



# Couple Effect of Loading Frequency and Uniformity Coefficient on the Liquefaction Resistance of Sand

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**Abstract.** The uniformity coefficient ( $C_u$ ) of sand and the cyclic loading frequency ( $f$ ) is unclear in affecting soil's cyclic undrained behavior. This note presents experimental research on the combined effect of  $f$  and  $C_u$  on the liquefaction resistance of the sand. A series of constant-volume, stress-controlled, cyclic direct simple shear tests (CDSS) were performed on silica sands mixed with different particle proportions to make both poorly ( $SP$ ) and well-graded ( $SW$ ) samples.  $SP$  and  $SW$  samples are deposited in a medium-density dry state, consolidated to vertical stress of 100 kPa, and cyclically loaded under a cyclic stress ratio of 0.1 with various loading frequency ( $f = 0.03, 0.05, 0.1, 0.2, \text{ and } 0.5$  Hz). In  $SP$  sand, the number of cycles to cause liquefaction ( $N_{cyc}$ ) remains unchanged when the load frequency rises from 0.03 Hz to 0.1 Hz, and increases when the load frequency rises from 0.1 Hz to 0.5 Hz. It can be stated that  $SP$  sand's liquefaction resistance is affected by the high loading frequency. On the contrary, the effect of loading frequency on the  $N_{cyc}$  of  $SW$  sand is negligible. Furthermore,  $SP$  sand is more resistant to liquefaction than  $SW$  sand. According to this study,  $C_u$  and  $f$  should be included in the sand's liquefaction resistance analysis.

**Keywords:** Liquefaction Resistance · Loading Frequency · Uniformity Coefficient · Poorly graded sand · Well-graded sand

## 1 Introduction

The frequency ( $f$ ) of the actual earthquake motions is random and varies from a few seconds to a few hertz. On the other hand, due to the limitation of the devices, laboratory tests were performed at lower frequencies (usually 0.1 Hz). Hence,  $f$  plays an important effect in laboratory testing results.

The effects of cyclic loading frequency on the liquefaction resistance of sand are summarized in Table 1. Some early studies found that increasing  $f$  had little impact on sand liquefaction resistance, but some more recent studies found that liquefaction resistance increased as  $f$  increased. Interestingly, some findings indicated that liquefaction resistance decreased with increased  $f$  or remained unchanged with a small  $f$  and

increased with  $f$ . Therefore, the  $f$  effect on sand's liquefaction resistance is an interesting topic that needs further investigation.

A literature review of the effect on liquefaction resistance of the uniformity coefficient ( $C_u$ ) also is shown in Table 2, which cannot be drawn to a consistent conclusion.

**Table 1.** The literature review of the effect of  $f$  on liquefaction resistance [1]

Authors	Test Method	$D_r$ (%)	$\sigma'$ (kPa)	Failure criteria	F (Hz)	Effect on liquefaction resistance
Peacock and Seed (1968)	CDSS	–	–	–	1/6–4	small
Lee and Fitton (1969)	CTX	50, 75	100	$DA = 5, 10, 20\%$	0.17, 1	increase with increases $f$
Yoshimi and Oh-Oka (1975)	RTT	37	95.2	$r_u = 1$	1–12	nearly independent
Chang et al. (1982)	CTX	16.93–43.11	–	–	0.0001–1	+No effect when $f$ is less than 0.01 Hz and increases with $f > 0.01\text{Hz}$
Tatsuoka et al. (1983)	CTX	50 - 85	98	$DA = 10\%$	0.05, 0.5	non effect
Tatsuoka et al. (1986)	CTX	50, 80	100	$DA = 10\%$	0.05, 1	rather insensitive
Polito (1999)	CTX	74	100	$r_u = 1$	0.5, 1	no difference
Aghaei Araei et al. (2012)	CTX	–	–	–	0.1–10	less dependent
Dash and Sitharam (2016)	CTX	54	100	$r_u = 1$	0.1–0.5	decrease with increases $f$
Nong et al. (2020)	CDSS	40, 80	50, 100, 200	$DA = 7.5\%$	0.05–1	increase with increases $f$
Zhu et al. (2021)	CTX	70	100	$SA = 3\%$	0.01–5	increase with increases $f$
Zeybek (2022)	CTX	39, 90	100	$r_u = 1$	0.1, 1	increase with increases $f$

