Prediction of Discharge Coefficients for Trapezoidal Labyrinth Weir with Half-Round (HR) and Quarter-Round (QR) Crest

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Abstract A labyrinth weir allows more flow for a given head and channel width relative to other linear weirs due to the additional crest length. Earlier research explored the correlation between discharge magnification ratio to head to weir height ratio for a different configuration. Discharge coefficient depends on head to weir height ratio, crest shape, crest thickness, apex configuration, and sidewall angle. Continued efforts have been made to develop an equation for discharge coefficient in terms of these parameters. Several equations relating discharge coefficient with head to weir height ratio using polynomial fit up to sixth order for each side wall angle are available in literature. Some investigators related the coefficients of polynomial with side wall angle resulting in a complex form of equation for the discharge coefficient. An attempt has been made to develop a relatively simple discharge coefficient equation involving lesser number of coefficients in terms of head to weir height ratio and sidewall angle. Some of the salient features of the study are described in the present paper.

Keywords Hydraulic structure · Hydraulic design · Coefficient of discharge · Trapezoidal labyrinth weir

Notations

N	Number of cycles
P	Weir height
g	Acceleration due to gravity
h	Head over weir
t	Sidewall thickness
A	Apex inside width (HRL and QRL)
α	Side wall angle

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L_1	Weir side leg length
D	Apex outside width
C_d	Coefficient of discharge
Q	Volumetric flow rate
$\eta = (H_T/P)$	Head to weir height
H_T	Total head
β	Angle of approach flow
θ	Cycle arc angle
L_c	Centreline length of sidewall
M_r	Magnification ratio

1 Introduction

A labyrinth weir is a linear weir symmetrically folded in a plan for providing a longer crest length. These types of weirs require less freeboard in the upstream reservoir than that in linear weir which facilitates flood routing and increases reservoir storage capacity. The typical layout of the labyrinth weir is shown in Fig. 1.

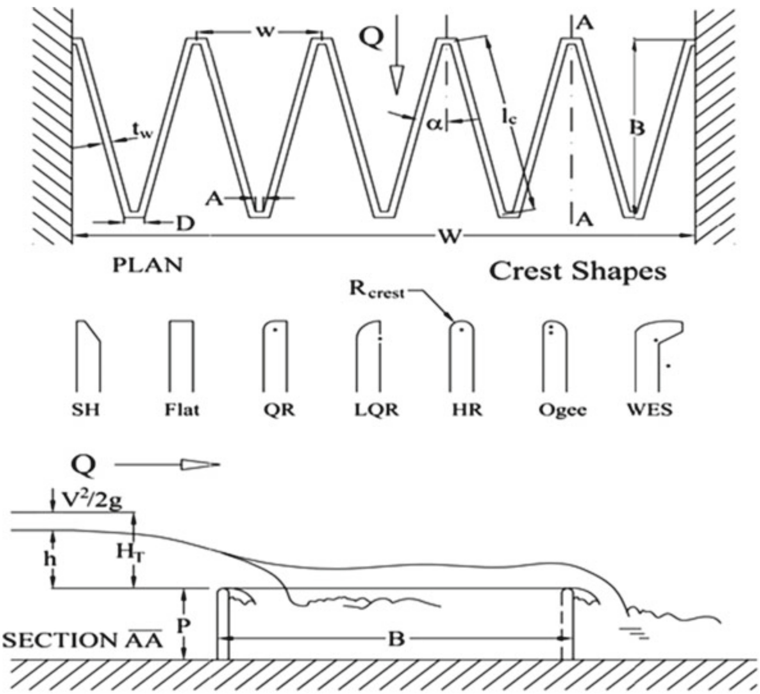

Fig. 1 Labyrinth weir geometric and hydraulic variables [5]

Discharge, Q over labyrinth weir is computed using linear weir equation [1].

$$Q = \frac{2}{3} C_d L \sqrt{2g} {H_T}^{3/2}, \tag{1}$$

where C_d is discharge coefficient, g is the acceleration due to gravity, L is effective weir crest length, and H_T is total head above weir crest including velocity head. The capability of the weir is affected by various factors like the angle of sidewalls (α), apex width, approach channel condition, head to weir height ratio ($\eta = H_T/P$), and vertical aspect ratio (W/P), where W is width of one cycle. Taylor et al. [2] conducted experiments on labyrinth weir with triangular, trapezoidal, and rectangular plan forms. They presented results in the form of graphs between magnification ratio ($m_r = Q_{\text{labyrinth}}/Q_{\text{linear}}$) and η. They concluded that m_r decreases with an increase in η. Darvas [3] used the data of model studies of Worona and Avon labyrinth weirs to develop a set of curves for designing labyrinth weirs. Houston [4] studied a scale model of the UTE dam's labyrinth weir.

