



Re-Calibrating of Dissipation Models for Breaking Wave Based on Parametric Wave Approach Using Root-Mean-Square Height and Mean Wave Height

Loc Xuan Luu^{1,2}, Nga Thanh Duong^{1,2(✉)}, Linh Hoang Tran^{1,2},
and Truong Quang Nguyen^{1,2}

¹ Faculty of Civil Engineering, Ho Chi Minh City University of Technology (HCMUT), 268 Ly Thuong Kiet Street, District 10, Ho Chi Minh City, Vietnam

dt.hnga@hcmut.edu.vn

² Vietnam National University Ho Chi Minh City, Linh Trung Ward, Thu Duc City, Ho Chi Minh City, Vietnam

Abstract. Wave height transformation plays a crucial role in the investigation of beach deformation and coastal structure design. Parametric wave approach is a widely used method for calculating the transformation of root-mean-square wave height only. The accuracy of existing models depends on wave and beach conditions. Due to the advantage and computational efficiency of parametric wave approach, it would be useful if this method could also be used for modeling mean wave height transformation. This paper focuses on improving the accuracy of existing parametric wave models in modeling root-mean-square wave height transformation under a wide range of wave and beach conditions. Additionally, the paper also explores the applicability of the parametric wave approach in predicting mean wave height transformation by recalibrating coefficients of existing models. Seven parametric wave models are examined and calibrated using both laboratory and field data to predict both root-mean-square and mean wave height transformation. The findings of the study show that the accuracy of root-mean-square wave height transformation prediction is enhanced after modifying coefficients. Furthermore, the paper demonstrates that the parametric wave approach can be effectively employed to predict mean wave height transformation. With adjusted coefficients, parametric wave models estimate mean wave height with great accuracy.

Keywords: Parametric wave approach · Root-mean-square wave height · Mean wave height · Energy dissipation · Wave height transformation

1 Introduction

In coastal engineering, waves are an important field of study that receives significant attention. As waves travel from offshore to the shoreline, they break near the shoreline, resulting in a gradual reduction in wave energy and wave height. The transformation

of waves from offshore to shoreline involves a natural mechanism that is not yet fully understood by humans. However, it is known that wave height plays a crucial role in studying beach deformation and designing coastal structures, as waves approaching the shoreline can cause significant changes and damage to the coast and coastal infrastructure. Therefore, many researchers focus on calibrating wave height [4, 5, 10, 11, 16]. Wave height is calculated based on the energy conservation law using wave energy dissipation models. Various methods, such as the representative method, parametric wave method, and probability method, can be used to develop energy dissipation models. The parametric wave method is simple and popular for determining root-mean-square wave height (H_{rms}). Most energy dissipation models are developed using a semi-empirical approach based on theory and experimental data. Consequently, the accuracy of the computed results depends on the quality of the dataset used, which includes data size and wave and bottom conditions. To ensure the accuracy of the models, it is necessary to propose models based on large datasets and various wave and seabed conditions. However, existing models have been developed based on small datasets and poor experimental conditions. Therefore, the verification and calibration of existing models using a large amount of updated data are vital to improve the accuracy of calculating wave height.

Parametric wave approach is applied to H_{rms} only. Nevertheless, due to its advantages, it is convenient if this method can be applied to mean wave height (H_m). Currently, there is limited study on calculating mean wave height using this method. Hence, it is necessary to examine the feasibility of calculating mean wave height using parametric wave approach.

The present research aims to verify and calibrate existing parametric wave models for predicting H_{rms} under a wide range of experimental conditions. Besides, the application of parametric wave approach for estimating mean wave height is assessed. A large amount of experimental and field data is used for calculation. Seven existing parametric wave models are collected for verification and calibration of their capacity to predict root-mean-square and mean wave height.

2 Data Collection and Existing Models

To confirm the predictive capacity of existing parametric wave models, a large amount of data, including 918 data points of root-mean-square wave height and 979 data points of mean wave height, was collected from four sources as shown in Table 1. These data belong to small-scale [12], large-scale [7], and field-scale experiments [6, 15].

