

# Damage Prediction by Using Nonlinearity of Damping



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**Abstract** The lack of rapid, simple, and reliable techniques in damage detection of large civil engineering structures has hindered the frequent application of available vibration-based structural health monitoring and damage detection methods. The basics of baseline data of undamaged structure detailed finite element models, or the need for numerous expensive sensors have further distanced the technique from practice. Thus, in this study, the application of a simple, baseline free time domain damage detection technique using acceleration data is discussed. The investigation is made using analysis of nonlinearity in damping extracted from ambient vibration data for identification of the existence of damage in a structure. It is known that the dominant mechanism of energy dissipation in the presence of structural defects such as cracks, defective connections, etc. is due to dry Coulomb friction and this type of damping is considered nonlinear. Contrary to this, in the undamaged state of structures, the dissipation of energy is mostly due to material damping which is considered a macroscopically viscous and constant type of damping. Thus, analysis of nonlinearity in damping and identification of the contribution of Coulomb friction in modal damping could reveal the existence of damage or defects in a structure. This study presents the application of the method to an experimental system at the laboratory to show the competency of the method. The experimental estimates obtained from the proposed method illustrate the effectiveness and efficiency of the method in portraying the existence of the damage and approximate quantification.

**Keywords** Nonlinear damping · Damage detection · Coulomb damping · Viscous damping · Ambient vibration

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## 1 Introduction

Detection of the occurrence of damage in a structure is the initial step of every structural health monitoring (SHM) system. Such information will facilitate engineers in planning detailed inspection activities to assess the structural integrity and performance of structures such as buildings, bridges, dams, pipe networks, etc. [1, 2].

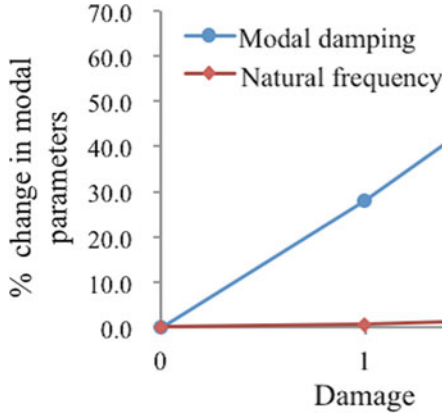
The existing structural damage detection methods can be categorized into two groups; local inspection methods and global detection methods [2]. Local inspection method includes visual inspections of the structure by an expert supplemented by localized testing based on ultrasonic or [1, 3] acoustic methods, x-ray methods, magnetic field methods, eddy-current methods, thermal field methods, etc. [4–6]. The visual inspection and testing methods are highly localized in nature and the outcome is highly dependent upon the inspectors' experience and knowledge. Further, the methods require the portion of the structure to be investigated to be readily accessible for testing [1]. Unfortunately, on most occasions, this requirement is not guaranteed, since most of the civil engineering structures especially bridges, are large and complex in nature, or their operating environment is hostile. In this situation, the development of efficient and effective methods for the detection of the occurrence of damage in structures is crucial.

The vibration-based structural damage detection method can effectively avoid the limitations of local inspection methods because it does not require that the vicinity of damage is known a priori, nor that the structural portion of interest is readily accessible for testing [7].

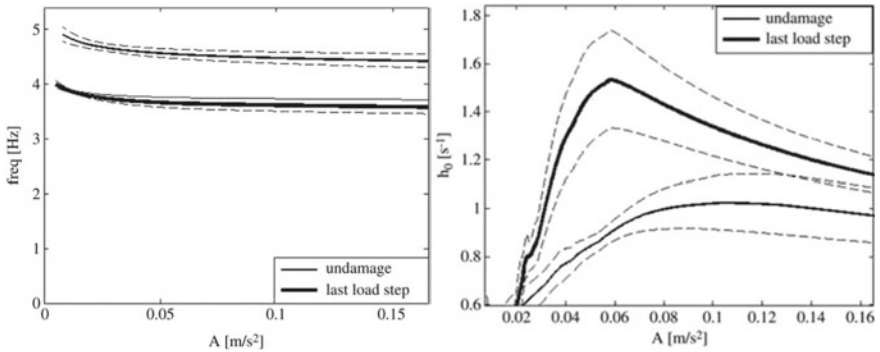
Methods found in the literature mostly propose damage detection, localization, and assessment using either natural frequencies or mode shapes and their derivatives [2]. It is generally acknowledged that natural frequencies have low sensitivity to damage and can usually be used to reflect damage to a moderate degree. Mode shapes and their derivatives have shown greater sensitivity to damage but capturing the mode shape and its changes requires numerous sensors distributed on the structure and proper placement. This situation has made utilizing mode shapes and their derivatives in practical applications of structural damage detection less attractive.

