Sensitivity of Angle Parameters in the Modelling of Bistable Variable Stiffness Laminates

B. Danish, K. S. Suraj, P. M. Anilkumar, and B. N. Rao

1 Introduction

Morphing structures capable of adapting their behavior in response to surrounding environmental stimuli are highly desirable in various engineering applications. In recent years, unsymmetrical bistable laminate has become an ideal candidate for morphing application due to its ability to exhibit two stable configurations in room temperature conditions. The shape change between stable equilibrium states can be made by applying sufficient energy to trigger the snap-through action [1]. Apart from the established aerospace applications, bistable laminates finds its position in other fields like wind and solar energy harvesting, robotics, and foldable structures [2]. The energy required for snap-through can be provided by either mechanical load [3, 4] or using smart materials like shape memory alloys and piezoelectric macro fiber composite (MFC) actuators [5–7].

The energy needed to trigger snap-through is crucial to the development of multistable structural components. If the snap-through energy requirements are high, the shape-transition becomes infeasible. Energy requirements for the snap-through transition can be reduced by spatially tailoring the stiffness of the individual bistable plate [8]. By changing the orientation of the fibres, it is possible to vary the spatial stiffness, producing laminates with variable stiffness (VS). Sousa et al. [8] proposed the idea of VS laminate in morphing applications. Later, Haldar et al. [9] explored the possibility of expanding the design space by tailoring VS angle parameters. Further, Anilkumar et al. [10] performed parametric studies on the snap-through and snap-back behaviors by varying the angle parameters of the VS laminate where the snap-through has been triggered using MFC actuators. In the studies mentioned, stable

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shapes and snap-through requirements were predicted using semi-analytical models based on the Rayleigh–Ritz minimization method. In addition, finite element (FE) frameworks have been widely adopted to study the behavior of bistable composites due to its improved accuracy in predicting the snap-through behavior in comparison with semi-analytical models. The variable stiffness laminates are getting paramount of attraction due to their ability to ensure the continuity of the smooth fiber between the two regions to avoid the stress concentrations along with the advantage of lower snap-through energy requirements, which is beneficial in morphing applications.

Although curvilinear path alignments to produce variable stiffness laminates can be accomplished through the Automated Fibres Placement technique, however manufacturing laminate layups with the precise VS parameters may not be easy. Uncertainties in the angle parameters due to the manufacturing constraints can lead to inaccurate evaluation of snap-through requirements. In order to access the tolerance limits of VS angle parameters, the sensitivity needs to be investigated. Studies on sensitivity analysis and uncertainty quantification due to uncertainties in design variables have been reported only on straight fibres laminates [11–14]. However, the VS laminates may or may not behave in the similar way as reported for straight fiber laminates. Additionally, there is still room for research into how variations in the VS angle factors affect the bistable behavior. Therefore, the goal of the current study is to comprehend the significance of minor perturbations of VS angle parameters on the bistability of VS unsymmetrical laminates, where sensitivity analysis on the deformation and snap-through load of bistable VS laminates due to uncertainty in angle parameters are investigated. Uncertainties in the snap-through loads and out-of-plane displacements due to the individual variations of ϕ, T_0, and T_1 has been investigated. The analysis has been performed on a commercially available finite element package, *ABAQUS*. Finally, design contour charts exploiting the change in characteristic parameters by altering the orientation angle for a family of VS laminate are prepared from the finite element analysis.

2 Variable Stiffness Laminate

The variable stiffness laminate defines by the three angle parameters ϕ, T_0, and T_1 as shown in Fig. 1. Modelled VS laminate fibres follow the curvilinear path suggested by Gürdal et al. [15]. The fiber orientation θ is defined as follows:

$$\theta(x') = \phi + \frac{(T_1 - T_0)|x'|}{d} + T_0, \quad \text{where,} \quad x' = x\cos\phi + y\sin\phi$$

