Adaptive Kriging Metamodel Based Reliability Analysis of Tunnel

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1 Introduction

The design and analysis of tunnel is a complex task due to the inadequacy of the knowledge of ground behaviour under excavation procedure and insufficient data on the state of stress of the ground condition. Tunnel is subjected to uncertainties caused by the innate variations in the rockmass and the imprecision of measurement and modelling. In case of conventional design, the unpredictability is generally taken in a deterministic way by assigning a value with safety factor, which uses the mean values of deformation characteristics of rock/soil-mass. A deterministic analysis gives the margin of safety to a very limited extent because the parametric uncertainties and their effect on the design are not taken into account. On the other hand, a probabilistic analysis approach provides a rational perspective to such problem. It also distinguishes between minor and major uncertainties. Therefore, the proposed research study focussed on safety assessment of tunnel in probabilistic format with due importance to the various parameters that affects the tunnel behaviour. Oreste presented a probabilistic numerical approach applicable for the design of primary tunnel supports based on the hyper-static reaction method by Monte Carlo simulation (MCS) [1]. Mollon presented response surface method (RSM) based reliability analysis of a shallow circular tunnel driven by pressurized shield in soil following Mohr–Coulomb (M–C) failure criterion [2]. Lü and Low implemented first order reliability method (FORM) to calculate the reliability index of a circular tunnel under hydrostatic stress field and compared with the result achieved by MCS method. They used an approach based on RSM and second order reliability method (SORM) to find the reliability of

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the tunnel based on M–C failure criterion and Hoek–Brown (H–B) yield criterion [3]. Lu et al. [4] performed probabilistic ground-support interaction analysis of deep rock excavation using artificial neural network (ANN) and uniform design (UD) based on the convergence–confinement method. The least squares support vector machines (LS-SVM) based RSM combined with FORM [5] has also been used in tunnel reliability analysis of elastic-perfectly-plastic rockmass. The moving least square method (MLSM) and FORM/SORM are also used for probabilistic analysis of rock tunnel excavation [6]. Apart from the mentioned method, Kriging model is known to be very efficient and flexible in dealing with cases involving numerical modelling [7]. The accuracy of the metamodel can be further improved by adding a new sample by means of a learning function, the augmentation of which can be stopped by means of a stopping criterion or stopping function. Bichon et al. [8] introduced an active learning method on the threshold of Kriging metamodel, the learning function called the Expected Feasibility Function (EFF). Echard et al. [9] introduced an efficient active learning function for probabilistic analysis combining Kriging and MCS for structural. In the AK-MCS, a candidate sample set was introduced to represent the whole simulation domain for an approximate result to update the Kriging model. The sampling concept for active learning in AK-MCS is further modified for rare events e.g., brute force MCS is replaced by importance sampling in AK-IS [10] and by subset simulation in AK-SS [11]. Learning functions implementing an active sparse polynomial chaos expansion applicable to system reliability analysis are also developed [12]. AK-MCS method are also adapted for small failure probabilities [13]. The method however is seldom used in safety assessment of deep tunnel. Following the same framework, an adaptive technique is proposed in the present work based on the maximin distance criterion from the reduced space considering the prediction uncertainties of Kriging model. The application potential of the proposed approach is elucidated by considering reliability analysis of a tunnel example problem. The accuracy and efficiency of the proposed approach is studied by comparing the reliability results obtained by the proposed approach with the most accurate reliability estimates obtained by brute force MCS technique.

2 Performance Function

The convergence confinement method is a simplified and rational approach for analysing the ground-support interaction [14]. The simplicity of the approach is due to the hypothesis based on which the method is based. For example, the tunnel is assumed to be circular and deep, subjected to hydrostatic lithospheric stress, in a continuous and homogeneous/isotropic soil/rock mass condition. Thus, the problem is reduced to a two-dimensional plain strain problem. The approach is extensively used as a basic tool for estimation of support requirement for stabilization and for final convergence estimate of tunnel wall. The underground tunnel is assumed to be subjected to hydrostatic insitu stress. During construction, with the progress of excavation, the rockmass is subjected to redistribution of stress due to loss of confinement

