Study of Bubble Growth on a Heated Vertical Surface: Influence of Axial Flow Vibration

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Abbreviations

Nomenclature

R_b	Bubble radius (mm)
α	Thermal diffusivity (m^2/s)
C_p	Specific heat (kJ/kg-K)
D_h	Hydraulic diameter (m)
G	Mass flux (kgm^{-2} s^{-1})
q_w	Wall heat flux (kWm^{-2})
d_w	Contact diameter (kk)
h_{lv}	Latent heat (kJkg^{-1})
t	Time (s)
d_b	Bubble departure diameter (mm)
S	Suppression factor
T	Temperature (K)
T_τ	Frictional temperature (K)
u^*	Frictional velocity (m/s)
k	Thermal conductivity (Wm^{-1} K^{-1})
Jd	Jacob number
Re	Reynolds number
Y	Wall distance (m)
y^+	Nondimensional wall distance

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h	Convective heat transfer coefficient ($Wm^{-2}\ K^{-1}$)
g	Gravitational acceleration (ms^{-2})
ID	Internal diameter (mm)
OD	Outer diameter (mm)

Greek Letters

α	Advancing contact angle
β	Receding contact angle
ϕ	Inclination of bubble
θ	Inclination of heated surface
ρ	Density of fluid (kgm^{-3})
μ	Dynamic viscosity of fluid ($kgm^{-1}\ s^{-1}$)
σ	Surface tension (N/m)
τ	Shear stress

Subscripts

conv	Convection
d	Departure
nb	Nucleate boiling
sat	Saturation
sub	Sub-cooling
w	Wall

1 Introduction

Heat transfer enhancement is of interest to a large number of different industrial applications. Coolant phase change is one of the approaches to improve the heat transfer rate from the heated surface to the coolant. Coolant phase change in the forced convection flow is known as flow boiling of the coolant. As the heated surface temperature crosses the coolant saturation temperature, the sufficient wall superheat triggers the nucleation of vapour bubbles at different sites called nucleation sites on the heated surface. The vapour bubbles form at the nucleation sites, grow in size and depart from the heated surface. This cycle is termed as bubble ebullition cycle, and during this process, coolant extracts additional heat from heated surface in the form of latent heat. The improved heat transfer rate significantly depends on the factors, such as subcooling, wall superheat, bubble growth rate, departure diameter and departure

frequency. Among these parameters, controlled phase change phenomenon can be achieved by varying the subcooling and by decreasing the bubble departure diameter. One such approach is by vibrating the heated surface. The additional forces acting on the bubble result in its early departure from the heated surface. As the bubble departure diameter reduces, the number of bubbles that are generated, i.e., bubble departure frequency increases which results in enhanced heat transfer rate. Therefore, it is important to study the bubble growth rate phenomenon on the vibrating surface.

The important aspect in flow boiling is to predict the departure diameter of the bubble. Experiments were conducted for predicting the bubble diameter during its growth and departure for different subcooling and pressure conditions. Several researchers have developed numerical models to accurately predict the bubble dynamics during its life cycle. However, there is no accurate numerical model to predict the bubble diameter. For the calculation of bubble growth rate, heat flux partitioning method is employed. In this method, the applied total wall heat flux ($q_w = q_{ml} + q_{sl} + q_c$) is considered to contribute for (i) heat transfer due to the evaporation of microlayer (q_{ml}) that gets formed beneath the bubble and the heated surface, (ii) heat transfer from superheated layer (q_{sl}) and (iii) heat transfer due to the condensation (q_c) from the top portion of the bubble that is exposed to the subcooled bulk liquid phase. Situ et al. [7] accounted for only superheated layer heat transfer similar to the model proposed by Zuber [9] for the calculation of bubble radius and compared their predictions with the experimental data.

A force balance-based approach is often widely used for predicting the bubble departure diameter. However, as described by [1], these model predictions are very sensitive and not robust. [4] developed the analytical model based on force balance approach where bubble departure diameter is calculated with forces acting on it, i.e., surface tension, buoyancy, unsteady drag, static drag, contact pressure force, hydrodynamic force and lift force. These forces are resolved to act on the bubble in both x- and y-directions. If the sum of the forces either in x-direction or in y-direction becomes greater than zero, then the bubble is considered to be departed either by sliding along or by lift-off from the heated surface. The corresponding bubble size is termed as the bubble departure diameter.