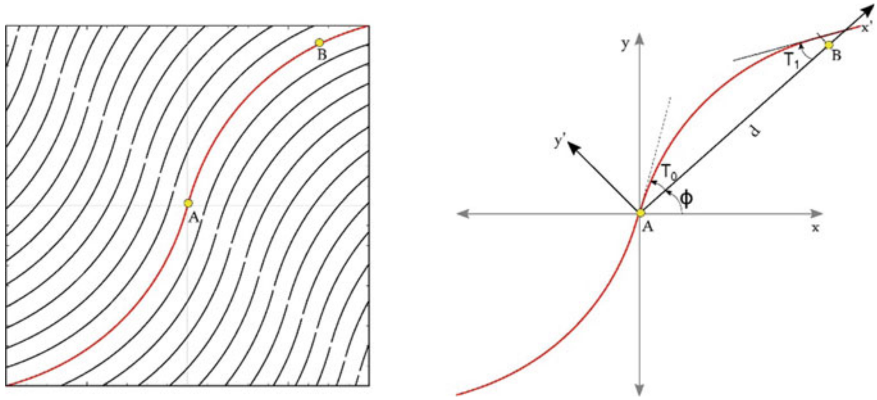

Fig. 1 Curvilinear path description of VS laminate

3 Finite Element Model

FE analyses of VS composite laminates considering geometric nonlinearities are performed in a commercially available FE package, *ABAQUS*. To take non-linear geometric behavior into consideration, the large deformation theory based on the NLGEOM option has been used. Four-node quadrilateral (S4R) shell elements with reduced integration have been used to model the laminates. A balance between computational speed and accuracy has been achieved by carefully studying mesh convergence and choosing the appropriate mesh sizes. The procedures that were used in the analysis are explained below.

3.1 Step 1: The Initial Step

In this initial step, thermal loading given as curing temperature of 180 °C is imposed on the laminate. The laminate has been clamped from its geometric centre by restraining the centre most node.

3.2 Step 2: The Cool-Down Step

The temperature of the laminate has been lowered to 20 °C in this step, where residual stresses are introduced into the laminate. The composite laminates deviate from their initial alignment in one of their potential stable shapes as a result of temperature differences. To prevent the appearance of unstable saddle shapes, imperfections are added to the proposed laminate model.

3.3 Step 3: The Snap-Through Step

In this stage, a static snap-through process has been simulated. At the four corners of the laminate, transverse point loads are applied to cause snap-through. In order to arrive at a converged equilibrium solution, automatic stabilization with a specific dissipated energy fraction has been imposed during the analysis. To deform the laminate into another stable shape, a load greater than the snap-through requirement must be applied.

3.4 Step 4: The Stability Check Step

After achieving the second stable state, the concentrated loads at the corners of the laminate are removed so that the laminate rests in one of its stable states (Fig. 2).

