Chapter 5
Truss Structure Optimization Using Constrained Version of Variations of Cohort Intelligence

Ishaan R. Kale, Ayush Khedkar, and Mandar S. Sapre

Abstract An especially significant class of structurally constrained optimization problems is truss design. This study presents a constrained version of two variations of the Cohort Intelligence (CI) algorithm. In this work, discrete variable truss structures with six bars and two cases with ten bars are studied using follow-best and follow-better approaches, as well as the self-adaptive penalty function (SAPF). These problems are associated with two linear constraints: tensile/compressive stress and deflection. Algorithm efficiency is evaluated by counting the function evaluations, computing CPU time, and determining the total weight of the truss structure. Compared to follow-better and other contemporary optimizers from literature, follow-best performs significantly better.

Keywords Self-adaptive penalty function · Discrete variables · Variations of CI · Design of trusses

5.1 Introduction

Truss structure problems are structural constrained optimization problems consisting of continuous, discrete, or mixed variables. The constraints are usually nonlinear in nature. There have been several techniques inspired by nature to solve truss structures problems. Genetic Algorithm (GA), Firefly Algorithm (FA) (Gandomi et al. 2011), Particle Swarm Optimization (PSO) (Li et al. 2009), and Artificial Bee Colony (ABC) (Sonmez 2011) are few optimizers from literature applied in this domain.

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In an earlier study, Kale and Kulkarni (2018) used Cohort Intelligence (CI) with a static penalty function (SPF). Some limitations were observed when CI-SPF was used for solving the constrained problems. SPF approach is associated with a penalty parameter which needed to be set for every problem. To set the appropriate penalty parameter, certain preliminary trials are required. This may increase the initial computational cost. Self-adaptive penalty functions (SAPFs) have been proposed as a solution to overcome these limitations (Kale and Kulkarni 2021). SAPF-based constraint handling with CI algorithm facilitates the solution of constrained problems involving variables of discrete, continuous, and mixed nature. Furthermore, the hybrid CI-SAPF-CBO, refined the results.

Patankar and Kulkarni (2018) developed seven variations of CI. These were applied to mesh smoothing of complex objects (Sapre et al. 2019) and for optimizing the abrasive water jet machining process (Gulia and Nargundkar 2019). Two variations of CI are applied in this paper to solve three test problems from the truss structural domain, namely a six-bar test problem and two ten-bar test problems. These are the follow-best and follow-better approaches. For constrained problems, other rules such as roulette, alienation and random selection, follow-worst, and follow-itself are not effective. Round-off integer sampling is used to handle discrete variables, and SAPF is used to handle constrained variables. The results obtained from follow-best and follow-better approaches are compared with those from GA, CI-SAPF, CI-SAPF-CBO, ABC, Adaptive Dimensional Search Algorithm (ADS), and Probability Collectives (PC).

The work is organized as follows: The mechanism of follow-best and follow-better approach using CI-SAPF is explained in Sect. 5.2. The solution to the truss structure problems follows next. In Sect. 5.4, the results are analyzed and discussed in details. The last section represents conclusion and future directions.

5.2 Mechanism of Follow-Best and Follow-Better Approach with SAPF

In follow-best approach, the candidate follows other candidates in the cohort situated at the best behavior. This assists the individuals to learn faster and achieve the cohort goal within less computational efforts. In follow-better approach, the candidate follows subsequent candidate exhibiting a better behavior than itself. The pseudocode of variations of CI using follow-best and follow-better mechanism incorporated with SAPF approach (Kale and Kulkarni 2021) is presented in Fig. 5.1.