Several parametric wave models have been developed for modeling energy dissipation as listed in Table 2. Battjes et al. [2] and Thornton et al. [14] developed models based on the energy dissipation of regular waves and the probability of wave breaking. Other studies [1, 3, 8, 13] modified formulas from previous studies to improve the accuracy of wave height prediction. These models are typically based on the bore concept. Rattanapitikon et al. [9] proposed a wave energy dissipation model based on the stable energy concept. Bore concept is based on the similar in mechanism of hydraulic jump and breaking wave. Meanwhile, stable energy concept assumes that energy flux is proportional to the difference in energy between local waves and stable wave.

Wave height transformation is computed from collected data and parametric wave model by the energy flux conservation equation (Eq. 1).

$$\frac{\partial(Ec_g)}{\partial x} = -D_B \quad (1)$$

$$E = \frac{1}{8} \rho g H_{rms}^2 \quad (2)$$

$$c_g = \frac{L}{T_p} \left(\frac{1}{2} + \frac{kh}{\sinh(2kh)} \right) \quad (3)$$

where x is the distance in cross-shore direction, E is the linear wave energy calculated from Eq. 2, c_g is the group velocity calculated from Eq. 3, k is the wave number, h is the depth of water, ρ is the water density, g is the gravity acceleration, T_p is the wave period, L is the wavelength, and D_B is energy dissipation computed from energy dissipation models which are listed in Table 2. In this table, H_b is the breaker height, Q_b is the fraction of the breaking wave, H_{rms0} and L_0 are the root-mean-square wave height and wavelength at deep water, and c is the wave velocity. In this study, energy due to wave fraction is ignored. After substituting D_B equation into Eq. 1, the energy flux conservation equation is solved by numerical integration from offshore to shoreline to determine wave height. x , h , and incident wave (H_{rms} and T_p) are input data, other parameters are determined based on linear wave theory from input data.

Table 1. Collected experimental data

Sources	No. of cases	No. of data	
		H_{rms}	H_m
Smith and Kraus [12] (SK90)	6	40	161
Kraus and Smith [7] (KS94)	50	800	800
Hotta et al. [6] (HMI82)	3	18	18
Thornton and Guza [14] (TG86)	4	60	–
Total	63	918	979

3 Model Examination

In this study, both H_{rms} and H_m are calculated from seven parametric models. to evaluate the prediction capacity of seven existing models, the root-mean-square relative error is used as Eq. 4.

$$RMSE = 100 \sqrt{\frac{\sum_{i=1}^N (H_{c_i} - H_{m_i})^2}{\sum_{i=1}^N H_{m_i}^2}} \quad (4)$$

Table 2. Existing energy dissipation models

Sources	K	Formulas
Battjes and Janssen [2] (BJ78)	$K_1 = 0.91$	$D_B = Q_b \frac{\rho g H_b^2}{4T_p} \frac{1 - Q_b}{-\ln Q_b} = \left(\frac{H_{rms}}{H_b} \right)^2$ $H_b = 0.14 L \tanh(K_1 kh)$
Thornton and Guza [14] (TG83)	$K_2 = 0.42$	$D_B = 0.51 \frac{3\sqrt{\pi}}{4} \left(\frac{H_{rms}}{H_b} \right)^2 \left\{ 1 - \frac{1}{[1 + (H_{rms}/H_b)^2]^{2.5}} \right\} \frac{\rho g H_{rms}^3}{4T_p h}$ $H_b = K_2 h$
Battjes and Stive [3] (BS85)	$K_3 = 0.57$ $K_4 = 0.45$ $K_5 = 33$	$D_B = Q_b \frac{\rho g H_b^2}{4T_p}$ $H_b = 0.14 L \tanh \left\{ \left[K_3 + K_4 \tanh \left(K_5 \frac{H_{rms} 0}{L_0} \right) \right] kh \right\}$
Southgate and Nairn [13] (SN93)	$K_6 = 0.39$ $K_7 = 0.56$ $K_8 = 33$	$D_B = Q_b \frac{\rho g H_b^3}{4T_p h}$ $H_b = h \left[K_6 + K_7 \tanh \left(K_8 \frac{H_{rms} 0}{L_0} \right) \right]$
Baldock et al. [1] (BHV98)	$K_9 = 0.39$ $K_{10} = 0.56$ $K_{11} = 33$	$D_B = \begin{cases} \exp \left[- \left(\frac{H_b}{H_{rms}} \right)^2 \right] \frac{\rho g (H_b^2 + H_{rms}^2)}{4T_p} & \text{for } H_{rms} < H_b \\ \exp[-1] \frac{2\rho g H_b^2}{4T_p} & \text{for } H_{rms} \geq H_b \end{cases}$ $H_b = h \left[K_9 + K_{10} \tanh \left(K_{11} \frac{H_{rms} 0}{L_0} \right) \right]$
Rattanapitikon and Shibayama [9] (RS98)	$K_{12} = -1.5$ $K_{13} = 15$	$D_B = 0.10 Q_b \frac{c\rho g}{8h} \left[H_{rms}^2 - \left(\text{hexp}(-0.58 - 2.0 \frac{h}{\sqrt{L H_{rms}}}) \right)^2 \right]$ $H_b = 0.1 L_0 \left\{ 1 - \exp \left[K_{12} \frac{\pi h}{L_0} \left(1 + K_{13} m_b^{4/3} \right) \right] \right\}$
Rattanapitikon and Sawanggun [8] (RS08)	$K_{14} = 0.57$ $K_{15} = 0.45$ $K_{16} = 33$	$D_B = \frac{\rho g H_b^2}{4T_p} \left[2.096 \left(\frac{H_{rms}}{H_b} \right)^2 - 1.601 \left(\frac{H_{rms}}{H_b} \right) + 0.293 \right]$ $H_b = L \tanh \left\{ \left[K_{14} + K_{15} \tanh \left(K_{16} \frac{H_{rms} 0}{L_0} \right) \right] kh \right\}$