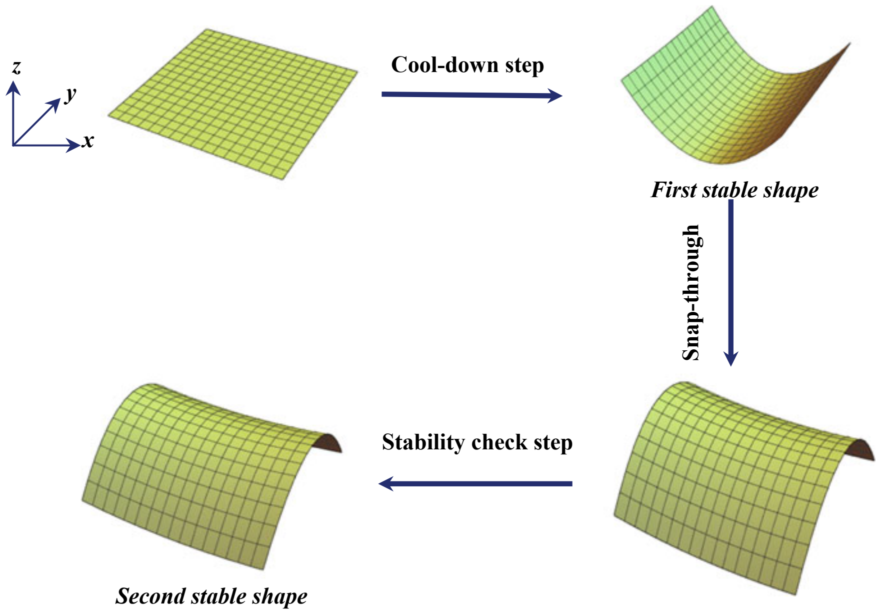

Fig. 2 Flowchart of steps followed in snap-through process

4 Result and Discussion

A square Cycom 977-2 VS lamina of 0.210 mm thickness is considered for the present investigation. A square laminate with a side length of 135 mm is considered, the material properties and the angle parameters are given in Table 1. Effects of individual perturbations on the angle parameters and all possible combinations of T_0 and T_1 are investigated.

4.1 Effect of Individual Perturbation in the Angle Parameters

The VS family following $\phi = 45°$, $T_0 + T_1 = 90°$ has been taken to investigate the sensitivity of ϕ, T_0, and T_1 parameters on the out-of-plane displacements and snap-through loads. The proposed finite element framework is used to perform a parametric study. Schematic representation of the individual perturbation of ϕ parameters is shown in Fig. 3. Similarly, the schematic representation of perturbation of T_0 and T_1 parameters are given in Fig. 4. In the perturbation studies, ϕ, T_0, and T_1 have been modified as $\phi + \delta\phi$, $T_0 + \delta T_0$, and $T_1 + \delta T_1$ respectively, where the range of $\delta\phi$, δT_0, and δT_1 limited to $\pm$ 5° in this study. In order to compare the results effectively, the output response of the model for the perturbed parameters has been normalised with the output response of the model with standard parameters. In the graphical illustrations, the x-axis denotes X_n/X_s where X_n is the changed property and X_s is the standard property. Similarly, the y-axis denotes Y_n/Y_s where Y_n is the output response for the changed property and Y_s is the output for the standard properties.

The result obtained from individual perturbation in the angle parameters is shown in Figs. 5 and 6. Among the VS parameters, the perturbation in ϕ is highly sensitive to the out-of-plane displacement profile, where $\pm$ 5° perturbation in ϕ leads to a change of − 19 to + 19% in the out-of-plane displacement. The individual perturbation of T_0 and T_1 parameters also has been studied and it is found that the perturbation in these parameters are less sensitive on the displacement profile variation. For − 5°

Table 1 Material and angle properties of VS laminate used

Property	Value
E_{11}	156 GPa
E_{22}	8.35 GPa
υ_{12}	0.33
G_{12}	4.2 GPa
α_{11}	-0.29×10^{-6} (°C)
α_{22}	24×10^{-6} (°C)
ϕ	45°
T_0	45°
T_1	45°