Let

K	Candidate Count in the cohort $(k = 1, \ldots, K)$
S	Sampling space
r	Sampling space reduction factor
P	Penalty function
g	A violation of a constraint measure

Initialize K, S, r

While

1. Generate the random solutions $\mathbf{X}$ using uniform distribution method.
2. Determine the value of the function $f(\mathbf{X}^k)$ for each candidate.
3. Use SAPF approach to evaluate a pseudo-objective function / behavior:
   $\Phi(\mathbf{X}^k) = f(\mathbf{X}^k) + P(\mathbf{X}^k)$
   where $P(\mathbf{X}^k) = f(\mathbf{X}^k) \times \sum_{k=1}^{K} g\,(\mathbf{X}^k)$
4. Determine the probability p^k of each candidate k in the cohort as: $p^k = \frac{1/\Phi^*(\mathbf{X}^k)}{\sum_{k=1}^{K}[1/\Phi^*(\mathbf{X}^k)]}$
5. (i) Using follow-best approach every candidate k follows the behavior with highest probability.
   (ii) Using the follow-better strategy, all candidates with a lower probability of selection than the candidate itself are discarded, and the remaining candidates are selected at random. As a result, every candidate follows themselves or a candidate who is better than themselves.
6. The sampling interval $\boldsymbol{S}^c$ in the neighborhood of each candidate, shrinks/ expands using sampling space reduction parameter r:
   $\left[S^{k,lower}, S^{k,upper}\right] = \left[\mathbf{X}^k - \left\|\frac{S^{upper} - S^{lower}}{2}\right\| \times r,\ \mathbf{X}^k + \left\|\frac{S^{upper} - S^{lower}}{2}\right\| \times r\right]$
7. **If**: Function value $\Phi^*(\mathbf{X}^k)$does not significantly improve, there is a state of saturation in the solution.
   Every candidate k expands/ shrink iteratively the sampling interval $\boldsymbol{S}^k$ to its original interval $\boldsymbol{S}$
   Accept $\Phi(\mathbf{X})$ as current behavior of cohort with the associated attributes $\mathbf{X}$.
   Else
   Go to Step 3
   End If

End While

Fig. 5.1 Pseudocode of variations of CI using follow-best and follow-better rule

5.3 Truss Structure Test Problems

This work is investigation of application of constrained version of variations of CI with SAPF approach in truss design. The six-bar and ten-bar examples were solved in the literature using GA (Nanakorn and Meesomklin 2001), CI-SAPD, CI-SAPF-CBO (Kale and Kulkarni 2021), ABC (Sonmez 2011), ADS (Hasançebi and Azad 2015), PC (Kulkarni et al. 2016). The mathematical formulation is shown in Eq. (5.1) as follows:

$$\begin{array}{ll} \text{Minimize} & W = \sum_{i=1}^{N} \rho A_i l_i \\ \text{subject to} & \begin{array}{l} |\sigma_i| \le \sigma_{\max} i = 1, 2 \ldots N \\ |u_i| \le u_{\max} j = 1, 2 \ldots M \end{array} \end{array} \tag{5.1}$$

where

W Objective function (Weight)

A_i Design variables—Cross section area of ith truss member where, $i = 1, 2, \ldots, N$
ρ Material density
l_i Length of each truss member i, $i = 1, 2, \ldots, N$
$\sigma_{\max}$ Maximum allowable stress.
$u_{\max}$ Maximum allowable displacement.

Weight reduction of the truss structure is the goal with the maximum allowable tensile and compressive stresses at every node, as well as maximum allowable displacements as the limitations. There are as many variables as members in a truss problem. So, a six-bar truss has six variables. Each link of these trusses is a separate entity. In both the cases, distinct discrete set is utilized for the selection of variables.

Test Problem-1: Six-Bar Truss Structure

The six-bar truss structure (refer to Fig. 5.2) problem was formerly discussed by Nanakorn and Meesomklin (2001) Kale and Kulkarni (2021). There are six design variables (cross-sectional area) equal to number of truss members. Here, $A_i \in \{1.62,$ 1.80, 1.99, 2.13, 2.38, 2.62, 2.63, 2.88, 2.93, 3.09, 3.13, 3.38, 3.47, 3.55, 3.63, 3.84, 3.87, 3.88, 4.18, 4.22, 4.49, 4.59, 4.80, 4.97, 5.12, 5.74, 7.22, 7.97, 11.50, 13.50, 13.90, 14.20, 15.50, 16.00, 16.90, 18.80, 19.90, 22.00, 22.90, 26.50, 30.00, $33.50\}$ in^2. The allowable stress is given as 25,000 psi, and allowable deflection is given as 2 in. The weight density of the material is 0.1 lb/in^3, and the modulus of elasticity is 10^7 psi.