Compared to natural frequencies and mode shapes, damping has been proven more sensitive to damages even portraying small, visually undetectable damages (Figs. 1 and 2). More evidence of the efficiency of damping in portraying damage in structures could be found in studies of [8–10].

Generally, the damping of vibration in a large structural system is due to various forms of energy dissipation mechanisms. It is commonly identified as viscous damping and Coulomb damping. Typically, in engineering practice, a linear viscous damping model is used for the sake of simplicity as it lends to a linear equation of motion [11, 12]. It has been understood that when a structure is undamaged, most of the energy dissipation is due to material damping which is categorized as viscous, i.e., proportional to the velocity of motion. However, when the structure is damaged, the most significant energy dissipation mechanism within the cracks or in a defective connection can be represented by Coulomb friction [13].



**Fig. 1** Variations in modal damping and natural frequency with increased severity of corrosion damage [10].  $U_0$ ,  $U_1$ , and  $U_2$  denote damage states 0 (undamaged), 1, and 2, respectively.



**Fig. 2** Instantaneous undamped natural frequency (a) and instantaneous damping coefficient  $h_0(t)$  (b) along the amplitude of vibration [8]

Bachmann and Dieterle [14] observed that energy dissipation due to excessive movements in joints, energy dissipation in opening and closing of cracks, slipping at interfaces, sliding in supports, etc. generate Coulomb type of damping with coexisting viscous type material damping.

Experiments on precast reinforced concrete slab panels [9] showed that the presence of small, visually undetectable cracks caused a considerable increase in damping. Further, they noted that the energy dissipation mechanism in a crack is mainly due to dry Coulomb friction. Further, they noted that visually undetectable cracks have very little change in natural frequencies and mode shapes, while there is a significant change in damping characteristics of the structure.

Frizzarin et al. [15] presented work on the theoretical-experimental identification technique for structural damage detection based on analysis of non-linear damping of measured structural vibration response. The experimental verifications

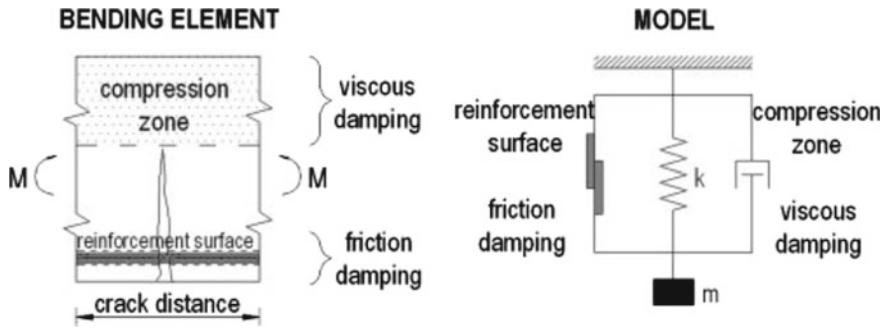


Fig. 3 Representation of cracked bending element and corresponding model [15]

were conducted using a large-scale concrete bridge model. They presented a model (Fig. 3) for a cracked bending element, where both the viscous and friction-damping phenomena co-existed at the same time. It shows that in the cracked zone where there is friction in the reinforcement surface, the most significant dissipation mechanism is friction damping. On the contrary, in the compression zone, it can be assumed that only material (viscous) damping is present. In the model,  $k$  represents the bending stiffness of the element, while  $m$  is the relevant mass.