Based on the brief literature review presented above, it can be inferred that the bubble growth models for the vibrating heated surfaces have not received the necessary attention. To this end, the present study focuses on analyzing the effect of the vibration of heated surface along the flow direction, to study bubble growth rate. The influence of vibration frequency as well as degree of subcooling are also investigated.

2 Methodology

In the present study, the flow boiling through a heated channel is studied. The annular test section with ID 38 mm, OD 19 mm, thus the equivalent hydraulic diameter is 19 mm [7]. Conservation of energy for the vapour bubble is used for calculating its growth rate, and a force balance method is used for calculating the bubble departure

diameter. The detailed description of these models are presented in the following sub-sections.

2.1 Nucleating Bubble

The growth rate of the vapour bubble is calculated by applying conservation of energy principle. The energy balance equation for the nucleating bubble can be written as follows:

$$\dot{E}_{stored} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} \tag{1}$$

As no heat is generated by the bubble, so $\dot{E}_{gen} = 0$ and $\dot{E}_{stored}$ is equal to the latent heat of vapourization (h_{lv}), where the subscripts l and v denote the liquid and vapour phases respectively (Fig. 1).

For simplification, the bubble is considered to be spherical in shape. Further, the heat transfer to the bubble from the superheated layer is only considered. Now, the bubble radius can be defined as [7]:

$$\frac{d(R)}{dt} = \left(\sqrt{\frac{3}{\pi}}\right) Ja * a^{0.5} t^{-0.5}, \tag{2}$$

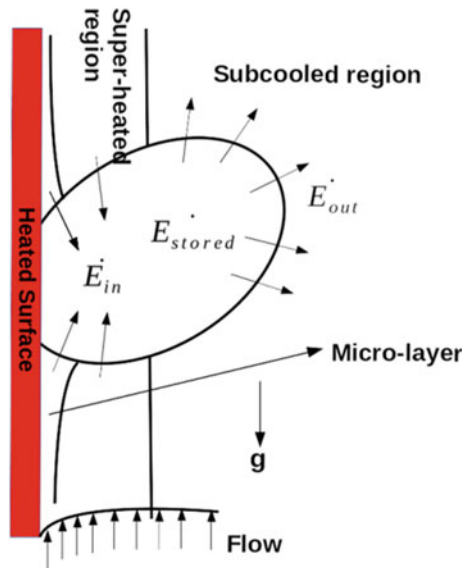**Fig. 1** Schematic of energy balance in the growing bubble

Table 1 Models for wall temperature prediction

	Correlation
Chen [2]	$q_w = q_{\text{conv}} + q_{\text{nucleate}}$ $q_{\text{conv}} = h_{\text{conv}}(T_{\text{wall}} - T_{\text{bulk}})$ $q_{\text{nucleate}} = h_{\text{nb}}(T_{\text{wall}} - T_{\text{sat}})$ $h_{\text{conv}} = 0.023 \text{Re}_l^{0.8} \, \text{Pr}^{0.4} \, \frac{k_L}{D_h} F$ $h_{\text{nb}} = 0.00122 \left(\frac{k_l^{0.79} C_{pl}^{0.45} \rho_l^{0.49}}{\sigma^{0.5} \mu_l^{0.29} h_{lv}^{0.24} \rho_g^{0.24}} \right) b$ $b = \Delta T_{sat}^{0.24} \Delta P^{0.75} S$ $F = 1 \& S = \frac{1}{1 + 2.53 \times 10^{-06} \text{Re}_{tp}^{1.17}},$

where Ja is Jacob number $\left(\frac{\rho_l C_{pf}(T_{\text{wall}} - T_{\text{sat}})}{\rho_v h_{lv}} \right)$, α is thermal diffusivity and R_b is the bubble radius which is a function of time (t) only.

2.2   Wall Temperature

Chen's correlation [2] is used for the calculation of the wall temperature of the heated surface, and the detailed correlation is given in Table 1.

Where q_w is the total applied wall heat flux, q_{conv} is the convective heat transfer by single-phase liquid and q_{nucleate} is the heat transfer from nucleate pool boiling. S and F are the suppression and the enhancement factors, respectively, considering $F = 1$ for the subcooled flow boiling.

2.3   Force Balance Model

For the calculation of bubble departure diameter, force balance approach is used. Figure 2 shows the schematic of different forces acting on the bubble, which can be resolved in both x-direction and in y-directions. If the sum of the forces in any direction crosses zero ($\sum F_x > 0$ or $\sum F_y > 0$), then the corresponding bubble diameter is called as bubble departure diameter.