Test Problem-2: Ten-Bar Truss Structure

The next example is shown in Fig. 5.3 and was previously discussed in (Nanakorn and Meesomklin 2001, Li et al. 2009; Sonmez 2011; Hasançebi et al. 2015). A ten-bar truss structure made of aluminum 2024-T3 is used in the analysis. The material density ρ is 0.1 lb/in^3, and the modulus of elasticity E is 10, 000 ksi. As shown in Fig. 5.3, a represents the longest length of the truss member. The maximum allowable tensile and compressive stresses $\sigma_{\max}$ on every member i are $\pm$ 25 ksi. The maximum allowable horizontal and vertical displacement $u_{\max}$ at every node are

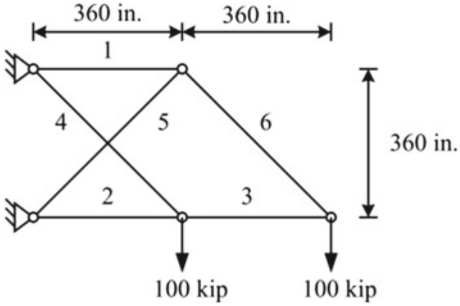

Fig. 5.2 Planar six-bar truss structure

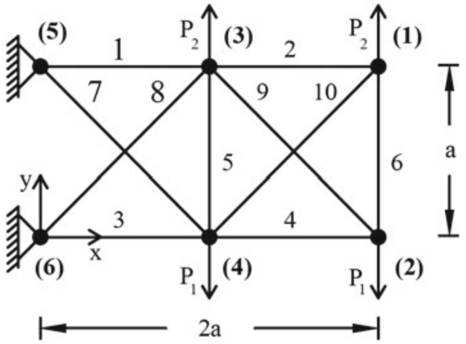

Fig. 5.3 Planar ten-bar truss structure, $a = 360$ in

± 2 in. The applied forces are $P_1 = 100$ kips and $P_2 = 0$. This problem involves ten design variables and two sub-cases.

Case 1: $A_i \in${1.62, 1.80, 1.99, 2.13, 2.38, 2.62, 2.63, 2.88, 2.93, 3.09, 3.13, 3.38, 3.47, 3.55, 3.63, 3.84, 3.87, 3.88, 4.18, 4.22, 4.49, 4.59, 4.80, 4.97, 5.12, 5.74, 7.22, 7.97, 11.50, 13.50, 13.90, 14.20, 15.50, 16.00, 16.90, 18.80, 19.90, 22.00, 22.90, 26.50, 30.00, 33.50}in^2.

Case 2: $A_i \in${0.1,0.5,1.0,1.5,2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5, 6.0, 6.5, 7.0, 7.5, 8.0, 8.5, 9.0, 9.5, 10.0, 10.5, 11.0, 11.5, 12.0, 12.5, 13.0, 13.5, 14.0, 14.5, 15.0, 15.5, 16.0, 16.5, 17.0, 17.5, 18.0, 18.5, 19.0, 19.5, 20.0, 20.5, 21.0, 21.5, 22.0, 22.5, 23.0, 23.5, 24.0, 24.5, 25.0, 25.5, 26.0, 26.5, 27.0, 27.5, 28.0, 28.5, 29.0, 29.5, 30.0, 30.5, 31.0, 31.5} in^2.