Tullis et al. [1] performed experiments to investigate the effect of η and α on the performance of linear labyrinth weirs and concluded that the efficiency of a labyrinth weir decreases with an increase in the head to weir height ratio. Using regression analysis, they proposed equations for C_d with η employing fourth order polynomial fit for each side wall angle (6°–35°). Willmore [6] conducted an experimental study on quarter-round, half-round, and ogee crest-shaped trapezoidal labyrinth weirs and plotted a graph between C_d versus η for a different configuration. He further obtained best-fit C_d equations using a polynomial of third to sixth order. Using the design curve of Tullis et al. [1], Ghare et al. [7] plotted a graph between $C_{d\max}$ versus η for different α and proposed equation for maximum discharge coefficient for the design of labyrinth weir. Ghodsian [8] conducted experiments on two cycle triangular labyrinth weir having different crest shapes (half-round, quarter-round and, sharp, flat top). He proposed a discharge coefficient equation in terms of weir head to weir height ratio and weir side leg length to width ratio (L_1/W) of one cycle. Kumar et al. [9] also related C_d with H_T/P and α for one cycle of sharp crested triangular planform weir. Carollo et al. [10] investigated outflow over sharp crested two cycle triangular labyrinth weir with different side wall angles and proposed an equation for discharge magnification. Crookston et al. [11] investigated the labyrinth weir nappe interference that decrease discharge efficiency, including local submergence. They demonstrated parametric methods for determining the size of the nappe interference region as a function of weir geometry and flow parameters. Khode et al. [12] carried out experimental studies on trapezoidal labyrinth weirs for side wall angles 8°, 10°, 20°, and 30° covering a wider range of flow conditions. They defined discharge coefficient in two forms, i.e. C_{dl} = discharge coefficient per unit length of labyrinth weir and C_{dw} = discharge coefficient per unit width of labyrinth weir and obtain relationship between discharge coefficient and head to weir height ratio using fourth degree polynomial for each side wall angle. Seamons [13] performed experiments on eight models of labyrinth weir with different upstream apex widths to investigate the effect of nappe interference. Vatankhah [14] proposed a general equation incorporating α

in the following form.

$$C_d = \left(k_0 + k_1\eta^{k_2\eta^{k_3}+k_4}\right)\left[k_5 + k_6(\sin\alpha)^{k_7}\right] + \left(k_8 + k_9\eta^{k_{10}} + k_{11}\eta^{k_{12}}\right)^{k_{13}}\left[k_{14} + k_{15}(\sin\alpha)^{k_{16}}\right]^{k_{17}}. \quad (2)$$

He developed two different regression equations for quarter-round and half-round trapezoidal labyrinth weir using the data collected by Tullis [1]. Based on the comparison of both the regression equations, he found that for small values of α from 6° to 20°, the curves of the discharge coefficient ratio $C_{d\mathrm{HR}}/C_{d\mathrm{QR}}$ show similar trends with reasonable similarity, while different behaviour was found for $\alpha = 35°$ with a lower discharge coefficient ratio. He suggested exploring the reason for this phenomenon using the supplementary data for α in the range of 20°–35°. This issue is considered important to improve the flow conditions for weirs and to use a more efficient crest shape for trapezoidal labyrinth weirs. The optimal value of η and α was also found to be $\eta = 0.101$ and $\alpha = 20.54$, which correspond to a maximum value of $C_{d\mathrm{HR}}/C_{d\mathrm{QR}} = 1.194$. Bilhan et al. [14] investigated the effect of nappe breakers in trapezoidal labyrinth weirs using a support vector regression (SVR) and extreme learning machine (ELM). Employing the data of the experimental study of Bilhan et al. [14], they proposed a fifth-degree polynomial fit equation for C_d. Some of the equations for discharge coefficient proposed by various researchers are summarized in Table 1.

Literature review on labyrinth weir indicates that magnification ratio is related to head to weir height ratio for a different configuration. Discharge coefficient is a function of η, α, and the crest shape, and it is expressed in the polynomial form for η (up to sixth degree) and coefficients of polynomial were related to α. It is worthwhile to note that using a polynomial of degree n for labyrinth weirs having m configurations (different side wall angles) requires $m^*(n + 1)$ coefficients leading to a complex form of equation for estimation of discharge coefficient. In the present study, an attempt has, therefore, been made to develop a relatively simple discharge coefficient equation involving few coefficients in terms of head to weir height ratio and sidewall angle.

2 Generalized Discharge Coefficient Equation

Discharge over a labyrinth weir depends on various parameters like head, weir height, sidewall angle, the shape of a crest, number of cycles, etc.

