Design of Teaching Model for Intuitive Imagination Development Supported by NetPad

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Abstract. As one of the core literacies in mathematics, intuitive imagination is essential in learning geometry in middle school. Recently, BOPPPS, which is a teaching model that featured clear objectives, has gradually become an effective course design model for cultivating students' intuitive imagination. Meanwhile, NetPad is a browser-based dynamic mathematical software that supports online mathematical experiments and dynamically displays geometric changes. In this paper, we propose a model called N-BOPPPS, which combines advantages of the BOPPPS teaching model and NetPad, aiming at developing students' intuitive imagination. This model consists of six procedures supported by NetPad, i.e., bridge-in, objective, pre-assessment, participatory learning, post-assessment, and summary. It supports classroom instructions such as the inquiry-based model and student-engaged model. Besides, it encourages students to carry out experimental investigations and innovative practice during the learning process, and then their intuitive imagination is improved. At last, teaching practice is conducted under the guidance of N-BOPPPS and the effectiveness of this model is verified.

Keywords: Intuitive imagination · NetPad · Teaching models

1 Introduction

In June 2020, the Ministry of Education promulgated the General High School Mathematics Curriculum Standards (2017 edition revised in 2020), which for the first time condensed and proposed the six core literacies of mathematical abstraction, intuitive imagination, mathematical modeling, logical reasoning, mathematical operations, and data analysis in mathematics [1]. In particular, intuitive imagination is quite helpful for exercising the mathematical mind. It can facilitate the composition of the abstract mind and transform it into a figurative mind in the process of mathematical geometry problem-solving. The inclusion of intuitive imagination in core literacy shows its significance of it in the development of students' learning and abilities.

W. Hong and Y. Weng (Eds.): ICCSE 2022, CCIS 1812, pp. 146–156, 2023.
https://doi.org/10.1007/978-981-99-2446-2_14

In the era of continuous development of science and technology, to ensure that we can effectively guide students to develop their core literacy, we must introduce new technologies and update our teaching sessions [2]. We could apply information technology to create an information-based environment to promote students' core literacy. Therefore, using information technology helps better develop students' intuitive imagination. NetPad is an online dynamic mathematical geometry software that uses mathematical logic, and graphical geometric constraint construction to draw graphs and create interactive digital resources. Furthermore, NetPad distinguishes itself from other dynamic geometry systems by being an open, internet-based, sharing-oriented intelligent system [3]. By integrating NetPad in class, we can achieve the integration of information technology and the subject.

At present, there are still some common problems in developing intuitive imagination in our mathematics education. For example, teachers are not proficient in using mathematical models in the classroom to develop students' problem-solving skills with graphs and failed to effectively guide students to think outside the box to let them try to use multiple graph drawing methods to solve the problem; the teacher; educators failed to utilize targeted IT software in the classroom, such incorporating dynamic geometry system (DGS) to assist in developing intuitive imagination to experience rich visuals, and teaching students to learn to use DGS in mathematical problem-solving [4, 5]. To overcome the above problems, educators should explore efficient teaching modes to cultivate students' intuitive imagination and create a lively atmosphere in the classroom to attract students to participate in learning. In addition, the BOPPPS teaching model has an effective teaching structure, which is a model that uses six instructional sessions such as effective introduction, timely assessment, and participatory learning, and it enhances teacher-student interaction and emphasizes student participation. In recent years, the BOPPPS model has been gradually introduced and practiced by scholars in China, with its practices showing that the model is an effective teaching model that can completely reflect the main position of students in the classroom, pay attention to the cultivation of students' thinking, and help teachers improve their effectiveness of teaching [6].

To sum up, based on the combination of the BOPPPS and NetPad, this paper proposes a teaching model called N-BOPPPS and its teaching practice that verified the effectiveness of N-BOPPPS. In Sect. 2, a review of the value of intuitive imagination and its concrete manifestation in math will be described. Also, the introduction of the BOPPPS model and the characteristics of the NetPad will be introduced. In Sect. 3, the N-BOPPPS teaching models will be proposed and introduced in detail. In Sect. 4, the process of teaching will be designed and practiced guided by the N-BOPPPS teaching model. In Sect. 5, we will conduct a summary analysis of the effectiveness of the practice.