5.4 Results and Discussion

The use of follow-best and follow-better approaches for discrete variable problems is pioneered for the first time ever for truss structural problems. CI's follow-best version is much more efficient than other algorithms. In CI-SAPF and CI-SAPF-CBO, the candidate's follow other candidate in a cohort probabilistically due to which there was a possibility of following even the worst behavior of the candidate. This may require a greater number of learning attempts (iterations) for the convergence. In CI-SAPF, the performance of this approach is dependent on roulette wheel approach as well as the value of r. However, in these proposed approaches, solution value is driven by setting a suitable value of r (Kale and Kulakrni 2018). This model incorporates SAPF to handle linear constraints associated with test problems.

The comparison is shown in Table 5.1. The standard deviation using the follow-best approach is 9.7666, average function evaluation count is 615, while the average CPU time is 0.64 sec. In terms of function evaluations and computational time, follow-best approach has shown much better performance in comparison with follow-better, CI-SAPF, CI-SAPF-CBO, and GA. The convergence trend can be observed from Figs. 5.4 and 5.5, respectively.

Table 5.1 Comparative analysis of optimizers for six-bar truss structures

Design variables (in^2)	GA (Nanakorn and Meesomklin 2001)	CI-SAPF (Kale and Kulkarni 2021)	CI-SAPF-CBO (Kale and Kulkarni 2021)	Follow-best	Follow-better
A_1	30	30	30	30	30
A_2	19.9	19.9	19.9	19.9	19.9
A_3	15.5	15.5	15.5	15.5	15.5
A_4	7.22	7.22	7.22	7.22	7.22
A_5	22	22	22	22	22
A_6	22	22	22	22	22
Truss weight W (lb)	4962.0966	4962.0966	4962.0966	4962.0966	4962.0966
Function evaluations	–	2250	1740	615	1865
Time	–	–	–	0.64	0.26

[NA] Not Available

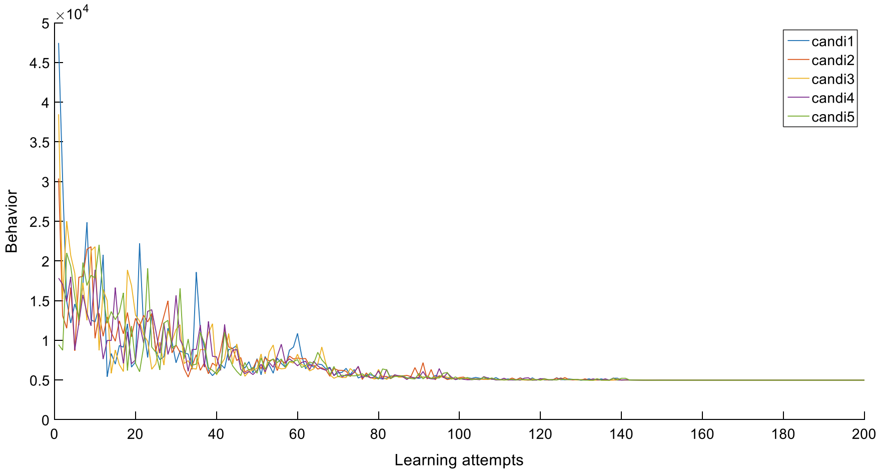

Fig. 5.4 Convergence trend of follow-best for solving six-bar truss problem

As compared to ABC (Sonmez 2011) and ADS (Hasançebi and Azad 2015) algorithms, the follow-best approach successfully solved Case 1 with a very small computational effort (refer to Table 5.2). The average count of function evaluations is 1855, standard deviation is 54.2289, average computational time required is 6.21 s. The function evaluations are very less as compared to other compared algorithms except ADS. This results to lower down CPU time as well. On the other hand, the