Surface tension force (F_s) tries to oppose the detachment of the bubble from the surface which is defined in Eqs. (3, 4)

$$F_{\text{sx}} = -1.25 d_w \sigma \frac{\pi (\alpha - \beta)}{\pi^2 - (\alpha - \beta)^2} (\sin \alpha - \sin \beta), \tag{3}$$

$$F_{\text{sy}} = -d_w \sigma \frac{\pi}{(\alpha - \beta)} (\cos \beta - \cos \alpha), \tag{4}$$

Fig. 2 Schematic of forces
on a growing bubble
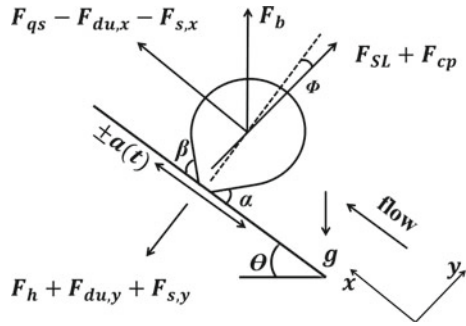
where F_{sx} and F_{sy} are the surface tension force in x-direction and in y-direction, α
and β are the advancing and receding contact angles and d_w is the contact width.

Shear lift force F_{sl}, acts perpendicular to the heated surface and supports the
detachment of the bubble. This shear lift force was given by [4].

$$F_{sl} = 0.5\rho_l U_l^2 \pi R^2 \left[3.87 G_s^{0.5} \left(\mathrm{Re}^{-2} + 0.118 G_s^2 \right)^{0.25} \right] \tag{5}$$

$$G_s = \frac{\mathrm{d}U_l}{\mathrm{d}y} \frac{R}{U_l} \tag{6}$$

$$\mathrm{Re} = \frac{\rho_l U_l d_b}{\mu_l} \tag{7}$$

The fluid velocity defined near the heated wall surface is assumed as single-phase
turbulent velocity profile.

$$\begin{cases} u^* = y^+, & y^+ \leq 5 \\ u^* = 5\ln(y^+) - 3.05, & 5 < y^+ < 30, \\ u^* = 2.5\ln(y^+) + 5. & 5, y^+ \geq 30 \end{cases} \tag{8}$$

where $y^+ = \frac{yu^*}{\vartheta}$, $u^* = \sqrt{\frac{\tau_{\mathrm{wall}}}{\rho_l}}$ and $\tau_{\mathrm{wall}} = \frac{f_l \rho_l v_l^2}{8}$, f_l is the friction factor and v_l is the
average velocity of the fluid.

Unsteady drag force is the force exerted by the growing bubble on the surrounding
fluid which is defined as:

$$F_{du} = -\rho_l \pi R^2 \left[R\ddot{R} + 1.5\dot{R}^2 \right], \tag{9}$$

where R_b is the bubble radius, $\dot{R}_b$ is the rate of change of bubble size (radius) with
time and $\ddot{R}_b = \frac{d\dot{R}_b}{dt}$.

Quasi-static drag force is the force applied on the bubble, considering it as a
stationary body as defined by Mei and Klausner [5, 6]. The drag force for the turbulent
flow is defined as:

$$\frac{F_{qs}}{6\pi \mu_l U_l R} = \frac{2}{3} + \left[\left(\frac{12}{\mathrm{Re}}\right)^n + 0.769^n\right]^{-1/n}, \tag{10}$$

where n = 0.65 and R_b is the bubble radius.

Buoyancy force acts on the bubble due to density difference between the liquid and the vapour phases. This force tries to detach the vapour bubble from the heated surface. The contact pressure force arises due to difference in pressure inside and outside the bubble, whereas the hydrodynamic pressure force acts on the bubble in the direction perpendicular to the heated wall. The above listed forces are defined as:

$$F_b = \frac{4}{3}\pi R^3(\rho_l - \rho_v)g, \tag{11}$$

$$F_{cp} = \frac{\pi d_w^2 2\sigma}{4 r_r}, \tag{12}$$

$$F_h = \frac{9}{8}\rho_l U^2 \frac{\pi d_w^2}{4}, \tag{13}$$

where ρ_l and ρ_g are the densities of surrounding liquid and vapor phases.