As a result, damage to the system can be detected if the envelope of free decay of a system is examined and it is determined that dry Coulomb friction is the main source of energy dissipation.

## 2 Objective and Scope

The objective of this paper is the present investigation on using analysis of time-dependent damping behaviour in the prediction of damage existence of a structure based on the work by [15]. Experimental application of the method was conducted using ambient vibration data from the experimental system at the laboratory. The knowledge obtained in this investigation is useful in readily applying the vibration-based method for damage detection in real structures which will be conducted in the next stage.

## 3 Damping Models and Damping Estimation

Typically, in engineering practice, a linear viscous damping model is used for the sake of simplicity as it lends to a linear equation of motion. But it is important to note that in real structures the damping behaviour is more complex and often nonlinear. As mentioned by [16], commonly used damping models to describe the real damping

behaviour can generally be presented by the Equation,

$$f_d(x, \dot{x}) = a\dot{x}|\dot{x}|^{\theta-1} \tag{1}$$

where,  $f_d(x, \dot{x})$  is the damping force,  $\dot{x}$  is the velocity and  $a$  is the damping coefficient. The value of  $\theta$  determines the damping model, e.g.

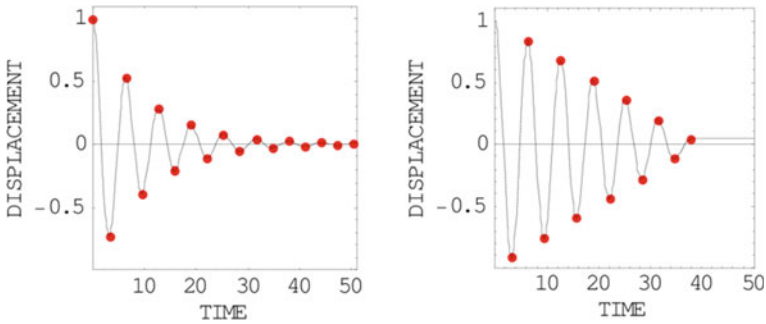
linear viscous damping ( $\theta = 1$ )

$$f_d(x, \dot{x}) = c\dot{x} \tag{2}$$

Coulomb (Dry friction) damping ( $\theta = 0$ )

$$f_d(x, \dot{x}) = \mu \frac{\dot{x}}{|\dot{x}|} = \mu \text{sign}(\dot{x}) \tag{3}$$

where,  $c$  is the viscous damping coefficient,  $\mu$  is the Coulomb damping coefficient. The free decay envelope of a system under viscous damping is seen as an exponential decay and is linear for a Coulomb-damped system as shown in Fig. 4. For these ideal systems, [17] provided individual functions of coulomb, viscous and quadratic damping acting on a Single Degree of Freedom System (SDOF) as given in Table 1.



**Fig. 4** Linear viscous damping and Coulomb (dry friction) damping [18]

**Table 1** Coulomb, viscous and quadratic damping mechanisms [23]

Damping type	Governing equation (Homogeneous form)	Envelop signal	Equivalent viscous damping coefficient
Coulomb	$\ddot{x} + \mu \frac{\dot{x}}{ \dot{x} } + \omega_n^2 x = 0$	$a_c(t) = \frac{-2\mu}{\pi\omega_n}t + y_0$	$C_{eq,c}(t) = \frac{2\hat{\mu}}{\pi\omega_n^2 a_0 - 2\hat{\mu}\omega_n t}$
Viscous	$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0$	$a_v(t) = e^{-\zeta\omega_n t}$	$C = 2\zeta\omega_n$
Quadratic	$\ddot{x} + \varepsilon \dot{x} \dot{x} + \omega_n^2 x = 0$	$a_q(t) = \frac{3\pi a_0}{3\pi + 4\varepsilon\omega_n a_0 t}$	$C_{eq,q}(t) = \frac{4\hat{\varepsilon}a_0}{3\pi + 4\hat{\varepsilon}\omega_n a_0 t}$

From above, quadratic damping is mostly associated with air damping, which occurs due to the air resistance of a moving structure. In large civil engineering structures, this type of damping is not significant and is not considered in this study.