Note  $D_r$  – relative density;  $\sigma'$  – confining stress; CTX—Cyclic Triaxial Test; RTT—Ring Torsion Test;  $r_u$ —excess pore pressure ratio;  $DA$ —double amplitude axial strain;  $SA$ —single amplitude axial strain

**Table 2.** The literature review of the effect of  $C_u$  on liquefaction resistance

Authors	Test Method	$C_u, C_c$	$D_{10}, D_{30}, D_{50}, D_{60}$ (mm)	Findings about liquefaction resistance
Vaid et al. (1990) [2]	CTX	$C_u = 1.5; 3; 6$	$D_{50} = 0.42$	+ $D_r$ low: poorly-graded sand less than well-graded sand + $D_r$ high: opposite trend
Kokusho et al. (2004) [3]	CTX	$C_u = 1.44\text{--}13.1$	$D_{50} = 0.14; 0.4; 1.15$	+Minor differences in cyclic liquefaction resistance
Yilmaz et al. (2008) [4]	CTX	$C_u = 1\text{--}12$ $C_c = 1\text{--}10$	$D_{10} = 0.04\text{--}1.02$ $D_{30} = 0.05\text{--}1.05$ $D_{60} = 0.06\text{--}1.1$	+No relationship between $C_u, C_c$ , and $CRR$
Monkul et al. (2021) [5]	CDSS	–	–	+No relationship between $C_u$ and $CRR$

This study used two types of Silica sand with the same mean diameter ( $D_{50} = 0.64$ ) but varying  $C_u$  and  $C_c$ , classified into *SP* sand and *SW* sand, to investigate the coupling effect of  $C_u$  and  $f$  on the liquefaction resistance of the sand.

## 2 Material and Testing Method

### 2.1 Materials

Silica sand with a specific gravity of 2.6 was used as a base sand, and several grain sizes of this sand were separated using the dry sieving technique. Two gradation curves for the poorly-graded sand (*SP*) and the well-graded sand (*SW*) were established by mixing various grain size groups with different proportions. The material properties of sand and particle-size distribution curves are shown in Table 3 and Fig. 1.

**Table 3.** Material properties of Silica sand

Sand type	$G_s$	$D_{10}$ (mm)	$D_{30}$ (mm)	$D_{50}$ (mm)	$D_{60}$ (mm)	$C_u$	$C_c$
<i>SP</i>	2.6	0.128	0.28	0.64	0.68	5.31	0.87
<i>SW</i>	2.6	0.106	0.3	0.64	0.85	8.02	1

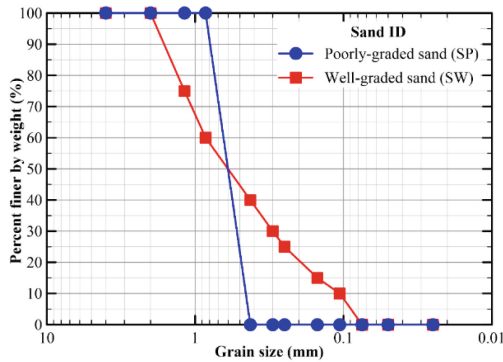


Fig. 1. Particle-size distribution curve of *SP* and *SW* sand

## 2.2 CDSS System and Sample Preparation

The testing program was conducted based on the CDSS system manufactured by Geocomp Corporation [1]. *SP* and *SW* silica sands in a dry state are poured into a cylinder-shaped steel-reinforced membrane by the dry deposition method to prepare a cylindrical sample with an initial relative density ( $D_r$ ) of 60% (medium state). The typical samples have an initial height of 25 mm and an initial diameter of 63.5 mm.