where *RMSE* is the root-mean-square relative error, H_{c_i} is the *i*th predicted wave height, H_{m_i} is the *i*th observed wave height, and *N* is the total data point number.

Table 3 displays the errors for each experiment and the average error of seven existing models for predicting root-mean-square wave height. the table indicates that the errors of the models range from 5.1% to 27.91% when considering each experiment. RS08 shows the Lowest Error with 7.81% and 13.22% for SK90 and TG86, respectively. In the case of KS94, RS98 is the best formula for estimating H_{rms} with an error of 5.48%. Meanwhile, the data of H82 is best predicted by BJ78 (5.1%). In terms of average RMSE, RS98 with an error of 7.69% is the most accurate model for predicting root-mean-square wave height. The formula of SN93 shows the worst predictive capacity (11.01%). These results indicate that data from different sources cannot be well predicted by a single model.

The results of applying parametric wave models to calculate mean wave height are shown in Table 4. In this case, the *RMSE* ranges from 5.17% (RS98) to 22.41% (TG83). BS85, RS98, and BJ78 exhibit great performance with the data from SK90 (6.89%), KS94 (5.17%), and H82 (5.52%), respectively. As for the average *RMSE*, the difference

in error among different models is slight. The model with the highest accuracy is RS98 (5.47%), followed by RS08 (6.96%), BJ78 (7.11%), BS85 (7.33%), SN93 (7.41%), TG83 (7.75%), and BHV98 (7.83%).

Table 3. Error of seven models with default coefficient for computing root-mean-square wave height

Models	SK90 (%)	KS94 (%)	H82 (%)	TG86 (%)	Avg. RMSE (%)
BJ78	10.48	7.35	5.10	27.38	10.66
TG83	26.01	8.89	9.34	17.96	10.13
BS85	8.35	7.49	13.81	15.18	9.27
SN93	14.14	6.73	11.77	27.91	11.01
BHV98	14.32	6.47	18.62	16.47	9.74
RS98	12.05	5.48	9.97	16.48	7.69
RS08	7.81	6.61	13.88	13.22	8.44

Table 4. Error of seven models with default coefficient for computing mean wave height

Models	SK90 (%)	KS94 (%)	H82 (%)	Avg. RMSE (%)
BJ78	8.07	7.29	5.52	7.11
TG83	22.41	7.83	6.60	7.75
BS85	6.89	6.71	10.94	7.33
SN93	16.32	7.10	9.23	7.41
BHV98	16.28	5.97	15.89	7.83
RS98	9.66	5.17	7.26	5.47
RS08	8.73	6.18	11.21	6.96

4 Model Calibration

In this section, the coefficients (K_1 - K_{16}) are recalibrated to improve the accuracy of seven energy dissipation models for estimating H_{rms} and H_m . The values of K_1 - K_{16} are adjusted until the lowest *RMSE* is obtained.