Dimensional analysis carried out in earlier studies indicated that discharge coefficient is mainly a function of head to weir height ratio. Side wall angle α plays an important role in the interference of nappe which ultimately affects the discharge. Therefore, in the present study, an attempt has been made to correlate the discharge coefficient with η and α. For this purpose, experimental data of Willmore [6] and

Table 1 Discharge coefficient predictors reported by some investigators

Author	Proposed equation	Remarks	Weir type	Crest type
Tullis et al. [1]	$C_d = 0.49 \pm B\left(\frac{H_T}{P}\right) - C\left(\frac{H_T}{P}\right)^2 + D\left(\frac{H_T}{P}\right)^3 - E\left(\frac{H_T}{P}\right)^4$	$0.05 \leq H/P \leq 1$	Trapezoidal–4 cycles	SC-FT-QR-HR
Ghodsian [8]	$C_d = 1.06\alpha^{1.5}\left(\frac{H}{P}\right)^{-0.606}\left(\frac{L}{W}\right)^{-0.237}$	$0.3 \leq H/P \leq 0.7$	Triangular- 2 cycles	SC-QR-HR-FT
Kumar et al. [9]	$C_d = (-0.065\theta^3 + 0.318\theta^2 - 0.537\theta + 1.190) + (0.090\theta^3 - 0.570\theta^2 + 1.460\theta - 1.670(h/p)$	$0 < h/p < 0.7$	Triangular plan form	SC
Khode et al. [12]	$C_w = B_\circ + B_1\left(\frac{H_T}{P}\right) + B_2\left(\frac{H_T}{P}\right)^2 + B_3\left(\frac{H_T}{P}\right)^3 + B_4\left(\frac{H_T}{P}\right)^4$	$0.1 \leq H/P \leq 0.7$	Trapezoidal–2 cycle	FT
	$\frac{Q}{Q_n} = \left[1 + \frac{\left(\frac{\iota}{\sqsupseteq} - 1\right)}{5.988\left(\frac{H}{\sqsupseteq}\right)^{1.419} + 1}\right]$	0 $.1 < h/p < 0.7$	Trangular labyrinth weir	SC
Crookston and Tullis [15]	$C_{d(\alpha)} = \alpha\left(\frac{H_T}{P}\right)^{b\left(\frac{H_T}{P}\right)^C} + d$	$0.05 \leq H/P \leq 0.9$	Trapezoidal–2 and 4 cycles	HR-QR
	$C_d = A \times (\eta)^{B \times (\eta)^C} + D$	0 $.1 < h/p < 0.7$	Trapezoidal labyrinth weir	HR and QR
	$C_d = i + j(\eta) + k(\eta)^2 + \iota(\eta)^3 + m(\eta)^4 + n(\eta)^5$	0 $.1 < h/p < 0.7$	Trapezoidal labyrinth weir	

Seamons [15] have been utilized to develop a generalized discharge coefficient equation for trapezoidal labyrinth weir with half-round (HR) and quarter-round (QR) crest shapes with η ranging from 0.04 to 0.8 and α from 6° to 35°.

Considering the following form of discharge coefficient:

$$C_d = a_1 + a_2(\eta)^{b_1} + a_3(\alpha)^{b_2}, \tag{3}$$

where a_1, a_2, a_3, are coefficients and b_1 and b_2 are exponents. Using Minitab software, the values of coefficients and exponents were determined. The following best-fit equation was obtained:

$$C_d = -0.118 - 0.486(\eta)^{0.72} + 0.559(\alpha)^{0.169}. \tag{4}$$

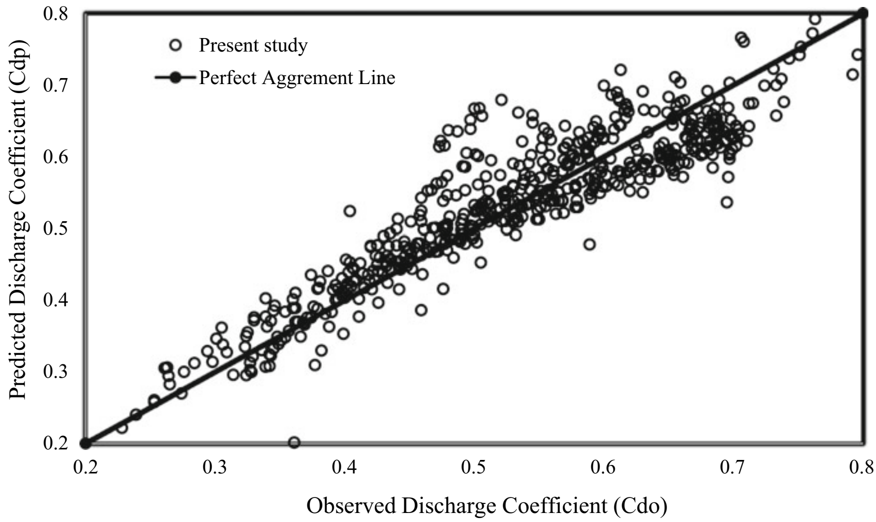

Fig. 2 Comparison of predicted and observed coefficient of discharge (C_d)

Using Eq. (4), discharge coefficient was calculated for known values of η and α from the data set of Willmore [6]; Seamons [15] and a graph is prepared between the predicted discharge coefficient C_{dp} and observed discharge coefficient C_{do} as shown in Fig. 2. A perusal of Fig. 2 indicates that the majority of data points lie on the perfect agreement line.