Then, the decay envelope of a system with combined viscous and coulomb types of damping should be characterized by a linear summation of the individual decay functions defined by [17]. Thus, the mass normalized equation of motion that defines a system with viscous and coulomb damping is shown in Eq. (4).

$$\ddot{x} + 2\zeta\omega\dot{x} + \mu[\text{sign}(\dot{x})] + \omega^2x = 0 \quad (4)$$

$$x(t) = x_0 \left[ \left( 1 + \frac{\gamma}{\xi} \right) e^{-\xi\omega t} - \frac{\gamma}{\xi} \right] \quad (5)$$

Based on Eq. (5), it is understood that the envelope of a free decay gives information about the energy dissipation mechanisms acting on the system. This equation contains values of  $\xi$  and  $\gamma$ . Then, as given by [15], assuming that the total loss of energy ( $\Delta E_{\text{Pot}}$ ) is the simple sum of energy loss due to viscous ( $\Delta E_{\text{Visc}}$ ) and friction ( $\Delta E_{\text{Frict}}$ ) energy dissipations, the relationship for decay envelope for the combined system (shown in Eq. 4) was obtained in terms of initial amplitude  $x_0$ , the natural frequency of the system  $\omega$  and the two damping ratios  $\xi$  for viscous damping and  $\gamma$  for friction damping as given in Eq. (5): information about the presence of each type of damping. Simultaneously, it contributes to each damping mechanism on total energy dissipation.

Value for  $\gamma = 0$  suggests that there is no dry friction type of energy dissipation and the decay envelope is purely exponential, indicating only a viscous type of energy dissipation acting on the system. In contrast to this, when no viscous type of damping is present, the decay envelope is linear and  $\xi = 0$ . This indicates entirely the presence of coulomb friction type of damping. Values in between these extremes indicate the type of damping acting on the system, contributing to oscillation decay. Different values for  $\gamma$  and  $\xi$  contribute to each damping mechanism on total damping.

Therefore, identifying the presence and contribution of friction damping in a system using this method is feasible and could be directly correlated to the occurrence of damage in the system.

## 4 Experimental Investigation

To study the applicability of the above method, an experiment was designed at the laboratory. Experiments were conducted using flexural vibration of cantilever beams since it is recognized as one of the most favoured techniques to study different damping characteristics of structural systems by many researchers, Paimushin et al. [19–23] among others. In this method, one end of the specimen was held in place,

while the other end of the specimen was allowed to move freely to respond to manual displacement or induced vibration.

The experimental setup used in the investigation is shown in Fig. 5. The cantilever beam was a mild steel rectangular box section in which one end was clamped to a large concrete base with the dimensions of 600 mm × 600 mm × 600 mm (Appx, 550 kg weight). The length of the mild steel tube was 1790 mm. A 40 mm × 80 mm box section with a thickness of 3.0 mm was used. Dry frictional resistance was applied using a mechanism attached close to the free end of the cantilever beam. The mechanism could apply different levels of frictional resistance to the vibration oscillation.

The acceleration measurements were started with some delay which is necessary for the transition from the initial state to the first, lowest vibration mode of the specimen. The measured acceleration response was filtered and used to derive the corresponding displacement responses using a freely available signal processing tool. Based on these response measurements, decay envelopes were obtained.

Figure 6 shows a signal of the acceleration response of the free vibration of the cantilever without any dry friction damping (Force “N” absent). Then the accelerometer data were band-pass filtered from 1 to 15 Hz since it is known that the fundamental frequency of the system is lower than 15 Hz as calculated. Figure 6 also shows the overlaid acceleration response after applying the bandpass.