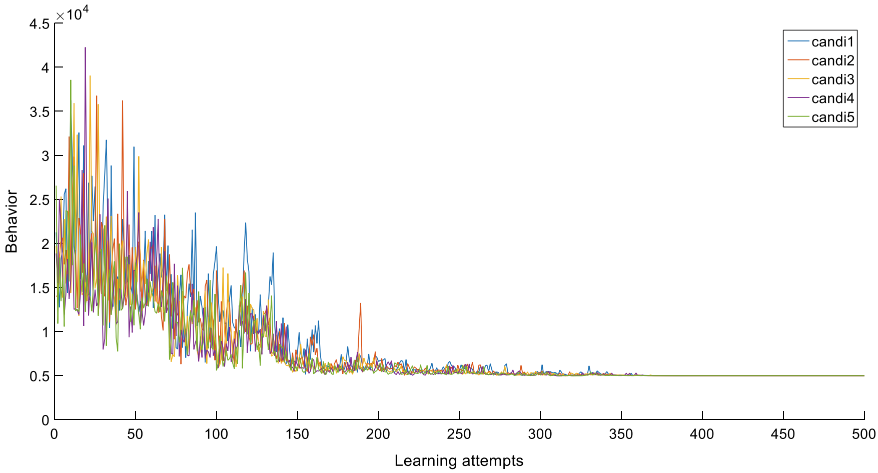

Fig. 5.5 Convergence trend of follow-better for solving six-bar truss problem

follow-better approach failed to obtain comparable solution. The convergence trend can be observed from Figs. 5.6 and 5.7, respectively.

It has been shown in Table 5.3 that the follow-best method of CI results is superior to PSO, PSOPC, and HPSO in solving case 2 (Li et al. 2009), marginally worse than CI-SAPF and CI-SAPF-CBO (Kale and Kulkarni, 2021) algorithms, and completely worse than PC (Kulkarni et al. 2016). The standard deviation with the follow-best approach was 21.5726, average function evaluation count is 2070, and average CPU time was 6.62 sec. The convergence trends for ten-bar Case 2 can be observed from Figs. 5.8 and 5.9, respectively.

5.5 Conclusions and Future Directions

Follow-best and follow-better versions of CI are successfully applied and validated for solving discrete variable truss structures with linear constraints in two cases of 6 bars and two cases of 10 bars. An integer sampling approach is used to handle discrete variables. In contrast, SAPF is used to manage the constraints associated with the problems. It must be noted that the CI variations doesn't require any preliminary trials as SAPF approach is self-supervised. The sampling space reduction factor is one of the solution driving factors; however, it is pre-defined within the range [0.95, 0.98] for these problems. The follow-best approach has obtained better results than follow-better approach due to the higher probability of following a good candidate/behavior from the set-in follow-best approach. There is a scope of following a worse solution in follow-better approach. We intend to apply this approach for complex 3-D spatial truss structure problems. The follow-best mechanism with SAPF approach could be

Table 5.2 Comparative analysis of optimizers for ten-bar case 1 truss structure

Design variables (in^2)	GA (Nanakorn and Meesomklin 2001)	ABC (Sonmez 2011)	ADS Hasançebi and Azad 2015	PC (Kulkarni et al. 2016a)	CI-SPF (Kale and Kulkarni 2018)	CI-SAPF (Kale and Kulkarni 2021)	CI-SAPF-CBO (Kale and Kulkarni 2021)	Follow-best	Follow-better
A_1	33.5	33.5	33.5	33.5	33.5	33.5	33.5	33.5	33.5
A_2	1.62	1.62	1.62	1.62	1.62	1.62	1.62	1.62	1.62
A_3	22.9	22.9	22.9	22.9	22.9	22.9	22.9	22.9	22.9
A_4	15.5	14.2	14.2	14.2	14.2	13.9	13.9	13.9	15.5
A_5	1.62	1.62	1.62	1.62	1.62	1.62	1.62	1.62	1.62
A_6	1.62	1.62	1.62	1.62	1.62	1.62	1.62	1.62	1.62
A_7	7.22	7.97	7.97	7.97	7.97	7.97	7.97	7.97	7.22
A_8	22.9	22.9	22.9	22.9	22.9	22.9	22.9	22.9	22.9
A_9	22	22	22	22	22	22	22	22	22
A_{10}	1.62	1.62	1.62	1.62	1.62	1.62	1.62	1.62	1.62
Truss weight W (lb)	5499.3000	5490.7400	5490.7400	5490.7378	5490.7378	5490.6020	5490.6020	5490.6021	5499.3258
Function evaluations	–	25,800	1000	1,852,059	19,250	16,940	14,160	1855	1900
Time	–	–	–	–	–	–	–	6.21	7.86

