

# Stackelberg Game Based Adaptive Value Evaluation Strategy



Chen-Ye Huang, Yi-Zhan Chen, and Xin-Lin Huang

**Abstract** In the process of recycling and reusing household appliances, implementing extended producer responsibility (EPR) has become increasingly important. Therefore it is necessary to design a value evaluation model to solve optimal pricing problems in closed-loop supply chain to drive producers to fulfil EPR. Based on Stackelberg Game and various recycle chain members, this paper provides two kinds of value evaluation models with the same structure but different parameters. Compared to three basic models, these two models are more concrete and practical. Besides, the profit of recycle chain in the first model is relatively high but the recycle rate is fixed at 12.5%. On the contrary, the profit of recycle chain in the second model is lower but the recycle rate can vary freely with the parameters and its total profit is a little higher. So these two models can be applied according to different situations.

**Keywords** Value evaluation · Extended producer responsibility (EPR) · Stackelberg game · Waste home appliance recycling

## 1 Introduction

In recent years, with the continuous development of global society and economy, the life standard of residents is constantly improving. Correspondingly, the electrical and electronic products industry is also expanding at a high speed and the products are constantly updated and iterated, which results in the greatly shortened service life of electronic products. These products have become waste electrical and electronic equipment prematurely and the quantity is huge, causing serious pollution and waste to the environment and resources. Furthermore, there are still problems existing in the recycling industry such as poor operation of recycling system and low reusing rate of products [1]. Therefore, some measures need to be executed to curb this situation.

Extended producer responsibility (EPR) is an effective means to recycle waste electronic products. It emphasizes that producers should not only consider the impact

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C.-Y. Huang · Y.-Z. Chen · X.-L. Huang (✉)

Department of Information and Communication Engineering, Tongji University, Shanghai, China  
e-mail: [xlhuang@tongji.edu.cn](mailto:xlhuang@tongji.edu.cn)

on the environment caused by the production process in the forward supply chain, but also assume the corresponding responsibility for the recycle process in the reverse supply chain [2]. The implementation of EPR also helps to improve the profits of supply chain members, enhance the corporate image and promote sales volume [3]. Moreover, EPR also encourages producers to incorporate environmental considerations into the design of their products [4].

Based on EPR, this paper will study the optimal sales and recycle price of each member in the closed-loop supply chain and solve the conflict among each member in the system, which will optimize the benefit and profit distribution of the closed-loop supply chain and minimize the extra cost of producers.

## 2 Research Method

### 2.1 Stackelberg Game

The pricing model of this paper is based on Stackelberg Game, a dynamic game model with complete information which reflects the asymmetric competition between enterprises [5]. In this model, there is a decision order among the members and these members make continuous decisions. The party that makes the decision first is called the “leader” and the party that makes the decision later is called the “follower”. In the decision-making process, the leader will revise its decision according to the follower’s possible decision and the follower will also consider its decision in the same way. These two sides will repeatedly consider their final decisions according to this logic until they reach Nash equilibrium.

### 2.2 Backward Induction

Stackelberg game model is usually solved by backward induction: In dynamic games like Stackelberg game, the leader should consider the decision chosen by the followers when making its decision. Only the last follower will not be affected by subsequent decisions and can directly make the optimal decision. Therefore, when the decision of last follower is determined, the game is solved. That is to say, it is assumed that all the possible decisions of each member in the game are known to all the others. Then the follower who makes the final decision starts to analyse all the possible decisions of its leaders and obtains the optimal expected decision based on these decisions. Next, according to the above logic, all the optimal decisions can be determined, so that the Nash equilibrium of the whole game can be obtained. When solving a game, we can also draw a relation tree to understand game relation and decision order more intuitively. It is important to note that in the whole game, all the members should have symmetric and complete information and they must fully understand the inter-



**Fig. 1** Flow chart of backward induction

est function of each member and the order structure of the game. In addition, all members should be rational and they must know that other members are also rational, which is the prerequisite and ideal condition to solve the game. The process of backward induction is shown in Fig. 1.

### 3 Model Building

#### 3.1 Assumptions and Parameters

This model sets three basic subjects in the closed-loop supply chain, namely producers, sellers and consumers and it also refines the recycle chain. Recycle chain in the general model is broken down into individual and enterprise recyclers, disintegrators and reproducers, among which the enterprise recyclers need to sell the recycled waste products to the disintegrators for further dismantling into usable materials. And the disintegrators will sell them to the reproducers. Individual recyclers, on the other hand, combine recycling and dismantling in one process, selling materials directly to the reproducers.