2 Related Work

2.1 The Definition and Value of Intuitive Imagination

To understand intuitive imagination, we need to first understand geometric intuition and spatial imagination separately, for these two parts are pointed out by the New Curriculum as the factors that make up intuitive imagination. The definition of intuitive imagination in the New Curriculum Geometric intuition is the literacy of using geometric figures to analyze and solve mathematical problems, it can use simple graphs to represent the abstract concepts in mathematical problems or ideas to solve problems, and achieve the idea of "visualization" effect. According to the research of Tang P., geometric intuition has the advantage of visualization and simplicity, which can facilitate mathematical problem-solving. As for the performance of spatial imagination, by using the existence of objects in a certain space and their characteristics, positions, and other properties, we can imagine in our minds what already exists or create things that do not exist [7]. It can be understood as a dynamic graphical transformation process, such as the transformation and matching of relationships between geometry and three views and expansion diagrams. Moreover, the relationship between geometric intuition and spatial imagination is inseparable and interrelated. Geometric intuition is a process belonging to human cognitive activity, while spatial imagination is a method in this procedure, and the two have an interactive relationship in human cognitive activity [8].

As educators cultivate students' intuitive imagination, they need to focus on both geometric intuition and spatial imagination learning. Therefore, educators need to develop students' ability to use graphs to describe, understand, explore and solve mathematical problems, promote their learning to make connections between numbers and shapes, and help students build intuitive models of mathematical problems while building geometric intuition. Furthermore, In the process of developing students' spatial imagination skills, students are expected to formulate the cognition of three-dimensional space and master the symbols for describing spatial figures. Students should learn to match three-dimensional shapes with flat graphs and build the ability to disassemble and combine three-dimensional shapes, and finally provide new directions for problem-solving.

2.2 BOPPPS Model

The BOPPPS teaching model is the main teacher training model adopted by the Canadian ISW program, which is highly operable and has clear teaching objectives, and is gradually being developed in China, profoundly affecting teachers and students [9]. Moreover, the BOPPPS teaching model divides teaching sessions into six modules: bridge-in, objective, pre-assessment, participatory learning, post-assessment, and summary (see Fig. 1).

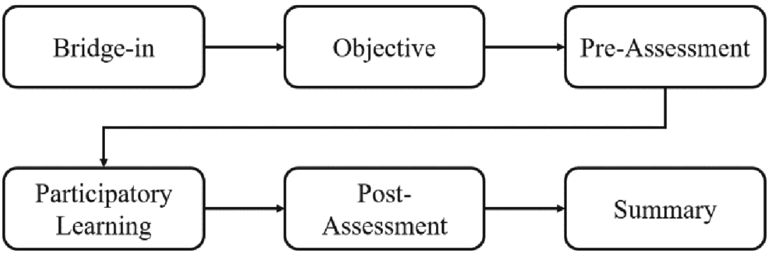


Fig. 1. BOPPPS model.

Bridge-in is the session that uses videos, questions or stories to attract students' attention, it can trigger students' curiosity and then introduce new content learning; Objectives make it clear to students that learning objectives or outcomes which the course needs to achieve; Pre-assessment is the session for teachers to know students' knowledge reserve and grasp their learning ability, so that they can adjust the content and progress of subsequent teaching sessions; Participatory learning reflects the concept of "student-centered" teaching and learning, which stimulates students' enthusiasm for learning by adopting active learning strategies and further deepens their understanding of the teaching content and finally achieve the learning objectives; Post-assessment is an important part of determining whether students have met expectations to see how well they have accomplished the learning objectives of the course; Summary can further enhance the students' impressions and consolidate the learning objectives by summarizing the knowledge points and the knowledge chain of this course.