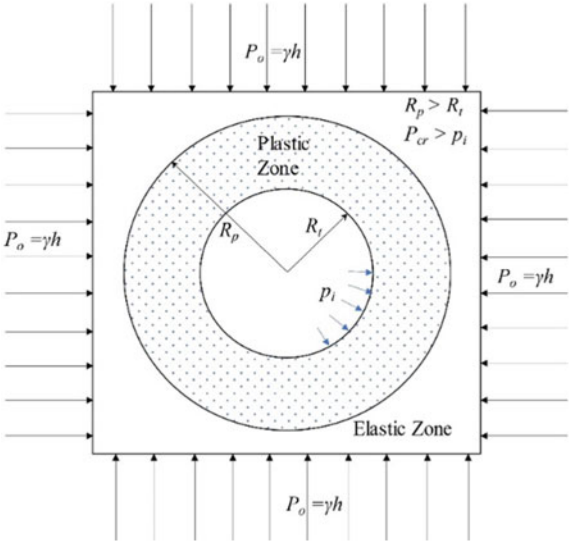


Fig. 1 A circular deep tunnel subjected to hydrostatic insitu stress

caused by excavation. The failure during the initial phase can be broadly classified into structurally influenced instability considering discontinuity of rockmass and stress induced instability considering continuity of rockmass. In the present study, the continuity of rockmass is assumed and according to the stress induced instability criteria, two possible failure mechanism are considered. A circular tunnel section subjected to hydrostatic insitu stress po and applied internal stress pi, having an internal radius of R_t and effective plastic zone radius R_p is shown in Fig. 1.

The M–C failure criterion presents the failure of an isotropic material by a set of equations in terms of principal stress neglecting the intermediate principal stress [15]. The criterion may be expressed as the relationship between the principal stresses or in terms of shear and normal stress on the failure plane. In terms of shear stress (τ), and normal stress (σ), the equation is given by Labuz and Zang [16] as,

$$|\tau| = c + \sigma \tan\phi \tag{1}$$

The given equation can be written in terms of principal stresses, σ_1 and σ_3 as,

$$(\sigma_1 - \sigma_3) = (\sigma_1 + \sigma_3)c + 2c.\cos\varphi \tag{2}$$

In terms of radial stress (σ_r) and circumferential stress (σ_θ) in cylindrical co-ordinate system, the above equation can be written as,

$$(\sigma_\theta - \sigma_r) = (\sigma_\theta + \sigma_r)\sin\varphi + 2c.\cos\varphi \tag{3}$$

Here, c is the cohesion of rockmass and φ is the friction angle of the rock mass.

At the plastic-elastic interface where $r = R_p$ (from Eqs. (2) and (3) and), we get:

$$\sigma_r = p_{cr} \tag{4}$$

Inserting the values in the equation for failure criterion:

$$p_{cr} = \frac{2p_o - \sigma_{cm}}{1 + K_p} \tag{5}$$

where $K_p = \frac{1+\sin\varphi}{1-\sin\varphi}$ and, $\sigma_{cm} = \frac{2c.\cos\varphi}{1-\sin\varphi}$

The radius of the plastic zone [3] is given by:

$$R_p = R_t\left(\frac{2(p_o(K_p - 1) + \sigma_{cm})}{(K_p + 1)(K_p - 1)p_i + \sigma_{cm}}\right) \tag{6}$$

The displacement in the plastic zone [3] is given by:

$$u_{rp} = \frac{R_t}{2G_r}\left[2(1-\nu)(p_o - p_{cr})\left(\frac{R_p}{r}\right)^2 - (1-2\nu)(p_o - p_i)\right] \tag{7}$$

G_r is the shear modulus and ν is Poisson's ratio of the rockmass, respectively.

The performance functions of circular unlined tunnel can now be constructed according to the above two solutions (Eqs. (6) and (7)) as followings,

$$g_1(x) = \lambda - \frac{R_p}{R_t} \tag{8}$$

$$g_2(x) = \varepsilon - \frac{u_{rp}}{R_t} \tag{9}$$

The performance threshold λ in Eq. (8) is the maximum value of the ratio between the radius of plastic zone and the tunnel opening radius. It depends directly on the maximum radius of the plastic zone which is in face derived by applying the least internal stress i.e., zero. In Eq. (9), ε is the ratio of the maximum radial convergence of the tunnel wall and the radius of tunnel, which is achieved by not applying any internal stress in the tunnel wall.

