^e$.

Case 5: $t_i < S_{k-1}^e$ **and** $t_{k-1} < S_k^e$.

$$u_0\left(\theta_k, S_k^e\right) + u_0\left(\theta_{k-1}, S_{k-1}^e\right) - u_0(\theta_{k-1}, t_{k-1}) \geq u_0\left(\theta_k, S_k^e\right).$$

The inequality holds because $u_0(\theta_{k-1}, t)$ is increasing when $t < S^e_{k-1}$.

In summary, type-θ_k PHs always prefer π_k over π_i.

Next, we show that $A \leq B$.

If $t_{j-1} > S^e_j$,

$$\begin{aligned} A &= \frac{1}{I}\left[u_0(\theta_j, t_{j-1}) - u_0(\theta_j, t_j)\right] \\ &\leq \frac{1}{I}\left[u_0(\theta_{j-1}, t_{j-1}) - u_0(\theta_{j-1}, t_j)\right] = B. \end{aligned}$$

The inequality follows from Lemma 3.

If $t_{j-1} \leq S^e_j$,

$$\begin{aligned} B &= \frac{1}{I}\left[u_0(\theta_{j-1}, t_{j-1}) - u_0(\theta_{j-1}, t_j)\right] \\ &\geq \frac{1}{I}\left[u_0(\theta_{j-1}, S^e_j) - u_0(\theta_{j-1}, t_j)\right] \geq A. \end{aligned}$$

A.2 Prove the Concavity of $f_i(t_i)$

Prove that

$$f_i(t_i) = \begin{cases} f^1_i(t_i) \text{ if } t_i < S^e_{i+1}, \\ f^2_i(t_i) \text{ otherwise}, \end{cases}$$

is a concave function.

Proof. For $f^{1''}_i < 0$ and $f^{2''}_i < 0$, $f^{1'}_i(t_i)$ and $f^{2'}_i(t_i)$ is decreasing. Furthermore, $f^1_i(S^e_{i+1}) = f^2_i(S^e_{i+1})$ and $f^{1'}_i(S^e_{i+1}) = f^{2'}_i(S^e_{i+1})$. But $f^{1''}_i(S^e_{i+1}) \neq f^{2''}_i(S^e_{i+1})$. Thus, $f'_i(t_i)$ is defined but $f''_i(t_i)$ is undefined when $t_i = S^e_{i+1}$.

To show that $f_i(t)$ is concave, we need to show that $\forall x_1, x_2$ and $\forall \lambda \in [0, 1]$,

$$\lambda f_i(x_1) + (1-\lambda) f_i(x_2) \leq f_i(\lambda x_1 + (1-\lambda) x_2).$$

Without loss of generality, we assume that $x_1 \leq x_2$. We distinguish four cases:

Case 1: $x_1 < S^e_{i+1}$ **and** $x_2 < S^e_{i+1}$.
It is intuitive because f^1_i is concave.

Case 2: $x_1 \geq S^e_{i+1}$ **and** $x_2 \geq S^e_{i+1}$.
It is also intuitive because f^2_i is concave.

Case 3: $x_1 < S^e_{i+1}$, $x_2 \geq S^e_{i+1}$ and $\lambda x_1 + (1-\lambda)x_2 < S^e_{i+1}$.
We write $x_0 = \lambda x_1 + (1-\lambda)x_2$. Then,

$$x_1 - x_0 = (\lambda - 1)(x_2 - x_1), x_2 - x_0 = \lambda(x_2 - x_1).$$

$$\begin{aligned}
&\lambda f_i\left(x_1\right)+(1-\lambda) f_i\left(x_2\right)-f_i\left(x_0\right) \\
=\ &\lambda\left[f_i^1\left(x_1\right)-f_i^1\left(x_0\right)\right]+(\lambda-1)\left[f_i^1\left(x_0\right)-f_i^1(S_{i+1}^e)\right] \\
&+(\lambda-1)\left[f_i^2(S_{i+1}^e)-f_i^2\left(x_2\right)\right] \\
=\ &\lambda f_i^{1\prime}(a)(x_1-x_0)+(\lambda-1) f_i^{1\prime}(b)(x_0-S_{i+1}^e) \\
&+(\lambda-1) f_i^{2\prime}(c)(S_{i+1}^e-x_2) \\
=\ &(\lambda-1)\left[f_i^{1\prime}(a)-f_i^{2\prime}(c)\right](x_2-S_{i+1}^e) \\
&+(\lambda-1)\left[f_i^{1\prime}(a)-f_i^{1\prime}(b)\right](S_{i+1}^e-x_0).
\end{aligned}$$

According to Mean Value Theorem, $a \in [x_1, x_0]$, $b \in \left[x_0, S_{i+1}^e\right]$, and $c \in \left[S_{i+1}^e, x_2\right]$. Because $f_i^{1\prime}(t)$ and $f_i^{2\prime}(t)$ is decreasing, $f_i^{1\prime}(a) \geq f_i^{1\prime}(b) \geq f_i^{1\prime}(S_{i+1}^e) = f_i^{2\prime}(S_{i+1}^e) \geq f_i^{2\prime}(c)$. Furthermore, we have $x_2 \geq S_{i+1}^e > x_0$ and $\lambda \leq 1$. Thus,

$$\lambda f_i\left(x_1\right)+(1-\lambda) f_i\left(x_2\right)-f_i\left(x_0\right) \leq 0,$$

meaning that $f_i(t)$ *s.t.* the condition for concavity in this case.

Case 4: $x_1 < S_{i+1}^e$, $x_2 \geq S_{i+1}^e$ and $\lambda x_1 + (1-\lambda)x_2 \geq S_{i+1}^e$.

We note that $f_i^2(t) \geq f_i^1(t)$ always holds. Thus,

$$\begin{aligned}
&\lambda f_i\left(x_1\right)+(1-\lambda) f_i\left(x_2\right)-f_i\left(x_0\right) \\
=\ &\lambda f_i^1\left(x_1\right)+(1-\lambda) f_i^2\left(x_2\right)-f_i^2\left(x_0\right) \\
\leq\ &\lambda f_i^2\left(x_1\right)+(1-\lambda) f_i^2\left(x_2\right)-f_i^2\left(x_0\right) \\
\leq\ &0.
\end{aligned}$$

The last line follows because f_i^2 is concave.

Therefore, we prove that $f_i(t_i)$ is a concave function.

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