The Fast Fourier Transform (FFT) of the response showed the first natural frequency of the system as 11.719 Hz. Thus, it could be concluded that the system tends to vibrate in its first flexural vibration mode under the excitation and is well separated from other higher modes.

Once the acceleration response was processed, the processed signal was used to derive the displacement response using the same signal processing tool. The decay curve was then obtained using the peaks of displacement response.

The experiments were conducted with external dry frictional resistance to beam vibration. The frictional resistance was controlled by adjusting the normal force between the sliding surfaces while the coefficient of friction was constant in this experiment setup due to no change in the mating surface. These externally applied different magnitudes of frictional forces represent the different degrees of damage

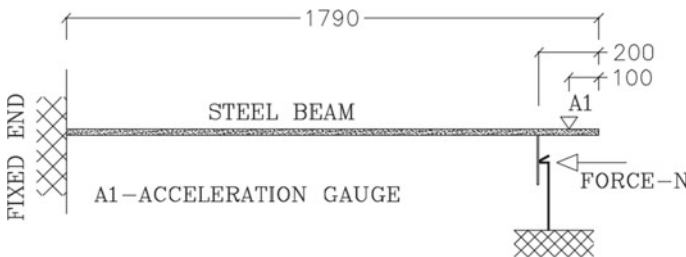
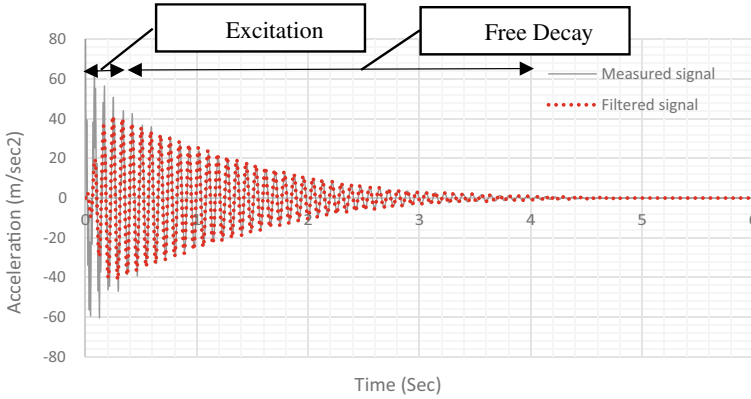


Fig. 5 Experimental cantilever beam model



**Fig. 6** Measured and filtered acceleration response

scenarios. Lower frictional resistance mimics a lower degree of damage in the system and increasing frictional resistance mimic increased damage scenarios.

Table 2 provides the details of the experiment scheme. Test 1 (T1) is without any dry coulomb friction and represents no damage scenario in the system and expected only viscous type of damping.

Other tests (T2, T3, T4) represent a system with different degrees of damage varying from low to high degrees. The displacement time history data with decay envelopes for the fundamental mode of the cantilever beam for tests T1, T2, T3, and T4 are shown in Fig. 7. The influence of an increase in dry friction force on the displacement amplitude decay envelope of free vibration is visible in the time histories.