3 Kriging Model

Let, the input variable x is l-dimensional vector with n number of sample sets. The variable is written in $n \times l$ matrix form with l being the number of variables and n being the total sample set generated. The Kriging model can be written as,

$$g(x_i) = \sum_{j=1}^{p} f_j(x_i)\beta_j + Z(x_i) = (f(x_i))^T \beta + Z(x_i) \tag{10}$$

where $\beta^{\mathrm{T}} = [\beta_1, \beta_2, \beta_3, \ldots, \beta_p]$ is the vector of regression-coefficient, very similar to the regression-coefficient vector used in polynomial model in response surface method and $f_j(x_i)_{i=1}^{p}$ is a set of known function. $Z(x)$ is a stationary Gaussian process (an assortment of random variable such that every finite collection from the collection follows a multivariate normal-distribution). The following statistical property is used to define the Gaussian process:

$$E(Z(\boldsymbol{x})) = 0 \text{ and } \mathrm{Var}(Z(\boldsymbol{x})) = \sigma_Z^2$$

$$\mathrm{Cov}[Z(\boldsymbol{x}), Z(\boldsymbol{w})] = \sigma_Z^2 R_\theta(\boldsymbol{x}, \boldsymbol{w})$$

σ_Z^2 is the unknown variance between two points of **x** and **w** space of the stochastic field $Z(\mathbf{x})$. R is the correlation matrix of dimension $(n \times n)$ and as the correlation-parameter vector of length n. There are variety of functional form defining the correlation [17, 18]. The following correlation model (anisotropic Gaussian model) is considered here:

$$R_\theta(\boldsymbol{x}, \boldsymbol{w}) = \prod_{i=1}^{n} \exp\left[-\theta_i |x_i - w_i|^2\right] \tag{11}$$

x_i and w_i are the ith co-ordinate point of **x** and **w**. The values of β and σ^2 are evaluated by Jones et al. [19],

$$\hat{\beta} = \frac{F^T R^{-1} g}{F^T R^{-1} F} \tag{12}$$

$$\hat{\sigma}^2 = \frac{(g - F\beta)^T R^{-1} (g - F\beta)}{n} \tag{13}$$

Since the value of $\hat{\beta}$ and $\hat{\sigma}^2$ are dependent upon the value of θ, hence θ is first evaluated using the maximum likelihood estimation by minimising $\Psi(\theta) = |R(\theta)|^{\frac{1}{n}} \sigma(\theta)^2$. The achieved predictor $G(x)$ with parameters: $\beta = \hat{\beta}$; $\sigma^2 = \hat{\sigma}^2$ and $\theta = \widehat{\theta}$; is known as the maximum likelihood empirical 'best linear unbiased predictor' (BLUP) $\widehat{G}(\boldsymbol{x})$, and is evaluated by,

$$\widehat{G}(\boldsymbol{x}) = \beta + r_o^T R_\theta^{-1} (g - \beta) \tag{14}$$

where $r_o = \{R(\boldsymbol{x}, \boldsymbol{x}_1), R(\boldsymbol{x}, \boldsymbol{x}_2), \ldots, R(\boldsymbol{x}, \boldsymbol{x}_n)\}$. Here, the optimal choice of the parameters θ is obtained as the maximum likelihood estimator using the "dacefit" algorithm of the DACE toolbox [17]. The least value of the mean square error between the predicted value $\widehat{G}(\boldsymbol{x})$ and the response value G(x), also known as the Kriging variance $\sigma^2_{\widehat{G}}(\boldsymbol{x})$ is given by,

$$\sigma^2_{\widehat{G}}(\boldsymbol{x}) = \widehat{\sigma}^2\left[1 + \boldsymbol{u}^T(F^T R^T F)^{-1}\boldsymbol{u} - r_o^T R_\theta^{-1} r_o\right] \quad (15)$$

where $\boldsymbol{u} = F^T R^{-1} r_o - f(\boldsymbol{x})$.