After the calibration of coefficients, the new coefficients and errors are listed in Table 5. It can be observed that the coefficient and error of BJ78 remain the same as in the existing model. For the other formulas, the coefficients change, and the errors are improved after modification. With the modified coefficients, BS85 and BJ78 show the best performance with respect to the data of SK90 (6.79%) and H82 (5.1%), respectively. In terms of SK94 and TG86, RS98 and RS08 provide the lowest errors with 5.2% and

10.4%, respectively. The table also reveals that the average *RMSE* reduces by 0.48% to 2.64% when recalibrating coefficients (except for BJ78). RS98, with an error of 6.33%, is still an excellent formula for estimating H_{rms} . However, BJ78, with an unchanged error, has the highest error when compared with the rest of the models. In general, modifying the coefficients leads to an enhancement in the *RMSE* in predicting H_{rms} .

Table 6 lists the modified coefficients, *RMSE* of each experiment, and average *RMSE* of all experiments when using the parametric wave model for computing mean wave height. BS85, RS98, and BJ78 are the top models providing the lowest accuracy when considering each experiment, namely that the error of BS85 and RS98 is 7.86% and 5.13%, respectively, for predicting wave height of SK90 and KS94, and the error of BJ78 and BS85, in terms of H82, is 5.3%. The average error of all experiments ranges from 5.18% to 7.08%. It reduces by 0.03% to 2.51% after modifying the coefficients. RS98, predicting mean wave height with the best accuracy (average *RMSE* = 5.18%), is followed by BHV98 (5.32%) and SN93 (5.68%). These are the top three models in estimating mean wave height. The application of BJ78 results in the worst error (7.08%).

It can be observed from the results of predicting H_{rms} and H_m that there is no single best model for all experiments. Besides, RS98 is the best model for both wave height predictions, whether before or after recalibrating coefficients. In this model, the author changed the energy dissipation formula of BJ78 from the bore concept to the stable concept. For the rest of the models, the formulas are developed based on the bore concept. It seems that the application of the stable concept to develop the energy dissipation formula leads to better results in estimating wave height when compared with the bore concept.

Table 5. Error of seven models with calibration coefficient for computing root-mean-square wave height

Models	K	SK90 (%)	KS94 (%)	H82 (%)	TG86 (%)	Avg. RMSE (%)
BJ78	$K_1 = 0.91$	10.48	7.35	5.10	27.38	10.66
TG83	$K_1 = 0.51$	19.31	7.30	5.42	22.79	9.65
BS85	$K_3 = 0.59; K_4 = 0.64; K_5 = 33$	6.79	6.61	10.47	16.20	8.39
SN93	$K_6 = 0.42; K_7 = 0.27; K_8 = 316$	12.62	5.44	5.59	27.13	9.67
BHV98	$K_9 = -0.50; K_{10} = 1.28; K_{11} = 693$	12.46	5.45	5.22	16.18	7.10
RS98	$K_{12} = -2.6; K_{13} = -21.8$	9.60	5.20	6.86	12.18	6.33
RS08	$K_{14} = 0.29; K_{15} = 0.69; K_{16} = 245$	8.48	6.28	5.21	10.40	6.66

Table 6. Error of seven models with calibration coefficient for mean wave height

Models	K	SK90 (%)	KS94 (%)	H82 (%)	Avg. RMSE (%)
BJ78	$K_1 = 0.89$	8.40	7.28	5.30	7.08
TG83	$K_2 = 0.48$	18.83	7.08	4.91	6.90
BS85	$K_3 = 0.45$; $K_4 = 0.48$; $K_5 = 162$	7.86	6.58	5.30	6.44
SN93	$K_6 = 0.59$; $K_7 = 0.05$; $K_8 = 36$	13.27	5.62	5.90	5.68
BHV98	$K_9 = 0.28$; $K_{10} = 0.45$; $K_{11} = 300$	12.98	5.27	5.47	5.32
RS98	$K_{12} = -1.89$; $K_{13} = -$ 3.3	9.44	5.13	5.46	5.18
RS08	$K_{14} = 0.75$; $K_{15} = 2.3$; $K_{16} = 2.33$	8.60	6.40	6.63	6.43