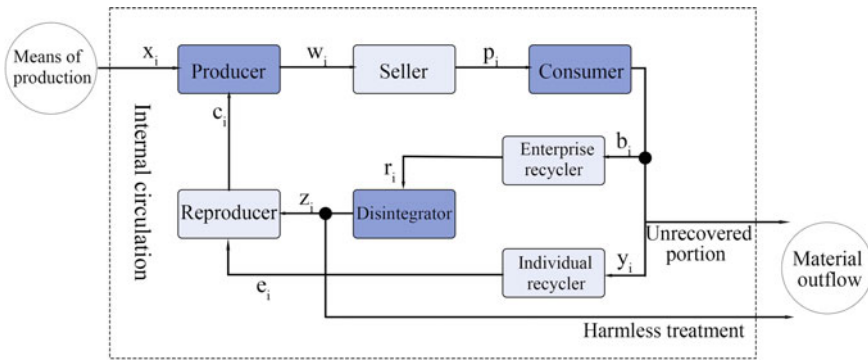
This model eliminates the cost of miscellaneous affairs such as production, operation and transportation of whole process, which is incorporated into the purchase cost and selling price, because these costs are not affected by the price optimization and adjustment strategy. In addition, it is necessary to assume that the market's supply and demand are completely balanced, which means that the quantity of products sold in the closed-loop supply chain is consistent with the actual demand of the home appliance market. The parameters are set as shown in Table 1, in which  $\beta_{rp}$  means sensitivity coefficient of reproducer to price and  $\beta_r$  means sensitivity coefficient of consumer to recycle price. These two parameters will divide the model into two different cases for calculation and discussion.

#### 3.2 Model Building

In the supply chain of this paper, the producer is the leader of Stackelberg game, while the seller and recycler are the followers. The order of the game is: the producer decides the wholesale price and recycle price, then the seller decides the retail price. The model legend is shown in Fig. 2.

**Table 1** Symbols and meanings

Symbols	Meaning	Symbols	Meaning
$x$	Cost of new means of production	$Q$	Quantity of waste products
$w$	Producer's price to seller	$\lambda$	Recycle rate of waste products
$p$	Seller's price to consumers	$\beta_{rp}$	Sensitivity of reproducer to price
$y$	Individual recycler's price to consumers	$\beta_r$	Sensitivity of consumer to recycle price
$b$	Enterprise recycler's price to consumers	$\pi_m$	Producer's profit
$z$	Disintegrator's price to reproducer	$\pi_{er}$	Enterprise recycler's profit
$r$	Enterprise recycler's price to disintegrator	$\pi_{ir}$	Individual recycler's profit
$e$	Individual recycler's price to reproducer	$\pi_s$	Seller's profit
$c$	Reproducer's price to producer	$\pi_d$	Disintegrator's profit
$a$	Demand for home appliances	$\pi_r$	Reproducer's profit
$\beta_s$	Consumer sensitivity to sales price	$\pi$	Total profit



**Fig. 2** Material circulation in the manufacturing and recycling process of household appliances

In the first model involving  $\beta_{rp}$ , both the enterprise and individual recyclers decide the recycle price to the consumers and the price to the disintegrator and reproducer based on market condition. The disintegrator will also determine the price to the reproducer. In fact, this model can be roughly regarded as two Stackelberg games with a similar structure. One is the game in supply chain and the other is the game in recycle chain. This separation is due to the fact that there is no clear direct quantitative relationship between the quantity of waste products and the sales status of products in the current market. Therefore, there is no game relationship between the consumer and recycler. The intersection point of the two chains is between the reproducer and the producer because there are transactions between them. But the price is determined

by the producer, so the game relationship still does not exist. Therefore, this model needs to analyse and solve two simple Stackelberg games.

In the second model involving  $\beta_r$ , each member of recycle chain decides its purchase price rather than its selling price, so there is no game existing in the recycle chain. Therefore, the optimal price can be calculated in turn according to the decision order. While the game in supply chain can be solved according to the method of the first model.

In order to make the model conform to the actual situation, the result of each subtraction operation in the profit function of the two models must be greater than zero.

## 4 Calculation

### 4.1 The First Model

The profit functions of members in the first model are shown in the following formulas. All of the profit functions are greater than 0 in order to be realistic. The reproducer's pricing should be less than the cost of new means of production to enable the producer to fulfill EPR.

$$\begin{aligned}\pi_m &= \{a - \beta_s p - [Q - \beta_{rp}(z + e)]\} (w - x) + [Q - \beta_{rp}(z + e)] (w - c) \\ \pi_{er} &= [Q - \beta_{rp}(z + e)] \frac{rb - b^2}{b + y}, \quad \pi_{ir} = [Q - \beta_{rp}(z + e)] \frac{ey - y^2}{b + y} \\ \pi_s &= (a - \beta_s p) (p - w), \quad \pi_d = [Q - \beta_{rp}(z + e)] \frac{b(z - r)}{b + y} \\ \pi_r &= [Q - \beta_{rp}(z + e)] \left( c - \frac{bz}{b + y} - \frac{ye}{b + y} \right), \quad \lambda = \frac{Q - \beta_{rp}(z + e)}{Q} \\ z &< c \leq x\end{aligned}\tag{1}$$