[NA] Not Available

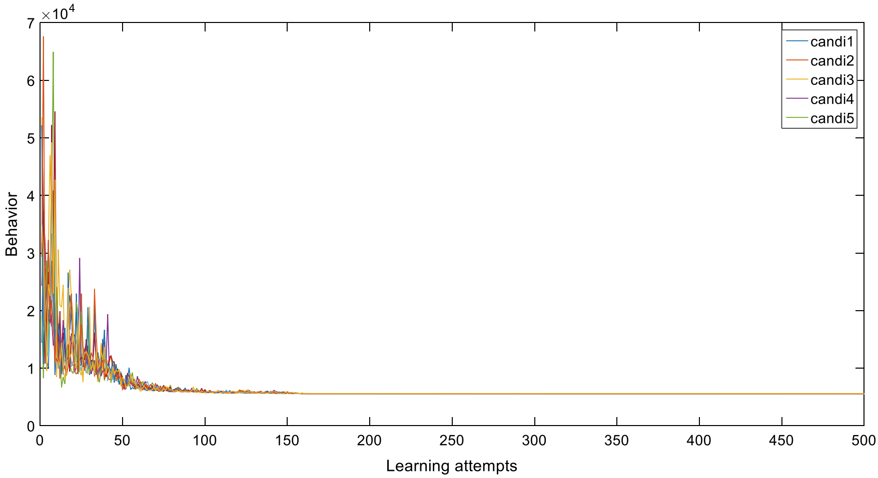

Fig. 5.6 Convergence trend of follow-best for case 1 of ten-bar truss problem

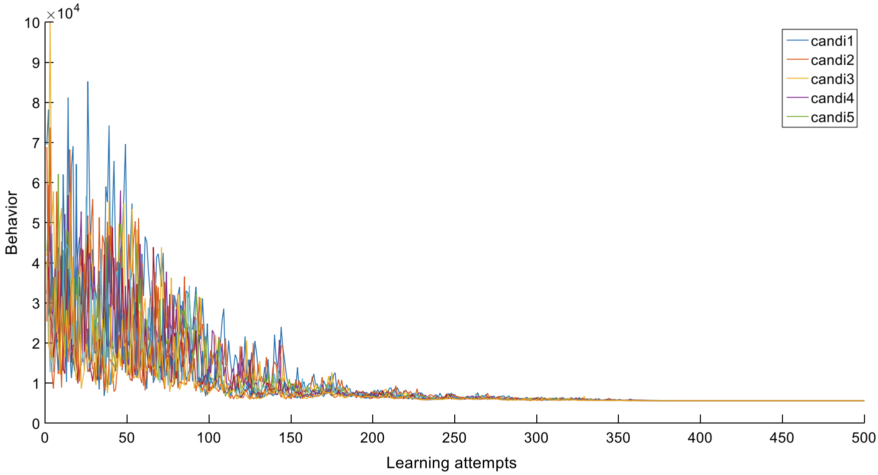

Fig. 5.7 Convergence trend of follow-better for case 1 of ten-bar truss problem

used to solve the scheduling and transportation problems as well as mixed variable design engineering problems.