If the sinusoidal motion is given to the heated surface, the displacement, acceleration and mass flux variation with respect to the sinusoidal motion of the heated surface is defined as:

$$y(t) = y_m \sin(2\pi f t), \tag{14}$$

$$a(t) = -4\pi^2 f^2 y_m \sin(2\pi f t), \tag{15}$$

where $y(t)$ is the displacement of the heated surface in the vertical direction, $a(t)$ is the acceleration of the plate and y_m is the maximum amplitude of the displacement. The gravitational acceleration is modified as:

$$g' = g - a(t). \tag{16}$$

Also due to variation in acceleration with respect to time, velocity of the incoming liquid, mass flux and finally the buoyant force will change, which is defined as:

$$u'_f = u_f + \Delta u_f \omega(t), \tag{17}$$

$$\omega(t) = \sin\left(\frac{2\pi t}{T}\right), \tag{18}$$

$$\Delta u_f = 53{,}000\left(\frac{(4\pi^2 y_m/T^2)}{g}\right)\mathrm{Re}^{-1.45} u_f, \tag{19}$$

$$\tau'_{\text{wall}} = 0.5 C_f \rho_f u_f'^2, \tag{20}$$

$$F_b' = \frac{4}{3} \pi R^3 (\rho_l - \rho_v) g', \tag{21}$$

where g', u_f', f, F_b' are the updated gravitational acceleration, liquid velocity, frequency with which the heated plate vibrates and the updated buoyant forces exerted on the bubble. R_b is the bubble radius.

The summation of the forces in both x- and y- directions are written as follows:

$$\sum F_x = F_{sx} + F_{qs} + F_{dux} + F_b + F_a, \tag{22}$$

$$\sum F_y = F_{sy} + F_{duy} + F_{sl} + F_h + F_{cp}. \tag{23}$$

If $\sum F_x > 0$ then the bubble slides along the heated surface. Whereas, if $\sum F_y > 0$ then the bubble lifts-off from the heated surface. In both the cases, either the bubble slides or lifts-off from heated surface is considered as departure from the heated surface. The corresponding bubble size is identified as the bubble departure diameter.

3   Results and Discussion

This section presents the validation studies for both the static and vibrating heated surface cases. Further, the study is extended to investigate the influence of vibrating frequency and the degree of subcooling on the bubble growth rate and the departure diameter.

3.1   Validation

The present model developed based on the energy and force balance for the calculation of bubble growth rate and the departure is validated with the experimental dataset of Situ et al. [7] and Sugrue et al. [8]. The corresponding experimental conditions are mentioned in Table 2. The present model accounts for only conduction heat transfer from the superheated layer and the predictions are compared with the experimental data as shown in Figs. 3, 4, 5, 6 and 7. A good agreement is noticed with a mean percentage error of 30%.

Figures 3, 4 present the comparison of bubble departure diameter predictions against the measured data for the static plate subjected to subcooled flow conditions. The same model is extended to predict the bubble growth rate and the departure diameter when the heated plate is subjected to a vibration. When the heated plate is

Table 2 Experimental data for validation

	Situ et al. [7]	Sugrue et al. [8]
Fluid	Water	Water
D_h (mm)	19.1	16.7
$G\left(\text{kgm}^{-2}\text{s}^{-1}\right)$	466–900	250–400
$\Delta T_{sub}(°C)$	1.5–20	10–20
$q\left(kWm^{-2}\right)$	60.7–206	50–100
Orientation	Vertical $(90°)$	$0° - 90°$

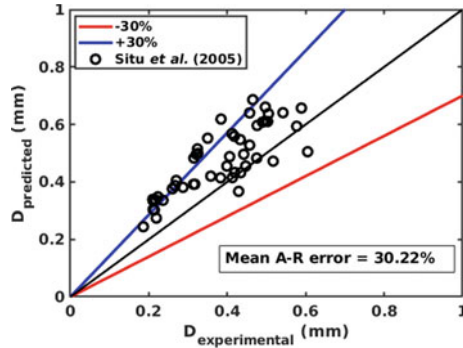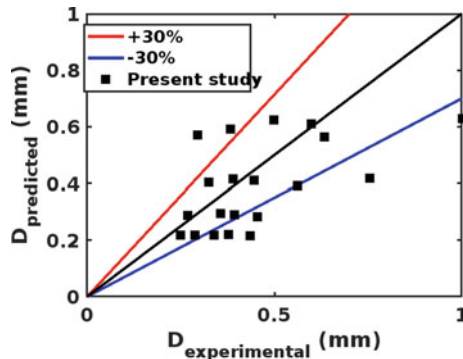**Fig. 3** Comparison of predicted bubble departure diameter against the experimental data of Situ et al. [7]

Fig. 4 Comparison of predicted bubble departure diameter with the experimental data of Sugrue et al. [8]

subjected to sinusoidal motion, the vibration results in additional acceleration and hence mass flux also changes as the heated surface is moving along the direction of the incoming liquid flow. As a result, additional forces are exerted on the bubble due to the variation in the acceleration, as defined in [3]. The variation of displacement, acceleration and mass flux are compared with the experimental data of Hong et al. [3] as shown in Fig. 5.

