Chapter 2
Displacement and Velocity Analysis

2.1 Introduction

The motion of a lever is expressed in terms of the linear displacements, linear velocities, and linear accelerations of its individual particles. However, the motion of a lever can also be determined based on the angular displacements, angular velocities, and angular accelerations of moving lines with the desired rigid lever. No matter what method is used to analyze the leverage, it is always necessary to determine the angular positions of the members before analyzing the velocity. Similarly, we need the angular velocities of the members before acceleration analysis. The kinematic analysis of a lever should always be performed as follows: position analysis, velocity analysis, and acceleration analysis. In addition to displacement analysis, several methods for determining velocities in mechanisms will be presented in this chapter.

2.2 Velocity Equations for the Curve Motion

Measuring and describing the motion of objects relative to a stationary coordinate system is called absolute motion analysis. The motion analysis will be relative if this analysis is performed on a moving device. If we denote the position of a particle moving on a straight line from the origin of coordinates with x, then we can write

$$\overline{V} = \frac{\Delta x}{\Delta t} \tag{2.1}$$

where Δx is the displacement in meters, Δt is the time interval in seconds, and $\overline{V}$ is the average velocity in meters per second.

A. Abbasimoshaei, T. A. Kern, *Machine Dynamics*,
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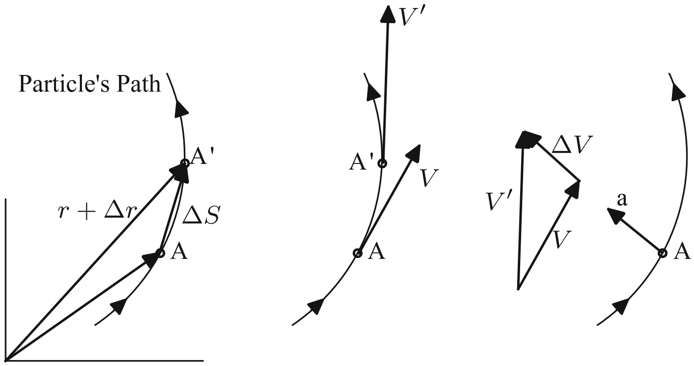


Fig. 2.1 A particle that moves continuously on a plane curve

Also, if V is the instantaneous velocity in meters per second, we have

$$V = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = \dot{x} \tag{2.2}$$

Note Velocity is the rate at which location changes relative to time. If the displacement is positive, the velocity is positive, and if it is negative, the velocity is negative.

The motion of a particle that travels along a curved path is called a curved transmission. Consider a particle that moves continuously on a plane curve, as shown in Fig. 2.1.

The average velocity of a particle between A and A' is defined as $\overline{v} = \Delta r/\Delta t$, in which r represents the location vector, and Δr represents the displacement of the particle over time Δt. Instantaneous velocity v, by definition, is the limit of average velocity when the time interval Δt converges to zero. Therefore,

$$v = \frac{dr}{dt} = \dot{r} \tag{2.3}$$

Note The value of v, a scalar quantity, is called speed.

In the orthogonal coordinate system $(x - y)$, the curve motion of the particle is determined by summing the x and y components of vectors of location, velocity, and acceleration. For this type of coordinate system, we have

$$r = x\hat{i} + y\hat{j} \tag{2.4}$$

$$\overrightarrow{v} = \overrightarrow{\dot{r}} = \dot{x}\hat{i} + \dot{y}\hat{j} \tag{2.5}$$

In the vertical-tangential coordinate system $(n - t)$, where the unit vector e_n is defined in the direction n and the unit vector e_t is defined in the direction t, we can write

$$\overrightarrow{v} = v\hat{e}_t \tag{2.6}$$

Note that t specifies the direction of motion and n the direction perpendicular to the motion path.

Also, in the polar coordinate system $(r - \theta)$, where the unit vector e_r is in the positive direction r, and the unit vector e_θ is in the positive direction θ, the velocity vector equation is as follows:

$$\overrightarrow{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta \tag{2.7}$$

Note Component v in direction r represents the rate of increase of vector r in direction θ due to the rotation.

Figure 2.2 shows the orthogonal and polar coordinate systems with unit vectors on the $x - y$ plane.

These equations can also be generalized for spatial curve motion. Figures 2.2 and 2.3 show the three coordinate systems of orthogonal $(x - y - z)$, cylindrical $(r - \theta - z)$, and spherical $(R - \theta - \varnothing)$ with unit vectors.

For orthogonal coordinates in the three-dimensional motion, we only need to add coordinate z and its derivatives to the equations of two-dimensional motion:

$$\overrightarrow{R} = x\hat{i} + y\hat{j} + z\hat{k} \tag{2.8}$$

$$v = \dot{R} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k} \tag{2.9}$$

Note that we show the location vector with the capital letter R in three-dimensional motion instead of r.

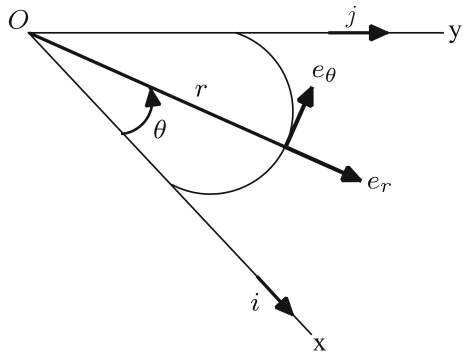


Fig. 2.2 Orthogonal and polar coordinate systems with unit vectors on the $x - y$ plane

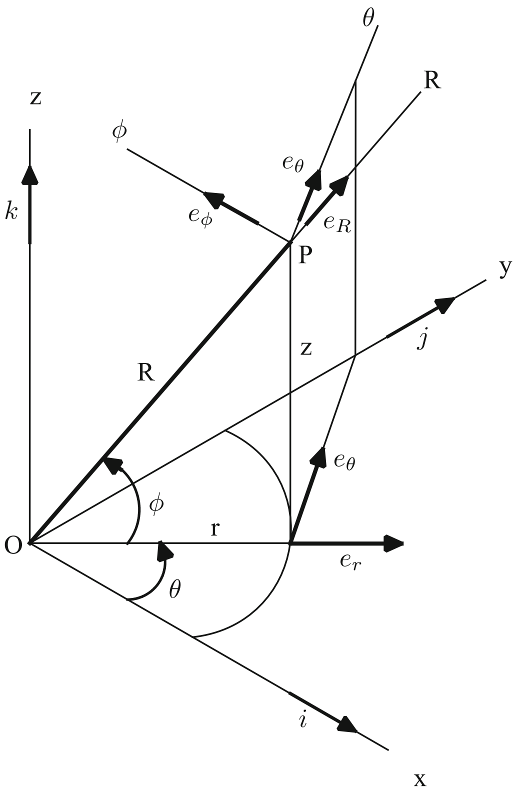


Fig. 2.3 Cylindrical and spherical coordinate systems in 3D

In the case of cylindrical coordinates, we only need to add coordinate z to the equations of motion in polar coordinates. Therefore,

$$\overrightarrow{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta + \dot{z}\hat{k} \tag{2.10}$$

Spherical coordinates also use a radial distance and two angles to determine the position of a particle. Unit vector e_R is in the direction of motion in which R increases but θ and $\varnothing$ are constant. Unit vector $e_\varnothing$ is in the direction of motion in which $\varnothing$ increases, but R and θ remain constant. Finally, unit vector $e_\varnothing$ is in the direction of motion in which $\varnothing$ increases and R and θ are constant. So we have

$$\overrightarrow{v} = \dot{R}\hat{e}_R + R\dot{\theta}\cos\varnothing\hat{e}_\varnothing + R\dot{\varnothing}\hat{e}_\varnothing \tag{2.11}$$

Example In Fig. 2.4, the piston shaft of the hydraulic jack moves to the left at a constant velocity of v. We denote $\overline{OA}$ by r. The values of $\dot{r}$ and $\dot{\theta}$ are

1) $\dot{r} = -v\cos\theta, \dot{\theta} = \frac{v}{r}\cos\theta$ 2) $\dot{r} = v\sin\theta, \dot{\theta} = \frac{v}{r}\cos\theta$
3) $\dot{r} = -v\cos\theta, \dot{\theta} = \frac{v}{r}\sin\theta$ 4) $\dot{r} = -v\cos\theta, \dot{\theta} = \frac{-v}{r}\sin\theta$

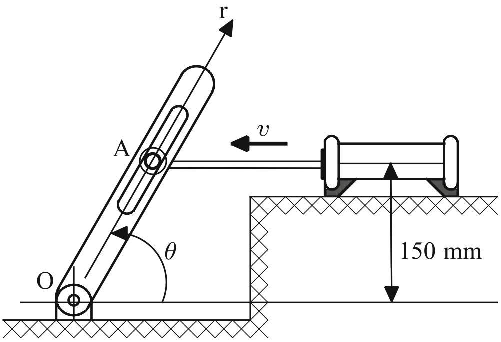


Fig. 2.4 A hydraulic jack

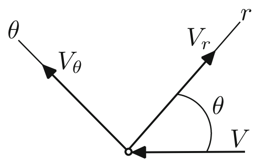


Fig. 2.5 Polar coordinate

Solution The motion is of the plane type. Using the motion equations in polar coordinate, we can plot the known velocity v of point A in directions r and θ (Fig. 2.5).

$$\vec{v} = v_r\hat{e}_r + v_\theta\hat{e}_\theta = -v\cos\theta\hat{e}_r + v\sin\theta\hat{e}_\theta$$

$$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta \quad \text{(velocity equation for polar coordinates)}$$

From the above two equations, we have

$$\dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta = -v\cos\theta\hat{e}_r + v\sin\theta\hat{e}_\theta \Rightarrow \begin{cases} \dot{r} = -v\cos\theta \\ \dot{\theta} = \frac{v}{r}\sin\theta \end{cases}$$

Option (3) is correct.

2.3 Angular Motion

Angular velocity and acceleration are the first and second derivatives of angular displacement θ of a line relative to time t, respectively. In machine analysis, the angular motion of each lever is expressed by the angular motion of a hypothetical line connected to it.

Note The angular motion of a lever may be similar to or different from the angular motion of the radius of the path curvature of individual particles of the lever.

An important concept in mechanisms is that only transmission is possible for a particle that is a point of infinitesimally small size, and the particle cannot rotate. Angular motion is the motion of a line, and since a particle is a point and not a line, angular motion is inconceivable for it. This concept must be well understood to understand the relative motion between particles. In the following, various methods of velocity analysis are examined.

2.4 Analytical Method of Velocity Analysis

It is not possible or appropriate to use a fixed coordinate system to study all motions. There are many geometric problems in which motion analysis is easier using measurements obtained from a moving coordinate system. By combining these measurements with the absolute motion of the moving coordinate system, the desired absolute motion can be determined. This method is called relative motion analysis.

Note An object only has relative motion relative to another object when their absolute motions are different from each other.

The position of an object like A relative to an object like B is equal to the absolute position A minus the absolute position B. A similar interpretation is used for velocity and acceleration. Thus, for velocity, we can write

$$V_{A/B} = V_A - V_B \tag{2.12}$$

or in other words

$$V_A = V_B + V_{A/B} \tag{2.13}$$

This study relative to the moving device is limited to devices with a transmission motion and no rotational motion. If the moving device also has a rotation velocity of ω, we will have

$$V_A = V_B + \omega \times r + V_{rel} \tag{2.14}$$

V_{rel} has no rotation velocity relative to the moving device, and to find it, stop the device's rotation and find $V_{A/B}$.

Note The difference between the relative velocities in rotating and non-rotating axes is in $\omega \times r$.

Note The relative velocity of the points that match at the point of contact of two rolling members is zero.

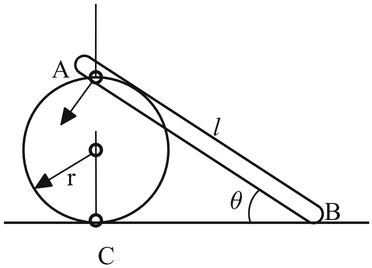


Fig. 2.6 A disk with pure rolling movement

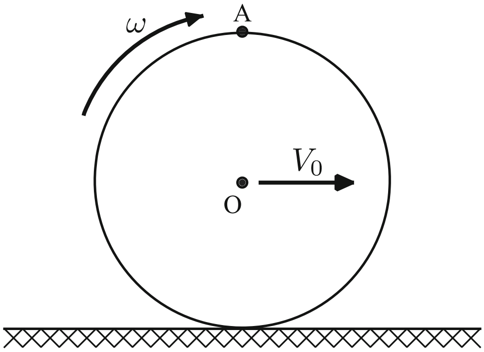


Fig. 2.7 A disk on the ground

Example A disk with pure rolling moves in Fig. 2.6, and the bar AB is jointed to it at point A. Obtain velocity B if the rotational velocity of the disk is ω.

1) Zero
2) $2r\omega$
3) $(2r + l\cos\theta)\omega$
4) The information is not enough.

Solution At this point, the velocity of point A is horizontal and equal to $2r\omega$. The velocity of point B is also always horizontal due to ground contact. So since the direction of the velocity of two different points of a rigid body is the same, it can be said that the object has no angular velocity and only has a transmission motion at that moment. Therefore the velocity vectors of all its points are equal to each other. So $V_B = V_A = 2r\omega$.

Option (2) is correct.

Note For two points located on a lever of a mechanism, $V_{rel} = 0$.

Example The disk's center shown in Fig. 2.7 moves at a velocity $V_0 = 1m/s$. If the disk has a radius $R = 10cm$ and an angular velocity $\omega = 20rad/s$, what is the velocity of point A at the top of the disk at the shown moment in meters per second?

1) 1
2) 2
3) 3
4) 4

Solution If we set the center of the moving device to O, we will have

$$V_A = V_O + \omega \times r + V_{rel}$$

Given that the points O and A are on one object, $V_{rel} = 0$ and we have

$$V_A = V_O + \omega \times |OA| \Rightarrow V_A = 1\hat{i} + (20 \times 0.1)\,\hat{i} = 3\hat{i}$$

Note that the disk has a sliding motion on the ground, and its motion is not pure rolling. In the case of pure rolling, the velocity of point O would be $2m/s$. Also, if the velocity of the point of contact with the ground is calculated, this velocity will not be zero.

Option (3) is correct.

2.5 Graphical Method for Velocity Analysis

Velocity polygons are good tools for determining the velocity of mechanisms. These polygons can be solved by drawing, analyzing, or combining the two. The main method used in graphical lever analysis is to work with one or two points at a known velocity to find one of the unknown velocities using the relation between the velocities of two points belonging to one lever in equation (2.14). Rotational joints form the transmission points because the mentioned points belong to two different members. Thus, the velocity of the rotating joint can be obtained by considering it as a point on one of the members to which it is attached.

In the mechanisms studied in machine dynamics and the method used, because the two points under analysis belong to the same lever, $V_{rel} = 0$, and this equation is simplified as follows:

$$\overrightarrow{V}_B = \overrightarrow{V}_A + \overrightarrow{\omega} \times \overrightarrow{r}_{B/A} \tag{2.15}$$

Equation 2.15 can be graphically represented as a vector triangle, as shown in Fig. 2.8.