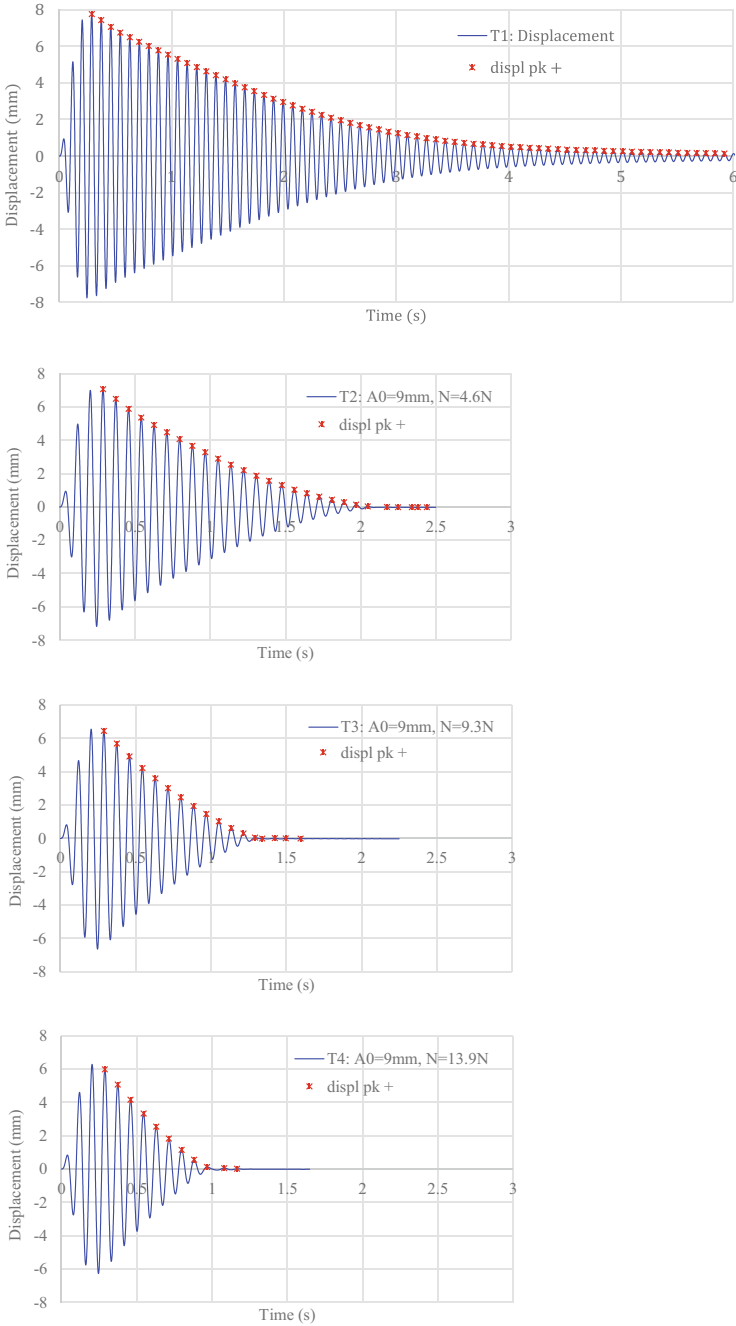
It is known that the logarithmic decrement ( $\delta$ ) of the decay envelope is related to energy dissipation [23]. For an exponential decay or decay envelope of a system with only a viscous type of damping, the logarithmic decrement is considered constant over time. When dry friction damping is present, the decay envelope’s logarithmic decrement ( $\delta$ ) becomes nonlinear [24]. To observe this with the results of the experiment, instantaneous logarithmic decrements for T1, T2, T3 and T4 are plotted against time. The variation of logarithmic decrement under different dry Coulomb friction amplitudes is presented in Fig. 8.

In Fig. 8, it is observed that the log decrement is constant when no coulomb friction is present in the test system indicating only a viscous type of damping [24].

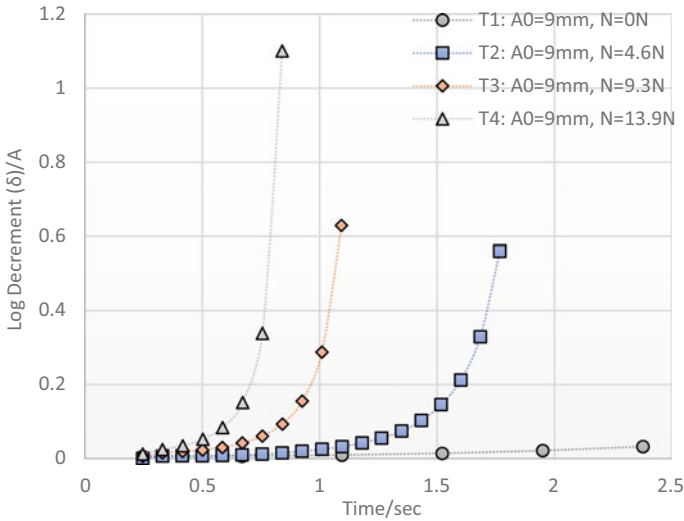
**Table 2** Experiment scheme for Coulomb damped free vibration of a cantilever