All samples are consolidated to confining stress of 100 kPa (i.e.,  $\sigma' = 100$  kPa). Then the samples were loaded under harmonic form loading under constant volume conditions at cyclic stress ratios, *CSR* of 0.1 with a wide range of loading frequency,  $f$  (0.03, 0.05, 0.1, 0.2, and 0.5 Hz). The *CDSS* tests can be terminated when the double amplitude shear strain ( $DA$ ) has reached 7.5%.

## 3 Result and Discussion

### 3.1 CDSS Testing Result

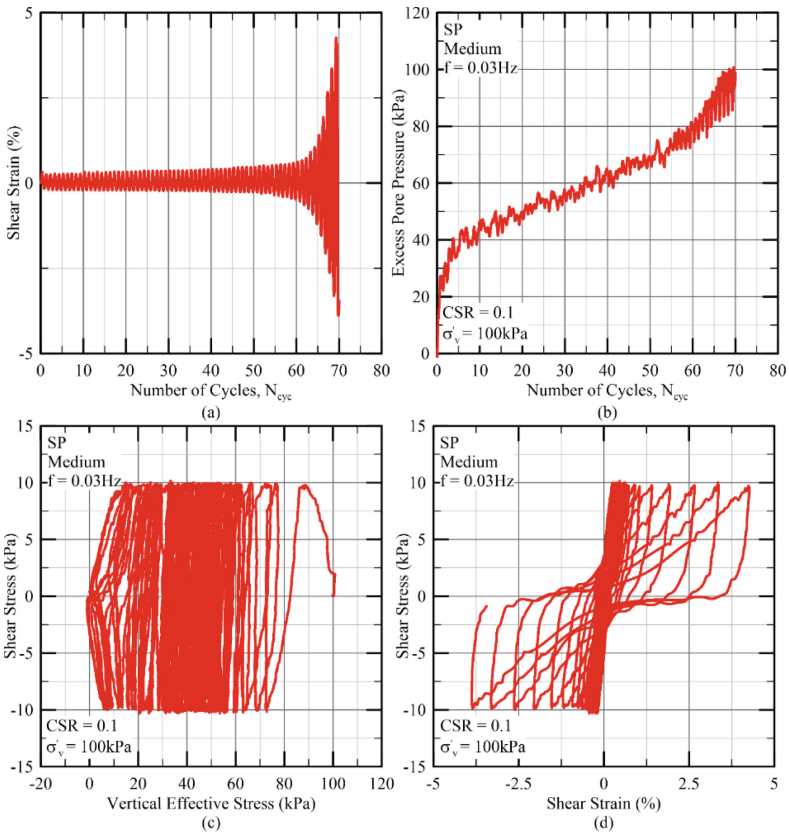
Table 4 summarizes the *CDSS* test results and the typical cyclic response of *SP* sand at the medium state with *CSR* of 0.1 and  $f$  of 0.03 Hz is illustrated in Fig. 2. While the test was performed until liquefying, the shear strain oscillated almost symmetrical at zero. The excess pore pressure rapidly increased in the first few cycles, then developed uniformly, leading to the effective vertical stress decreasing to zero at liquefy state.

### 3.2 Effect of $F$ on $N_{cyc}$

Figure 3 illustrates the relationship between  $N_{cyc}$  and  $f$  for *CSR* of 0.1. For *SP* sand, at *CSR* = 0.1,  $N_{cyc}$  remains unchanged ( $N_{cyc} = 70$ ) when frequency increases from 0.03 Hz to 0.1 Hz and rises from 70 to 111 when  $f$  increases from 0.1 Hz to 0.5 Hz. In general, *SP* sand's liquefaction resistance is affected by high  $f$ . By contrast, for the *SW* sand, the effect of  $f$  on  $N_{cyc}$  can be neglected. Moreover, the  $N_{cyc}$  of *SW* sand is higher than that of *SP* at  $f$  less than 0.2 Hz, which leads to the fact that the liquefaction resistance of *SW* sand is higher than that of *SP* sand.