The bubble growth rate predictions at a single nucleation site on the heaving heated surface are shown in Fig. 6. A good agreement is observed when compared
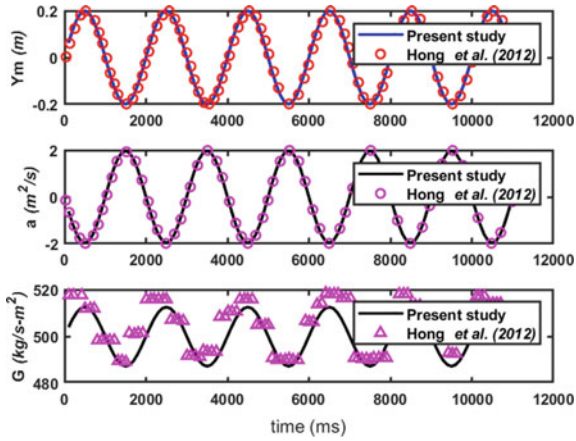
Fig. 5 Temporal variation of mass flux due to an imposed axial vibration Hong et al. [3]
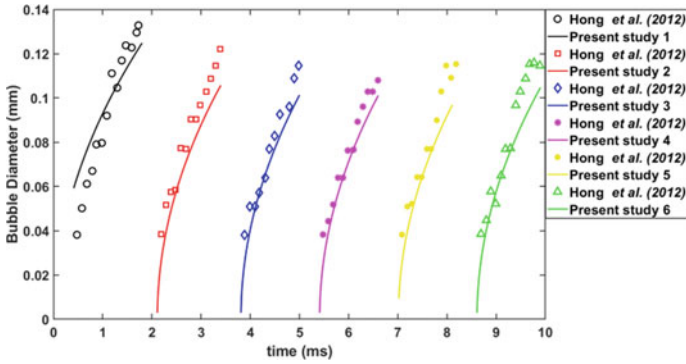

Fig. 6 Bubble growth rate over six different time periods against the experimental study of Hong et al. [3]
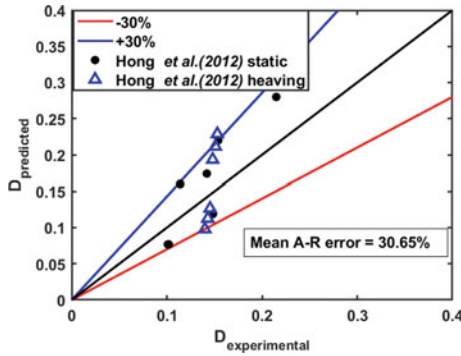

Fig. 7 Bubble departure diameter for static and vibrating plate against the experimental data of Hong et al. [3]

with the experimental data of Hong et al. [3]. Further, the bubble departure diameter is calculated for the dataset of Hong et al. [3] for the static and heaving heating surfaces. A good agreement with a mean relative error of around 30% is observed as shown in Fig. 7. Further, the mass flux also varies as a sinusoidal function because of the heated surface motion. As a result of the fluctuating mass flux, bubble departure diameter also changes with time. If the mass flux is greater than its mean value for the same heat flux and subcooling, then the bubble departure diameter is observed to be smaller compared with the static case. Whereas, if the mass flux is smaller than its mean value then, the bubble departure diameter is noticed to be larger than the static case. These predictions are in line with the experimental observation of Hong et al. [3].