Since the BOPPPS teaching model has been introduced to China for a relatively short period, there are fewer studies about its application to the cultivation of core literacy in mathematics [10]. However, by applying the BOPPPS model to the development of core literacy in other subjects, we can learn that this model has positive effects on developing students' core literacy compared with the traditional teaching model, and can be applied to the teaching of developing intuitive imagination skills.

2.3 NetPad

NetPad is a cross-platform Dynamic Geometry System with rich dynamic mathematical resources and powerful geometric drawing capabilities [11]. NetPad offers a 2D drawing area (Fig. 2) and a 3D drawing area (Fig. 3). It allows not only geometric drawing and symbolic computation but also automatic reasoning about geometric shapes, seamless integration with other systems, as well as providing a convenient platform for sharing and communication.

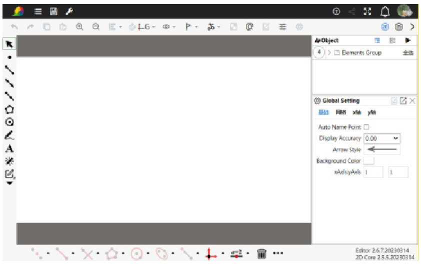


Fig. 2. 2D drawing area on NetPad.

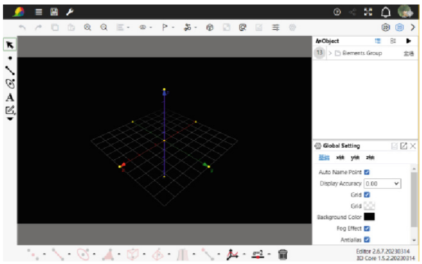


Fig. 3. 3D drawing area on NetPad

Many Chinese researchers have discussed the meaning and role of NetPad and given the advantages of NetPad in teaching mathematics. According to the research of Wang J.X., the picture preview method of the web board can provide teachers and students with accurate interactive educational resources quickly and easily [12]. What's more, based on the CCEMS-based evaluation model, it is effectively verified that the NetPad pays attention to user experience and that it can satisfy both teachers' and students' needs [13].

In the era of informatization in education, the advantages of NetPad such as the large user community and rich teaching resources provide alternative teaching tools and teaching materials of various types levels to promote the integration of information technology and courses. Thus, teachers can create contexts, set inspiring questions, and guide practice in teaching through NetPad. Moreover, students are also able to form a systematic knowledge system of mathematics with the support of NetPad, develop their problem-solving skills and IT operational skills, and finally improve core literacy.

3 N-BOPPPS Teaching Model

Based on the BOPPPS teaching model and NetPad, this paper proposes the N-BOPPPS teaching model. The model involves the entire teaching and learning activities, which would enhance students' creative awareness, problem-solving skills, and intuitive imagination (see Fig. 4).

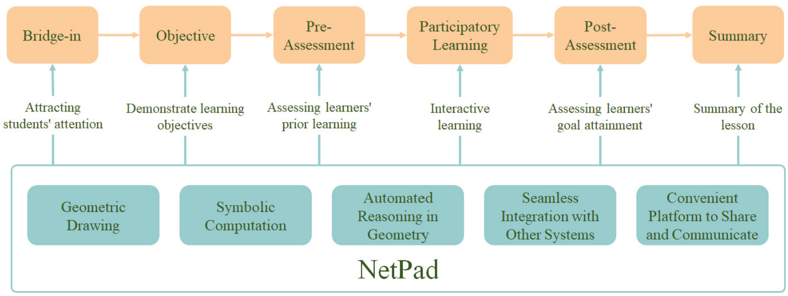


Fig. 4. N-BOPPPS teaching model.

Bridge-in: An orientation session before formal teaching. The teacher uses specialized NetPad cases to attract students' attention and raises questions to guide students' thinking as well as interact with them, gradually moving to the core part of the teaching. Furthermore, teachers can use what they have already learned in the last class to make organic connections with what they will be learning, which can be visually displayed through NetPad.