Solving by backward induction can get the result as follows:

$$y = b = \frac{Q}{8\beta_{rp}}, \quad e = r = \frac{3Q}{8\beta_{rp}}, \quad z = \frac{Q}{2\beta_{rp}}, \quad w = \frac{a}{2\beta_s} + \frac{x}{2}, \quad p = \frac{3a}{4\beta_s} + \frac{x}{4}\tag{2}$$

$$\begin{aligned}\pi_m &= \frac{a^2}{8\beta_s} - \frac{ax}{4} + \frac{\beta_s x^2 + Qx - Qc}{8}, \quad \pi_{er} = \pi_{ir} = \frac{Q^2}{64\beta_{rp}} \\ \pi_s &= \frac{(a - \beta_s x)^2}{16\beta_s}, \quad \pi_d = \frac{Q^2}{128\beta_{rp}} \\ \pi_r &= \frac{Q}{8}c - \frac{7Q^2}{128\beta_{rp}} > \frac{Q^2}{128\beta_{rp}}, \quad \lambda = 12.5\%\end{aligned}\tag{3}$$

**Fig. 3** Seller recycling mode block diagram



### 4.2 The Second Model

The profit functions of members in the second model are shown in the following formulas. Unlike the first model, there is an optimal solution for the pricing of all members.

$$\begin{aligned}
 \pi_m &= [a - \beta_s p - \beta_r(b + y)](w - x) + \beta_r(b + y)(w - c) \\
 \pi_{er} &= \beta_r b(r - b), \quad \pi_{ir} = \beta_r y(e - y) \\
 \pi_s &= (a - \beta_s p)(p - w), \quad \pi_d = \beta_r b(z - r) \\
 \pi_r &= \beta_r(b + y)c - \beta_r b z - \beta_r y e, \quad \lambda = \frac{\beta_r(b + y)}{Q}
 \end{aligned}
 \tag{4}$$

Solving by backward induction can get the result as follows:

$$b = \frac{x}{16}, y = r = \frac{x}{8}, z = e = \frac{x}{4}, c = \frac{x}{2}, p = \frac{3a + \beta_s x}{4\beta_s}, w = \frac{a + \beta_s x}{2\beta_s}
 \tag{5}$$

$$\begin{aligned}
 \pi_m &= \frac{(a - \beta_s x)^2}{8\beta_s} + \frac{3\beta_r x^2}{32}, \quad \pi_{er} = \frac{\beta_r x^2}{256}, \quad \pi_{ir} = \frac{\beta_r x^2}{64} \\
 \pi_s &= \frac{(a - \beta_s x)^2}{16\beta_s}, \quad \pi_d = \frac{\beta_r x^2}{128}, \quad \pi_r = \frac{3\beta_r x^2}{64}, \quad \lambda = \frac{3\beta_r x}{16Q}
 \end{aligned}
 \tag{6}$$

## 5 Example Analysis

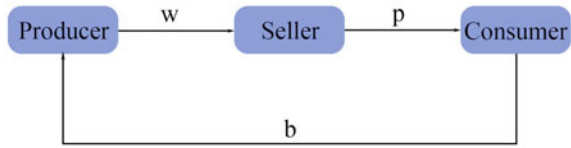
### 5.1 Three Basic Recycle Models

The block diagram of the seller recycling model and the optimal pricing and profit functions of its members are shown in Fig. 3. This is a model where the seller participates in recycling.

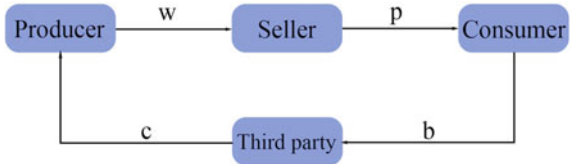
$$w = \frac{a + \beta_s x}{2\beta_s}, c = \frac{x}{2}, b = \frac{x}{4}, p = \frac{3a + \beta_s x}{4\beta_s}
 \tag{7}$$

$$\pi_m = \frac{(a + \beta_s x)^2}{8\beta_s} + \frac{\beta_r x^2}{8} - \frac{ax}{2}, \quad \pi_s = \frac{(a - \beta_s x)^2}{16\beta_s} + \frac{\beta_r x^2}{16}
 \tag{8}$$