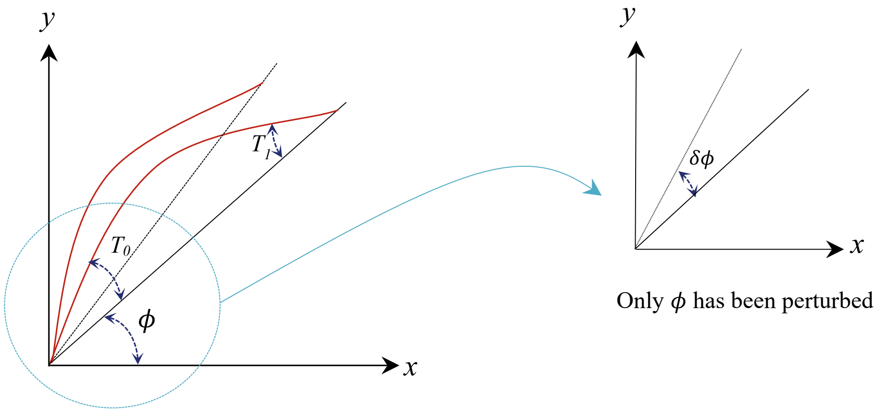

Fig. 3 Schematic representation of perturbation of ϕ

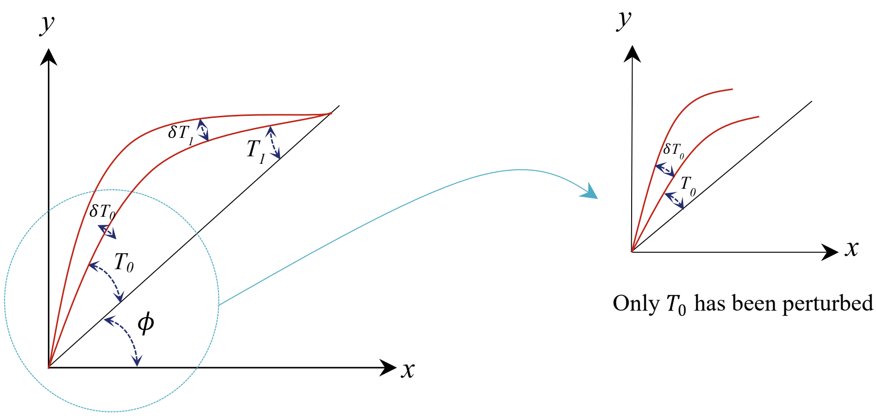

Fig. 4 Schematic representation of perturbation of T_0, and T_1

perturbation in T_0 and T_1 values, the maximum change in out-of-plane displacement is + 4.3%. Whereas, for 5° perturbations in T_0 and T_1 values, the maximum change in out-of-plane displacement is − 7.3%. A similar study has been performed to check the sensitivity of the snap-through loads, and the result obtained has been depicted in Fig. 6.

Among the VS angle parameters, the perturbation in T_0 and T_1 parameters are highly sensitive to the snap-through load, where ± 5° perturbation in T_0 leads to a change of + 3.2 to − 10.6%, and ± 5° perturbation in T_1 leads to a change of in − 6.5 to 10% in the snap-through load. The perturbation in ϕ is less sensitive to the snap-through load requirements, however, the change in snap-through load is within − 2.73% due to ± 5° perturbation in ϕ. It can be concluded from the study that the perturbation in T_0 and T_1 has a significant effect on the snap-through