**Table 4.** CDSS test results

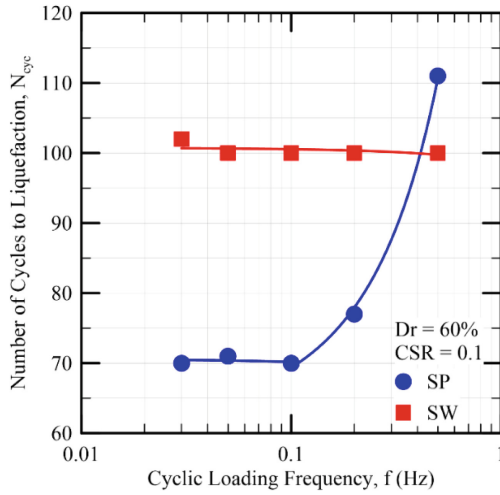
$\sigma'$ (kPa)	$D_r$ (%)	CSR	F (Hz)	$N_{cyc}$	
				SP	SW
100	60	0.1	0.03	70	102
			0.05	71	100
			0.1	70	100
			0.2	77	100
			0.5	111	100



**Fig. 2.** Undrained response of Poorly-graded sand (SP) in CDSS test with CSR of 0.1 and  $f$  of 0.03 Hz.

For practical use, the normalized number of cycles ( $NN_{cyc}$ ) is proposed by Eq. (1).

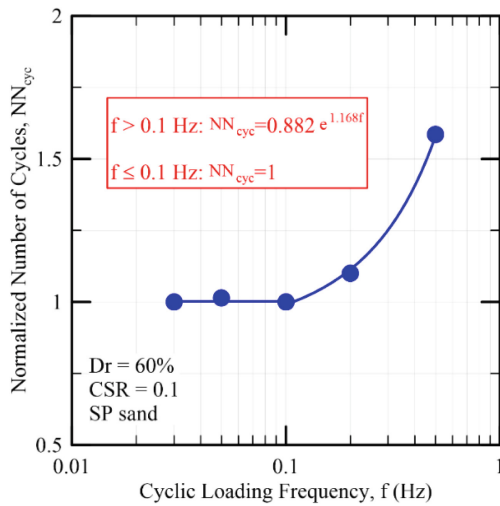
$$NN_{cyc} = N_{cyc}^{f=iHz} / N_{cyc}^{f=0.1Hz} \quad (1)$$



**Fig. 3.** Relationship between  $N_{cyc}$  and  $f$

where  $N_{cyc}^{f=iHz}$  is  $N_{cyc}$  at  $f$  of  $i$  Hz, and  $N_{cyc}^{f=0.1Hz}$  is  $N_{cyc}$  at  $f$  of 0.1 Hz.

Figure 4 displays a function of  $NN_{cyc}$  and  $f$ , and the relationship can be easily used in practice as the reference index.



**Fig. 4.** Relationship between  $NN_{cyc}$  and  $f$

## 4 Conclusion

The note presented a laboratory testing analysis on the effect of  $f$  on  $N_{cyc}$  of Silica sand with the same mean diameter and different particle size distribution ( $SP$  and  $SW$ ). The CDSS test results showed that when  $f$  increased from 0.03 Hz to 0.1 Hz,  $N_{cyc}$  remained unchanged, while it increased when  $f$  increased from 0.1 Hz to 0.5 Hz. It can be stated that  $SP$  sand's liquefaction resistance is affected by the high  $f$ . In contrast,  $f$  had little effect on the liquefaction resistance of  $SW$  sand. Furthermore,  $SP$  sand was more resistant to liquefaction than  $SW$  sand. According to this study,  $C_u$  and  $f$  should be included in the sand's liquefaction resistance analysis.

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