5 Conclusions

The current research examines and recalibrates coefficients to improve the accuracy of parametric wave models for estimating root-mean-square and mean wave height. Seven existing models are collected for the computational process under a large dataset of 918 data points for root-mean-square wave height and 979 data points for mean wave height. The results show that *RMSE* is improved after the modification of coefficients. The average *RMSE* of the models reduces by 0.48% to 2.64% and 0.03% to 2.51% when modeling root-mean-square and mean wave height after recalibration, respectively. With the new coefficients, the top three formulas showing the best error for estimating H_{rms} are RS98 (6.33%), RS08 (6.66%), and BHV98 (7.01%). In terms of H_m , the models that perform with excellent results are RS98 (5.18%), BHV98 (5.32%), and SN93 (5.68%). RS98 stands out as the outstanding model with the lowest error for estimating wave height in cases of both default and recalibrated coefficients. These top three formulas are recommended for use in modeling H_{rms} and H_m .

Acknowledgment. We acknowledge Ho Chi Minh City University of Technology (HCMUT), VNU-HCM for supporting this study.

References

1. Baldock TE, Holmes P, Bunker S, Van Weert P (1998) Cross-shore hydrodynamics within an unsaturated surf zone. *Coast Eng* 34:173–196. [https://doi.org/10.1016/S0378-3839\(98\)00017-9](https://doi.org/10.1016/S0378-3839(98)00017-9)
2. Battjes JA, Janssen JPFM (1978) Energy loss and set-up due to breaking of random waves. In: *Proceedings of the coastal engineering conference*. ASCE, pp 569–587
3. Battjes JA, Stive MJF (1985) Calibration and verification of a dissipation model for random breaking waves. *J Geophys Res Ocean* 90:9159–9167. <https://doi.org/10.1029/JC090IC05P09159>
4. Duong NT, Tran KQ, Luu LX, Tran LH (2023) Prediction of breaking wave height by using artificial neural network-based approach. *Ocean Model* 182:102177. <https://doi.org/10.1016/j.ocemod.2023.102177>
5. Elbisy MS, Elbisy AMS (2021) Prediction of significant wave height by artificial neural networks and multiple additive regression trees. *Ocean Eng* 230:109077. <https://doi.org/10.1016/j.oceaneng.2021.109077>
6. Hotta S, Mizuguohi M, Isobe M (1982) A field study of waves in the nearshore zone. In: *Coastal engineering proceedings*, pp 38–57
7. Kraus NC, Smith MJ (1994) SUPERTANK laboratory data collection project. Tech Rep CERC-94-3, WES, US Army Corps Eng Vols 1 2
8. Rattanapitikon W, Sawanggun S (2008) Energy dissipation model for a parametric wave approach based on laboratory and field experiments. *Songklanakarin J Sci Technol* 30:333–341
9. Rattanapitikon W, Shibayama T (1998) Energy dissipation model for irregular breaking waves. *Coast Eng J* 40:327–346. <https://doi.org/10.1061/9780784404119.007>
10. Rattanapitikon W, Shibayama T (2006) Breaking wave formulas for breaking depth and orbital to phase velocity ratio. *Coast Eng J* 48:395–416. <https://doi.org/10.1142/S0578563406001489>
11. Rattanapitikon W, Tran KQ, Shibayama T (2015) Estimation of Maximum Possible Wave Heights in Surf Zone. *Coast Eng J* 57:1550001-1-1550001–19. <https://doi.org/10.1142/S0578563415500011>
12. Smith ER, Kraus NC (1990) Laboratory study on macro-features of wave breaking over bars and artificial reefs. Tech Rep CERC-90-12, WES, US Army Corps Eng
13. Southgate HN, Nairn RB (1993) Deterministic profile modelling of nearshore processes. Part 1. Waves and currents. *Coast Eng* 19:27–56
14. Thornton EB, Guza RT (1983) Transformation of wave height distribution. *J Geophys Res* 88:5925–5938. <https://doi.org/10.1029/JC088iC10p05925>
15. Thornton EB, Guza RT (1986) Surf zone longshore currents and random waves: Field data and models. *J Phys Oceanogr* 16:1165–1178
16. Tran KQ, Duong NT, Luu LX, Tran LH, Rattanapitikon W (2023) Development of novel parametric wave model for irregular wave height transformation. *Ocean Eng* 278:114493. <https://doi.org/10.1016/j.oceaneng.2023.114493>