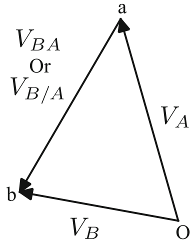


Fig. 2.8 Vector triangle

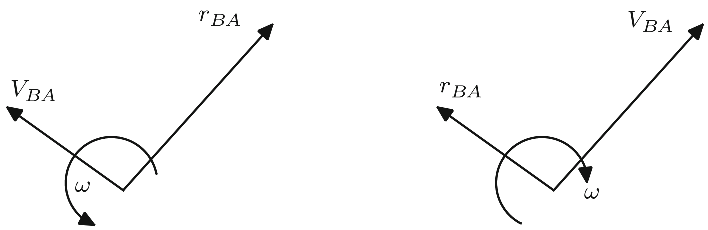


Fig. 2.9 Finding direction of the third vector

Note This triangle can always be solved by knowing the direction and size of one of the three vectors and the direction of the other two vectors. This is a common situation in planar velocity analysis.

According to Fig. 2.9, the vectors used in equation $\overrightarrow{\omega} \times \overrightarrow{r}_{B/A}$ are reciprocally orthogonal. Because we know the lines that the vectors must be in line with, the main issue is determining the direction of the lines and the size of each of the vectors. By knowing the direction of the two vectors, the direction of the third vector can be found by the rule of the right hand by observing the known directions.

For example, suppose lever 2 moves in the four-bar mechanism in Fig. 2.10, and its uniform angular velocity ω_2 is known. We want to find the velocity V_B of point B and the angular velocities ω_3 and ω_4. The known geometric parameters are also shown in the figure.

Since each vector has size m and direction d, we can easily reflect the information and unknowns of a vector equation in a table. Two scalar (numerical) unknowns

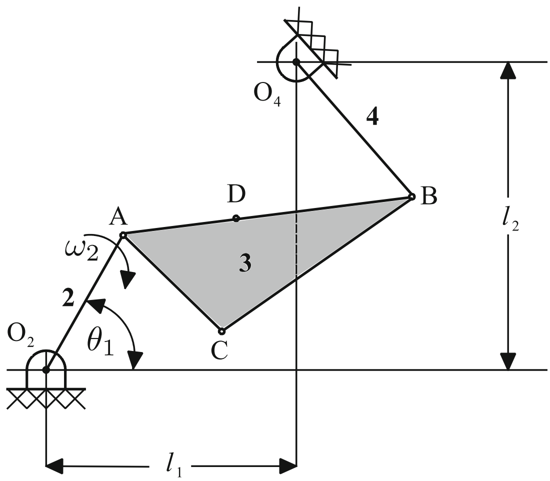


Fig. 2.10 Four-bar mechanism

(I).	$\boldsymbol{V_B}$ =	$\boldsymbol{V_A}$ +	$\boldsymbol{V_{BA}}$
Length (***m***)		✓	
Direction (***d***)	✓	✓	✓

Fig. 2.11 Velocity calculation parameters

can be obtained from a vector equation. The velocity equation can be written in Fig. 2.11.

wherein

$$V_A = \text{along the perpendicular to } O_2A \text{ of known size } V_A = |O_2A|\,\omega_2$$
$$V_{BA} = \text{along the perpendicular to } BA \text{ of unknown size}$$
$$V_B = \text{along the perpendicular to } O_4A \text{ with an unknown size}$$

We can plot a velocity polygon with only two unknowns of the vector equation. First, we consider the desired origin O_v and draw it with its size and direction V_A. On the other hand, the directions of V_B and V_{BA} are known. From the origin, we determine the direction of V_B with a dashed line. On the other hand, according to the equation $V_{BA} = V_B - V_A$, so the V_{BA} vector must start from the end of vector A and be connected to the end of vector B. So with the V_{BA} direction, we draw a dashed line from the end of the vector V_A in this direction. The intersection of the V_B and V_{BA} directions completes the polygons. Now we add the direction of the arrows V_B and V_{BA} so that the sum of the polygons matches the sum of the sentences of the velocity Equation (I). We will mark the tip of the V_B vector with B. The steps are shown in Fig. 2.12.

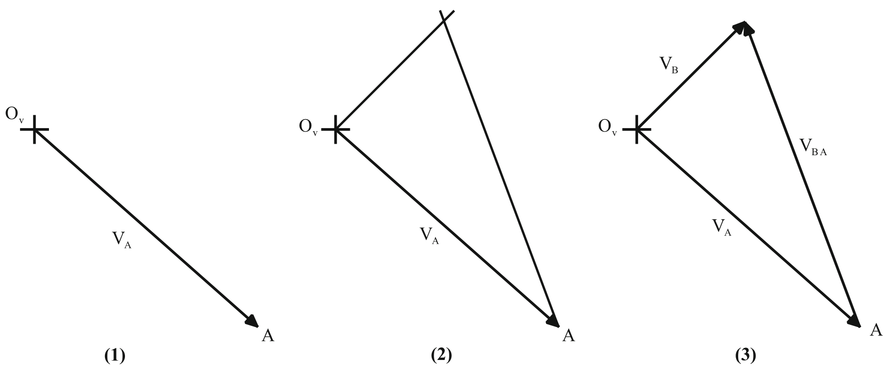


Fig. 2.12 Steps of finding the relative speed

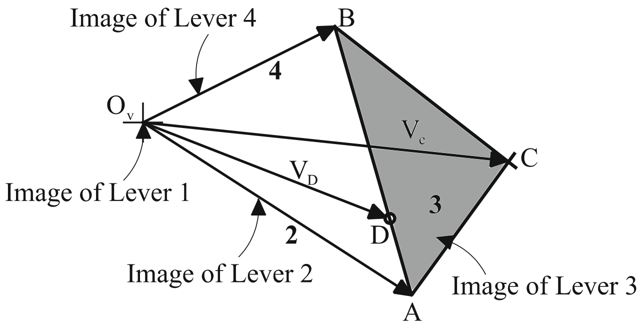


Fig. 2.13 Image of velocities

According to the figures, ω_3 and ω_4 can be obtained from V_{BA} and V_B, respectively. The size of ω_{32} and ω_{43} can also be determined. We have

$$\omega_3 = \frac{V_{BA}}{|BA|}\ ,\ \omega_4 = \frac{V_B}{|O_4B|}\ ,\ \omega_{32} = \omega_3 - \omega_2\ ,\ \omega_{43} = \omega_4 - \omega_3$$

We should use Equations (II) and (III), which express the relation between V_C, V_A, and V_B to determine V_C.

$$\text{(II) } V_C = V_A + V_{CA}$$

$$\text{(III) } V_C = V_B + V_{CB}$$

The V_{CA} and V_{CB} extensions are known according to the shape of the mechanism and are perpendicular to the sides CA and CB, respectively. Equation (II) is used, and the extension of the vector V_{CA} from point A is plotted in the following figure. Then, according to Equation (III), the extension of the vector V_{CB} is plotted from point 3. The intersection of the extensions V_{CA} and V_{CB} completes the polygon. The intersection of point C gives V_C. It should be noted that the sum of the vectors of polygons must be compatible with equations (II) and (III) (Fig. 2.13).

The hatched triangle ABC is called the image (projection) of the velocity of lever 3 and is similar to lever 3. By determining the position of any given point D of lever 3 on the velocity image, its velocity can be obtained. According to the figure, the vector drawn from O_v to D is V_D. The image of the velocity of lever 1 at origin is O_v because lever 1 is fixed, and its velocity is zero. The images of velocities of levers 2 and 4 are the lines O_vA and O_vB, respectively, which correspond to the O_2A and O_4B of the mechanism, respectively.

Note Having velocity images of all levers of a mechanism allows calculating the linear velocity of all lever points.

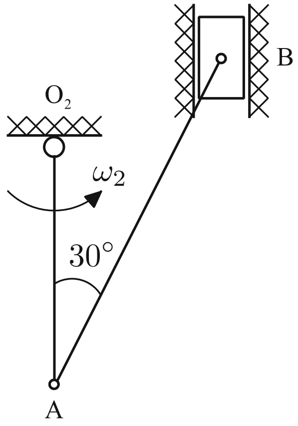


Fig. 2.14 Crank-slider mechanism

From the above analysis, we have

$$\omega_3 = \frac{V_{BA}}{|BA|}, \ \omega_3 = \frac{V_{CA}}{|CA|} = \frac{V_{CB}}{|CB|} = \frac{V_{DA}}{|DA|}$$

In other words, all the relative velocities of the points on a lever are proportional to the distances between these points.

Example Given the mechanism in Fig. 2.14 for ω_{AB} angular velocity, which of the following is true? (O_2A is parallel to the motion path B.)

1) The direction of ω_{AB} is the same as the direction of ω_2, but $|\omega_{AB}| > |\omega_2|$.
2) When O_2A is in line with AB, ω_{AB} is zero.
3) ω_{AB} equals twice the velocity B divided by the length AB.
4) ω_{AB} equals the difference of the absolute velocities A and B divided by the length AB.

Solution We draw the velocity polygon (Fig. 2.15) with a free scale using the relative velocity equations in the mechanisms.

$$V_A = V_A + V_{BA}$$

$$\omega_{BA} = \frac{V_{BA}}{|BA|}$$

From the polygon, velocity is $|V_B| = \frac{|V_{BA}|}{2}$, and by placing it in the above equation, we have

$$\omega_{BA} = \frac{V_{BA}}{|BA|} = \frac{2V_B}{|BA|}$$

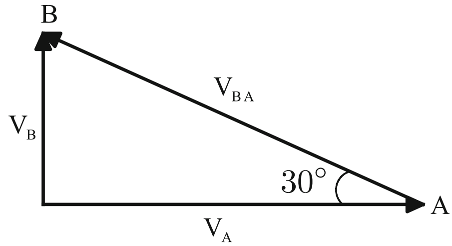


Fig. 2.15 Velocity polygon

So option (3) is correct. But about other options,

$$\omega_{BA} = \frac{V_{BA}}{|BA|} = \frac{2V_B}{|BA|}$$

Therefore, option (1) depends on the numerical value of $|O_2A|$. Option (2) is also incorrect. If the two levers, O_2A and AB, are in the same direction since the directions of velocities A and B will be different, there will also be angular velocities of ω_{AB}, and it will not be zero. But for option (4), we write from the polygon of velocity

$$\omega_{AB} = \frac{V_{BA}}{|BA|} = \frac{V_B - V_A}{|BA|} = \frac{V_B - V_B \cot 30}{|BA|} \neq \frac{2V_B}{|BA|}$$

It is observed that the statement of the option (4) cannot always be correct.

It should be noted that this question can be easily solved by finding the instantaneous center of rotation A and B.
Option (3) is correct.

Example If the velocity of point A is 2.5(m/s), what is the velocity of point B? (Fig. 2.16)

1) 1.5 m/s 2) 2.5 m/s 3) 7.5 m/s 4) 5 m/s

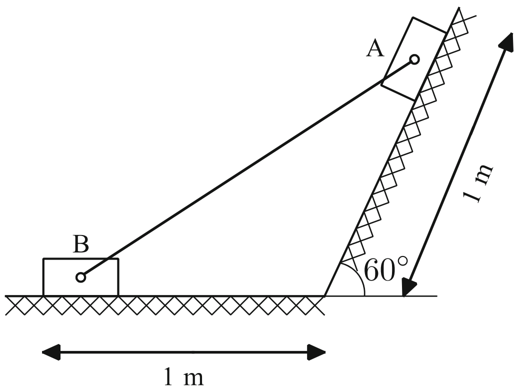


Fig. 2.16 Two points connected with one link

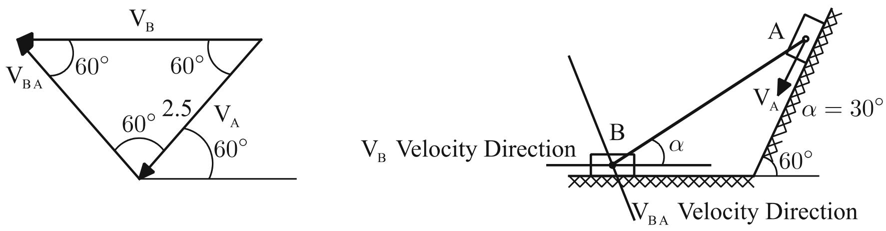


Fig. 2.17 Velocity triangle

Solution We have

$$V_B = V_A + V_{BA}$$

Given that the directions of V_B and V_{BA} are known, then the equation has two unknowns, magnitude of V_B and V_{BA}. By drawing the velocity triangle (Fig. 2.17), since the resulting triangle is equilateral,

$$V_B = 2.5 \text{ m/s}$$

Option (2) is correct.

2.6 Instantaneous Center of Rotation Method for Velocity Analysis

2.6.1 Instantaneous Center of Velocity

In the following discussion, another concept is used to determine the linear velocity of the mechanism particles, which is the concept of the instantaneous center of velocity. This concept is based on the fact that at a given moment, the velocities of a pair of matching points located on two moving levers are equal relative to a fixed lever, and therefore their relative velocities relative to each other are zero. At this point, each lever has only a pure rotation around the matching points relative to the other lever. A special case of this is when one lever is moving, and the other is fixed. Thus, the absolute velocity of a pair of matching points of these two levers is zero, and at this moment, the moving lever rotates around these matching points relative to the fixed lever. In both cases, the set of matching points is called the instantaneous center of velocity (or instantaneous center). Thus an instantaneous center is a point that

(a) Is located on both objects.
(b) The object has no relative velocity in it.

(c) At a given moment, one object can be imagined rotating around it relative to another object.

Note If a hinge connects two levers, their instantaneous center is the point of contact in the hinge because the velocity vector of the points of contact in the hinge is the same.

Even if the two levers are not directly connected, there will be an instantaneous center (geometric position) for them in any desired state.

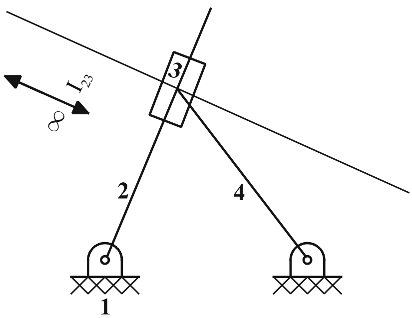
Note As the mechanism passes through different positions of a motion cycle, the position of the instantaneous center of one lever relative to the other lever changes over time due to the polygonal deformation of velocity, except for levers that have a pure rotation whose instantaneous center is fixed points.

The velocities of all points of the levers, which act as sliders in a single crank and slider mechanism and only have transmission, are parallel to each other. Also, their perpendiculars are parallel, and their intersection is at infinity. Thus the instantaneous center of a lever in the transmission is at an infinite distance from the lever and perpendicular to the transmission path. Therefore, if, as shown in Fig. 2.18, the slider (member 3) slides on a non-curved member (member 2), the instantaneous center of the two members is at an infinite distance from the point of sliding and on a line perpendicular from the sliding point to the member on which the slider slides (drawn dashed line).

Note If two members have sliding contact with each other, the instantaneous center of the two members is somewhere on the common perpendicular line of the two members drawn at the point of contact (Fig. 2.19A and B), and if one of the two members is a slider, the instantaneous center is on the center of curvature of the other member (Fig. 2.19C).

Note If two members have a pure rolling contact, their instantaneous center is at the same point of contact, because in pure rolling, the velocity vector of the contact points of the two members is the same.