4 Adaptive Kriging Approach of Reliability Analysis

The applications of Adaptive Kriging Based MCS (AK-MCS) for reliability analysis of structures are enormous. However, it is not applied in reliability analysis of tunnel. The present study attempts to explore an adaptive Kriging approach on the basis of the Max–min distance concept. The proposed approach is built primarily on the basis of AK-MCS approach. Thus, the AK-MCS based approach is explained in this section and the proposed adaptive Kriging is presented in the next section. The failure probability of a structural system having performance function $g(x)$ is given by:

$$P_f = \int_F f_X(x)\mathrm{d}x = \int_{\mathbb{R}^n} I_F(x) f_X(x)\mathrm{d}x \quad (16)$$

where $f_X(\mathbf{x})$ is the joint probability distribution function (PDF) of random variable $\mathbf{x} = \{x_1, \ldots, x_n\}$, F is the region of failure given by $F = \{\mathbf{x}|g(\mathbf{x}) < 0\}$, the indicator function, $I_F(\mathbf{x})$ becomes 1 if $\boldsymbol{x} \in F$ and 0 otherwise. For MCS, the failure probability is:

$$P_f \approx \widehat{P}_f = \frac{1}{N_{MCS}} \sum_{i=1}^{N_{MCS}} I_F(x_i) \quad (17)$$

For, $\{\mathbf{x}_i, i = 1, 2, \ldots, N_{MC}\}$ are samples drawn from the PDF. In case of MCS, the entire population are to be evaluated with the performance function. The tedious process of evaluating the entire population with the performance function is reduced by applying AK-MCS based on active learning method. The Kriging model is trained with fewer possible samples by applying the active learning function on the MCS population. The refined Kriging model then evaluates the MCS population instead of the predefined limit state function. The active learning process dynamically updates and refine the Kriging model based on adopted DOE which is iteratively enriched by adding new sample identified by an active learning function. The active learning function continues to enrich and refine the Kriging model until a prescribed stopping condition is attained. It is to be noted that the mean Kriging (i.e., the regression part in Eq. 10 contains a constant only) is considered. The steps followed in AK-MCS are outlined below.

1. *Generation of a standard Monte Carlo population in the prescribed design space*:

   The population *S* consists of *N* numbers of MCS samples following the respective PDF in the parameter space. The population S is only used as a pool for drawing and identifying the next best sample which is assessed on the defined performance function.
2. *Initial design of experiments (DOE) definition*:

   N_1 samples are randomly selected from the population S and evaluated with the true performance function. The DOE is generally preferred to be less to reduce the number of calls to the true performance function.
3. *Construction of the Kriging model based on initial DOE*:

   The Kriging model is constructed based on the current DOE.
4. *Prediction by Kriging and failure probability estimation*:

   Kriging predictions (Eq. (14)) are obtained using the DACE MATLAB toolbox. Then, the failure probability is assessed with the signs of these predictions as the ratio of the points in the population S with a negative Kriging prediction and the total number of points in *S*.
5. *Recognition of the suitable next point to evaluate on the defined performance function*:

   The next best sample is identified using the learning function. Here the learning function $U(x)$ [9] is used:

$$U(x) = \frac{|\mu_g(x)|}{\sigma_g(x)} \tag{18}$$

   The sample is given by:

$$\breve{x} = \arg\min_{x \in S} U(x) \tag{19}$$

   The sample with minimum $U(x)$ are either located near the limit state $((\mu_g(x) \approx 0))$ or have high prediction uncertainty $((\sigma_g(x) \gg 0))$, or both. The sample so drawn will have high potential to cross the prediction separator. Hence should be included in the DOE.
6. *Definition of stopping condition for learning*:

   The Kriging model is updated by the augmented DOE. The iteration process is stopped when the stopping condition is outreached. The stopping condition is defined as:

$$\min(U(x)) \geq 2, \forall x \in S \tag{20}$$

which means that the probability of executing a wrong sign prediction is $\Phi(-2) < 0.023$

7. *Update of the previous DOE with the best suitable point*:

   The active learning process is continued if the stopping condition given in step (6) is not satisfied. The best sample achieved is computed on the performance function and added to DOE. The method goes back to step (3) and the Kriging model is updated with the updated/augmented DOE.