Objective: Setting learning objectives based on the requirements of intuitive imagination. The teacher sets the three-dimensional objectives (Fig. 5) of teaching with a clear understanding of the core knowledge of the class, the needs and the abilities of the students, and the requirements of each level based on intuitive imagination. Importantly, these objectives explicitly state the requirements and levels that should be achieved through learning in terms of knowledge and skills, process and methods, as well as emotions and values.

Pre-assessment: The session of understanding students' interests and prior knowledge. The teacher utilizes well-designed cases to assess learners' prior knowledge and graphic

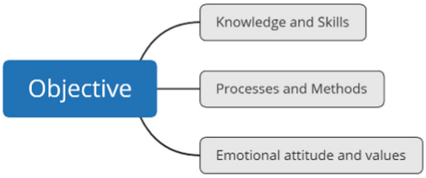


Fig. 5. Three-dimensional teaching objectives.

intuition and then guides the pace of the teaching process and subsequent arrangements. Through the interaction of information between teachers and students, teachers can understand students' interests and abilities to facilitate teaching and learning, while students can review previous knowledge points to lay the foundation of knowledge for the subsequent learning that takes place in class.

Participatory Learning: Teachers and students use NetPad to interactively learn the core content and enhance their intuitive imagination in the process. To reflect a student-centered position and motivate students, participatory learning is a core aspect of the N-BOPPPS model. The educator uses NetPad to create a simple and intuitive context, first showing the case and explaining the steps to guide students to actively participate in class. When most students have completed the basic drawing, the teacher gives students sufficient time to explore different solutions and encourages them to discuss and help each other in small groups to complete the work together and eventually lead them to achieve the objectives.

According to the combination of teaching theme and case, use the questioning method to develop students' thinking and lead them to reflect, then use NetPad to reproduce the process of solving the problem, so that students can deduce many things from one case. Students use the properties of graphs in class to explore mathematical patterns, discover the connections between graphs and quantities, and develop solutions to use graphs to solve problems.

Post-assessment: An assessment or test session to see if students have accomplished the learning objectives. The teacher checks students' knowledge understanding of the lesson by displaying some simple multiple-choice questions or creating problem situations and organizes students to conduct a comprehensive analysis to assess whether students' general quality and problem-solving skills have been improved. And then, the teacher collects students' work and allows them to present and share concepts of their work so that they could have a clear understanding and integration of what they have learned.

Summary: A conclusion of the lesson to help students deepen their knowledge. The teacher makes comments on students' work and praises their efforts as well as their learning achievements timely so that students can gain a sense of self-efficacy and accomplishment. Finally, the teacher uses NetPad to display a summary of knowledge points, organize and review the lecture content, provide timely feedback on the lesson, and guide students to further consolidate the learning objectives.

4 Teaching Practice Based on N-BOPPPS

Based on the guidance of the N-BOPPPS model and the knowledge related to square roots, this section implements the case of "$\sqrt{n}$ on the number axis" on NetPad and carries out instructional practice (see Fig. 6).

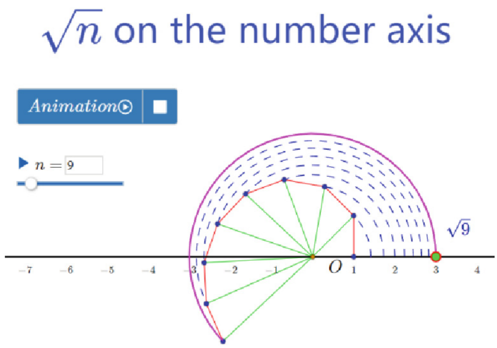


Fig. 6. $\sqrt{n}$ on the number axis.

Bridge-in: Since students have already mastered the position of simple rational numbers on the axis, teachers design problems that will be displayed in NetPad. The problem is taken as the core content that needs to be solved in this lesson so that students can think about the problem before the class starts, stimulate their interest in learning, establish the basic concept of graphical and quantitative relationships, and complete the thinking training of graphical description and induction.