Fig. 2.18 Slider on the connecting points of two links



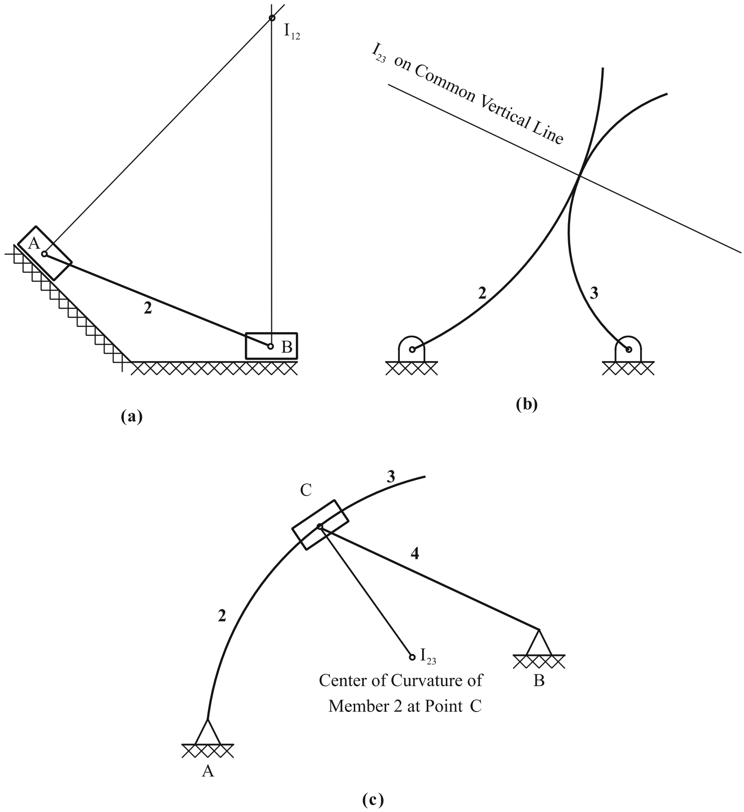


Fig. 2.19 Common perpendicular line of the two members drawn at the point of contact

Example In a five-bar mechanism, according to Fig. 2.20, what are the locations of instantaneous centers (moments) between members (4 and 5), (3 and 4), and (1 and 5)?

1) The instantaneous center makes no sense in this type of mechanism.
2) The instantaneous centers of 45, 34, and 15 are at infinity.
3) The instantaneous center 45 is at point B, 34 at point A, and 15 on the perpendicular line at infinity.
4) The instantaneous center 45 is at point B, 34 on the line perpendicular to the member at infinity, and 15 at infinity.

Solution The members 5 on 4, 3 on 4, and 5 on 1 have a straight sliding motion, so the instantaneous centers are at infinity.

Option (2) is correct.

The desired instantaneous center of lever 2 relative to lever 1 is denoted by 21 or 12, and the instantaneous center of lever 4 relative to lever 3 is displayed with 43

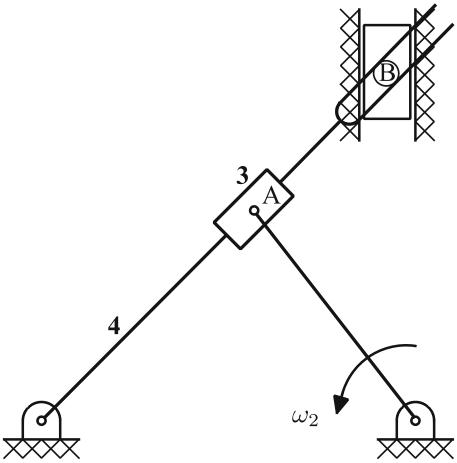


Fig. 2.20 Five-bar mechanism

or 34. In general, the instantaneous center of lever A relative to lever B is displayed with AB or BA. Sometimes it is also indicated by I_{AB} or I_{BA}.

Note The desired instantaneous center of lever A relative to lever B is on the desired instantaneous center of lever B relative to lever A.

2.6.2 Kennedy's Theorem

Kennedy's theorem for three independent objects with a generally planar motion states that their three instantaneous centers are on a common straight line. In a mechanism consisting of n levers, there are $n - 1$ instantaneous centers for each assumed lever. Thus for n lever, there will be $n(n - 1)$ instantaneous centers. But since the position of each instantaneous center is assigned to two centers, the total number of positions (N) is obtained from the following equation:

$$N = \frac{n(n-1)}{2} \tag{2.16}$$

To determine the instantaneous centers of a mechanism, we only need to write the numbers of all members on the perimeter of a circle at separate points. Then, if the instantaneous center of both members is known, connect the points related to those two members with a line so that with a simple look, it is determined which instantaneous centers are known and which are unknown.

Example In the mechanism shown in Fig. 2.21, according to the coordinate axes specified in the figure at point B, which square locates the instantaneous center I_{36}?

1) First 2) Second 3) Third 4) Fourth

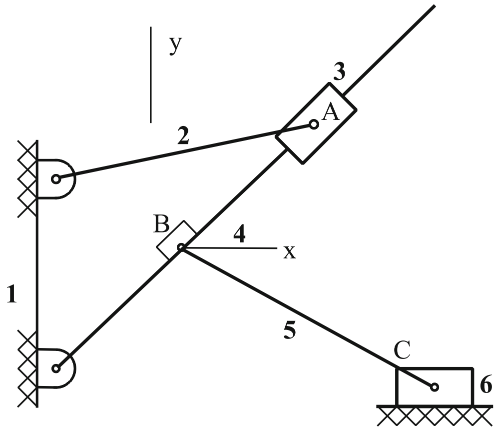


Fig. 2.21 Mechanism with three sliders

Solution According to the existing rotational and sliding joints, the instantaneous centers I_{12}, I_{34}, I_{45}, I_{56}, and I_{16} are known.

We draw a circle chart according to the available information (Fig. 2.22A). We use Kennedy's theorem for centers that are a little harder to find. In Fig. 2.22B, a dashed line is used to find the instantaneous center I_{13} that completes two triangles. Triangle 3.2.1 represents three centers (12, 23, and 13) of levers 1, 2, and 3, which are on a straight line according to Kennedy's theorem. The intersection of the two lines of the mechanism determines the center 13 that should be on both of these lines. The corresponding dashed line must be converted to a full line (Fig. 2.22C) to show the unknown center. Figure 2.22D shows the next step in which the position of center 46 is determined using triangles 6.4.1 and 6.5.4.

$$I_{13} \xrightarrow{\text{At the intersection of the connecting lines}} \begin{cases} I_{12} - I_{23} \\ I_{14} - I_{34} \end{cases}$$

$$I_{64} \xrightarrow{\text{At the intersection of the connecting lines}} \begin{cases} I_{16} - I_{14} \\ I_{65} - I_{54} \end{cases}$$

$$I_{63} \xrightarrow{\text{At the intersection of the connecting lines}} \begin{cases} I_{34} - I_{64} \\ I_{16} - I_{13} \end{cases}$$

We see that I_{63} is in the second area (Fig. 2.23).

Option (2) is correct.

Example In the mechanism of Fig. 2.24, where is the location of the instantaneous center between members 1 and 3?

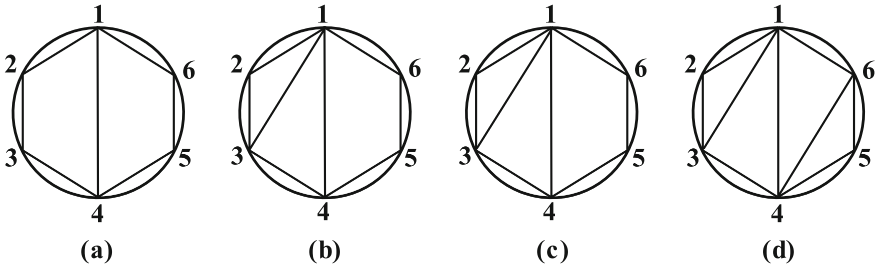


Fig. 2.22 Instantaneous centers finding

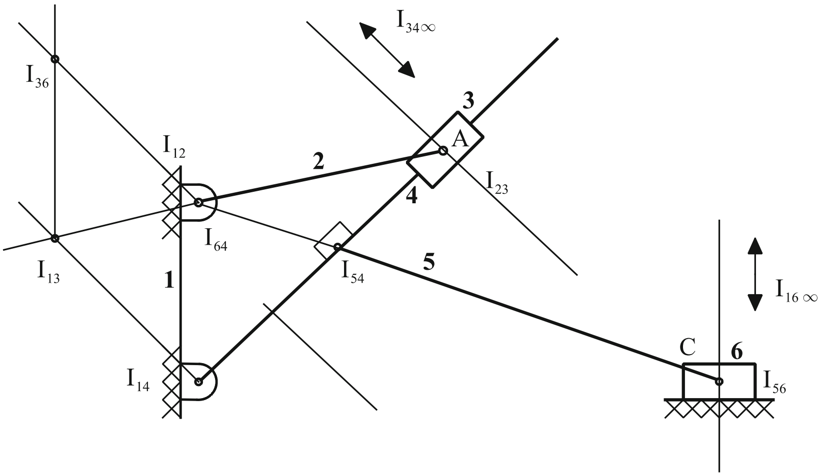


Fig. 2.23 Instantaneous centers

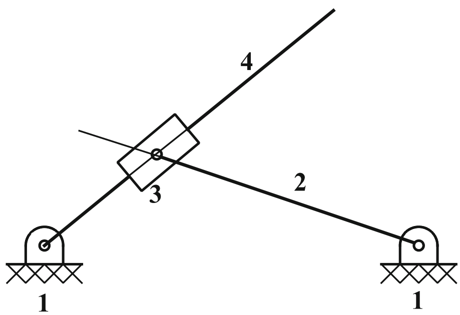


Fig. 2.24 Mechanism with slider in between

1) Along member 2 but not at infinity
2) Along member 2 and at infinity
3) On member 2
4) On the instantaneous center of members 1 and 2

Solution We know from Kennedy's theorem that for three members (1, 3, and 4), if the instantaneous centers of I_{34} and I_{14} are known, the instantaneous center of I_{13} will be somewhere on the connecting line of points I_{34} and I_{14}. For members 3 and 4, the instantaneous center is on a line perpendicular to the axis on which the slider slides and is at infinity (Fig. 2.25).

The line connecting the instantaneous centers 14 and 34 starts from the hinge connecting the lever 4 to the ground (I_{14}) and continues until the perpendicular and member 4 and infinity. On the other hand, the instantaneous center 13 will be along the line connecting the instantaneous centers 12 and 23, which is the extension of member 2. According to Fig. 2.26, it can be seen that the instantaneous center 13 is along the member 2 but is not at infinity.

Option (1) is correct.

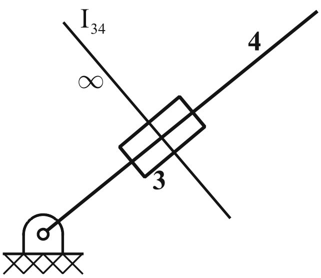


Fig. 2.25 Instantaneous centers of the mechanism with slider in between

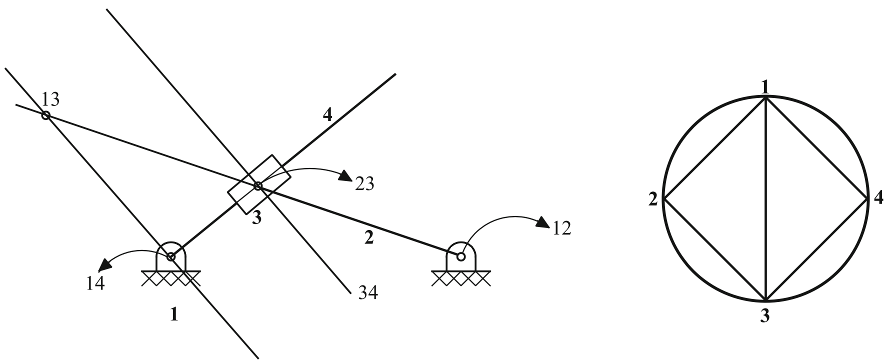


Fig. 2.26 Instantaneous center 13 is along member 2

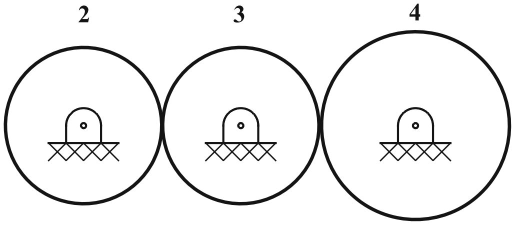


Fig. 2.27 A three-gear system

Note The instantaneous center of two levers is a point that has the same velocity if placed on either of them.

Example Where is the instantaneous center of rotation (24) in the three-gear system shown in Fig. 2.27?

1) In the middle of the center line of circles (2) and (4)
2) Point of contact of circles (2) and (3)
3) Intersection of the center line with the common internal tangent of circles (2) and (4)
4) Intersection of the center line with the common external tangent of circles (2) and (4)

Solution According to Kennedy's theorem, the instantaneous center 24 (I_{24}) is on the line connecting I_{12} and I_{14}. On the other hand, according to Fig. 2.28 and the point expressed, if the instantaneous center is a point of member 2 (or its extension), its velocity must be equal to when it is a point of member 4 (or its extension). The location of the instantaneous center is found by forming the velocity triangles of two objects and obtaining the point of common velocity.

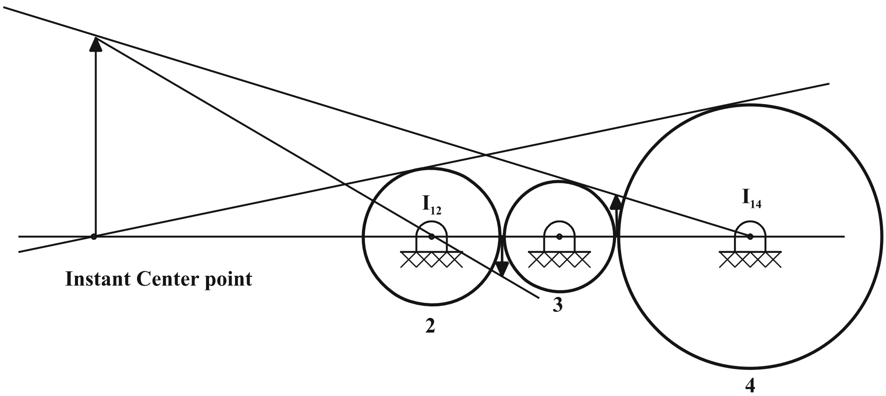
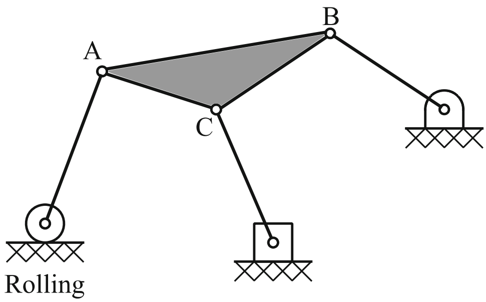


Fig. 2.28 Instantaneous centers finding of the three-gear system

Fig. 2.29 A rolling, slider, rotational joint mechanism



Option (4) is correct.

If the degree of freedom of the mechanism is more than one, determining all instantaneous centers is possible when the velocity characteristics of all its n members are known. For example, suppose the number of inputs is less than the number of degrees of freedom. In that case, the movement of the lever cannot be fully predicted, and only some of its instantaneous centers can be determined.

Note For a system of one degree of freedom, it is always possible to determine all instantaneous centers.

Example Which statement is correct for the shown mechanism in Fig. 2.29?

1) According to the available information, all instantaneous centers can be determined.
2) For this mechanism, more than three instantaneous centers of rotation are at infinity.
3) For such a mechanism, the instantaneous center of rotation cannot be defined.
4) According to the available information, some instantaneous centers can be determined.

Solution For this mechanism $n = 7$, $f_1 = 8$, and $f_2 = 0$ from Gruebler's equation, we have

$$\text{DOF} = 3\,(n - 1) - 2f_1 - f_2 = 3\,(7 - 1) - (2 \times 8) = 2$$

Only some instantaneous centers can be determined because the number of degrees of freedom is more than one.

Option (4) is correct.