3 Validation of Proposed Equation

The data of seven prototype dams having labyrinth shape with sidewall angle 9.14° $\leq \alpha \leq$ 23.6° have been selected for validation. Table 2 gives the data of the prototype dam with their discharge coefficients (C_{do}) along with the computed discharge coefficient (C_{dpp}) of the present study. The percentage error in estimation of C_d varies from − 11.06 to 9.27% with an average of − 0.20%. This table also includes the values of discharge coefficient provided by Khode et al. [16] having error variation of − 11.51–6.94% with an average of 0.69%. The quantile plot shown in (Fig. 3) indicates that the data points lie close to line of perfect agreement except one point corresponding to Carty USA dam.

Table 2 Comparison of coefficient of discharge of prototype dam with the coefficient of discharge predicted by empirical equations

Location	α	P	H_T	η	N	L	Q	C_{do}	C_{dpp}	C_{dpk}	Error (%)	
											Present study	Khode
Bartletts ferry, USA	14.5	3.43	2.44	0.644	20.5	1441	5920	0.365	0.405	0.407	– 11.06	– 11.51
Boardman, USA	19.44	3.53	1.80	0.500	2	109.2	387	0.497	0.510	0.491	– 2.61	1.20
Carty, USA	19.40	2.76	1.80	0.642	2	109.2	387	0.497	0.451	0.490	9.27	1.41
Dungo, Angola	15.20	4.30	2.40	0.560	4	115.5	576	0.454	0.447	0.434	1.67	4.45
Hyrum, USA	9.14	3.66	1.82	0.500	2	91.44	262	0.402	0.398	0.374	0.91	6.94
Navet, Trinidad	23.6	3.05	1.68	0.550	10	137	481	0.546	0.520	0.521	4.79	4.58
Ute dam, USA	12.15	9.14	5.8	0.630	14	1024	15,574	0.369	0.385	0.377	– 4.37	– 2.25
Average Percentage Error											– 0.20	0.69

Note A = Sidewall angle, P = Weir height, H_T = Total head, N = Number of cycle, L = Crest length, Q = Total flow, C_{do} = Coefficient of discharge observed, C_{dpp} = Coefficient of discharge of present study, C_{dpk} = Coefficient of discharge of Khode

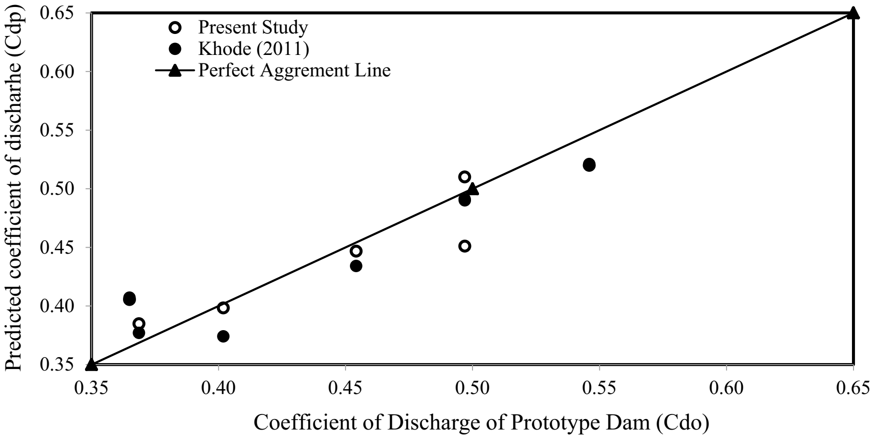

Fig. 3 Comparison of coefficient of discharge of prototype dam with the coefficient of discharge predicted by empirical equations

4 Conclusion

Labyrinth weirs in many cases are favourable design solutions to regulate upstream water elevation and increase flow capacity. Due to the complex design characteristic, the coefficient of discharge of labyrinth weir can be applied for the sidewall angle α in varying from 6° to 35°. A simple discharge coefficient equation in terms of η and α has been obtained using regression analysis in the present study. Discharge coefficients computed using proposed equation give an average error of $-$ 0.20% for prototype dams and comparable with the value reported in literature by Khode et al. [16].

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