- Question: We all know that there is a "one-to-one" relationship between the points and the numbers on the number axis, and know how to draw the numbers 1, 2, or -2, -4 on it. However, do you know how to quickly find the point of $\sqrt{2}$ on the number axis with NetPad? What about $\sqrt{17}$?

Objective: Before entering the formal lesson, use 2 or 3 min to communicate to students the learning objectives of the lesson. Then, allow students to begin by broadly framing what they will learn in class and developing an overall understanding. Ensure that students are clear about the purpose of the class.

The three-dimensional objectives are the following:

Knowledge and Skills: Can accurately describe the relationship between the position of $\sqrt{n}$ on the number axis and the characteristics of its distribution; can try to inductively verify the mathematical principle behind the position of $\sqrt{n}$ on the number axis as the Pythagorean Theorem; can be able to imitate display cases to produce graphs of the position of $\sqrt[3]{n}$ or n^2 on the number axis.

Processes and Methods: Master the methods of constructing visual displays of geometric figures and solving problems in mathematical problem situations; Experience the process of exploring questions and developing your own methods of learning and solving geometric problems.

Emotional attitude and values: Experience the rules of motion when the relationship between graphs and quantities changes through cases; through the spiral line shown in the case study, perceive the beauty of mathematical figures, which in turn stimulates the interest in learning math.

Pre-assessment: Using questions to guide students to test their knowledge and visual level of graphics before class, the teacher understands students' interests and motivation according to their answers and then adjusts the teaching progress in time, which helps to achieve the classroom teaching objectives through the interaction between teachers and students.

- Question: What are the elements of the number axis? (Students make a diagram to describe or answer directly)
- Question: What are the meaning and properties of square roots? How do you distinguish square roots from arithmetic square roots? (Students summarize the characteristics)
- Question: Where are the common square root numbers, e.g., $\sqrt{16}$, $\sqrt{49}$? located on the number axis? (Students quickly point to the corresponding positions or describe them orally.

Participatory Learning: Use NetPad to guide students to join the learning process, which reflects the concept of "student-centered" education. The use of problem-driven, cooperative learning and independent student inquiry in the teaching is very motivating for students.

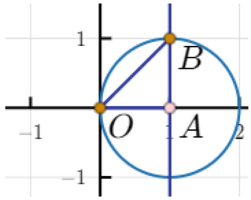
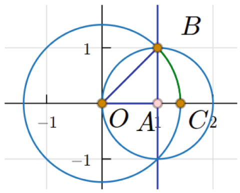
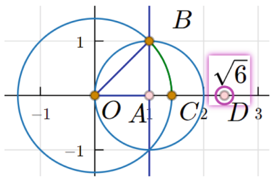
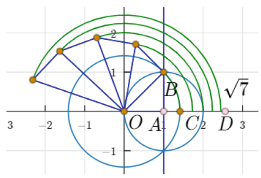
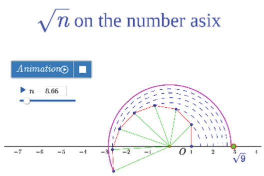
Perform the steps of the case on the drawing board, and combine them with questions to attract students' attention, such as "What is the next missing step", "What kind of graphical effect can be obtained from this step", "Why should we do this", etc. The specific steps are shown in Table 1.

After showing the steps of the case, students are guided to imitate the operation and reproduce the case. If students encounter difficulties, they are encouraged to communicate and discuss with their partners and then complete the work together. For students who are quick to complete their work, teachers can intentionally guide students to consider the patterns of quantity and graphical change or to use NetPad to create, such as an exploration of the location of $\sqrt[3]{n}$ or n^2 on the number axis.

Post-assessment: After completing the class, the teacher set up test questions that correspond to the learning objectives and target different dimensions to test the learning effect such as basic concepts, case design, and mathematical principles.