8. *Calculation of the coefficient of variation (COV_{pf}) of the probability of failure*:

   If the stopping condition in step (7) is achieved, the learning process is stopped and the metamodel is considered accurate enough on the performance function's signs of the N_{MC} points. The next step is to check whether the Monte Carlo population S is sufficiently large to give low COV_{pf} on the Kriging estimation of the failure probability (step (4)). Values of COV_{pf} less than 5% is taken considered acceptable.

$$COV_{\hat{P}_f} = \sqrt{\frac{1 - \hat{P}_f}{\hat{P}_f N_{MC}}} \tag{21}$$

9. *Updating of the initial population*:

   If the estimated COV_{pf} is high than prescribed value (5%), S is updated with new set of points from another Monte Carlo population (generated like in step (1)). It is then followed by (step (4)) to predict the new population and the active learning method continues till the stopping condition is achieved. No information about the previous evaluations is lost.

10. *End of the AK-MCS*:

    The COV_{pf} is calculated and if it is small enough then the method is stopped, the failure probability is assessed. The estimate is the final result of AK-MCS.

5 Proposed Adaptive Kriging Approach

An adaptive Kriging approach is proposed here based on the maximin distance concept. A reduced space is constructed first by the proposed approach. Then a new training point is selected based on the maxmin distance criterion. This process goes on iteratively, the details of which are given below.

A population S consisting of N_{MC} samples in the input parameter space is generated following the respective PDF. The selection of initial training samples and searching of the next best training sample are limited to the population space S only. To build an initial DOE, N_1 samples are selected randomly from the population space S and evaluated with the true performance function. The number N_1 is generally preferred to be less for reduction of the total number of calls on the true performance function. Once the initial DOE is built, the Kriging model is constructed based on it. Kriging prediction and its variance at N_{MC} samples are obtained using the Kriging model. Then, the failure probability is assessed with the signs of these predictions as the ratio of the points in the population S with a negative prediction and the total number of points in S.

Now, to construct the reduced space, first, $U(\mathbf{x})$ function is evaluated at all points in S based on the Kriging predictions and its variance obtained from the Kriging model. Then, samples, which are satisfying $U(\mathbf{x}) < 2$, are selected to construct a reduced space (denoted by R). This implies that all MCS samples having the probability of executing a wrong sign prediction greater than 0.023 (i.e., $\Phi(-2)$) is included in the reduced space. Like active learning-based AK-MCS method, the sign of predicted performance function at any point having probability of executing a wrong sign prediction less than 0.023 is considered as accurate. After that, a new training sample is selected by maximin distance criterion. For this, the scaled Euclidian distance of each point in the reduced space from its corresponding nearest training sampling is calculated as,

$$D(\boldsymbol{x}) = \|\boldsymbol{x}_r - \boldsymbol{x}_{nearest}\| \tag{22}$$

where $\| \; \|$ represents the scaled Euclidian distance, $\mathbf{x}_r$ is a point of the reduced space and $\mathbf{x}_{nearest}$ is the nearest training sample of the point $\mathbf{x}_r$. The point in the reduced space having the maximum value of such calculated distance is selected as the next training sample. The next training sample is given by:

$$\breve{\mathbf{x}} = \underset{\mathbf{x} \in R}{\arg\max}\, D(\mathbf{x}) \tag{23}$$

The sample $\breve{\mathbf{x}}$ is included into the DOE. The sample with maximum $D(\mathbf{x})$ improve the space-filling property of the new augmented samples. Hence increases the efficiency of the model. The Kriging model is updated by the augmented DOE. Subsequently, the Kriging prediction and its variance at MCS samples are updated. Based on the updated prediction, the updated failure probability is obtained. In addition, the reduced space R is reconstructed based on the updated value of U-function at the MCS points. Again, a new training point is selected based on Eq. (23) and Kriging model is updated. Thus, the failure probability is updated iteratively. It can be noted here that no new training point can be added if there is no new sample in the reduced space. Thus, this is treated as the stopping condition for adaptive sampling. Alternatively, the stopping condition can also be expressed as,

$$\min_{x \in S} U(\mathbf{x}) \geq 2 \tag{24}$$

The next step is to check whether the Monte Carlo population S is sufficiently large to give low COV_{pf} on the Kriging estimation of the failure probability. The $\mathrm{COV_{pf}}$ is calculated (ref. Eq. (21)) and the COV_{pf} value below 5% is considered to be acceptable. If the estimated COV_{pf} is higher than the prescribed value (5%), S is enriched with new set of N_{MC} points generated from the associated PDF of the input variables. The prediction of the new population is done and the adaptive sampling continues till the stopping condition is achieved. No information about the previous evaluations is lost. The COV_{pf} is calculated and if the value is very high, then the method again enriches the Monte Carlo population space S. Once, the COV_{pf} is small enough (i.e., less than 5%), the method is stopped, and the failure probability is assessed.