Test no.	Initial displacement ( $A_0$ ) (mm)	Normal force (N)
T1	9.0	0
T2	9.0	04.6
T3	9.0	09.3
T4	9.0	13.9





**Fig. 7** Displacement—time histories under different dry Coulomb friction amplitudes and constant viscous damping



**Fig. 8** Log decrement comparison for various dry friction amplitudes

When Coulomb friction is introduced to the system, the variation of log decrement becomes nonlinear. The nonlinearity is increased with an increment in the amplitude of the applied normal force.

Therefore, it is evident that the nonlinearity in the damping behavior exposes the existence of damage in the system. Nevertheless, quantification of nonlinearity provides evidence of the quantitative information of the damage in the system.

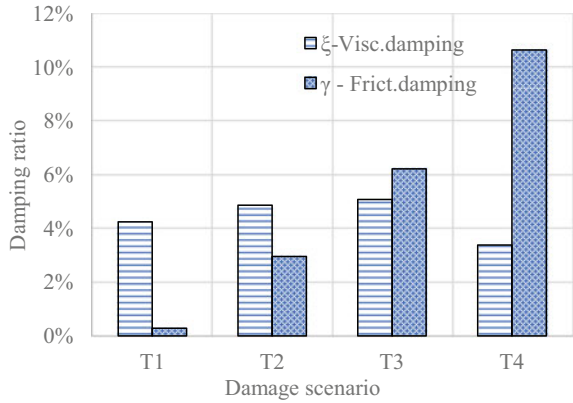
Thus, moving forward based on the analysis of decay envelopes, (based on Eq. 5), the parameters,  $\xi$ , and  $\gamma$  could be assessed for each test configuration using simple curve fitting techniques. Such an assessment was conducted for each test, T1, T2, T3, and T4. The result of the assessment is shown in Table 3. The same results were elaborated in Fig. 9 and represent the increase in friction damping component  $\gamma$  with increased friction resistance.

The numerical values in Table 2 and its graphical representation in Fig. 9 shows that the dominant damping mechanism in Test (T1) is the viscous type and it is constant over all four tests as expected due to the nature of the test setup. While, for

**Table 3** Variation of friction force and observed damage percentage

Test ID	T1	T2	T3	T4
F (N)	0	4.6	9.3	13.9
$\xi$ —visc. damping	0.042474	0.04872	0.050819	0.033814
$\xi\%$	4.2	4.9	5.1	3.4
$\gamma$ —frict. damping	0.00288	0.029573	0.062152	0.10648
$\gamma\%$	0.3	3.0	6.2	10.6

**Fig. 9** Comparison of observed damage percentage



other tests T2, T3, and T4, the dominant damping mechanism gradually changed into dry friction damping consistently with the increased frictional resistance. Further, it depicts the capacity of the proposed method in portraying the quantitative information of the damage.

## 5 Conclusions

This study discusses the application of a simple, baseline free time domain damage detection technique using acceleration data. The investigation was made using analysis of nonlinearity in damping extracted from ambient vibration data for identification of the existence of damage in a structure. Experiments were conducted using a simple cantilever beam set up under the influence of different dry Coulomb friction magnitudes to represent different degree damage scenarios.

In the investigation, a strong correlation between the increase in the normal contact force (Coulomb friction force) and the nonlinear behaviour of damping was observed. The nonlinearity of damping behaviour was seen to increase when the dry frictional force was increasing. Thus, it is evident that nonlinearity in damping could portray the existence of damage in a structure.

Further, it was shown that the parametric identification of the decay function of the combined system could provide quantitative information on the existing damage by distinguishing the contribution of each viscous type and friction type of damping based on a controlled laboratory environment.

Further research is needed to inspect the feasibility of the method to be applied in real structures.

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