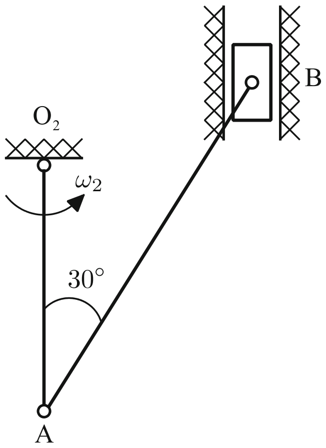


Fig. 2.30 Rotating slider mechanism

2.6.3 Determining the Velocity with the Help of Instantaneous Centers

The Kennedy theorem can be used as a suitable tool to directly determine the absolute velocity of any point in a mechanism without determining the velocity of the midpoints (such as the velocity polygon method).

Example According to the mechanism for angular velocity ω_{AB} shown in Fig. 2.30, which of the following statements is true? (O_2A is parallel to path B.)

1) The direction of ω_{AB} is equal to the direction of ω_2, but $|\omega_{AB}| > |\omega_2|$.
2) When O_2A is in line with AB, ω_{AB} is zero.
3) ω_{AB} equals twice the velocity B divided by the length AB.
4) ω_{AB} is equal to the difference of the absolute velocities A and B divided by the length AB.

Solution Because the velocity direction is known at two points, A and B, the instantaneous center I_{13} is also known, and it seems that object 3 is pinned around this point (Fig. 2.31).

$$V_A = |CA|\,\omega_{AB} \quad V_A = |O_2A|\,\omega_2 \qquad V_B = |CB|\,\omega_{AB}$$

$$\Longrightarrow \omega_{AB} = \frac{V_B}{|CB|} = \frac{V_B}{|AB|\sin 30} = \frac{2V_B}{|AB|}$$

The reason for the incorrectness of options (1), (2), and (4) was mentioned in solving this example in the drawing method for the velocity analysis.

Option (3) is correct.

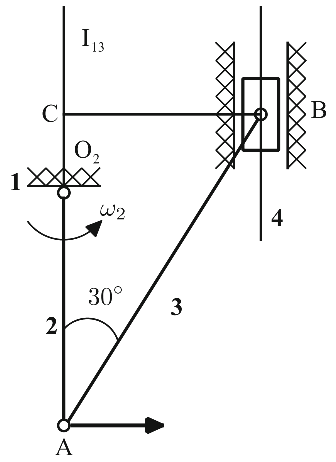


Fig. 2.31 Velocity direction analyses

2.7 Using the Transmission Line for Velocity Analysis

The distance between the points on a rigid object is always the same and does not change. Therefore, the velocity component of any two desired points of a rigid object in line with the connection of these two points is equal to each other. For example, if point A has the velocity component V_{At} along the line AB in Fig. 2.32, point B must also have the same velocity component in this direction. It is said that the velocity component of V_{At} is transmitted exactly to point B along line AB and line AB is called the transmission line.

The use of the transmission line is not limited to one member. When two rigid members are in contact with each other, the points of contact of both members must have the same velocity component in the direction of the common perpendicular at the point of contact. In this case, the common perpendicular of the two members at the point of contact is called the velocity transmission line between the two members. Using this concept in analyzing some problems is a much simpler and faster method than using velocity vector equations and other methods.

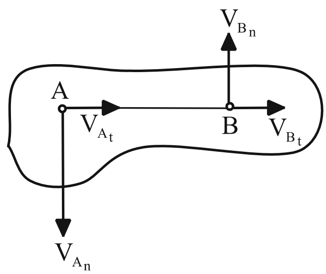


Fig. 2.32 Velocity along the connection line

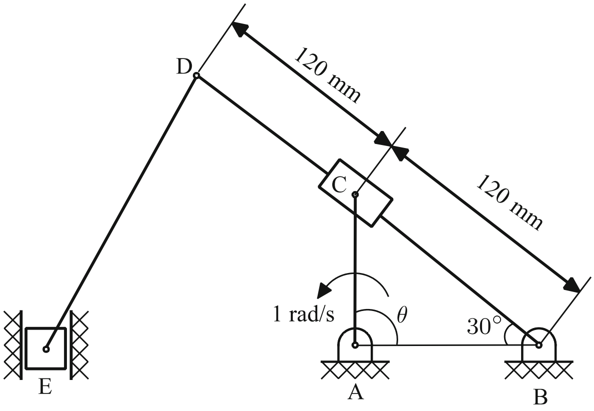


Fig. 2.33 A crankshaft bar mechanism

Example If the angular velocity of the AC crankshaft is constant and equal to $1(rad/s)$ (Fig. 2.33), at the moment when $\theta = 90°$, the angular velocity of the bar DB is equal to

1) $\frac{1}{4}$ rad/s 2) $\frac{1}{6}$ rad/s 3) $\frac{1}{8}$ rad/s 4) Neither

Solution The velocity of point C is known, and the line perpendicular to BD at point C acts as the transmission line (Fig. 2.34). Members 2 and 3 must have the same velocity component along the transmission line. We denote this component with V'. We have

$$\left.\begin{array}{r} V_C = |AC|\,\omega_4 \\ |AC| = 120 \sin 30 = 60 \text{ mm} \\ \omega_4 = 1 \text{ rad/s} \end{array}\right\} \Longrightarrow V_C = 60 \text{ mm/s}$$

$$V' = V_C \cos 60 = 30 \text{ mm/s}$$

$$\omega_2 = \frac{V'}{|BC|} = \frac{30}{120} = \frac{1}{4} \text{ rad/s}$$

Option (1) is correct.

Example Bar 3 makes an angle of 30° with the horizon surface and the ramp, and bar 5 is perpendicular to the ramp (Fig. 2.35). If $V_A = 1$ cm/s, then

1) $V_B = V_A = V_{A/B}$ 2) $V_B = 1 \cos 30° \sin 30°$
3) $V_B = 1 \cos 30° \cos 30°$ 4) $V_B = 1$ Parallel to the ramp

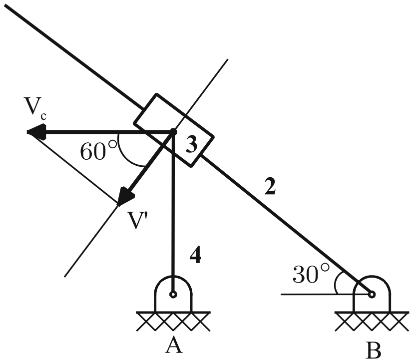


Fig. 2.34 Two bars and one slider

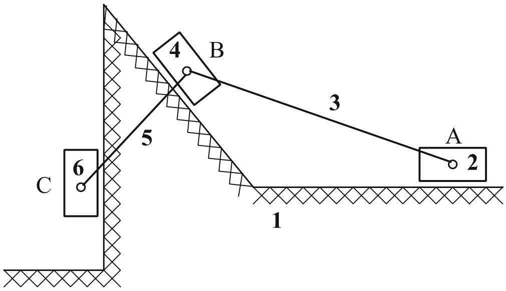


Fig. 2.35 Objects on the surface, connecting by two links

Solution Method 1: If we consider bar 3 as a velocity transmission line between its end points A and B, the velocity components of points A and B in the direction of bar 3 must be the same. On the other hand, points A and B also belong to sliders 2 and 4, so their velocity direction is parallel to their slide surface. Since the angle between the slide surfaces with bar 3 is both the same and equal to 30 degrees, therefore,

$$|V_A| \cos 30 = |V_B| \cos 30 \Longrightarrow |V_B| = |V_A| = 1 \text{ cm/s}$$

It is clear that velocity B is also parallel to the ramp.

Method 2: The instantaneous center of rotation of levers 2 and 4 with joints A and B forms an equilateral triangle (Fig. 2.36), and it can be written as

$$V_A = V_B = V_{A/B} = V_{B/A} = 1 \text{ cm/s}$$

Option (4) is correct.

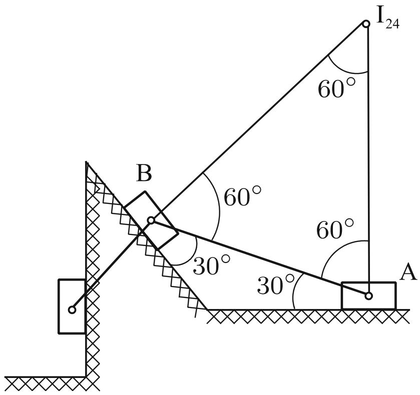


Fig. 2.36 Equilateral triangle

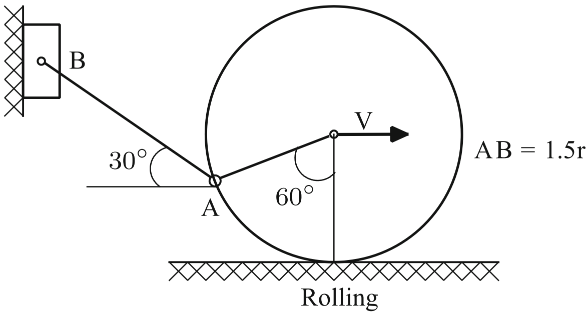


Fig. 2.37 A roller movement

Some Examples of "Displacement and Velocity Analysis"

1. In the shown mechanism in Fig. 2.37, the velocity of the roller center is V. What is the velocity of slider B?
   1) $\frac{1}{2}V$ 2) 0 3) V 4) $\frac{3}{2}V$
2. In the formed four-bar mechanism in Fig. 2.38, if the point S is assumed to belong to the AB interface, the velocity value of this point relative to the O_4B interface is equal to
   1) 0 2) $\overline{O_4S}.\omega_4$ 3) $\overline{SB}(\omega_4 - \omega_2)$ 4) $\overline{SB}\omega_4$
3. The eight-bar lever in Fig. 2.39 is under ω_2. Given the degree of freedom, which of the following equations is true for the value of the rotational velocity of the levers?

   1) $\omega_2 = \omega_3 = \omega_4$
   2) $\omega_5 = \omega_6 = \omega_7 = \omega_8 = 0$
   3) $\omega_2 = \omega_3 = \omega_4 = \omega_5 = \omega_6 = \omega_7 = \omega_8$
   4) $\omega_3 = \omega_5 = \omega_6 = \omega_7 = \omega_8$

4. In the six-bar mechanism in Fig. 2.40 for the positions $\theta_2 = \theta_6$ and $2O_4P_4 = O_4Q_4$, the velocity of slider 5 relative to slider 3 is
   1) 0 2) 0.5 3) 2 4) 1

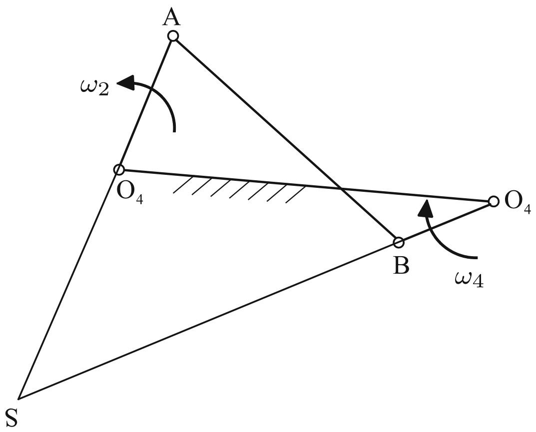


Fig. 2.38 A four-bar mechanism for velocity analysis

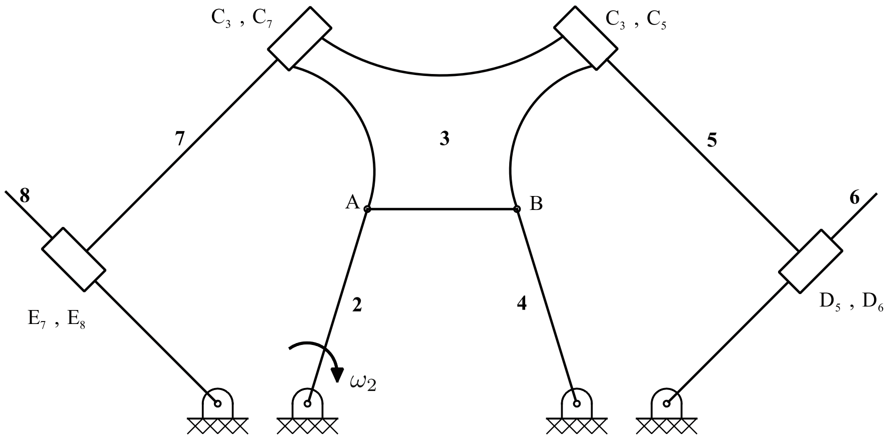


Fig. 2.39 An eight-bar lever

5. In the mechanism of Fig. 2.41, where is the instantaneous center of rotation of slider B relative to the ground (frame)?

1) Along the (p) and at ∞	2) At point Q
3) At point P	4) Along the (b) and at ∞

6. In the mechanism in Fig. 2.42, if the bar AB with a length of 10 cm moves with a clockwise rotational velocity of 1 rad/s, determine the velocity of the joint B and its direction at the shown moment. Do the rollers have both rolling and sliding movements?

1) 8.7 cm/s to the right
2) 5 cm/s to the left

Fig. 2.40 A six-bar mechanism

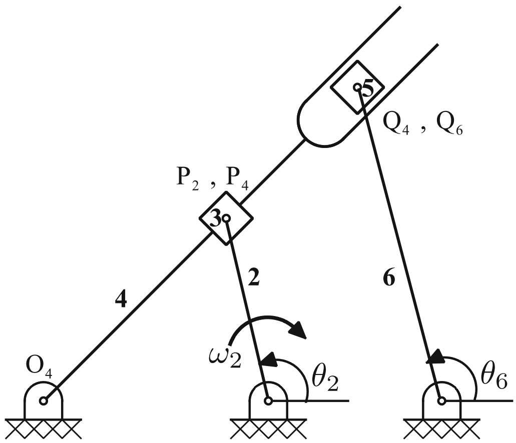
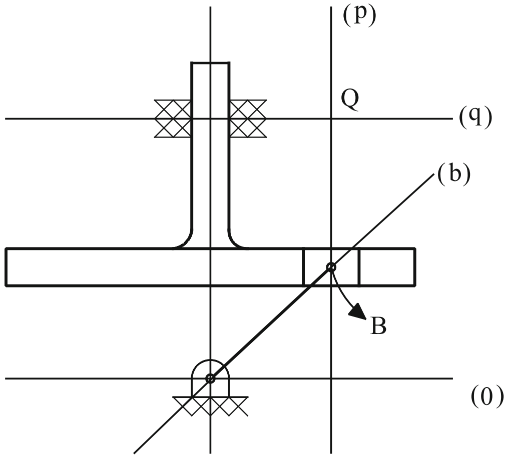


Fig. 2.41 Crank-slider



3) 5 cm/s to the right
4) It cannot be determined because the degree of freedom of the mechanism becomes 2.

7. In the six-bar lever, according to Fig. 2.43, if the velocity of point D, V_D is known, which of the following statements is correct?

1) At this point, the velocity of slider 6 is smaller than V_D.
2) At this point, the velocity of slider 6 is the same as the velocity of point D.
3) It is evident from the figure that the velocity of slider 6 is greater than V_D.
4) Because bar 4 carries a slider and is along the slider 6, the lever locks at this point.