6 Reliability Analysis of Unlined Circular Tunnel Subjected to Hydrostatic Insitu Stress

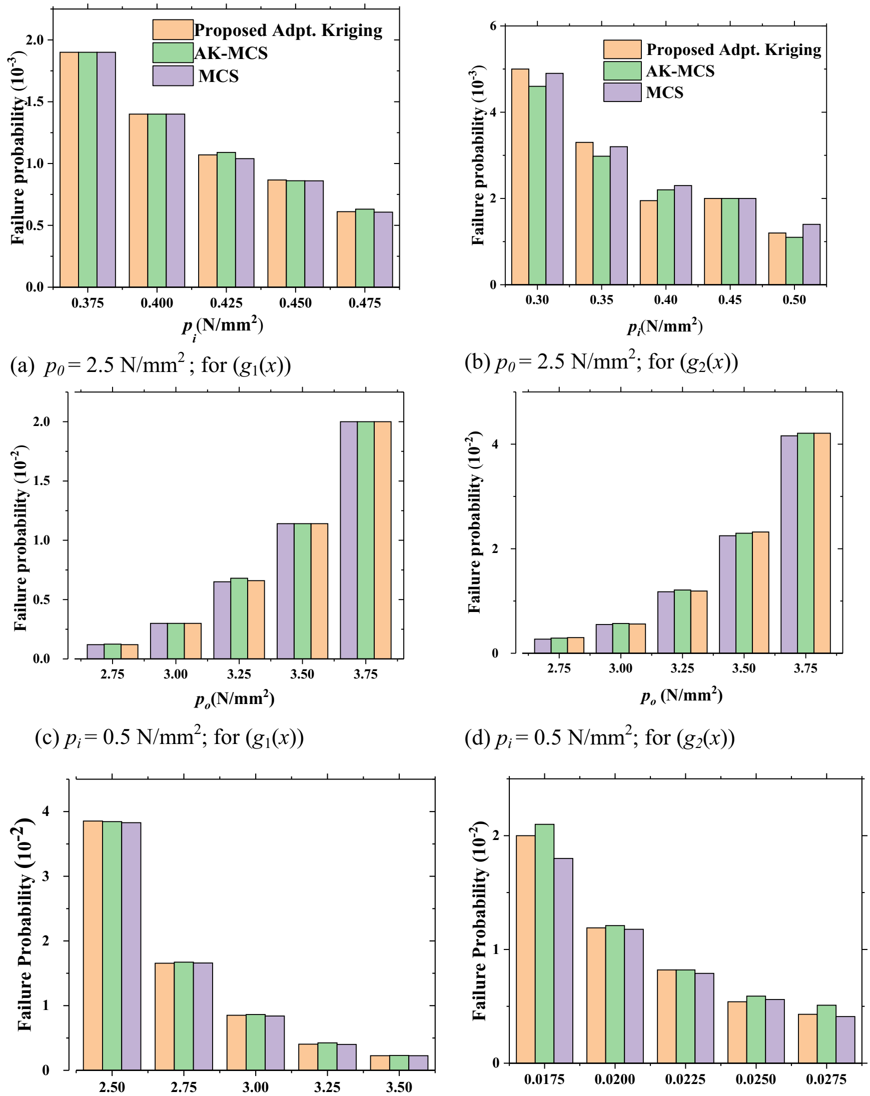
The effectiveness of the AK-MCS and the proposed adaptive Kriging approach for reliability analysis of underground tunnel is demonstrated by considering an unlined-circular tunnel subjected to hydrostatic insitu stress, p_o and applied internal stress, p_i. The reliability analysis is performed based on the analytical formulation framework defined in Sect. 2. The results of the AK-MCS and the proposed approach are compared and validated with the results obtained by MCS. The M–C failure criterion is applied to assess the plastic radius of the tunnel. The cohesion, elastic modulus, angle of internal friction and Poisson ratio of the rockmass defines the elasto-plastic behaviour of the tunnel. The statistical properties of the parameters considered to be random are provided in Table 1.