**Fig. 4** Producer recycling mode block diagram



**Fig. 5** Third party recycling model



The block diagram of the producer recycling model and the optimal pricing and profit functions of its members are shown in Fig. 4. In this model, only the producer is responsible for recycling.

$$w = \frac{a + \beta_s x}{2\beta_s}, b = \frac{x}{2}, p = \frac{3a + \beta_s x}{4\beta_s} \tag{9}$$

$$\pi_m = \frac{(a + \beta_s x)^2}{8\beta_s} + \frac{\beta_r x^2}{4} - \frac{ax}{2}, \pi_s = \frac{(a - \beta_s x)^2}{16\beta_s} \tag{10}$$

The block diagram of the third party recycling model and the optimal pricing and profit functions of its members are shown in Fig. 5. There is a third party responsible for recycling in this model.

$$w = \frac{a + \beta_s x}{2\beta_s}, c = \frac{x}{2}, b = \frac{x}{4}, p = \frac{3a + \beta_s x}{4\beta_s} \tag{11}$$

$$\pi_m = \frac{(a - \beta_s x)^2 + \beta_s \beta_r x^2}{8\beta_s}, \pi_s = \frac{(a - \beta_s x)^2}{16\beta_s}, \pi_r = \frac{\beta_r x^2}{16} \tag{12}$$

### 5.2 Examples

**Example of the models of this paper.**

Parameter setting:  $x = 10, a = 300, \beta_s = 15, \beta_{rp} = 20(\beta_r = \frac{50}{3}), c = 8, Q = 250.$

It can be seen from Table 2 that when the recycle rate is the same, total profit of the second model is slightly higher than that of the first model and the profit of the producer is much higher. While the profit of all members in the recycle chain is lower. Therefore, it can be inferred that the game of the recycle chain increases its members' profit, but decreases the total profit of whole closed-loop supply chain.

**Table 2** Example of two models in this paper

	$\pi_m$	$\pi_s$	$\pi_{er}$	$\pi_{ir}$	$\pi_d$	$\pi_r$	$\pi$	$\lambda$ (%)
Model 1	250	93.75	48.83	48.83	24.41	79.10	544.92	12.5
Model 2	343.75	93.75	6.51	26.04	13.02	78.13	561.20	12.5

**Table 3** Comparison of profit and recycle rate of all parties

	Seller recycling model	Producer recycling model	Third party recycling model	Model 2 in this paper
$\pi_m$	437.5	687.5	437.5	375
$\pi_s$	218.75	93.75	93.75	93.75
$\pi_r$	–	–	125	93.75
$\pi$	656.25	781.25	656.25	617.19
$\lambda$ (%)	20	40	20	15

**Example of three basic models and the second model in this paper.**

Parameter setting:  $x = 10, a = 300, \beta_s = 15, \beta_r = 20, Q = 250$ .

As can be seen from the results in Table 3, when the parameters of models are the same, the total profit and recycle rate of the optimal pricing strategy will decrease with the increase of the number of members of the recycle chain. This is because with the increase of members, the price of recycling waste products from consumers will gradually decrease in order to ensure the profits of each member. Moreover, in all models, producers account for the largest proportion of total profit, which is their advantage as the game leader and also improves their motivation to reproduce and fulfill EPR.

## 6 Conclusion

1. The first model of the paper is applicable to the situation where there is two games existing in both the recycle chain and the supply chain. The second model is applicable to the case that there is no game existing in the recycle chain.
2. The total profit of the second model is slightly higher than that of the first. But the profit of members in the recycle chain is quite low, which may lead to the decline of recycle efficiency. This shows that game can make the recycle chain gain more profits.
3. In order to maximize the profit of closed-loop supply chain and the producer, the number of members in the recycling chain should be reduced as much as possible to ensure that the waste products can be recycled from consumers at the highest possible price.



**Acknowledgements** This work was supported by the National Key R&D Program of China under Grant 2022YFB3305801.

## References

1. Han H, Fan X, Zhang Q, Du Y (2022) Situation and development trend for recycling of electrical and electronic equipment. *Comput Integr Manuf Syst* 28(7)
2. Jia J (2020) Study on recycling mode of waste electrical appliances and electronic products under extended producer responsibility system. Taiyuan University of Technology
3. Zhou J, Tao X (2016) Research on revenue-sharing contract of sales-recycling closed-loop supply chain with EPR. *Sci Decis Mak* 02:39–57
4. Subramanian R, Gupta S, Talbot B (2009) Product design and supply chain coordination under extended producer responsibility. *Prod Oper Manag* 18(3):259–277
5. Zhou L, Ma Y, Ki F (2021) Research on dual-model recycling pricing of waste household appliances based on Stackelberg game. In: 2021 international conference on E-commerce and E-management (ICECEM), pp 170–173