- Question: Which characteristic of the distribution of the position of $\sqrt{n}$ on the number axis does the case reveal? (Students answer)
- Question: What is the mathematical principle behind the distribution of $\sqrt{n}$ on the number axis? (Students answer and give a simple proof)
- Referring to this case, what should $\sqrt[3]{n}$ or n^2 look like on the number axis? (Students give a general description and work on the case after class)

Table 1. Steps of "$\sqrt{n}$ on the number axis".

Step	Result
Step 1: Make line segment AB and BO (see Fig. 6) a: Make the point A (1,0) and make a vertical line from the point A to the origin O. Make a circle of radius 1 with point A as the center, make the intersection of the circle and the vertical line B, make the line segment AB and the line segment BO;	 **Fig. 7.** The effect of step 1.
Step 2: Make point C (see Fig. 7) a: Take the point O as the center and the line BO as the radius to make a circle O, make the intersection of the circle O and the x-axis C; b: Introduce the variable n, make the arc BC with the point D (sqrt(n),0), "x-drag" for n.	 **Fig. 8.** The effect of step 2.
Step 3: Additional graphing (see Fig. 8) a: Parameter animation for variable n. Then, make free text and set the text property to "$$sqrt($$bl{n,0})$$"; b: Merge free text and point D. Use the [Attach/Separate/Merge] tool to merge free text and point D, then hide the text.	 **Fig. 9.** The effect of step 3.
Step 4: Iterative mapping (see Fig. 9) a: Iterate the graph. Set "original image" to point A, "reflection" to point B, "iteration depth" to n-2, "iterated objects" to point B, line AB, line OB, and arc BOC.	 **Fig. 10.** The effect of step 4.
Step 4: Beautify the graphics (see Fig. 10) a: hide the redundant elements (circle O, circle A, and the vertical line), and finally set the graphic color according to your preference.	 **Fig. 11.** The effect of step 5.

Summary: After evaluating and encouraging students' work, the teacher summarizes the relevant knowledge points. In this class, students will explore the problem of "$\sqrt{n}$ on the number axis" through a case demonstration on NetPad. Students experience the relationship between graphs and quantities while participating and creating cases in NetPad, then learn how to use the relationship between graphs and graphs, as well as between graphs and quantities to solve problems. Students further facilitate the formation of intuitive models of problem-solving by visualizing visual and imaginative stimuli through graphics.

5 Conclusion

Based on the BOPPPS teaching model, this paper proposes the N-BOPPPS, which aims at developing students' intuitive imagination, and finally carries out teaching practice supported by NetPad. During the practice, the teacher guided students to cooperate and explore not only to complete the case but also to verify the mathematical principles behind the case to summarize the distribution of $\sqrt{n}$ on the number axis.

With the N-BOPPPS teaching model, teachers can organize situational and structured knowledge in teaching sessions, while students learn to use the number axis to determine the positions of rational numbers, gain an initial understanding of spirals, and understand the relationships between different number systems with the help of models and knowledge organization. Students master the solution of the relationship between graphs and graphs or between graphs and quantities by working with group members. Finally, students will continue to improve their level of intuitive imagination skills, truly discover the role and significance of graphical intuition in learning, and develop the mindset of using geometric intuition to solve problems.

However, there are still some improvements needed in the teaching practice based on the N-BOPPPS model. In future research, we will pay more attention to the collection and analysis of experimental data, and use the data to support the detailed improvement of the N-BOPPPS model; we will combine effective assessment methods to test students' intuitive imagination level; we will conduct more N-BOPPPS-based teaching practices for different chapters of mathematic. In summary, the N-BOPPPS teaching model has a positive effect on the development of students' intuitive imagination literacy. More teaching practices should be conducted in the future to continuously improve N-BOPPPS and promote the practice on a large scale to help cultivate students' intuitive imagination effectively.

Acknowledgments. This work was supported by the China Postdoctoral Science Foundation (No. 2021M700920), the Guangzhou Academician and Expert Workstation (No. 20200115-9), and the Innovation Projects of Universities in Guangdong Province (No. 2020KTSCX215).

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