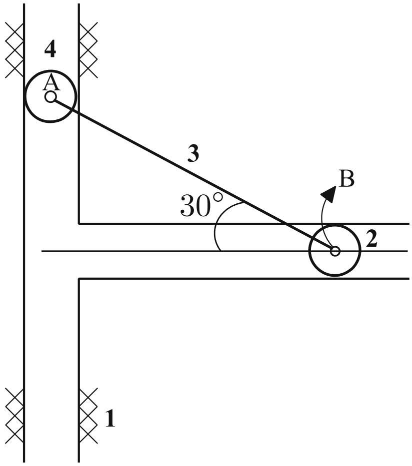


Fig. 2.42 Rollers with rolling and sliding movements

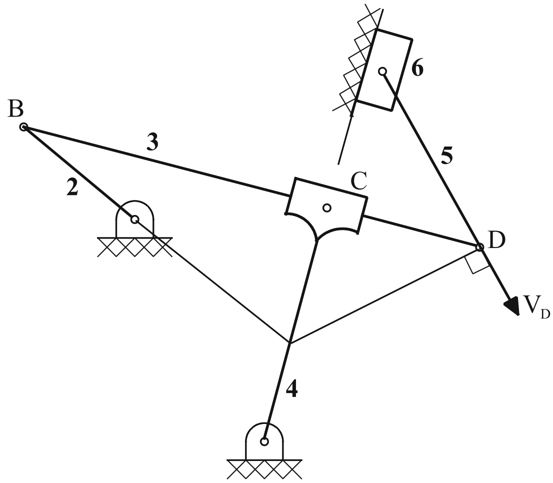


Fig. 2.43 A six-bar lever mechanism

8. In the four-bar mechanism in Fig. 2.44, if point C is the center of curvature of bar 2 at the point of contact with 3, which option is correct for instantaneous center 13?
   1) Point A 2) Point B 3) Point C 4) Point D
9. In the six-bar mechanism, according to Fig. 2.45, if V_Q is known for this moment, which of the following statements is correct?
   1) The velocities of sliders 4 and 6 are fractions of the velocity vector V_Q.
   2) The velocity of slider 4 depends on V_Q, but the velocity of slider 6 is independent.
   3) The velocity of slider 4 is equal to V_Q, and the velocity of slider 6 is determined by it.

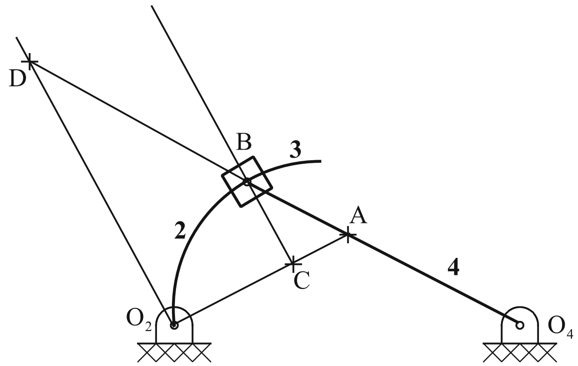


Fig. 2.44 A four-bar mechanism with a curved link

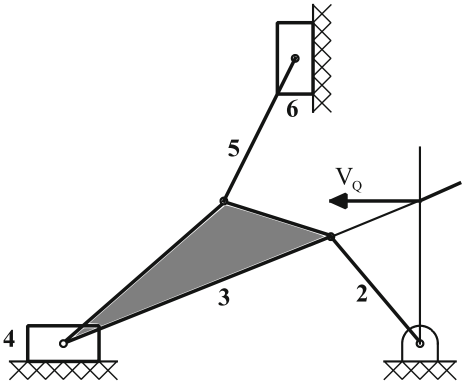


Fig. 2.45 A six-bar mechanism with two sliders

4) Sliders 4 and 6 have a rotational motion around the center at infinity, and their velocity has nothing to do with V_Q.

10. The value of ω_2 in the mechanism in Fig. 2.46 is equal to 10 rad/s and counterclockwise. Find the value of ω_4.

1) 8.3, counterclockwise
2) 10.7, moving counterclockwise
3) 15.9, counterclockwise
4) 20.6, clockwise

11. In a four-bar mechanism, according to Fig. 2.47, if bar 2 provides input movement, where is the center that slider 3 rotates around?

1) In terms of the type of lever, at point B
2) At the intersection of bar 4 with a line perpendicular to the groove from point O_2
3) At the intersection of bar 4 with a line perpendicular to the groove from point B
4) In terms of the lever type, it only has a sliding motion, and this center does not exist.

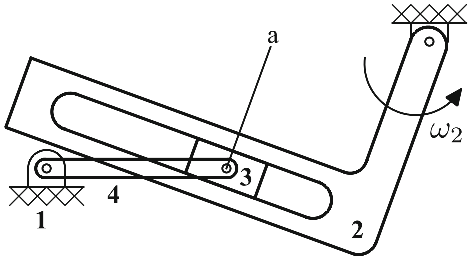


Fig. 2.46 A mechanism with counterclockwise input

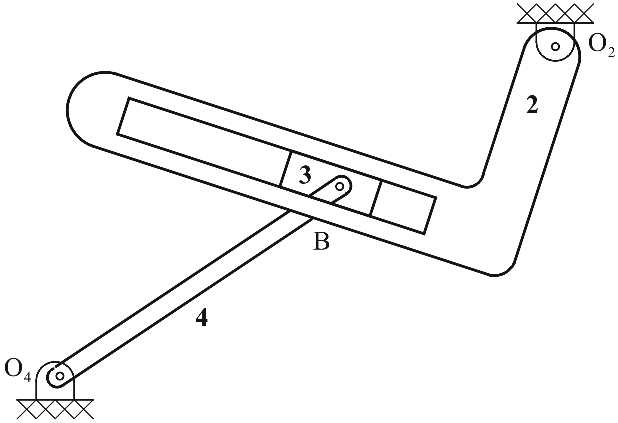


Fig. 2.47 A four-bar mechanism with rotating link in between

12. According to Fig. 2.48, the ascent velocity of the follower at the shown moment in centimeters per second is equal to N = 120 rpm, and the dimensions are in centimeters.
    1) 31.416 2) 37.25 3) 37.7 4) 43.53
13. In the mechanism in Fig. 2.49, if ω_2 is known, the mechanism has 21 instantaneous centers. How many of them can be determined?
    1) All 2) 9 3) 11 4) 10
14. In Fig. 2.50, obtain ω_{AB} in rad/s.
    1) 56.6 CCW 2) 56.6 CW 3) 34.1 CCW 4) 28.3 CCW
15. In the OAC slider-crank mechanism (Fig. 2.51), which answer is correct for the slider's velocity?
    1)$|V_C| = \omega_2.\overline{BC}$ 2)$|V_C| = \frac{\overline{OA}.\overline{BC}}{\overline{AB}}.\omega_2$
    3)$|V_C| = \frac{\overline{OA}.\overline{AC}}{\overline{BC}}.\omega_2$ 4)$|V_C| = \frac{\overline{BC}.\overline{AC}}{\overline{OA}}.\omega_2$
16. In the shown mechanism in Fig. 2.52, which statement is correct for the location of the instantaneous center of the velocity of members 4 and 5?

    1) It is along member 4.
    2) It is at point A.

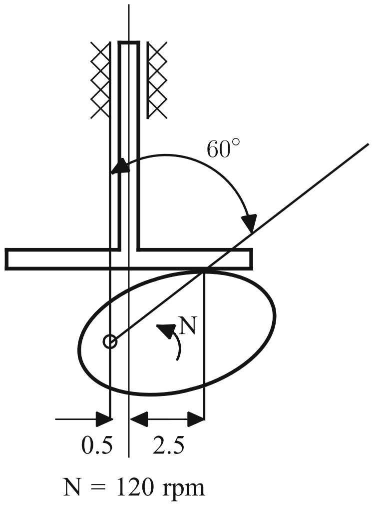


Fig. 2.48 A cam-follower mechanism

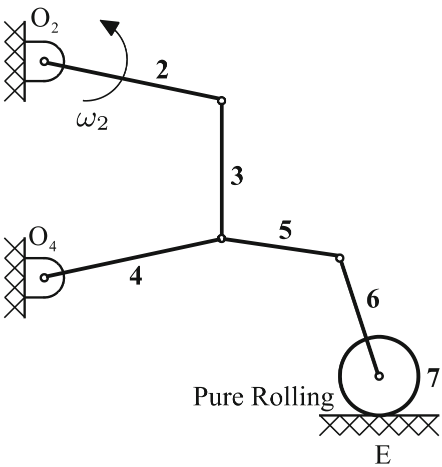


Fig. 2.49 Mechanism with 21 instantaneous centers

3) It is along the line perpendicular to member 4 from point A and at an infinite distance.
4) It is along the line perpendicular to member 4 from point A and at a finite distance.

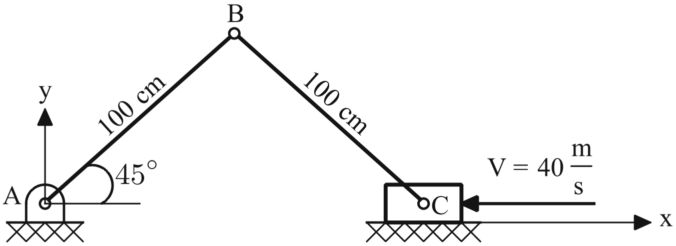


Fig. 2.50 Link AB rotational velocity

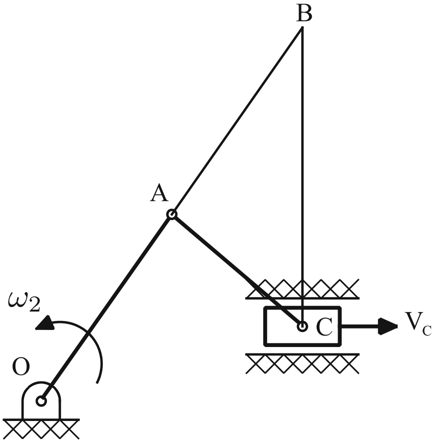


Fig. 2.51 A slider-crank mechanism

17. The bar AB rotates clockwise (Fig. 2.53) with angular speed $\omega = 3$ rad/s. The speed of slider D and angular speed of bar DB are

1) $\dot{\theta} = 3$ rad/s, $V_D = 2$ m/s
2) $\dot{\theta} = 3$ rad/s, $V_D = 4$ m/s
3) $\dot{\theta} = \frac{20}{3}$ rad/s, $V_D = 2$ m/s
4) $\dot{\theta} = \frac{20}{3}$ rad/s, $V_D = 4$ m/s

18. According to Fig. 2.54, the parallel plates are in contact with the cylinder, moving at certain velocities without sliding. Which of the following is <u>incorrect</u>?

1) Point $V_O = \frac{V_1}{4}$
2) Point$\omega = \frac{3}{2}\frac{V_2}{R}$
3) Point $\omega = \frac{3}{4}\frac{V_1}{R}$
4) Point $\omega = \frac{|V_1|-|V_2|}{2R}$

19. According to Fig. 2.55, what is the ascent velocity of the follower at the shown moment in centimeters per second? (N = 120rpm and sizes are in centimeters.)
1) 25.133 2) 29.25 3) 37.7 4) 45.533

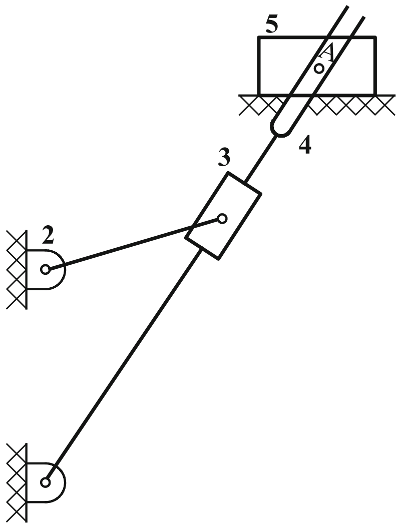


Fig. 2.52 A mechanism with some sliding members

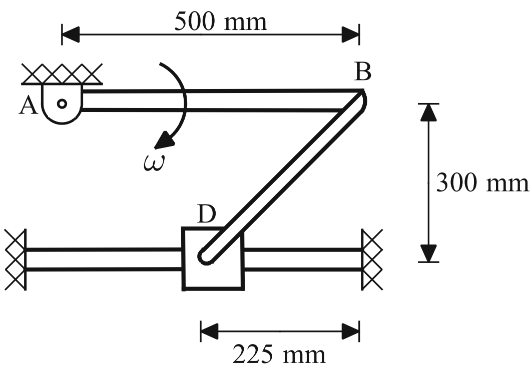


Fig. 2.53 Bar AB rotates clockwise as input

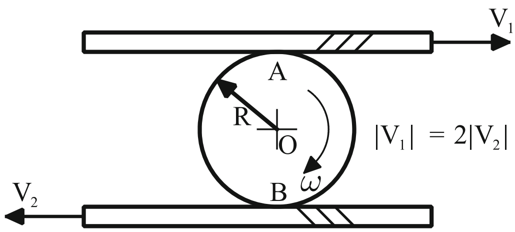


Fig. 2.54 Two parallel plates in contact with the cylinder

20. According to Fig. 2.56, what is the magnitude of the angular velocity of bar 3 at this moment in terms of radians per second? 1) 4.45 2) 6.36 3) 7.22 4) 8
21. In the mechanism shown in Fig. 2.57, the velocity of point *A* is given. Point *A* is marked on the center of the slider. The velocity of point *B* is equal to 1) 1.9 m/s 2) 2.9 m/s 3) 3.9 m/s 4) 4.9 m/s

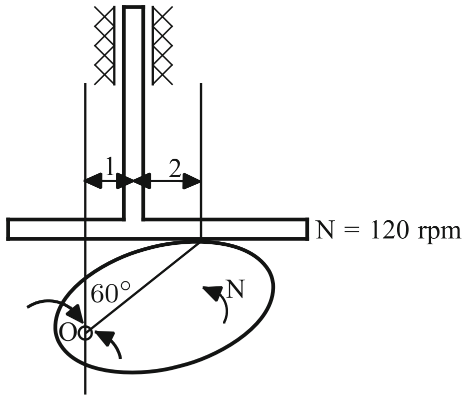


Fig. 2.55 Cam-slider mechanism

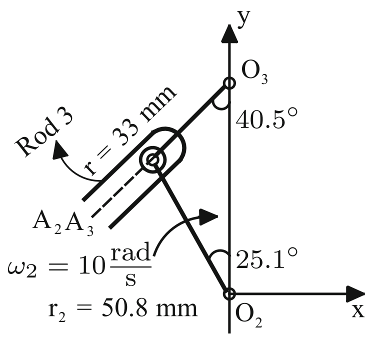


Fig. 2.56 Rod 3 has a slider

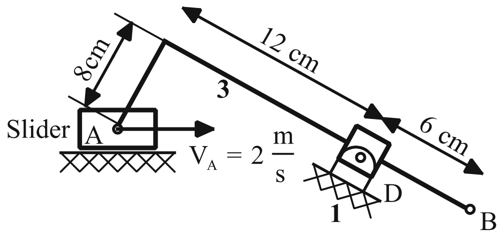


Fig. 2.57 Point A is sliding on the surface

22. What equation is correct for the four-bar mechanism in Fig. 2.58?

$$O_2A = 1 \quad O_4B = 4 \quad O_2O_4 = 4$$

1) $\omega_2 = 4\omega_4$ 2) $\omega_4 = 4\omega_2$ 3) $\omega_2 = 2\omega_4$ 4) $\omega_4 = 2\omega_2$

23. Given the angular velocity of the arm OB (member 2) (Fig. 2.59), which of the following equations is correct for finding the angular velocity of the arm BD (member 3)?