Table 5.3 Comparative analysis of optimizers for ten-bar case 2 truss Structure

Design variables (in^2)	PSO (Li et al. 2009)	PSOPC (Li et al. 2009)	HPSO (Li et al. 2009)	MBA (Sadollah et al. 2012)	PC (Kulkarni et al. 2016a)	CI-SAPF (Kale and Kulkarni 2021)	CI-SAPF-CBO (Kale and Kulkarni 2021)	Follow-best	Follow-better
A_1	24.5	31.5	31.5	29.5	23.5	31	31	30.5	29.5
A_2	0.1	0.1	0.1	0.01	0.1	0.1	0.1	0.1	0.1
A_3	22.5	23.5	24.5	24	26	23	23	23	24.5
A_4	15.5	18.5	15.5	15	14	15	15	15.5	14
A_5	0.1	0.1	0.5	0.01	0.1	0.1	0.1	0.1	0.1
A_6	1.5	0.5	0.5	0.05	2	0.5	0.5	0.5	1
A_7	8.5	7.5	7.5	7.5	12.5	7.5	7.5	7.5	8
A_8	21.5	21.5	20.5	21.5	13	21	21	21	21.5
A_9	27.5	23.5	20.5	21.5	2	21.5	21.5	21.5	21.5
A_{10}	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Truss weight W (lb)	5243.7100	5133.1600	5073.5100	5067.3300	4686.7729	5061.7597	5061.7597	5062.5325	5096.7936
Function evaluations	–	–	–	–	2,363,380	9450	8400	2070	2100
CPU time	–	–	–	–	–	–	–	6.62	10.45

[NA] Not Available

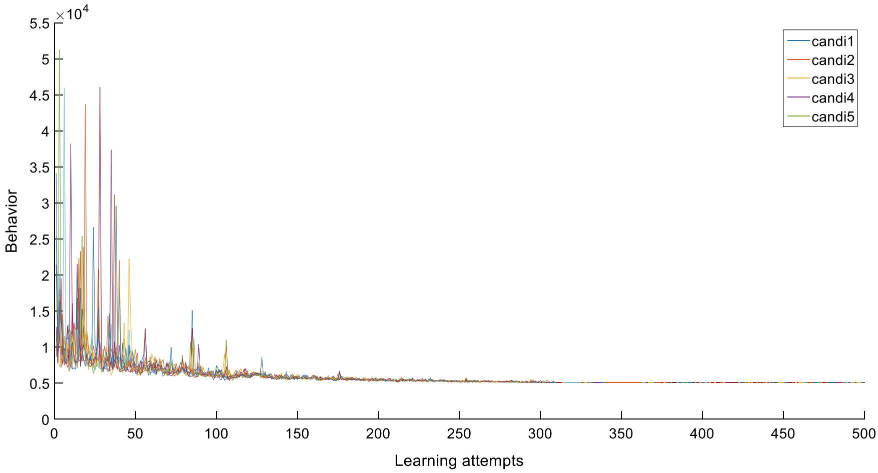

Fig. 5.8 Convergence trend of follow-best for case 2 ten-bar truss problem

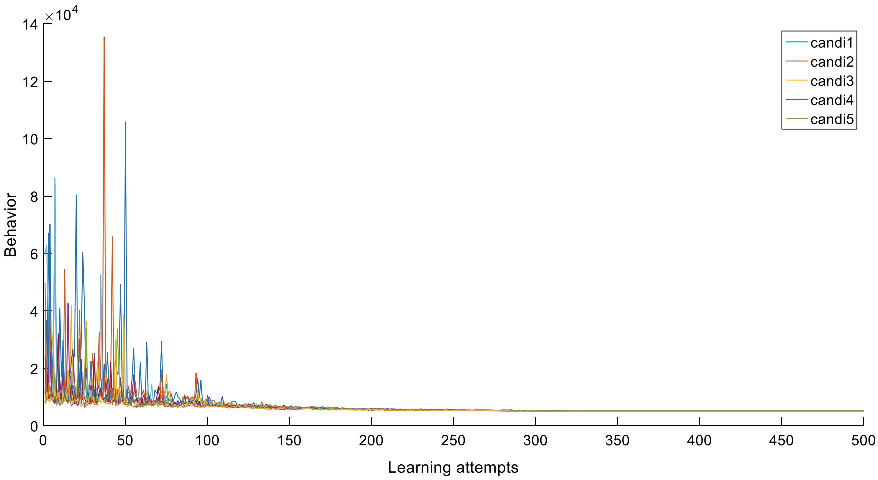

Fig. 5.9 Convergence trend of follow-better for ten-bar case 2 truss problem

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