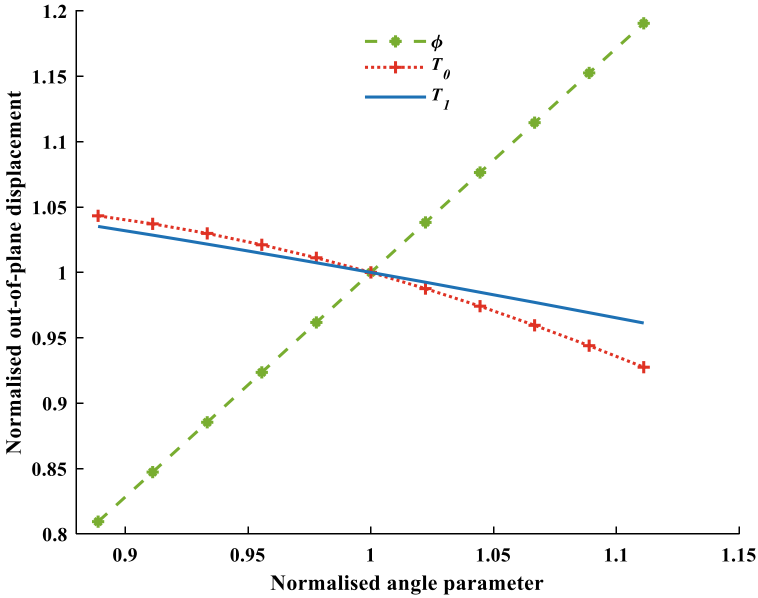

Fig. 5 Change in out-of-plane displacement with the change in ϕ, T_0 and T_1

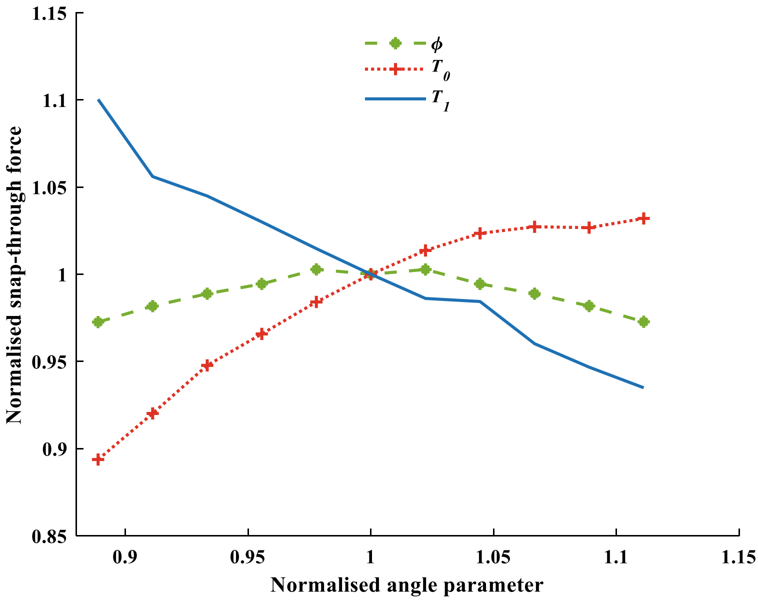

Fig. 6 Change in snap-through load with the change in ϕ, T_0 and T_1

load. Even though perturbation in these parameters does not have much effect on the out-of-plane displacement however perturbation in their combination may lead to a significant change in out-of-plane displacement and alter the deformation profile.

4.2 *Effect of Combined Perturbation in the T_0 and T_1 Parameters*

To investigate the combined perturbation effects in T_0 and T_1, a sampling plan has been suggested in Fig. 7. A $\pm 5°$ perturbation in δT_0 and δT_1 is selected for the present analysis. Lower limit and upper limit of T_0 and T_1 are 40° and 50° respectively. A set of points with all possible combination of T_0 and T_1 has been selected for the analysis. The contour plot on the sensitivity of out-of-plane displacement and snap-through load resulting from the combined perturbation in T_0 and T_1 is shown in Figs. 8 and 9. From the contour plot of out-of-plane displacement (Fig. 8), it is observed that the perturbation in the combination of T_0 and T_1 leads to a displacement change of $+$ 7.31% to $-$ 11.36%, where the maximum value of 14.67 mm occurs at $T_0 = T_1 =$ 40° and minimum value of 12.12 mm occurs at $T_0 = T_1 =$ 50°. It is evident from this study that both angle parameters alter the deformation profile in the same way, positive change in both angle parameters leads to negative change in out-of-plane displacement and vice-versa.

Figure 9 represents the contour plot of the variation of snap-through load due to combined perturbation in T_0 and T_1 parameters. The combined perturbation in these parameters leads snap-through to varies from $-$ 15.32 to 13.73% with maximum values of 4.74 N occurring at $T_0 =$ 50°, $T_1 =$ 40° and minimum value of 3.23 N occurs at $T_0 =$ 40° and $T_1 =$ 50°.