1) $\overline{V}_D = \overline{V}_B + \overline{V}_{D/B}, \omega_{BD} = \frac{V_{D/B}}{\overline{DB}}$
2) $\overline{V}_{C_3} = \overline{V}_B + \overline{V}_{C_3/B}, \omega_{BD} = \frac{V_{C_3/B}}{\overline{BC_3}}$

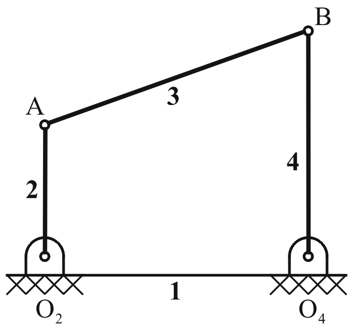


Fig. 2.58 Four-link mechanism

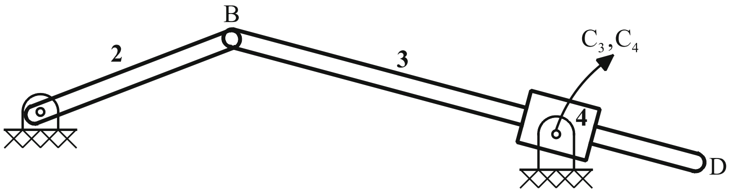


Fig. 2.59 Angular velocity of the arm OB is given

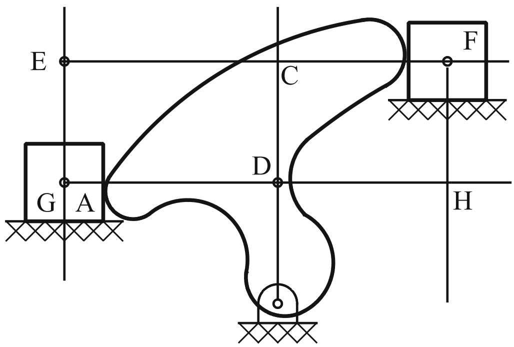


Fig. 2.60 Two sliding motions in two sides

3) $\overline{V}_{C_3} = \overline{V}_B + \overline{V}_{C_4/B}, \omega_{BD} = \frac{V_{C_4/B}}{\overline{C_4B}}$
4) $\overline{V}_{C_4} = \overline{V}_{C_3} + \overline{V}_{C_4/C_3}, \omega_{BD} = \frac{V_{C_4/C_3}}{\overline{C_4C_3}}$

24. What are the two points of the instantaneous centers of the non-primary rotation (other than 12, 13, and 14) in the mechanism of Fig. 2.60?
 1) A and B 2) C and D 3) F and G 4) F and E

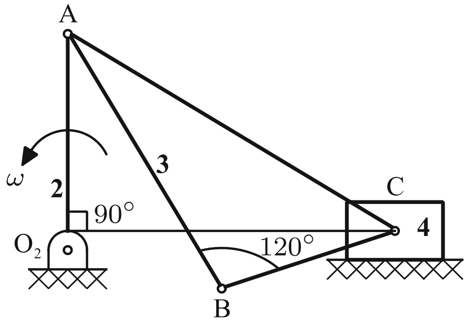


Fig. 2.61 A crank mechanism

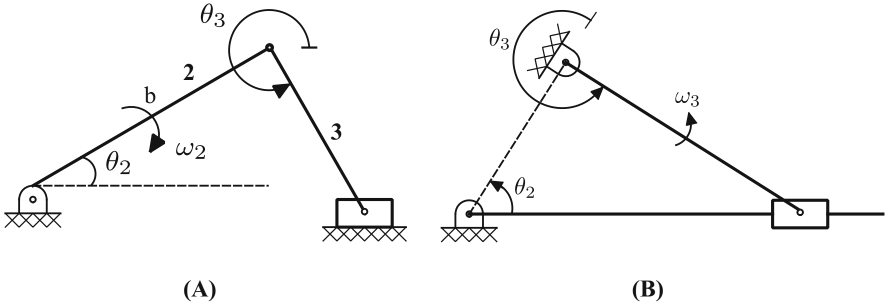


Fig. 2.62 Two crank-slider mechanisms

25. In the crank mechanism of Fig. 2.61, point B is a point on interface 3. In the shown state in the figure, what is the velocity of point B? ($AB = BC$)

    1) Zero
    2) Half the velocity of point A.
    3) Equal to the velocity of point A.
    4) Twice the velocity of point A.

26. If the angular velocity equation $\omega_3 = \frac{-b}{c}\frac{\cos\theta_2}{\cos\theta_3}\omega_2$ is correct for the sliding crank mechanism of Fig. 2.62A, which of the following equations about the rapid return mechanism of Fig. 2.62B is correct?

    1) $\omega_2 = \dfrac{-b}{c}\dfrac{\cos\theta_2}{\cos\theta_3}\omega_2$ 2) $\omega_3 = \dfrac{\omega_1}{1 + \frac{b}{c}\frac{\cos\theta_2}{\cos\theta_3}}$

    3) $\omega_1 = \dfrac{\omega_3}{1 + \frac{b}{c}\frac{\cos\theta_2}{\cos\theta_3}}$ 4) $\omega_3 = \dfrac{-b}{c}\dfrac{\cos\theta_2}{\cos\theta_3}\omega_2$

27. For the shown mechanism in Fig. 2.63, if the velocity of point D is known, which group of the following equations is sufficient to obtain the angular velocity of member 2?

    1) $V_E = V_{C_4} + V_{E/C_4}$ $V_B = V_E + V_{B/E}$ $V_E = V_D + V_{E/D}$

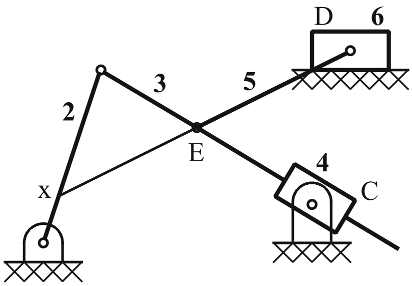


Fig. 2.63 Consider the velocity of point D is known

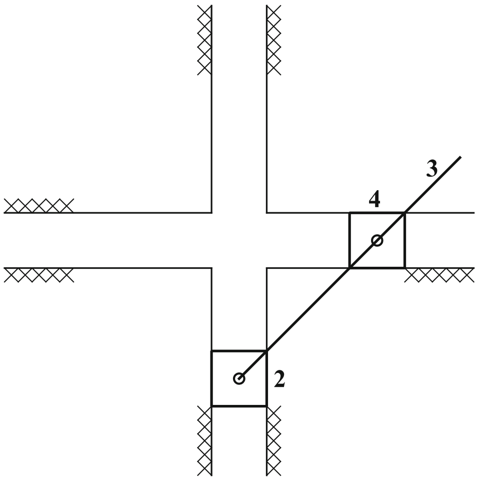


Fig. 2.64 An elliptical compass

2) $V_{C_4} = V_{C_3} + V_{C_4/C_3}$ $V_B = V_{C_4} + V_{B/C_4}$ $V_E = V_D + V_{E/D}$
3) $V_B = V_{C_3} + V_{B/C_3}$ $V_E = V_D + V_{E/D}$ $V_B = V_x + V_{B/x}$ $V_{C_3} = V_{C_4} + V_{C_3/C_4}$
4) $V_B = V_{C_3} + V_{B/C_3}$ $V_E = V_D + V_{E/D}$ $V_{C_3} = V_x + V_{C_3/x}$ $V_{C_3} = V_{C_3/C_4}$

28. Which of the following statements is true about the elliptical compass (Fig. 2.64)?

   1) It is a lever with four bars, one degree of freedom, and four instantaneous centers.
   2) It is a lever with six instantaneous centers, three of which are at infinity.
   3) The elliptical compass cannot be used to draw a circle.
   4) The elliptical compass can be used to draw a circle if the degree of freedom of the mechanism is changed.

29. In the mechanism in Fig. 2.65, the instantaneous centers 2 and 4 are

   1) Placed on component 3
   2) Along component 3 and at infinity
   3) Along component 3 but not at infinity
   4) Not defined for this mechanism

Fig. 2.65 Objects 2 and 4 are on the surface

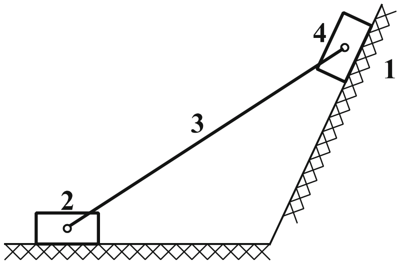
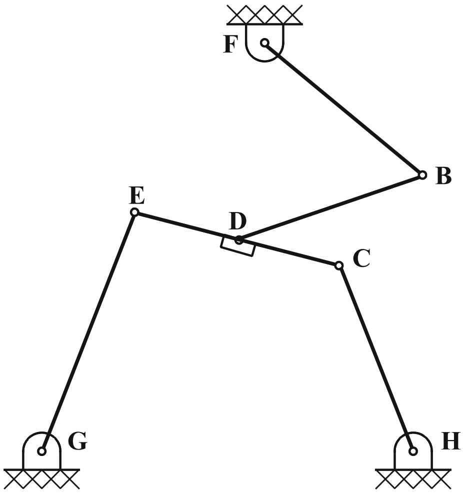


Fig. 2.66 A six-bar mechanism



30. Which of the following equations is not true about the six-bar mechanism in Fig. 2.66?

1) $\begin{cases} -V_{C/H} - V_{E/C} + V_{G/E} = 0 \\ -V_{B/F} - V_{D/B} - V_{E/D} + V_{G/E} = 0 \end{cases}$

2) $\begin{cases} -V_{C/H} - V_{E/C} + V_{E/G} = 0 \\ -V_{B/F} - V_{D/B} - V_{E/D} + V_{E/G} = 0 \end{cases}$

3) $\begin{cases} V_D = V_E + V_{D/E} \\ V_D = V_C + V_{D/C} \end{cases}$

4) $\begin{cases} V_C = V_H + V_{C/H} \\ V_C = V_D + V_{C/D} \end{cases}$

31. According to Fig. 2.67, a pin with diameter d is installed around disk A, and four grooves with width d have been created on the other disk. Disc A rotates evenly every second, and the motion is transferred to Disc B. In this case,

1) Disc B rotates only half a turn each time Disc A rotates.

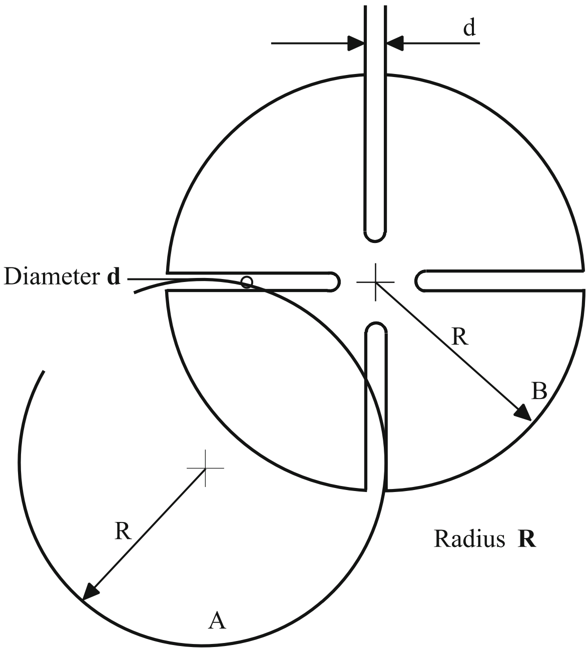


Fig. 2.67 Disc A rotates evenly every second

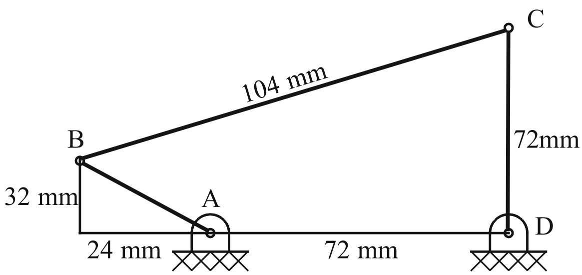


Fig. 2.68 Angular velocity of the member AB is equal to 20rad/s

2) Disc B Continues to rotate for only half a second for each rotation of disc A.
3) When disk B rotates, its angular velocity at any moment is equal to the angular velocity of disk A.
4) None.

32. In the previous problem, when the pins are in line with the centers of disks A and B, the angular velocity of disk B is equal to

1) 2π rad/s 2) $\frac{2\pi}{\sqrt{2}-1}$ rad/s 3) 4π rad/s 4) $\frac{1}{\sqrt{2}-1}$ rad/s

33. In the mechanism of Fig. 2.68, if the angular velocity of the member AB is equal to $20 rad/s$, the angular velocity of the member BC will be equal to

1) 5 rad/s 2) 10 rad/s 3) 15 rad/s 4) None

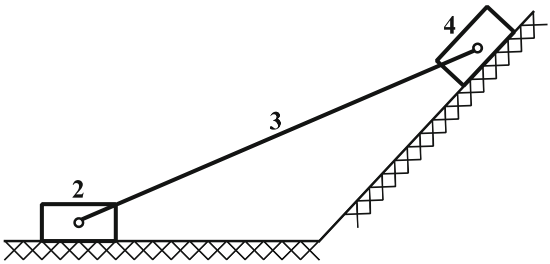


Fig. 2.69 Link 3 connects objects 2 and 4

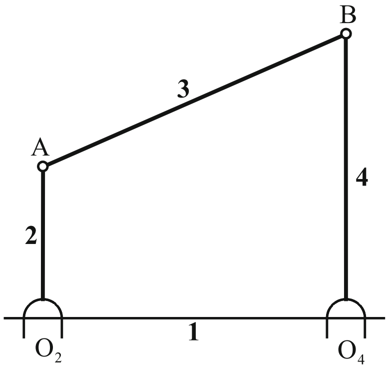


Fig. 2.70 A normal four-bar mechanism

34. In the mechanism shown in Fig. 2.69, the instantaneous centers 2 and 4 are
    1) Located on component 3
    2) Along component 3 but not at infinity
    3) Along component 3 and at infinity
    4) Not defined for this mechanism

35. Which equation is true in the four-bar mechanism in Fig. 2.70?

$$O_2A = 1 \quad O_4B = 4 \quad O_2O_4 = 4$$

1) $\omega_2 = 2\omega_4$ 2) $\omega_4 = 4\omega_2$ 3)$\omega_4 = 0$ 4)$\omega_2 = 4\omega_4$

Answers for the Examples of "Displacement and Velocity Analysis"

1. Option (2) is correct.
   The angular velocity of the roller is equal to $\omega = \frac{V}{R}$. If its rolling point is called O, we have

$$V_A = V_O + \omega \times r = 0 + \left(\frac{V}{R}\right) r = V$$

$\overrightarrow{V}_A$ is perpendicular to OA and has no component in the direction OA. Since OA and AB are also aligned, $\overrightarrow{V}_A$ has no component in the direction AB. Therefore, $\overrightarrow{V}_B$ also has no component in the direction AB. Note that the direction of velocity B must be in the direction of sliding (vertical direction). And since the component of this velocity in the direction AB is zero, $\overrightarrow{V}_B$ itself is also zero. That is, the velocity of slider B is zero.

2. Option (1) is correct.

   O_4 is the instantaneous center 14, B is the instantaneous center 34, A is the instantaneous center 23, and O_2 is the instantaneous center 12. It is easy to conclude from Kennedy's theorem that S is the instantaneous center 13, and therefore its instantaneous velocity is zero.