3.2 Influence of Subcooling and Vibration Frequency

Heat transfer enhancement from the heated surface can be achieved by disrupting the thermal boundary layer. Due to the disruption of the thermal boundary layer, the heated surface gets in to direct contact with the lower temperature liquid coolant which results in heat transfer enhancement. There are various ways for disrupting the formation of thermal boundary layer such as (a) providing a small sinusoidal displacement to the heated surface either along the fluid flow or normal to the flow and (b) by the nucleation of vapour bubble on the heated surface. Thermal boundary layer gets disturbed due to the bubble growth and departure from the heated surface. At the time of bubble departure, the bubble takes away the significant amount of heat from the surface. When the bubble leaves the nucleation site, the surrounding cold fluid rushes to that site and thus increases the heat transfer. Therefore, the bubble departure diameter and the frequency have a strong influence on the heat transfer from the heated surface. The bubble departure diameter can be varied to further increase the heat transfer by vibrating the heated surface with a sinusoidal displacement along the flow for subcooled flow boiling conditions. When a vapour bubble forms over the heated surface, during its growth, it extracts an additional amount of heat from the surface in the form of latent heat, and then departs from the nucleation site. In the present study, the heated surface is vibrated with a certain frequency along the coolant flow direction to study the growth rate of the bubble on the vibrating heated surface. The additional forces acting on the bubble due to the vibration of heated surface may result in the early departure of the vapour bubble. As a result, the heat transfer rate enhances compared with the heat transfer rate from the static heated surface. To this end, the influence of subcooling and vibration on the bubble growth rate and departure diameter is studied in the present work.

Figure 8 shows that with increase in frequency of vibration of the heated surface, the bubble departure diameter gradually decreases. The effect of subcooling on the bubble departure diameter is studied by considering two different values of subcooling, 20 and 30 K for a heat flux of 300 kWm^{-2}, mass flux of 250 kgm^{-2}s^{-1}

at 1 bar pressure. This shows that (Fig. 8), with decrease in bubble departure diameter with frequency, decreases the total ebullition time, due to which large amount of bubble will able to depart from the heated surface which in turn increases the heat transfer from the heated plate. Similarly, shown in Fig. 9 is the evolution of bubble with time by varying frequency of the heated plate. Figure 9 also shows that with increase in frequency, the bubble growth time and bubble departure diameter decrease.

Fig. 8 Variation of bubble departure diameter with the vibration frequency of the heated surface for two different subcoolings, 20 and 30 K
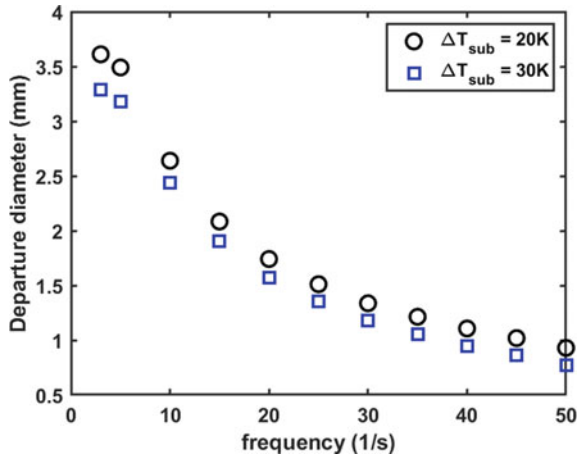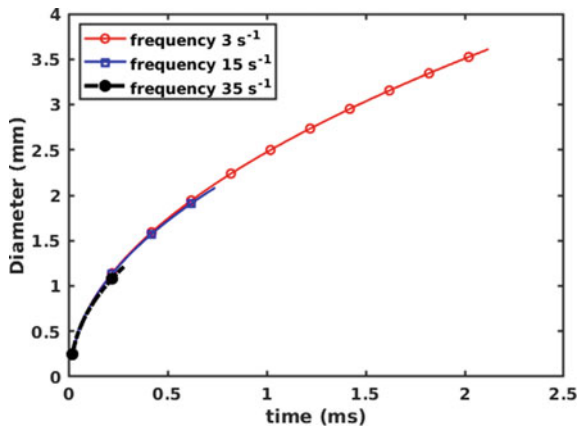

Fig. 9 Influence of the frequency of vibration of the bubble growth rate

4 Conclusions

Numerical simulations are carried out to determine the bubble growth rate and departure diameter under subcooled flow boiling conditions for both the static and vibrating surfaces. For the calculation of vapour bubble growth rate, only conduction heat transfer from the superheated layer is considered, whereas for calculating the bubble departure diameter force balance model was implemented. The following conclusions were made based on the present study.

- The present numerical model is validated against the experimental data for static and vibrating heated surfaces, and a good agreement was noticed.
- Due to the vibration of heated surface along the flow direction, the bubble departure diameter decreases compared with the static case.
- By increasing the degree of subcooling, the bubble departure diameter is found to decrease, which in turn increases the heat transfer rate from the heated surface.

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