The performance function is defined by Eqs. (8) and (9). The performance threshold λ and ε are taken as 3 and 0.02 [3]. The Poisson ratio is taken as 0.22. A parametric study is made by varying the applied internal stress, hydrostatic insitu stress, and performance threshold. The number of random samples (N_1) taken to initially start the Kriging model is 12. Three cases are considered for each performance function, in the first case (Case 1) the value of p_o is taken as 2.5 N/mm^2 with varying values of p_i. In the second case (Case 2), the value of p_i is taken as 0.5 N/mm^2 with varying values of p_o. In the third case (Case 3) the value of p_i and p_o are taken as 0.25 N/mm^2 and 2.5 N/mm^2 for $g_1(x)$ and 0.5 N/mm^2 and 3.25 N/mm^2, respectively for $g_2(x)$ with variation in the values of the performance functions. The variation of failure probability (p_f) for all the three cases are shown in Fig. 2.

It may be noted that both the AK-MCS and the proposed approach can estimate the failure probability with reasonable accuracy. The accuracy and efficiency of the

Table 1 Statistical properties of the parameters [3]

Sl. No.	Property		Units	Distribution	Mean
1	Elastic modulus	E	MPa	Normal	1185
2	Cohesion	C	MPa	Normal	0.28
3	Angle of internal friction	ϕ	Degree	Normal	23.7



(a) $p_0 = 2.5$ N/mm^2 ; for ($g_1(x)$)

(b) $p_0 = 2.5$ N/mm^2; for ($g_2(x)$)

(c) $p_i = 0.5$ N/mm^2; for ($g_1(x)$)

(d) $p_i = 0.5$ N/mm^2; for ($g_2(x)$)

(e) p_i = 0.25 N/mm^2; p_o = 2.5 N/mm^2; for ($g_1(x)$)

(f) p_i = 0.5 N/mm^2; p_o = 3.25 N/mm^2; for ($g_2(x)$)

Fig. 2 Variation in failure probability (p_f) due to change in **a** internal stress for $g_1(x)$; **b** internal stress for $g_2(x)$; **c** insitu stress for $g_1(x)$; **d** insitu stress for $g_2(x)$; **e** performance function λ in $g_1(x)$ and **f** performance function ε in $g_2(x)$

Table 2 Average number of samples required and average absolute percentage error for $g_1(x)$

Method	Case 1 (Fig. 2a)		Case 2 (Fig. 2c)		Case 3 (Fig. 2e)	
	Average no. of samples	Average % error	Average no. of samples	Average % error	Average no. of Samples	Average % error
Proposed adaptive Kriging	29	0.87	24	0.31	24	0.70
AK-MCS	27	1.78	22	1.76	21	2.29

Table 3 Average number of samples required and average absolute percentage error for $g_2(x)$

Method	Case 1 (Fig. 2b)		Case 2 (Fig. 2d)		Case 3 (Fig. 2f)	
	Average no. of samples	Average % error	Average no. of samples	Average % error	Average no. of samples	Average % error
Proposed adaptive Kriging	60	6.93	47	3.68	51	4.89
AK-MCS	61	7.75	47	3.43	48	10.60

proposed method and the AK-MCS is compared in terms of the percentage error in the failure probability with respect to direct MCS. The average percentage error and the average number of samples required are enlisted in Tables 2 and 3 for the first and second performance functions respectively.

Both the method shows good accuracy and efficiency for the first performance function. The total number of samples required are also similar for the two methods. For the second performance function, as the equation is highly non-linear, at some instances the accuracy is moderate for both the methods. However, the proposed method shows better accuracy than the AK-MCS in majority of the cases.

7 Conclusion

The proposed adaptive Kriging method is applied to assess the safety of tunnel based on the allowable plastic radius and tunnel wall displacement criteria. The proposed adaptive Kriging approach is noted to provide comparatively better accuracy. When highly nonlinear performance function is involved, the method showed moderate accuracy, similar to the AK-MCS. As random sampling was adopted in the both the methods, therefore it can be concluded that there is high chance of increasing the accuracy and efficiency of the proposed approach if better DOE framework is adopted. There is scope of improvement in the adaptive technique based on the stopping criteria. The applicability of the proposed adaptive Kriging method in reliability analysis of tunnel is studied for simple problem and needs to verify for more realistic tunnel reliability analysis problem involving finite element response analysis.

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