3. Option (4) is correct.

   To have a certain movement, the degree of freedom of the mechanism must be one, because otherwise, the calculation of all velocity parameters will not be possible. The contact of the sliders and bars is with weld, and it is possible to move through the sliders. Members 2, 3, and 4 form a four-bar mechanism that, with ω_2, the values of ω_3 and ω_4 will also be known. The rotation of member 3 moves bars 5 and 7 at the same rotational velocity (ω), and so bars 6 and 8 will rotate at the same rotational velocity. So we have

$$\omega_3 = \omega_5 = \omega_6 = \omega_7 = \omega_8$$

4. Option (2) is correct.

   First method:

   The velocity of member 5 in the direction perpendicular to member 4 is twice the velocity of member 3 in the direction perpendicular to member 4. It is equal to $O_4Q_4 = \omega_4$ and $O_4P_4 = \omega_4$, respectively. On the other hand, the velocities of both sliders 5 and 2 are parallel. Since the length of member 6 is twice the length of member 2 (according to the similarity of the triangles), the velocity of member 5 in the direction perpendicular to member 6 is twice the velocity of member 3 in the direction perpendicular to member 2. Therefore, their image along member 4 has the same ratio.

   Second method:

$$V_{P_2} = V_{P_4} + V_{P_2/P_4}$$

$$V_{Q_6} = V_{Q_4} + V_{Q_6/Q_4}$$

   It can be shown that these two vector equations form two similar triangles with a similarity ratio of 2. Therefore $V_{Q_6} = 2V_{P_2}$ and therefore $V_{Q_5} = 2V_{P_3}$.

5. Option (4) is correct.

   Using Kennedy's theorem, we can find the instantaneous center of rotation of slider B relative to the ground. According to Fig. 2.71,

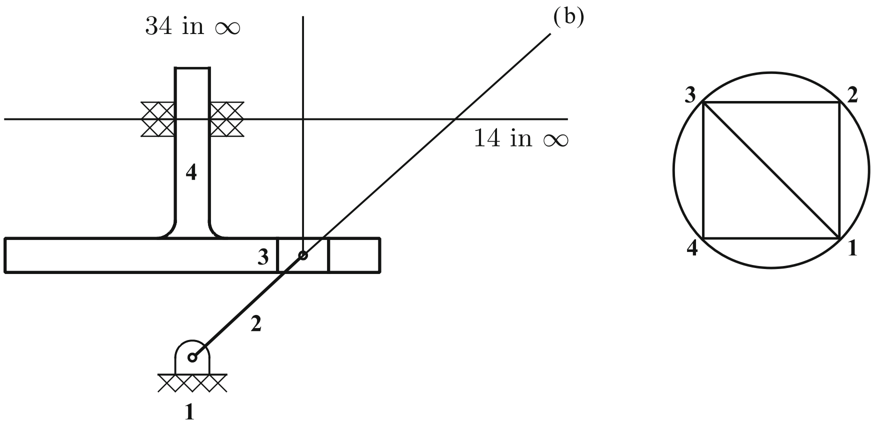


Fig. 2.71 Finding the instantaneous rotation centers

I_{12} is connected to I_{23}, which is a line in the direction (b). The junction of I_{14} and I_{34} is also at infinity. Thus the instantaneous center I_{13} is located along (b) and at infinity.

6. Option (2) is correct.

$$V_A = V_B + \omega \times R_{BA}$$

$$\Longrightarrow V_A \hat{j} = V_B \hat{i} + (1\hat{k}) \times 10(\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j})$$

Since the component $\hat{j}$ of the velocity B is zero, we consider only the component $\hat{i}$, and from the above equation, we have

$$V_B = -5\hat{i}$$

7. Option (3) is correct.

According to the transmission line discussion, the velocity component of the two ends of member 5 in the direction of connection of these two points is the same. On the other hand, point D has only one velocity component in the direction of member 5. Still, the other end of member 5 (the head connected to lever 6) and this component of velocity have another component in the direction perpendicular to member 5. Therefore, part of the velocity of slider 6 is equal to the total velocity of point D. It can be said that the total velocity of slider 6 is greater than the velocity D.

8. Option (1) is correct.

Instantaneous center 13 is located at the intersection of the connecting line between instantaneous centers 23–12 (O_2C) and instantaneous centers 34–43 (member 4). This intersection is point A.

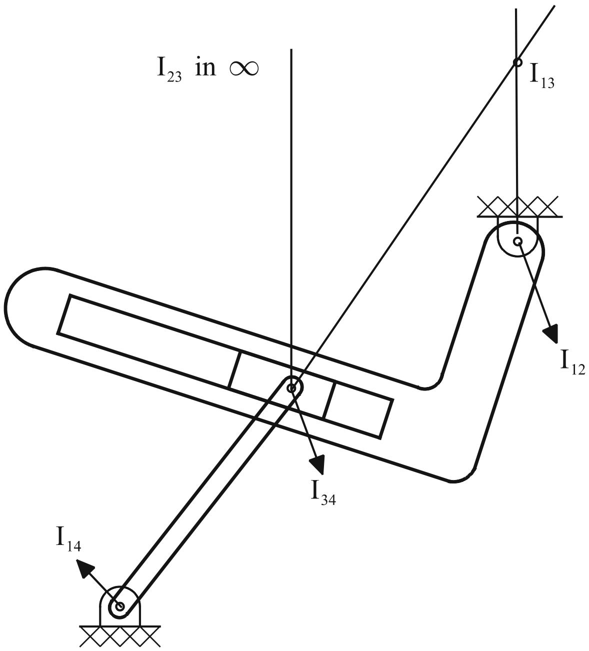


Fig. 2.72 Line connecting instantaneous centers of links 2 and 3 and links 1 and 2

9. Option (3) is correct.

   Point Q is located along the virtual extension of member 2. This point is located along the connecting line of I_{12} and I_{14}. On the other hand, this point is along the connecting line of I_{34} and I_{23}. Thus, this point is the instantaneous center of rotation 24 (I_{24}) according to Kennedy's theorem. So its velocity is equal to the velocity of slider 4.

10. Option (2) is correct.

    Due to the lack of dimensions and sizes, an exact solution cannot be provided. But considering Fig. 2.72, the distance of I_{12} to slider 3 is approximately equal to the distance of I_{14} to this slider. On the other hand, I_{12}, I_{14}, and slider 3 are approximately in the same direction. Therefore, the velocity of slider 3 perpendicular to this direction is calculated from the following equations:

$$\left.\begin{aligned} V_3 &= \omega_4 \times r_4 \\ V_3 &= \omega_2 \times r_2 \end{aligned}\right\} \Longrightarrow \omega_4 \simeq \omega_2$$

11. Option (2) is correct.

    Instantaneous center I_{13} is located along the line connecting instantaneous centers I_{23} and I_{12}, as well as instantaneous centers I_{14} and I_{34}.

12. Option (3) is correct.

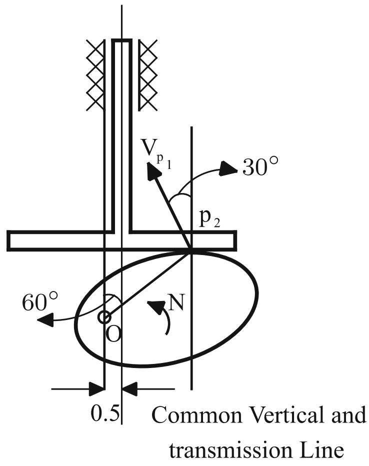


Fig. 2.73 Camshaft and follower at the point of contact

Assuming that P_1 and P_2 are the points belonging to the camshaft and the follower at the point of contact (Fig. 2.73), respectively:

$$V_{P_1} = |OP_1|\omega_1 \quad \omega_1 = \frac{120 \times 2\pi}{60} = 12.57 \text{ rad/s}$$

$$|OP_1| = \frac{3}{\sin 60} = 3.46\text{cm}$$

$$V_{P_1} = 3.46 \times 12.57 = 43.49\text{cm/s}$$

The image of the velocities of points P_1 and P_2 along the common vertical must be equal and in the same direction. And on the other hand, because the direction of V_{P_2} is also in the direction of the common vertical,

$$V_{P_2} = V_{P_1} \cos 30 = 43.49 \cos 30 = 37.7\text{cm/s}$$

Test method: The desired answer is the multiplication of the horizontal distance of the point of contact from the point O $(3cm)$ with ω_1.

$$V_{P_2} = 3 \times 12.57 = 37.7$$

10. Option (3) is correct.

By knowing w and ω_2, the velocity characteristics of members 2, 3, and 4 can be obtained. Therefore, all instantaneous centers related to the 4-3-2-1 mechanism, which are 6, can be calculated.

Also, due to the pure rotation of the disk, the instantaneous center I_{17} is the point of contact of the disk with the ground. Also, I_{67}, I_{56}, I_{35}, and I_{45} are quite clear. Then the other 5 instantaneous centers of the system are also identified. Because the system has two degrees of freedom, it is impossible to determine all instantaneous centers by knowing only one velocity quantity (ω_2). So $6 + 5 = 11$ instantaneous centers can be determined.

11. Option (4) is correct.

The velocity component of points B and C along the BC transmission line must be the same, so

$$V_{C/BC} = 40 \cos 45^\circ = \frac{40}{\sqrt{2}} = 28.28$$

This component is the velocity in the direction of BC, from C to B. But the velocity vector of point B is perpendicular to AB. Therefore due to the perpendicularity of BC to AB, it is in the direction of BC (away from C).

$$V_{B/BC} = V_B = 28.28 \text{ m/s}$$

On the other hand,

$$V_B = |AB|\,\omega_{AB} = \frac{100}{100}\omega_{AB} \Longrightarrow \omega_{AB} = V_B = 28.3$$

12. Option (2) is correct.

Point B is the instantaneous center of rotation and the instantaneous center of the bar AC with the ground, and it looks like this bar is pinned to the point B, so

$$\begin{pmatrix} V_A = |AB|\,\omega_{AC} \\ V_A = |OA|\,\omega_2 \end{pmatrix} \Longrightarrow \omega_{AC} = \frac{|OA|\,\omega_2}{|AB|}$$

$$V_C = |BC|\,\omega_{AC} = \frac{|BC|\,|OA|}{|AB|}\omega_2$$

Note that the dot symbol does not mean internal multiplication in this question.

13. Option (3) is correct.

Member 5 slides on member 4, and this slide is a straight line. So the instantaneous center is on the line perpendicular to member 4 and is at infinity.

14. Option (3) is correct.

You can see the directions of the velocities in Fig. 2.74.

$$|V_B| = |AB|\,\omega \qquad |V_B| = 0.5 \times 3 = 1.5 \text{ m/s} \qquad V_D = V_B + V_{D/B}$$

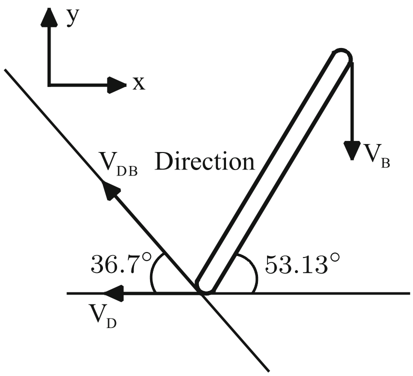


Fig. 2.74 Directions of velocities

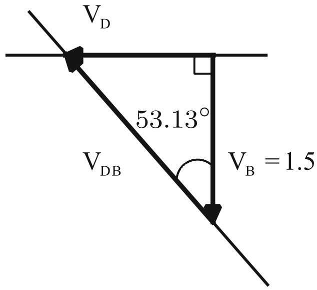


Fig. 2.75 Velocity triangle

According to the velocity triangle drawn in Fig. 2.75, we can write

$$\left|V_{D/B}\right| = \frac{|V_B|}{\cos 53.13} = 2.5 \text{ m/s}$$

$$\dot{\theta} = \frac{\left|V_{D/B}\right|}{|DB|} = \frac{2.5}{0.375} = \frac{20}{3} \text{ rad/s}$$

$$V_D = |V_B| \tan 53.13 = 2 \text{ m/s}$$

15. Option (4) is correct.

    The planes have non-slide contact with the disk, so their velocity is the same as the disk velocity at points A and B (Fig. 2.76). By obtaining the center of rotation C,

$$\omega = \frac{\left|V_{A/B}\right|}{|AB|} = \frac{3V_2}{2R} = \frac{3V_1}{4R}$$

$$V_1 = |CA|\,\omega \Longrightarrow |CA| = \frac{4R}{3} \Longrightarrow |CO| = \frac{R}{3}$$

$$V_0 = |CO|\,\omega = \frac{R}{3}.\frac{3}{4}\frac{V_1}{R} = \frac{V_1}{4}$$

So options (1), (2), and (3) are correct and option (4) is incorrect.

Fig. 2.76 Velocities of points A and B

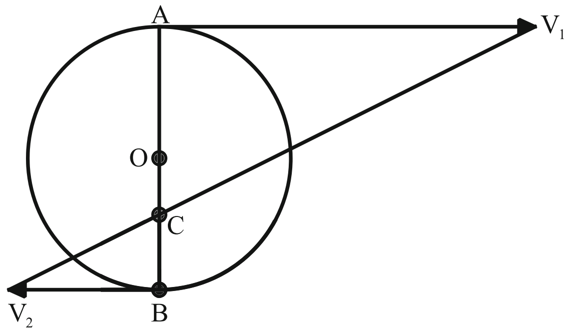
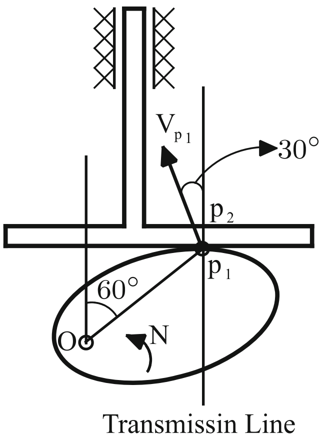


Fig. 2.77 Transmission line and common vertical direction



16. Option (3) is correct.

    Assuming that P_1 and P_2 are the points belonging to the camshaft and the follower (Fig. 2.77) at the point of contact, respectively:

$$V_{P_1} = |OP_1|\,\omega_1$$

$$|OP_1| = \frac{3}{\sin 60} = 3.46 \text{ cm} \qquad \omega_1 = \frac{120 \times 2\pi}{60} = 12.57 \text{ rad/s}$$

$$V_{P_1} = 3.46 \times 12.57 = 43.49 \text{ cm/s}$$

    The image of the velocities of points P_1 and P_2 in the common vertical direction must be equal and in the same direction. And on the other hand, because the direction of V_{P_2} is also in the common vertical direction,

$$V_{P_2} = \left|V_{P_1}\right| \cos 30 = 43.49 \cos 30 = 37.7 \text{ cm/s}$$

17. Option (2) is correct.

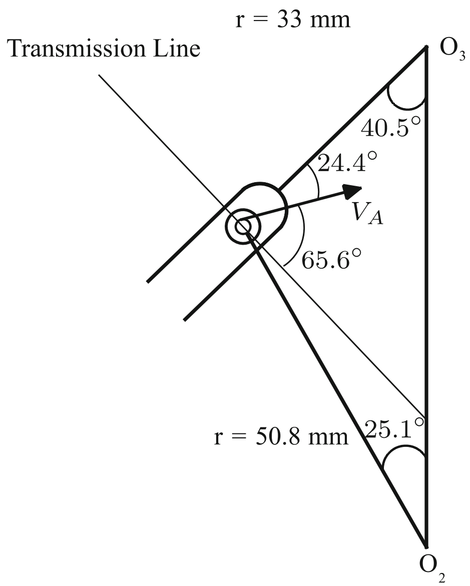


Fig. 2.78 The velocity of the center of the roller

The velocity of point A_2 in the center of the roller is as in Fig. 2.78:

$$\left|V_{A_2}\right| = r_2.\omega_2 \qquad \left|V_{A_2}\right| = 50.8 \times 10 = 508 \text{ mm/s}$$