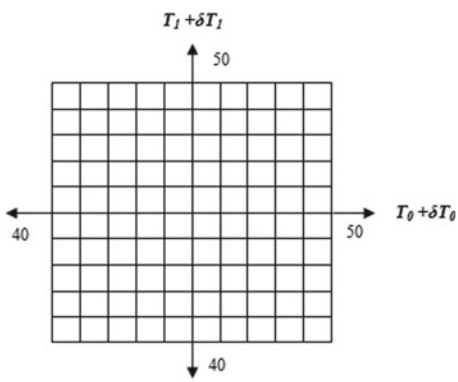

Fig. 7 Sampling scheme of combined perturbation in T_0 and T_1 parameters

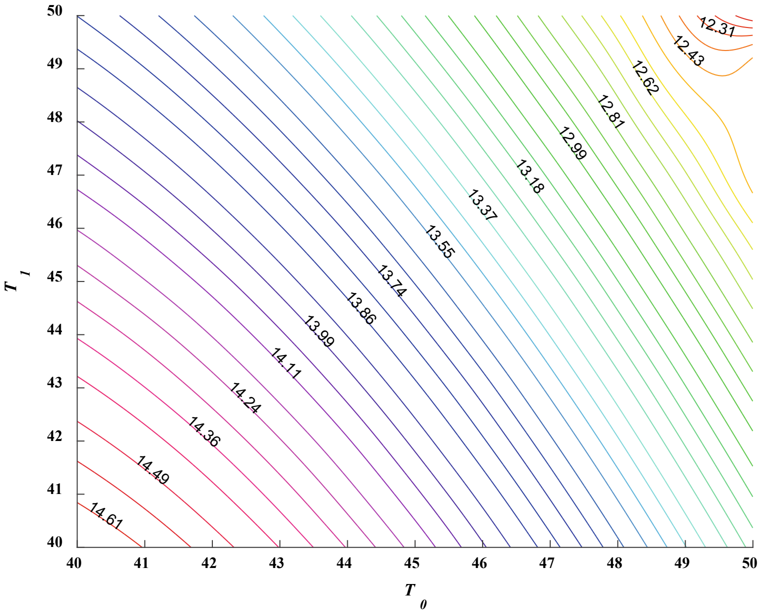

Fig. 8 Change in out-of-plane displacement with the change in combination of T_0 and T_1

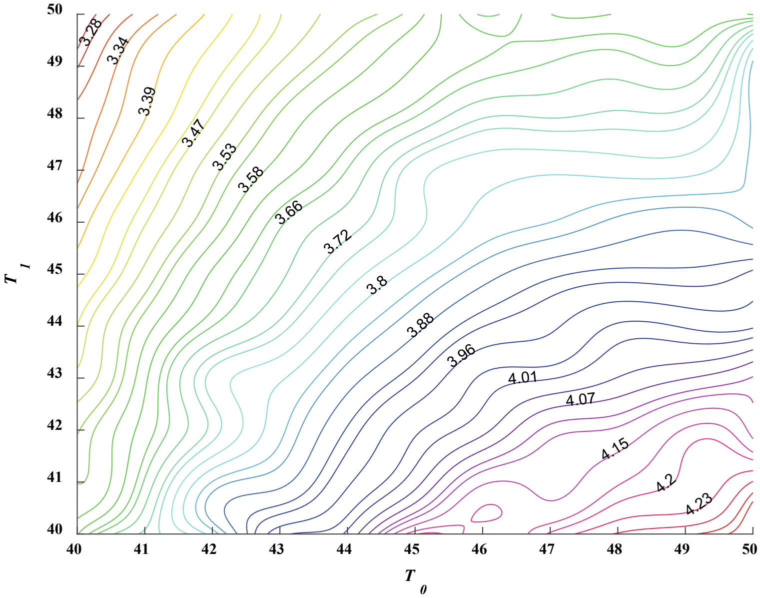

Fig. 9 Change in snap-through load with the change in combination of T_0 and T_1

5 Conclusion

The variable stiffness composites generated using curvilinear fiber alignments can be used as an alternative to conventional composites in bistable structures to tailor the deformations and snap-through requirements. However it could be challenging to create laminate layups with the precise designated VS angle specifications due to manufacturing constraints. A study on the impact of minor changes in design variables is needed to check for the tolerance limits of fibre angle alignments. In this paper, the effect of individual perturbation of ϕ, T_0, and T_1 and a set of combinations of perturbation in T_0, and T_1 are performed using a finite element-based model. The results reveal that angle parameter ϕ is highly sensitive while the parameters T_0, and T_1 are less sensitive to the deformation profile of the VS laminate. A $\pm 5^\circ$ change in ϕ leads to a maximum change of 19% in the out-of-plane displacement. The individual perturbation in T_0 and T_1 significantly affects the snap-through requirements. A $\pm$ 5° individual perturbation in T_0 and T_1 leads to a maximum change of $-$ 10.6 and 10% in the snap-through load. The perturbation in the combination of T_0 and T_1 also has been studied and it is found that even though the deformation profile is less sensitive to individual perturbation in these parameters however perturbation in their combination leads to a maximum change of 11.36% in out-of-plane displacement. Further perturbation in these combinations leads to a maximum change of 15.32% in snap-through load. The results can be used as a design tool while taking the tolerance limits of T_0 and T_1 parameters into account.

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