The image of the velocity of point A_2 in the common vertical direction (transmission line) gives us the velocity of point A_3 belonging to bar 3 at the point of contact.

$$\left|V_{A_3}\right| = \left|V_{A_2}\right| \cos 65.6 = 508 \cos 65.6 = 209.85 \text{ mm/s}$$

$$\omega_3 = \frac{\left|V_{A_3}\right|}{r_3} = \frac{209.85}{33} = 6.36 \text{ rad/s}$$

18. None of the options is correct.

If point D belongs to member 3, its velocity direction is known. On the other hand, points D and A belong to a rigid body, so the velocity components are the same along the connecting line (Figs. 2.79 and 2.80) (see the text of the transmission line section).

$$V_D \cos 33.6 = V_A \Longrightarrow V_D = 2.4 \text{ m/s}$$

$$V_{D/A} = V_D \sin 33.6 = 1.33 \text{ m/s}$$

$$\omega_3 = \frac{V_{D/A}}{|AD|} = \frac{1.33}{0.144} = 9.24 \text{ rad/s } c\omega$$

$$V_B = V_D + V_{B/D}$$

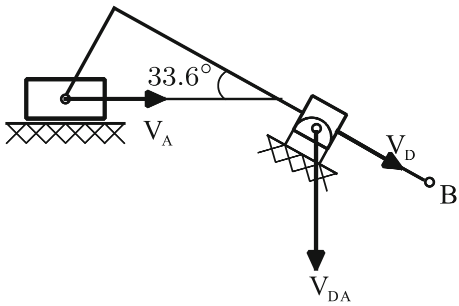


Fig. 2.79 Velocity components of points A and D

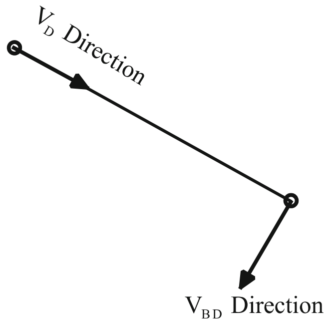


Fig. 2.80 Velocity of point D relative to point B

$$V_{B/D} = |DB|\,\omega = 0.06 \times 9.24 = 0.55 \text{ m/s}$$

$$V_B = \sqrt{V_D^2 + V_{B/D}^2} = \sqrt{2.4^2 + 0.55^2} = 2.46 \text{ m/s}$$

19. Option (1) is correct.

 The velocities of points A and B from member 3 are in the same direction, so ω_3 is zero.

$$V_{B/A} = |AB|\,\omega_3 = 0$$

$$V_B = V_A + V_{B/A} \Longrightarrow V_B = V_A$$

$$|O_2A|\,\omega_2 = |O_4B|\,\omega_4\omega_2 = 4\omega_4$$

20. Option (2) is correct.

 In the given mechanism, the velocity of point B and the velocity direction of point C_3 are known. At the same time, the velocity direction of $V_{C_3/B}$ is also known, so by solving the following equation, the size of $V_{C_3/B}$ and V_{C_3} can be obtained.

$$V_{C_3} = V_B + V_{C_3/B}$$

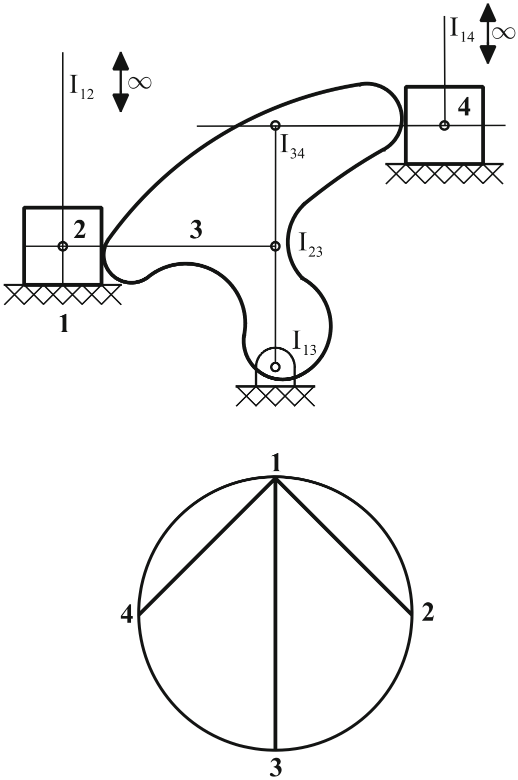


Fig. 2.81 Auxiliary circle

By obtaining $V_{C_3/B}$,

$$\omega_{BD} = \frac{V_{C_3/B}}{|BC_3|}$$

So option (2) is correct.

Option (1) is wrong because we know nothing about point D, the equations of option (3) are also wrong, and option (4) does not give us any specific information.

21. Option (2) is correct.

First, note that the velocities of components A and B must be equal at the instantaneous center I_{AB}. Therefore, I_{23} and I_{34} must be located on the common vertical (3,2) and (4,3), respectively.

By drawing an auxiliary circle (Fig. 2.81) and considering the direct contact of 2 to 3 and 3 to 4, we can conclude

$$I_{34} \xrightarrow{\text{At the intersection}} \begin{cases} I_{14} - I_{13} \text{ The connecting line} \\ \qquad \text{Common vertical} \end{cases}$$

$$I_{23} \xrightarrow{\text{At the intersection}} \begin{cases} I_{12} - I_{13} \text{ The connecting line} \\ \text{Common vertical} \end{cases}$$

So C and D are the instantaneous centers.

22. Option (3) is correct.

The direction of velocity at points A and C is clear. Because the two velocities are parallel, the instantaneous center of I_{13} is at infinity, meaning that all points on member 3 have the same velocity:

$$V_B = V_A$$

23. Option (3) is correct.

We must first note the difference between the two sets. The difference is in the fixed position of hinge 23 and the consequent fixing of bar 2. On the other hand, due to the similarity of the geometry of the two sets, the relative angular velocity between the members must remain constant. The given equation of angular velocity in relative terms is as follows:

$$(\omega_3 - \omega_1) = -\frac{b}{c}\frac{\cos\theta_2}{\cos\theta_3}(\omega_2 - \omega_1)$$

Because for the first mechanism, $\omega_1 = 0$.

In the second mechanism, $\omega_2 = 0$, so from the placement of $\omega_2 = 0$ in the above equation, we have

$$\omega_3 - \omega_1 = +\frac{b}{c}\frac{\cos\theta_2}{\cos\theta_3}\omega_1 \Longrightarrow \omega_1 = \frac{\omega_3}{1 + \frac{b}{c}\frac{\cos\theta_2}{\cos\theta_3}}$$

24. Option (3) is correct.

If C_3 and C_4 are points corresponding to C belonging to members 3 and 4, respectively:

In option (1) between E and C_4, in option (2) between B and C_4, and in option (4) between x and C_3, where the points belong to two different objects, the written velocity equations are not correct, but if the two points coincide, such as C_4 and C_3, the equation between the velocities is correct. The equations of the option (3), while regarding this note, use the points about which we have information, such as points B, D, and C_3, the velocity of which is known to us, while the last equation is an additional equation and there is no need for it.

25. Option (2) is correct.

$$n = 4 \Longrightarrow \text{Number of instantaneous centers} = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

$$n = 4,\ f_1 = 4,\ f_2 = 0 \Longrightarrow \text{DOF} = 3\,(n-1) - 2f_1 - f_2$$

$$= 3\,(4-1) - (-2 \times 4) = 1$$

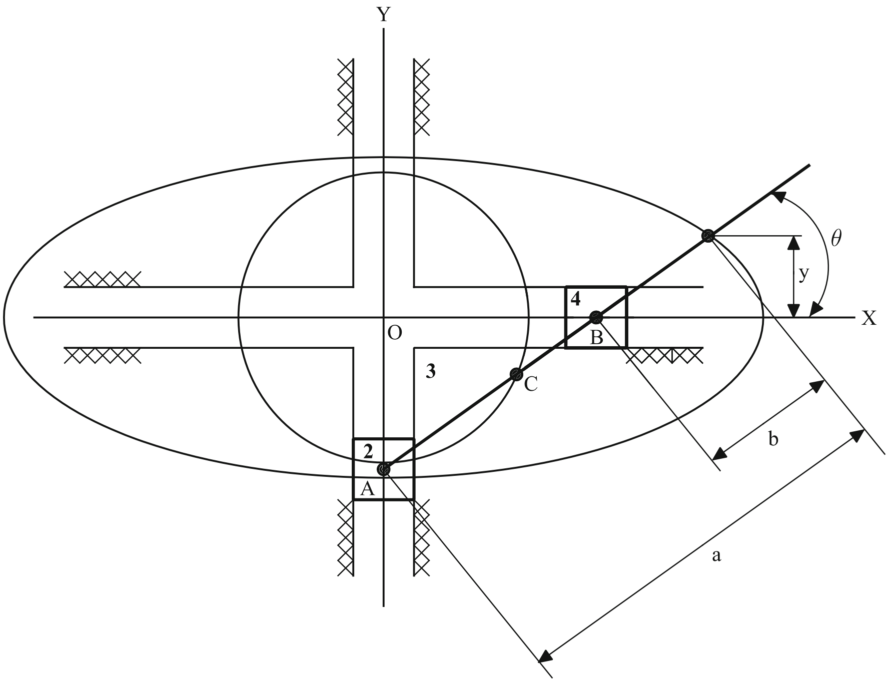


Fig. 2.82 Circle is a kind of ellipse

Option (3) is wrong because the circle is a kind of ellipse (Fig. 2.82). It should be noted that if the tip of the pen is in the middle of the distance between joints 23 and 34 (meaning when the pen P is at point C), the equation of the circle is obtained, and there is no need to change the degree of freedom of the mechanism.

26. Option (3) is correct.

    According to Kennedy's theorem, I_{24} is at the intersection of the connecting line of I_{23} and I_{34} with the connecting line of I_{12} and I_{14}, but I_{14} and I_{12} are at infinity, so the instantaneous center I_{24} is along component 3 and at infinity (Fig. 2.83).

27. Option (1) is correct.

    Options (3) and (4) represent the velocity equations between two points on a rigid body, so they are correct. In option (2), since points F, H, and G are fixed,

$$V_{B/F} = V_B \qquad V_{E/G} = V_E \qquad V_{C/H} = V_C$$

$$- V_{C/H} - V_{E/C} + V_{E/G} = -V_C - V_E + V_C + V_E = 0$$

$$- V_{B/F} - V_{D/B} - V_{E/D} + V_{E/G} = -V_B - V_D + V_B - V_E + V_D + V_E = 0$$

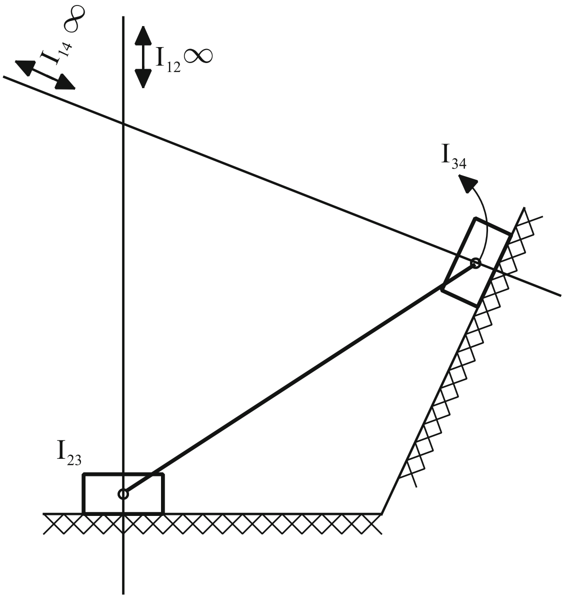


Fig. 2.83 Intersection of the connecting lines

Option (2) equations are also correct, so option (1) is wrong. Option (1) equations can be checked like option (2). Note that these equations are vectors.

28. Option (4) is correct.

    In the Genoa wheel mechanism, for one rotation of disc A, disc B rotates a quarter, and then disc B continues to rotate for a quarter of a second.

29. Option (2) is correct.

    The pin is a moving point pin on the coordinate system connected to disk B (Fig. 2.84). When the pin is aligned with the disc centers A and B, its relative velocity relative to the groove is zero. So if P_1 and P_2 are points belonging to disks 1 and 2 at the point of contact, then

$$V_P = V_{P_1} = V_{P_2} + V_{P_1/P_2} \qquad V_{P_1/P_2} = 0$$

$$V_{P_1} = V_{P_2} \qquad R_{\omega_1} = |C_2P|\,\omega_2$$

$$|C_1C_2| = \sqrt{2}R \qquad |C_2P| = \left(\sqrt{2}-1\right)R$$

$$\omega_1 = 2\pi \text{ rad/s}$$

$$R_{\omega_1} = \left(\sqrt{2}-1\right)R\omega_2 \Longrightarrow \omega_2 = \frac{2\pi}{\sqrt{2}-1}\text{ rad/s}$$

30. Option (1) is correct.

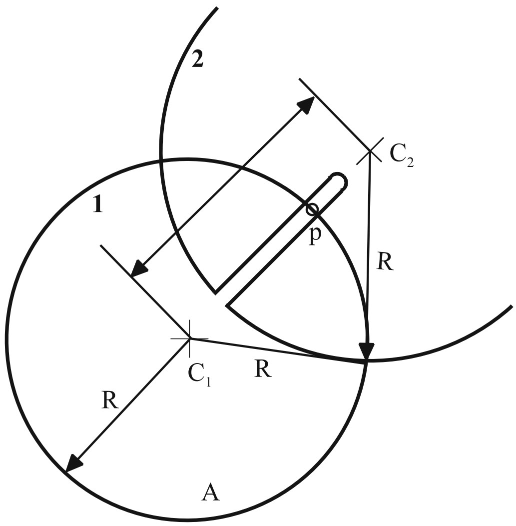


Fig. 2.84 P is the contact point

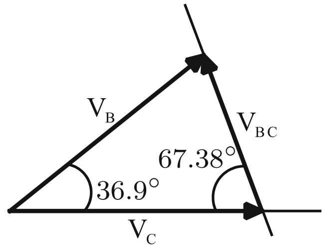


Fig. 2.85 Points B and C velocities

The direction of velocity V_C is known. Using vector equations and the geometry of the object (Fig. 2.85),

$$|AB| = 40 \text{ mm}$$

$$V_B = |AB|\,\omega = 40 \times 20 = 800 \text{ mm/s}$$

$$V_B = V_C + V_{B/C}$$

$$\frac{\sin 67.38}{|V_B|} = \frac{\sin 36.9}{\left|V_{B/C}\right|}$$

$$\left|V_{B/C}\right| = 520.36 \text{ mm/s}$$

$$\omega_{BC} = \frac{\left|V_{B/C}\right|}{|B_C|} = \frac{520.36}{104} = 5 \text{ rad/s}$$

31. Option (3) is correct.

According to Kennedy's theorem, I_{24} is at the intersection of the connecting line of I_{23} and I_{34} with the connecting line of I_{12} and I_{14}. But I_{12} and I_{14} are at infinity. So I_{24} is along lever 3 and at infinity.

32. Option (4) is correct.

The velocities of points A and B, from member 3, are in the same direction, so $\omega_3 = 0$. So we have

$$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}, V_{B/A} = |AB|\,\omega_3 = 0 \Longrightarrow V_B = V_A$$
$$\Longrightarrow |O_2A|\,\omega_2 = |O_4B|\,\omega_4 \Longrightarrow \omega_2 = 4\omega_4$$